

Digital Communication**Quiz 2**Date: 23rd Nov., 2018

Time: 45 Minutes

Max Marks: 20

Roll No.: 1611EC126

Name: Vishakha

Q1. Draw the block diagram of a phase locked loop (PLL). What is the role of loop filter and loop gain in the design of PLL? Show relevant calculation and indicate the condition to lock both incoming frequency and phase.

[5]

Q2. What are the methods you know for Carrier Recovery? Draw the block diagram of any of the methods and indicate the advantages and disadvantages of the method.

$$2 = \sum_{j=0}^m n_j$$

Q3. i) Define - Code Rate. ii) Indicate Hamming Bound and how will you define a Perfect Code. iii) Confirm the possibility of a (18,7) binary code that can correct up to three errors. Can this code correct up to four errors?

$$\sum_{j=0}^3 18C_j$$

$$d_{\min} = 2t+1$$

[1+2+2=5]

Q4. Use the Generator polynomial $g(x)=x^3+x+1$ to construct a systematic (7,4) cyclic code. If the received word is 1101100, determine the transmitted data word.

$$S(x) = \frac{f(x)}{g(x)}$$

$$g(x) = x^6 + x^5 + x^3 + x^2$$

$$g(x) = x^3 + x + 1$$

$$e = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$c = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{array}{r}
 x^3 + x + 1 \quad | \quad x^6 + x^5 + x^3 + x^2 \\
 \underline{x^6 + x^4 + x^3} \\
 x^4 + x^3 \\
 \underline{x^4 + x^2 + x} \\
 x^3 + x^2 + x \\
 \underline{x^3 + x^2 + x} \\
 0
 \end{array}
 \quad = x^{2,1}$$

$$e' = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Subject: Digital Communications [ECE325]**B. Tech., 3rd Year, ODD Semester, End-Term Examination**

Date: 06.12.2018

Time: 180 Minutes

Max Marks: 50

- Instructions:**
- 1) Please check there must be 4 questions in the paper, printed on both sides.
 - 2) All questions are compulsory. Answer to a question should be at one place.

1. (a) Two 4-ary signal constellations are shown below in Fig.1. It is given that φ_1 and φ_2 constitute an orthonormal basis for the two constellations. Assume that the four symbols in both the constellations are equiprobable. Let $N_0/2$ denote the power spectral density of white Gaussian noise.

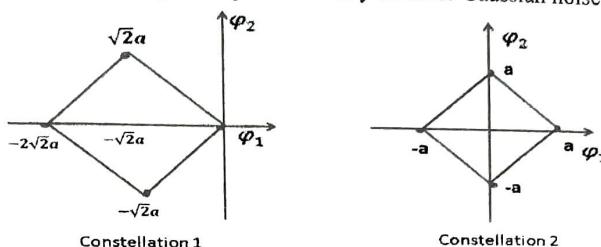


Fig. 1

What is the ratio of the average energy of constellation 1 to the average energy of constellation 2? Also, if these constellations are used for digital communications over AWGN channel then comment which is the more efficient constellation in terms of symbol error probability.

- (b) In a PCM system, the signal $m(t) = \sin[100\pi t + \cos(100\pi t)]$ is sampled at the Nyquist rate. The samples are processed by a uniform quantizer with step size 0.75V. Determine the minimum data rate of the PCM system in bits per second.
 (c) Bit 1 and 0 are transmitted with equal probability. At the receiver, the pdf of the respective received signals for both bits are shown in Fig.2.

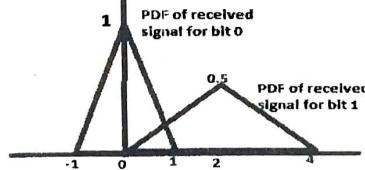


Fig. 2

Determine the BER, if the detection threshold is 1. Also find out the optimum threshold to achieve minimum BER.

- (d) For a QPSK signal configuration, assuming all the symbols are equiprobable, derive the symbol error probability of the optimum receiver for an AWGN channel.
 (e) The received signal in a binary communication system that employs antipodal signals is $r(t) = s(t) + n(t)$, where $s(t)$ is shown in the Fig.3 below and $n(t)$ is AWGN with power spectral density $N_0/2$ W/Hz.
 (i) Sketch carefully the impulse response of the filter matched to $s(t)$.
 (ii) Sketch carefully the output of the matched filter when the input is $s(t)$.
 (iii) Determine the variance of the noise at the output of the matched filter at $t = 3$.
 (iv) Determine the probability of error as a function of A and N_0 .

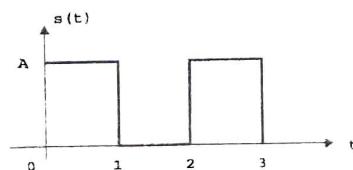


Fig. 3

$$(2+2+4)/5 = 15$$

2. (a) Draw the block diagram of a PLL and explain its operation by drawing the equivalent circuit of the same. How the choice of loop filter and loop gain controls the operation of a PLL? Indicate the condition to lock - i) incoming signal frequency and ii) incoming signal frequency and phase both, by showing detailed small-signal error analysis.
 (b) Suppose the loop filter of a PLL has the transfer function $H(s) = 1/(s + \sqrt{2})$. Determine the closed-loop transfer function and comment on the capability of this PLL to lock incoming signal frequency/ phase/ both frequency and phase.

[6+4 = 10]

3. (a) Explain the operation of "Squaring Loop" for carrier recovery by drawing suitable block diagram. Indicate the disadvantages of this method.
 (b) Why clock synchronizer is important to have in the digital communication system? Draw the block diagram and explain the operation of a bit synchronizer technique known to you.

