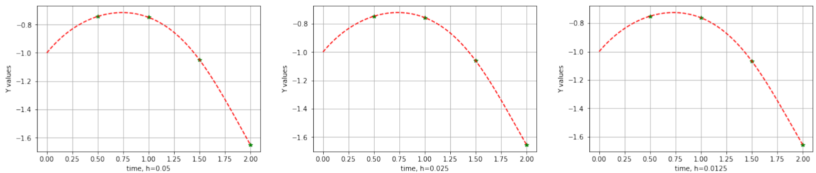
SINGLE-STEP METHODS

Forward Euler's Method

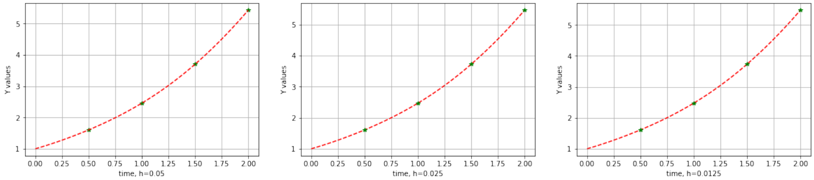
Equation 1 plots and results.



(a) h = 0.05 (b) h = 0.025 (c) h = 0.0125

Figure 1: Equation 1 plots using Forward Euler's Method

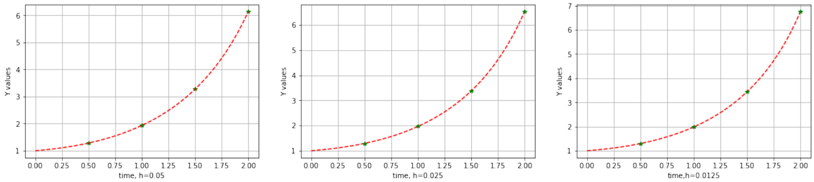
Equation 2 plots and results.



(a) h = 0.05 (b) h = 0.025 (c) h = 0.0125

Figure 2: Equation 2 plots using Forward Euler's Method

Equation 3 plots and results.

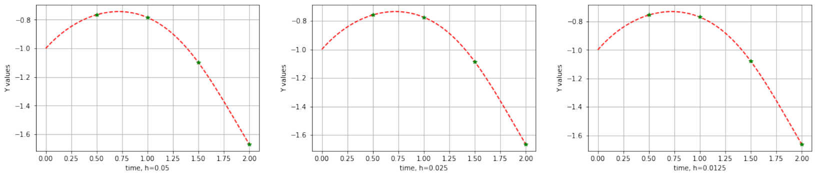


(a) h = 0.05 (b) h = 0.025 (c) h = 0.0125

Figure 3: Equation 3 plots using Forward Euler's Method

Backward Euler's Method

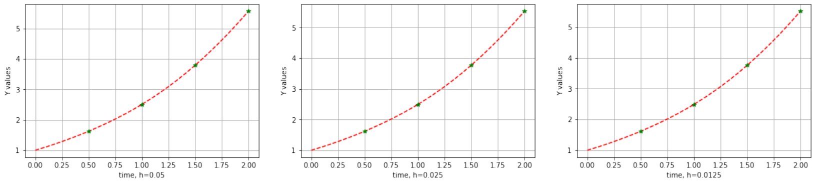
Equation 1 plots and results.



(a) h = 0.05 (b) h = 0.025 (c) h = 0.0125

Figure 4: Equation 1 plots using Backward Euler's Method

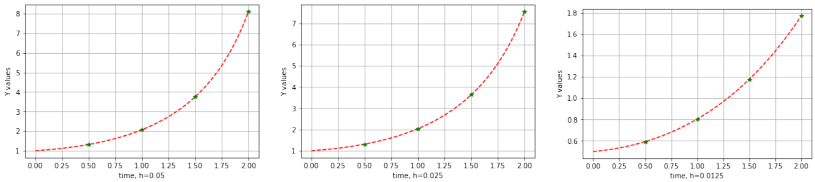
Equation 2 plots and results.



(a) h = 0.05 (b) h = 0.025 (c) h = 0.0125

Figure 5: Equation 2 plots using Backward Euler's Method

Equation 3 plots and results.

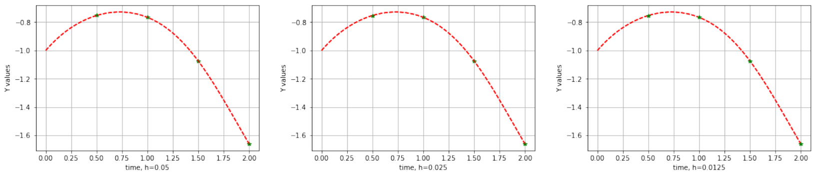


(a) h = 0.05 (b) h = 0.025 (c) h = 0.0125

Figure 6: Equation 3 plots using Backward Euler's Method

Heuns Method

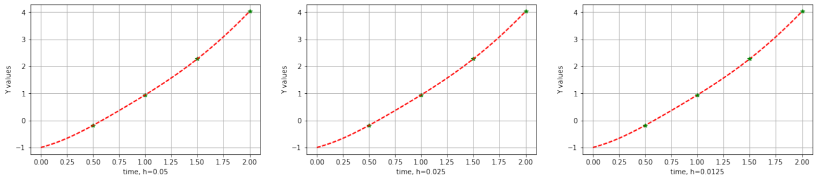
Equation 1 plots and results.



(a) h = 0.05 (b) h = 0.025 (c) h = 0.0125

Figure 7: Equation 1 plots using Heuns Method

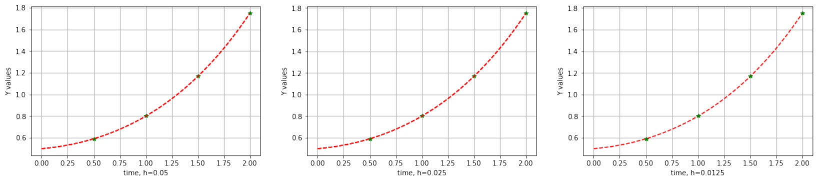
Equation 2 plots and results.



(a) h = 0.05 (b) h = 0.025 (c) h = 0.0125

Figure 8: Equation 2 plots using Heuns Method

Equation 3 plots and results.

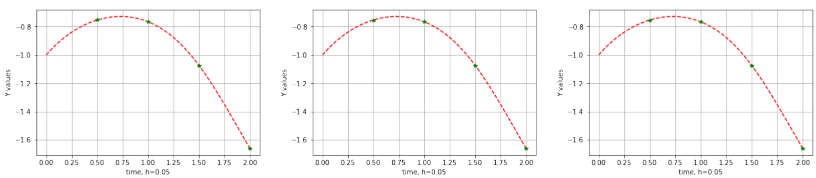


(a) h = 0.05 (b) h = 0.025 (c) h = 0.0125

Figure 9: Equation 3 plots using Heuns Method

Fourth Order Runge-Kutta Method

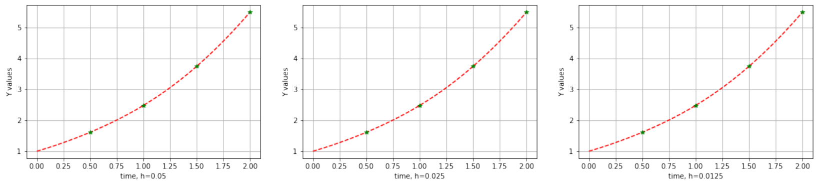
Equation 1 plots and results.



(a) h = 0.05 (b) h = 0.025 (c) h = 0.0125

Figure 10: Equation 1 plots using Fourth Order Runge-Kutta Method

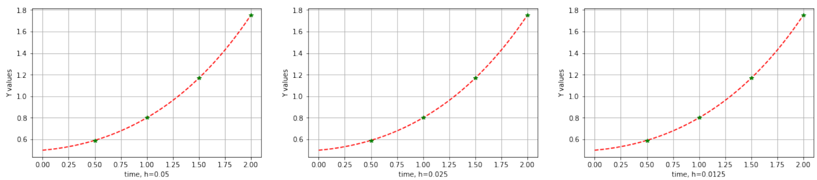
Equation 2 plots and results.