[5+5 = 10]

4. (a) The parity check matrix of a linear block code is given below:

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(i) Determine the Generator matrix for this code in systematic form.

(ii) How many code words are there in the code?

(iii) What is the minimum distance (d_{\min}) for this code?

(iv) How many random errors per codeword can be detected and corrected?

(b) Show how the code polynomial $c(x)$ in a systematic form can be obtained given the message $d = 1011$, where the generator polynomial is $g(x) = x^3 + x + 1$ of a (7,4) Hamming Code.

(c) Get the condition and define Hamming bound. If you are supposed to detect and correct up to three errors, what should be the value of k and n to construct a Perfect code? What will be the Code efficiency of such a code?

(d) For the convolution encoder shown below, in Fig.4, construct the code tree and find out the codeword for the incoming data pattern 110101. What is the constraint length and code rate of such a code?

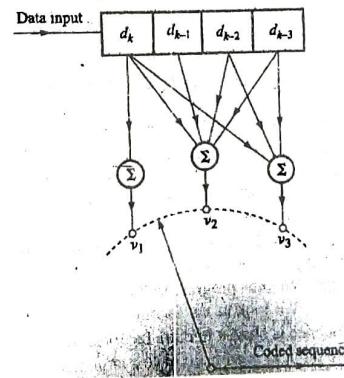


Fig. 4

[4+3+4+4 = 15]

110101000

CODE.....

11295
S. No.....

The LNM Institute of Information Technology, Jaipur

(Deemed to be University)

Instruction to Candidate (for examination)

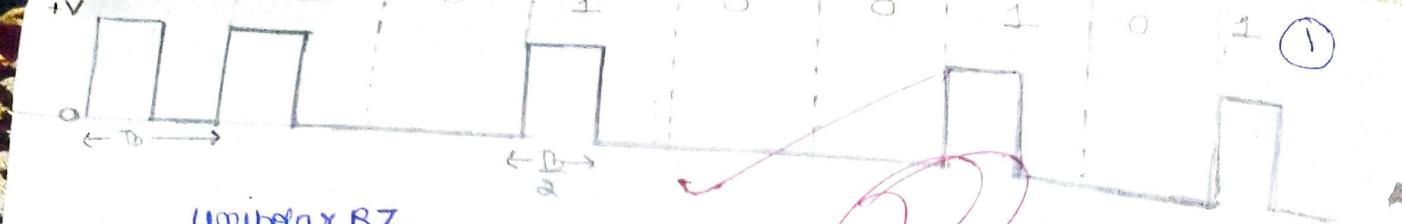
1. Immediately on receipt of the Test Booklet the candidate will fill in the required particulars on the cover page with Ball Point Pen only.
2. Candidates shall maintain perfect silence and attend to their Question Paper only. Any conversation or gesticulation or disturbance in the Examination Room / Hall shall be deemed as misbehaviour. If a candidate is found using unfair means or impersonating, it shall be treated as breach of code of conduct and the matter dealt with accordingly.
3. No candidate, without the special permission of the Invigilator concerned, will leave his/her seat or Examination Room until the full duration of the paper is over. Candidate should not leave the room / hall without handing over their Answer Sheets to the Invigilator on duty.
4. During the examination time, the invigilator will check ID Card of the candidate to satisfy himself / herself about the identity of each candidate. The invigilator will also put his her signature in the place provided in the Answer Sheet.
5. The Candidate shall fill the number of supplementary sheets attached, on the front page of the main answer sheet.
6. **Bringing cell phones /communication devices in the examination hall is strictly prohibited. Exam conducting authority will not be responsible for the custody of such articles. However, use of scientific calculator is permitted.**

Name of the student: Nishakha PhaniwamiRoll No.: 16UEC126Name of Examination: Mid TermSubject: Digital CommunicationDay & Date: Friday, 5/10/18

No. of Supplementary Sheets Attached:

W.A
Student's SignatureSB
Invigilator's Signature

Question No..	Marks Obtained
1	5
2	4
3	5
4	1
5	5
6	4.5
7	
8	
9	
10	
Total Marks	(24.5)

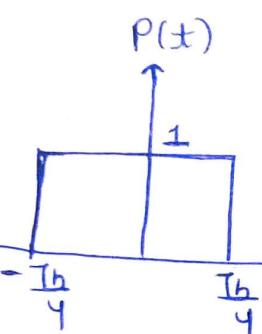
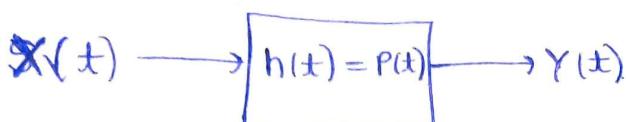


Unipolar RZ

$$Y(t) = \sum a_k P(t - kT_b)$$

$$X(t) = \sum a_k \delta(t - nT_b)$$

Q1(a)



$$= 1 \operatorname{rect}\left(\frac{t}{T_b/2}\right)$$

$$\longleftrightarrow 1 \times \frac{T_b}{2} \sum \frac{\sin\left(\omega T_b \frac{n}{4}\right)}{\omega T_b}$$

$$P(f) = \frac{T_b}{2} \sum \sin\left(\frac{\omega T_b}{4}\right) = \frac{T_b}{2} \sin\left(\frac{\pi f T_b}{2}\right)$$

$$|P(f)|^2 = \frac{T_b^2}{4} \sin^2\left(\frac{\pi f T_b}{2}\right)$$

$$S_Y(f) = |P(f)|^2 S_X(f)$$

$$S_X(f) = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} R_n +$$

$$S_X(f) = \frac{1}{T_b} \left[R_0 + \sum_{n=-\infty}^{\infty} R_n e^{-j 2\pi f n T_b} \right] \quad n \neq 0$$