(a) h = 0.05 (b) h = 0.025 (c) h = 0.0125

Figure 11: Equation 2 plots using Fourth Order Runge-Kutta Method

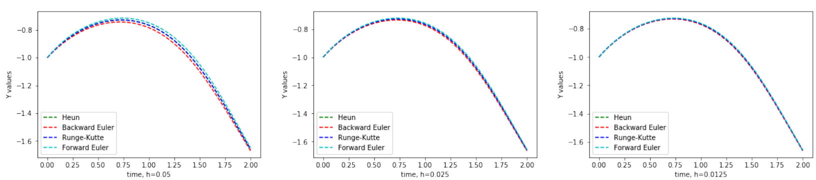
Equation 3 plots and results.



(a) h = 0.05 (b) h = 0.025 (c) h = 0.0125

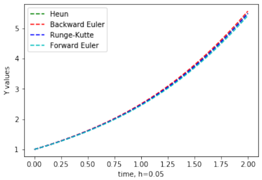
Figure 12: Equation 3 plots using Fourth Order Runge-Kutta Method

Combined plots for all the methods to compare the solutions of different methods visually.

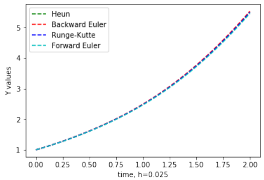


(a) h = 0.05 (b) h = 0.025 (c) h = 0.0125

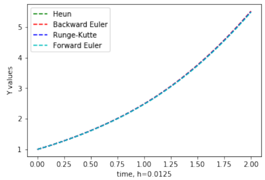
Figure 13: Plots for Equation 1



(a) h = 0.05

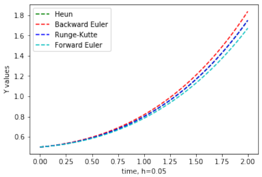


(b) h = 0.025

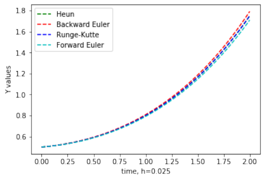


(c) h = 0.0125

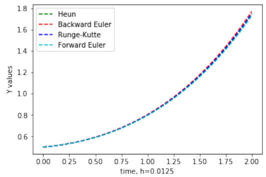
Figure 14: Plots for Equation 2



(a) h = 0.05



(b) h = 0.025



(c) h = 0.0125

Figure 15: Plots for Equation 3

We compare the numerical solutions. On increasing time error gets worse. because we replace the true derivative with a changed derivative. an error comes up at the first iterate. The errors accumulate in forward Euler. On comparing all the methods we see that Forward Euler gives the worst performance whereas Runge-Kutte method gives significantly better solution than that. Heuns method and Runge Kutte method have almost similar solutions. We can also see that Backward Euler's solution is better than Forward Euler's method in every case.

	Runge kutta method			
Equation 1	T = 0.5	T = 1.0	T = 1.5	T = 2.0
H = 0.05	-0.753152466115977	-0.7660129078714482	-1.073977076148927	-1.66021466899215
H = 0.025	-0.753152459195553	-0.7660129444554604	-1.07397716847706	-1.66021474838093
H = 0.0125	-0.753152458653264	-0.766012946663488	-1.07397717434056	-1.6602147539578

	Backward eulers method			
Equation 1	T = 0.5	T = 1.0	T = 1.5	T = 2.0
H = 0.05	-0.765084721890842	-0.784559294032681	-1.09959081469125	-1.67102196406876
H = 0.025	-0.758736235266817	-0.774815068460158	-1.08633741202135	-1.66534519886658
H = 0.0125	-0.75585173371123	-0.770300338258529	-1.08005252221462	-1.66272430149866

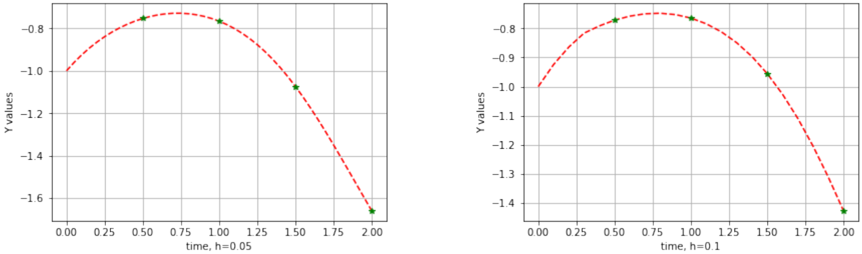
	Forward eulers method			
Equation 1	T = 0.5	T = 1.0	T = 1.5	T = 2.0
H = 0.05	-0.742376370613666	-0.749012746749339	-1.04907793208396	-1.64978104900298
H = 0.025	-0.747851829744917	-0.757586848410821	-1.0617795006843	-1.65515368601389
H = 0.0125	-0.750523194743634	-0.761818276284225	-1.06794057499058	-1.65771984718583