For unipolar line coding

$$R_0 = \frac{1}{N} \sum a_k^2 = \frac{1}{N} \left[\frac{N}{2}(1) + \frac{N}{2}(0) \right] = \frac{1}{2}$$

$$R_1 = \frac{1}{N} \left[\frac{N}{4}(1)(1) \right] = \frac{1}{4}$$

$$R_2 = \frac{1}{N} \left[\frac{N}{8}(1)(1) + \frac{N}{8}(1)(1) \right] = \frac{1}{4}$$

$$R_X(m) = \begin{cases} \frac{1}{2} & m=0 \\ \frac{1}{4} & m \neq 0 \end{cases}$$

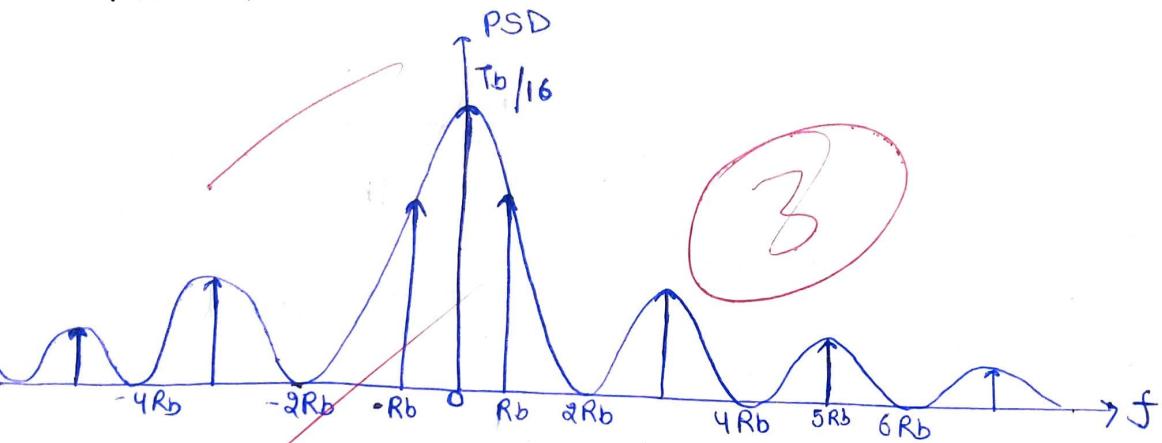
$$S_x(f) = \frac{1}{gT_b} + \frac{1}{4T_b} \sum_{n=-\infty}^{\infty} e^{-j2\pi f n T_b} \quad n \neq 0$$

$$= \frac{1}{4T_b} + \frac{1}{4T_b} \sum_{n=-\infty}^{\infty} e^{-j2\pi f n T_b}$$

$$S_x(f) = \frac{1}{4T_b} + \frac{1}{4T_b^2} \sum \delta\left(f - \frac{n}{T_b}\right).$$

$$S_y(f) = \left[\frac{T_b^2}{4} \sin^2 \left(\frac{\pi f T_b}{2} \right) \right] \left[\frac{1}{4T_b} + \frac{1}{4T_b^2} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right) \right]$$

Power spectral density of unipolar RZ



$$\frac{\pi f T_b}{2} = m\pi \rightarrow f = 2mR_b \quad (\text{null points})$$

Bandwidth = $2R_b$

Q2 bandwidth = $f_m = 4.5 \text{ MHz}$

Nyquist rate = $2f_m = 9 \text{ MHz}$

$$(a) f_s = \frac{35}{100} \times (9 \text{ MHz}) + 9 \text{ MHz}$$

$$= \frac{4}{4} \left(\frac{9}{4} + 9 \right) \text{ MHz}$$

$$= \frac{45}{4} \text{ MHz}$$

$$f_s = 11.25 \text{ MHz}$$

Sampling rate

$$f_s = 11.25 \times 10^6 \text{ samples/sec}$$

$\frac{3.6}{4.5}$

(3)

$$(b) L = 2048$$

$$n = \log_2 2048 = \log_2 2^{11} = 11$$

number of binary digits = 11

$$(c) \text{ Bit rate } (\frac{\text{Bits}}{\text{sec}}) = 11 \frac{\text{Bits}}{\text{sample}} \times 11.25 \times 10^6 \frac{\text{Samples}}{\text{sec}}$$

$$\text{Bit rate} = (\text{no of Bits}) (f_s)$$

$$\boxed{\text{Bit rate} = 123.75 \times 10^6 \text{ Bits/sec.}}$$

$$(d) f_s = 2f_m' \text{ (minimum)}$$

$$\Rightarrow \text{minimum bandwidth required} = f_m'$$

$$= \frac{f_s}{2}$$

$$= \frac{11.25 \text{ MHz}}{2}$$

$$\boxed{f_m' = 5.625 \text{ MHz}}$$

minimum bandwidth required

For minimum bandwidth

Sampling frequency = Nyquist rate

$$\boxed{f_s = 2f_m'}$$

Q3. 16 QAM \Rightarrow 2 (4 PAM)

$$X(t) = A_1 P_1(t) + A_2 P_2(t)$$

↓ ↓
order 4 order 4

$$E_s = E_{s1} + E_{s2}$$

$\sqrt{P_1} \text{ PAM}$ $\sqrt{P_2} \text{ PAM}$

$$E_s = \frac{A_1^2}{3} (m-1) + \frac{A_2^2}{3} (m-1)$$

$$E_s = \frac{2A^2}{3} (m-1)$$

Symbol energy.