	Heuns method			
Equation 1	T = 0.5	T = 1.0	T = 1.5	T = 2.0
H = 0.05	-0.753484099003156	-0.76649793325139	-1.07463430906214	-1.66037099688016
H = 0.025	-0.753234659853217	-0.766133849550775	-1.07413881260235	-1.66025095835605
H = 0.0125	-0.753172912860201	-0.766043120381276	-1.07401724923138	-1.66022348616215

MULTI-STEP METHODS

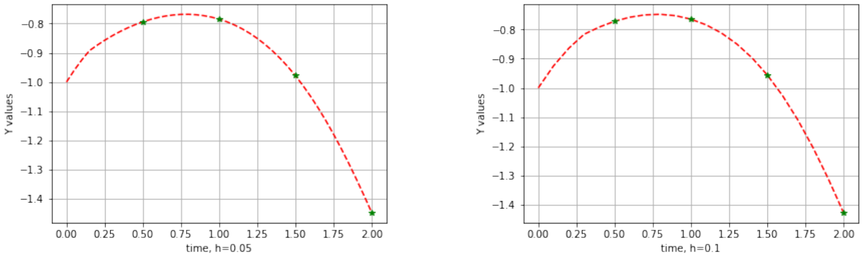
Equation 1 plots and results for all the three methods are shown below.

Equation 1	Adam Bashford Method	Adam Moulton Method	Predictor Corrector Method
H = 0.05	[-0.75316913] [-0.76602695] [-1.07400984] [-1.66019612]	[-0.79392758] [-0.78474849] [-0.97864397] [-1.44736878]	[-0.75315091] [-0.76601163] [-1.07397399] [-1.66021767]
H = 0.1	[-0.77174891] [-0.76455224] [-0.9555658] [-1.42730317]	[-0.77174891] [-0.76455224] [-0.9555658] [-1.42730317]	[-0.75313454] [-0.7659944] [-1.07392138] [-1.66029265]