$$A = \sqrt{\frac{3E_s}{2(m-1)}} \quad (\text{for m QAM})$$

$$E_s^{\text{MPAM}} = \frac{1}{m/2} \sum_{i=0}^{m-1} (2i+1)^2 A^2 = \frac{A^2 (m^2 - 1)}{3}$$

$$P_1(t) = \sqrt{\frac{2}{T}} \cos 2\pi f_c t$$

$$P_2(t) = \sqrt{\frac{2}{T}} \sin 2\pi f_c t$$

$$\boxed{E_{P1} = E_{P2} = 1}$$

$$P_c \text{ (Probability of correct transmission)} = \left(1 - P_e^{\sqrt{m} \text{ PAM}}\right) \left(1 - P_e^{\frac{\sqrt{m} \text{ PAM}}{2}}\right) \quad (4)$$

$$= \left(1 - P_e^{\sqrt{m} \text{ PAM}}\right)^2$$

$$P_e^{\text{MQAM}} = 1 - \left[1 - P_e^{\sqrt{m} \text{ PAM}}\right]^2$$

$$= 1 - \left[1 + (P_e^{\sqrt{m} \text{ PAM}})^2 - 2 P_e^{\sqrt{m} \text{ PAM}}\right]$$

$$= 2 P_e^{\sqrt{m} \text{ PAM}} - (P_e^{\sqrt{m} \text{ PAM}})^2$$

Assume $(P_e^{\sqrt{m} \text{ PAM}})^2 \ll 1$

$$\Rightarrow P_e^{\text{MQAM}} = 2 P_e^{\sqrt{m} \text{ PAM}} \rightarrow (a)$$

$$P_e^{\sqrt{m} \text{ PAM}} = \frac{m-2}{m} \quad \begin{array}{l} \text{Now For MQAM} \\ \text{let probability of each symbol} = \frac{1}{M} \end{array}$$

$$(m-2) \text{ cases}$$

$$P_e^{\text{MPAM}} = \frac{(m-2)}{m} 2 Q\left(\frac{A}{\sigma}\right) + \frac{2}{m} Q\left(\frac{A}{\sigma}\right) \quad \begin{array}{l} 2 \text{ extreme cases} \\ \text{2 cases} \end{array}$$

$$= 2 \left(1 - \frac{1}{m}\right) Q\left(\frac{A}{\sigma}\right)$$

$$P_e^{\sqrt{m} \text{ PAM}} = 2 \left(1 - \frac{1}{\sqrt{m}}\right) Q\left(\frac{A}{\sigma}\right)$$

$$\text{For MQAM} \quad A = \sqrt{\frac{3 E_s}{2(m-1)}} \quad \sigma = \sqrt{\frac{N_0}{2}}$$

From equation (a)

$$P_e^{\text{MQAM}} = 2 P_e^{\sqrt{m} \text{ PAM}} = 2 \times 2 \left(1 - \frac{1}{\sqrt{m}}\right) Q\left(\sqrt{\frac{3 E_s}{N_0(m-1)}}\right)$$

$$P_e^{\text{MQAM}} = 4 \left(1 - \frac{1}{\sqrt{m}}\right) Q\left(\sqrt{\frac{3 E_s}{N_0(m-1)}}\right)$$

SER for m = 16

(5)

$$\begin{aligned} &= 4 \left(1 - \frac{1}{\sqrt{16}}\right) Q\left(\sqrt{\frac{3 E_s}{N_0 (16-1)}}\right) \\ &= 4 \left(1 - \frac{1}{4}\right) Q\left(\sqrt{\frac{3 E_s}{15 N_0}}\right) \\ &= +3 Q\left(\sqrt{\frac{E_s}{5 N_0}}\right) \end{aligned}$$

$$\boxed{SER = 3 Q\left(\sqrt{\frac{E_s}{5 N_0}}\right)}$$

Symbol error Probability



N_0 = Noise Power

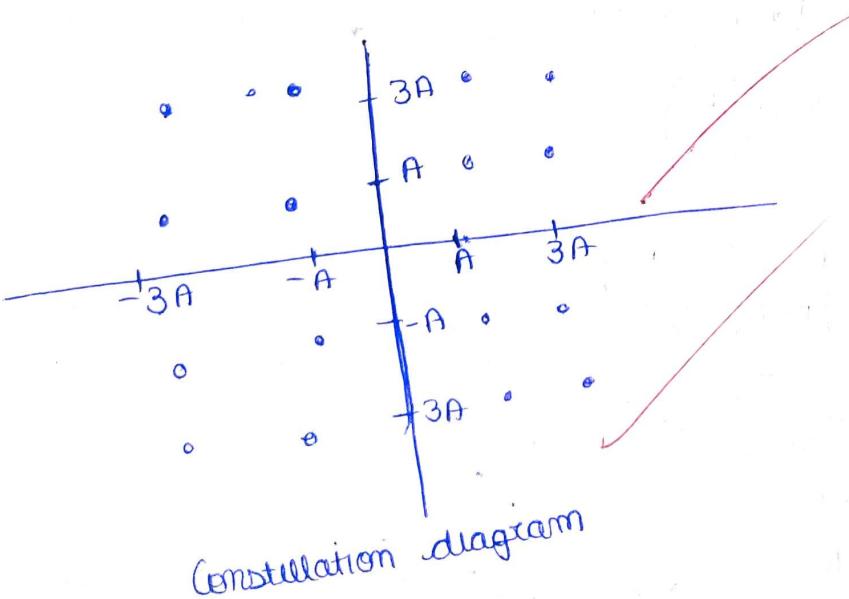
E_s = symbol energy

$$E_s = \log_2 m E_b = 4 E_b$$

$$SER = 3 Q\left(\sqrt{\frac{4 E_b}{5 N_0}}\right)$$

$$y(t) = a_1 p_1(t) + a_2 p_2(t) + n(t)$$

$\xrightarrow{h_1(t) = p_1(T-t)} r_1(t) \rightarrow r_1(T) \rightarrow a_1$
 $\xrightarrow{h_2(t) = p_2(T-t)} r_2(t) \rightarrow r_2(T) \rightarrow a_2$



6

$$\text{Q4} \quad n = 2.08 \times 10^6$$

$$P_b \leq 10^{-6}$$

$$S_n(w) = 10^{-8} \quad (\text{Noise Power spectral density})$$

To find \rightarrow Bandwidth
 \rightarrow Signal power at receiver

(a) BPSK

$$P_e^{\text{BPSK}} = Q\sqrt{\frac{2E_b}{N_0}} \leq 10^{-6}$$

$$\text{PSD} = \frac{1}{\sqrt{2\pi f^2}} e^{-\frac{f^2}{2\pi f^2}} \rightarrow N(0, \sigma^2)$$

Signal Power = E_b

$$\boxed{N_0 = 10^{-8}} =$$

$$\text{BER} = Q\sqrt{\frac{2E_b}{10^{-8}}} \leq 10^{-6}$$

$$Q\sqrt{\frac{E_b}{10^{-8}}} \leq 10^{-6}$$

$$Q\sqrt{E_b \times 10^8} \leq 10^{-6}$$

$$\text{if } \frac{N_0}{2} = 10^{-8}$$

$$\Rightarrow E_b \times 10^8 \approx 4.7$$

$$E_b = 4.7 \times 10^{-8}$$

$$\text{if } N_0 = 10^{-8}$$

$$Q\sqrt{\frac{2E_b}{10^{-8}}} \leq 10^{-6}$$

$$2 \times 10^{-8} \times E_b = 4.7$$

$$E_b = 2.35 \times 10^{-8}$$

Signal Power

Bandwidth =

???

(b) MPSK

$$\text{SER} = 2Q\sqrt{\frac{2E_s \sin^2(\frac{\pi}{16})}{N_0}}$$

$$E_s = (2.08 \times 10^6) E_b$$

Signal Power = E_s $E_s =$

$$\text{SER} = 2Q\sqrt{\frac{2E_s \sin^2(\frac{\pi}{16})}{N_0}}$$

$$= 2Q\sqrt{\frac{2E_s \sin^2(\frac{\pi}{16})}{10^{-8}}}$$

$$\text{SER} = 2Q\sqrt{\frac{2E_s \times 10^{-6} \times (3.42)^2}{10^{-8}}}$$

$$E_s = (2.08 \times 10^6) (4.7 \times 10^{-8})$$

$$E_s = 9.776 \times 10^{-2} \quad (\text{if } \frac{N_0}{2} = 10^{-8})$$

$$N_0 = 10^{-8}$$

$$\Rightarrow E_s = 4.888 \times 10^{-2}$$

$$\text{SER} = 2Q$$

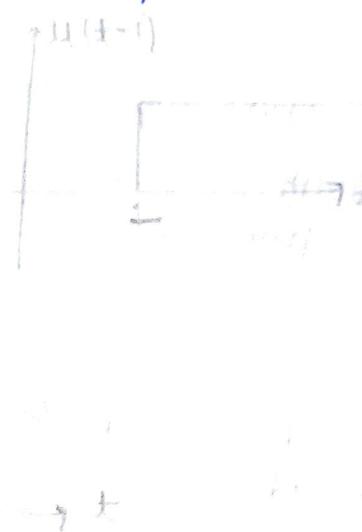
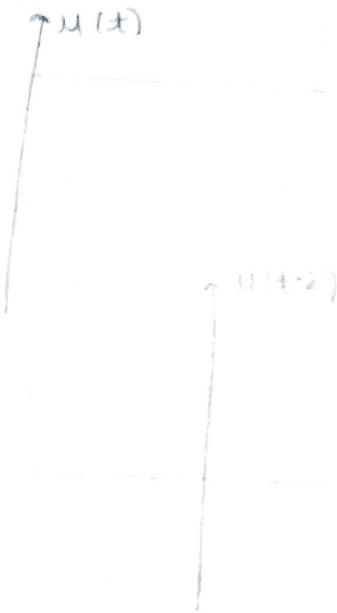
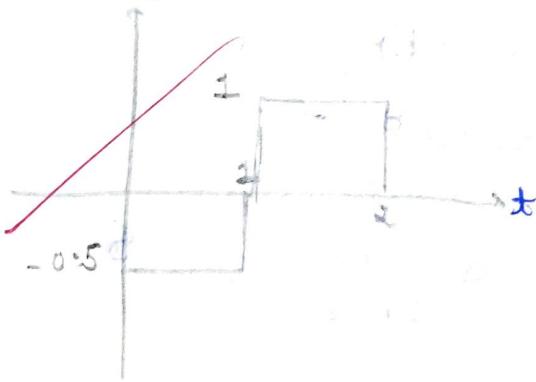
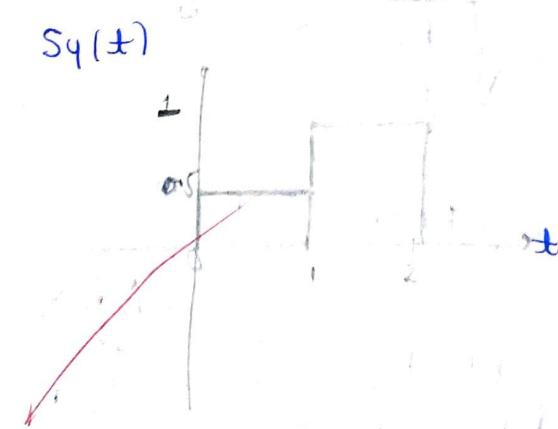
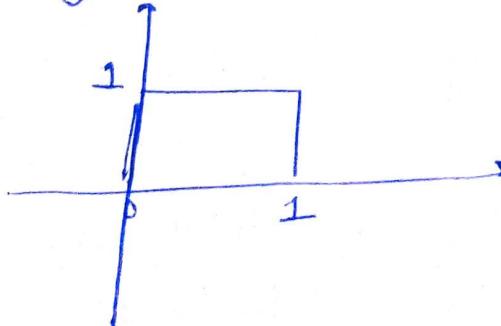
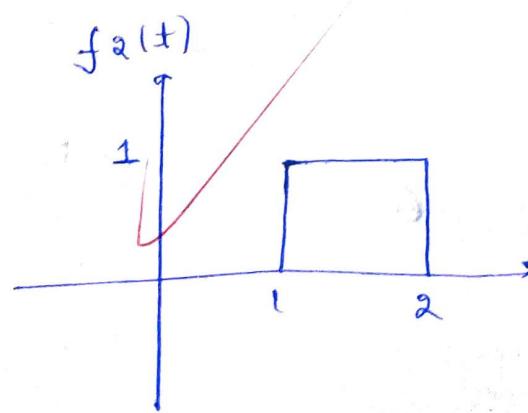
$$\sqrt{\frac{2E_s \times 11.74 \times 10^{-6}}{10^{-8}}}$$