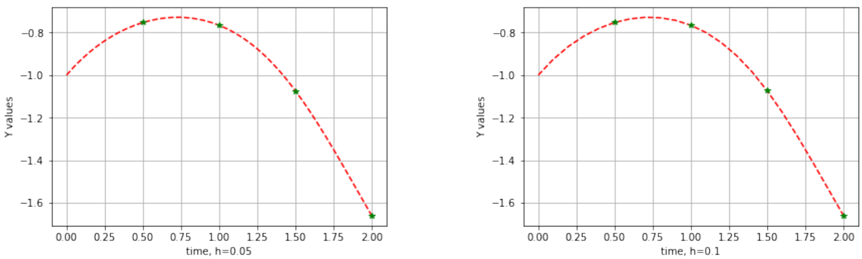
(a) h = 0.05 (b) h = 0.1

Figure 16: Equation 1 plots using Adam Bashford Method



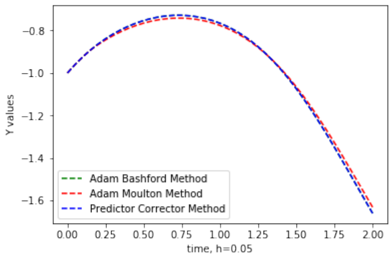
(a) h = 0.05 (b) h = 0.1

Figure 17: Equation 1 plots using Adam Moulton Method

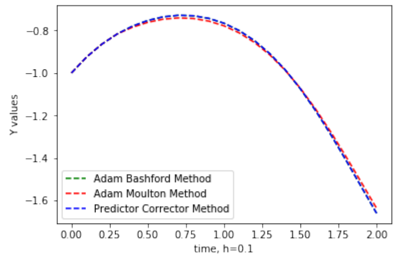


(a) h = 0.05 (b) h = 0.1

Figure 18: Equation 1 plots using Predictor Corrector Method



(a) h = 0.05

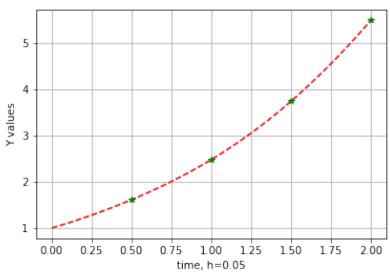


(b) h = 0.1

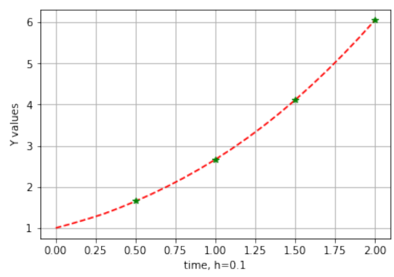
Figure 19: Equation 1 plots

Equation 2 plots and results for all the three methods are shown below.

Equation 2	Adam Bashford Method	Adam Moulton Method	Predictor Corrector Method
H = 0.05	[1.61262049] [2.48091954] [3.74548959] [5.49587967]	[1.66254756] [2.67230405] [4.11730667] [6.05495304]	[1.61262231] [2.48090936] [3.74547865] [5.49587243]
H = 0.1	[1.64977813] [2.66333076] [4.10896269] [6.04663004]	[1.64977813] [2.66333076] [4.10896269] [6.04663004]	[1.61262152] [2.48089534] [3.74546353] [5.4958613]