???

05

$$S_1(t) = u(t) - 1.5u(t-1) + 0.5u(t-2)$$

7

 $S_1(t)$  $S_2(t)$  $S_3(t)$ $S_4(t)$  $f_1(t)$  $f_2(t)$ 

(8)

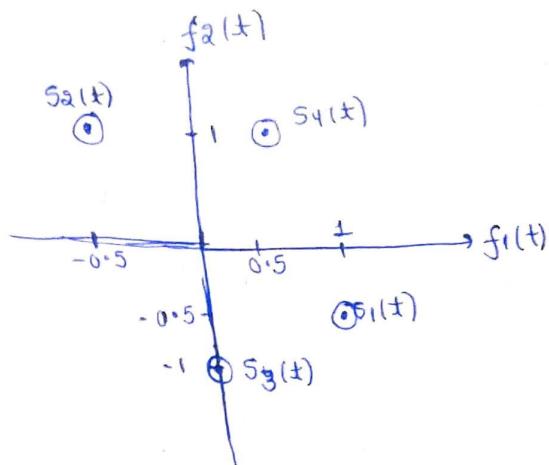
$$\langle s_1, f_1 \rangle = \int_0^1 1 dt = 1$$

$$\langle s_1, f_2 \rangle = \int_1^2 -0.5 dt = -0.5$$

$$s_1(t) = s_{11}f_1 + s_{12}f_2 \\ = 1f_1 - 0.5f_2$$

$$s_1(t) = f_1(t) - 0.5f_2(t)$$

$$s_1(t) = [1, -0.5] \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$



$$s_{21} \quad \langle s_2, f_1 \rangle = \int_0^1 -0.5 dt = -0.5$$

$$\langle s_2, f_2 \rangle = \int_1^2 1 dt = 1$$

$$s_2(t) = [-0.5, 1] \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

$$s_2(t) = -0.5f_1(t) + f_2(t)$$

$$\langle s_3, f_1 \rangle = 0$$

$$\langle s_3, f_2 \rangle = \int_1^2 (-1) dt = -1$$

$$s_3(t) = [0, -1] \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}$$

$$s_3(t) = -f_2(t)$$



$$\langle s_4, f_1 \rangle = \int_0^1 0.5 dt = 0.5$$

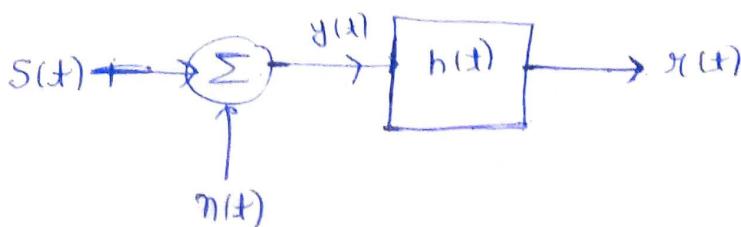
$$\langle s_4, f_2 \rangle = \int_1^2 1 dt = 1$$

$$s_4(t) = 0.5f_1(t) + f_2(t)$$

$$s_4(t) = [0.5, 1] \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}$$

$$\textcircled{Q6} \quad (a) \quad S_n(f) = \frac{N_0}{2}$$

⑨



$$r(t) = (S(t) + n(t)) * h(t)$$

$$\text{Let } S(t) = a_0 P(t).$$

$$r(t) = (a_0 P(t) + n(t)) * h(t)$$

$$r(t) = \int_{-\infty}^t (a_0 P(t-\tau) + n(t-\tau)) h(\tau) d\tau$$

$$r(t) = \int_{-\infty}^t a_0 P(t-\tau) h(\tau) d\tau + \int_{-\infty}^t n(t-\tau) h(\tau) d\tau$$

\downarrow
 $s_o(t)$

\downarrow
 $n_o(t)$.

$$\text{Power of } s_o(t) = E \left[\left| \int_{-\infty}^t a_0 P(t-\tau) h(\tau) d\tau \right|^2 \right]$$

$$= E \left[a_0^2 \left| \int_{-\infty}^t P(t-\tau) h(\tau) d\tau \right|^2 \right]$$

$$= E \{ a_0^2 \} \left[\left| \int_{-\infty}^t P(t-\tau) h(\tau) d\tau \right|^2 \right]$$

Pd

$$\text{Power of noise } n_o(t) = E[\tilde{n} \cdot \tilde{n}]$$

$$= E \left[\int_{-\infty}^t n(t-\tau) h(\tau) d\tau \int_{-\infty}^t n(t-\tilde{\tau}) h(\tilde{\tau}) d\tilde{\tau} \right]$$

$$= \int_{-\infty}^t \int_{-\infty}^t h(\tau) h(\tilde{\tau}) E[n(t-\tau) n(t-\tilde{\tau})] d\tau d\tilde{\tau}$$

$$= \int_{-\infty}^t \int_{-\infty}^t h(\tau) h(\tilde{\tau}) \frac{N_0}{2} \delta(\tilde{\tau}-\tau) d\tau d\tilde{\tau}$$

$$= \frac{N_0}{2} \int_{-\infty}^{\infty} h(\tau) \left(\int_{-\infty}^{\infty} \delta(\tilde{\tau} - \tau) h(\tilde{\tau}) d\tilde{\tau} \right) d\tau \quad (10)$$

$$= \frac{N_0}{2} \int_{-\infty}^{\infty} h(\tau) h(\tau) d\tau$$

$$= \frac{N_0}{2} \int_{-\infty}^{\infty} h(\tau)^2 d\tau$$

$$SNR = \frac{P_d \left(\int_{-\infty}^{\infty} P(T-\tau) h(\tau) d\tau \right)^2}{N_0 \int_{-\infty}^{\infty} h^2(\tau) d\tau}$$

(Cauchy-Schwarz inequality)

$$\left(\int_{-\infty}^{\infty} P(T-\tau) h(\tau) d\tau \right)^2 \leq \int_{-\infty}^{\infty} P^2(T-\tau) d\tau \cdot \int_{-\infty}^{\infty} h^2(\tau) d\tau.$$

~~Equality holds when~~

$$K P(T-\tau) = h(\tau)$$

③

To maximize SNR Ratio (let K=1)

$$\boxed{h(\tau) = P(T-\tau)}$$

$$\Rightarrow \boxed{h(t) = P(T-t)}$$

$$SNR = \frac{P_d \int_{-\infty}^{\infty} P^2(T-\tau) d\tau}{\frac{N_0}{2}}$$

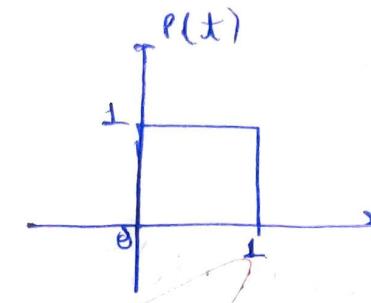
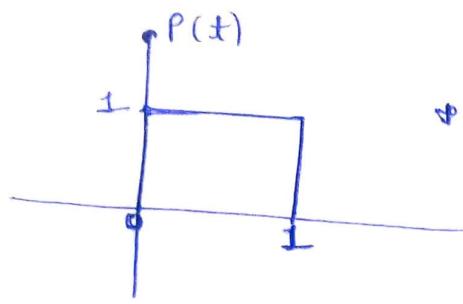
(11)