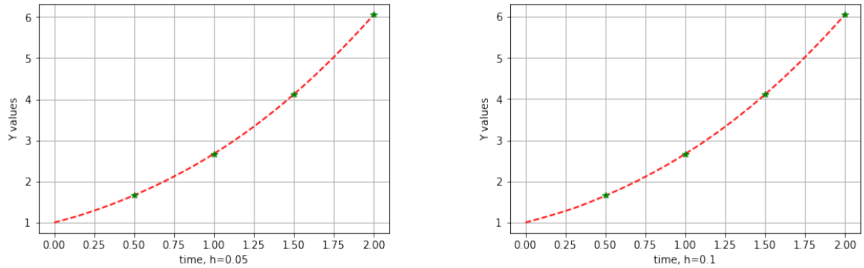


(a) h = 0.05



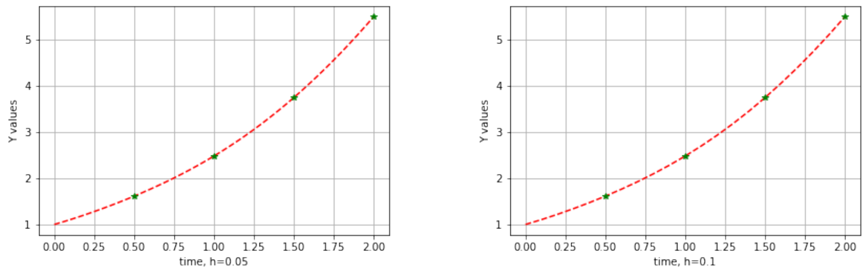
(b) h = 0.1

Figure 20: Equation 2 plots using Adam Bashford Method



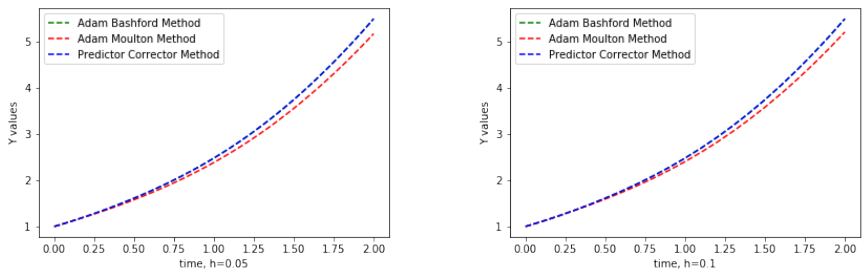
(a) h = 0.05 (b) h = 0.1

Figure 21: Equation 2 plots using Adam Moulton Method



(a) h = 0.05 (b) h = 0.1

Figure 22: Equation 2 plots using Predictor Corrector Method

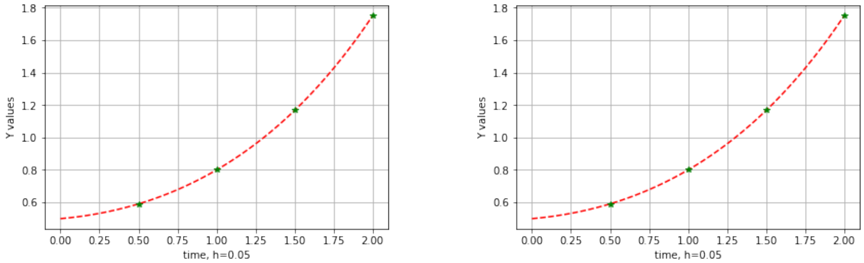


(a) h = 0.05 (b) h = 0.1

Figure 23: Equation 2 plots

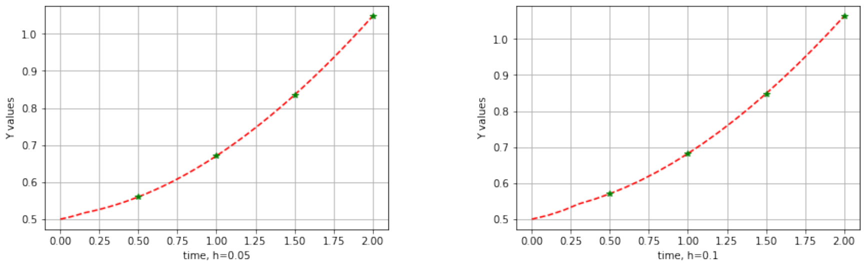
Equation 3 plots and results for all the three methods are shown below.

Equation 3	Adam Bashford Method	Adam Moulton Method	Predictor Corrector Method
H = 0.05	[0.59090898] [0.79999956] [1.16666544] [1.74999692]	[0.56010973] [0.67078128] [0.83503278] [1.04737121]	[0.5909091] [0.80000003] [1.16666675] [1.75000019]
H = 0.1	[0.57048693] [0.68198396] [0.84796152] [1.06282028]	[0.57048693] [0.68198396] [0.84796152] [1.06282028]	[0.59090918] [0.80000033] [1.16666754] [1.75000211]



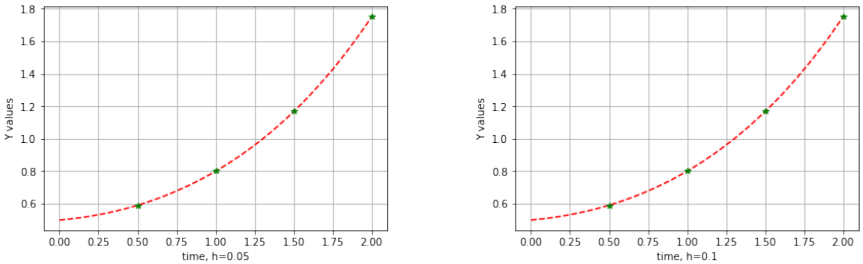
(a) h = 0.05 (b) h = 0.1

Figure 24: Equation 3 plots using Adam Bashford Method



(a) h = 0.05 (b) h = 0.1

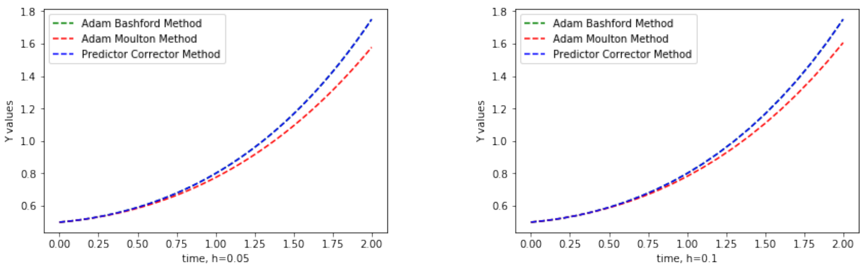
Figure 25: Equation 3 plots using Adam Moulton Method



(a) h = 0.05 (b) h = 0.1

Figure 26: Equation 3 plots using Predictor Corrector Method

Combined plots



(a) h = 0.05 (b) h = 0.1

Figure 27: Equation 3 plots

We can conclude that upon using predictor corrector method, by using adam bashford and adam moulton for prediction and correction steps yields us better results than the individual methods. and on increasing the time steps solutions of various methods come closer.