$$\text{(b)} \quad \theta(t) = p(t) * r(t)$$

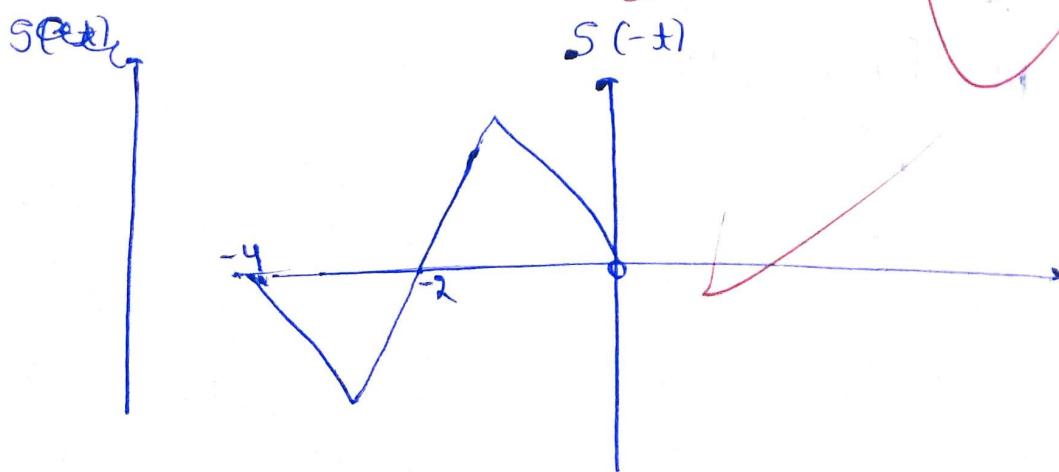
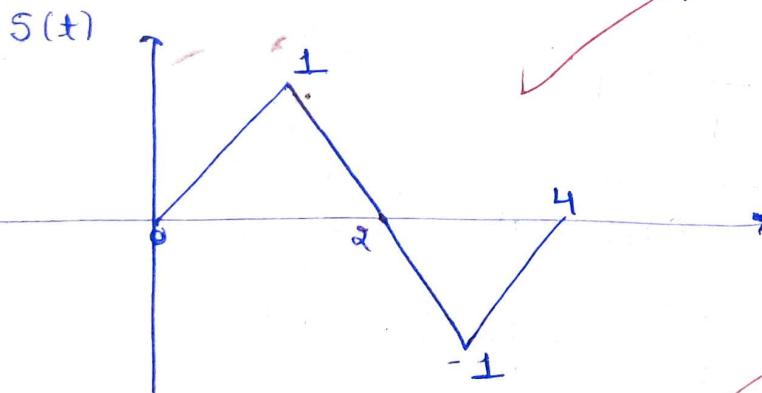
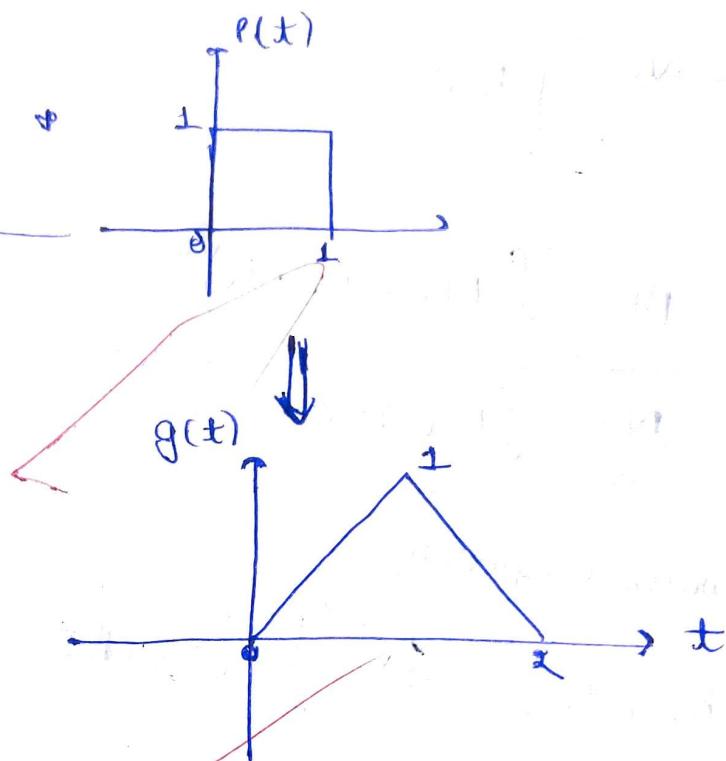
$$S(t) = g(t) - [s(t-2) * \theta(t)]$$

$$S(t) = g(t) - g(t-2)$$

$$h(t) = S(T-t)$$

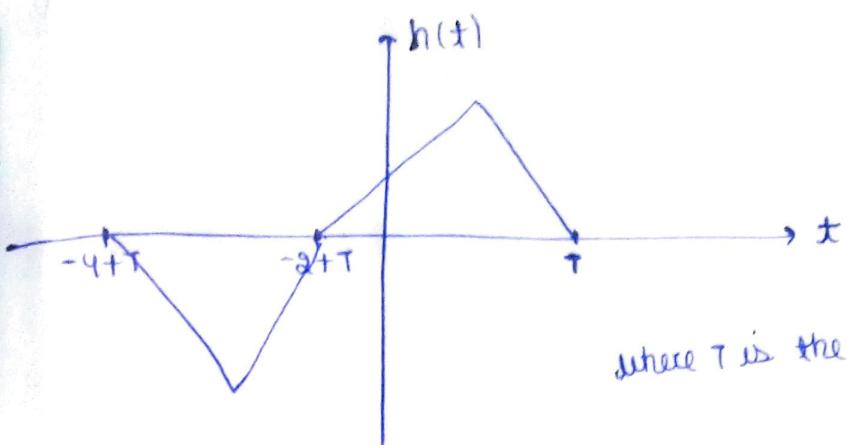


$$P(t) = u(t) - u(t-1)$$



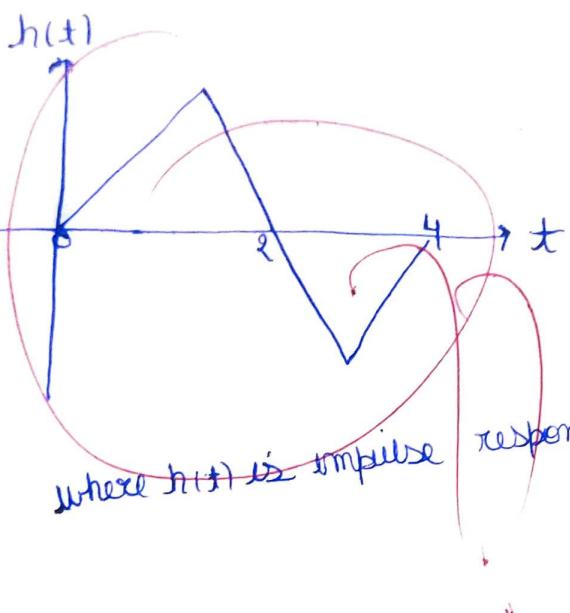
$$h(t) = S(T-t)$$

(12)



where T is the time period of $S(t)$

For $T=4$



where $h(t)$ is impulse

response of filter matched to $S(t)$