Lecture "Digital Signal Processing"

Prof. Dr. Dietrich Klakow, Summer Term 2021

Assignment 6

Submission deadline: 31 May 2021, 23:59

Submission Instructions:

You have one week to solve the assignments.
The code should be well structured and commented. Do not use any Matlab-Toolbox or Python external libraries if it is not mentioned that you can use them.

- You are required to hand in your solutions in a group of two students.
- There are two parts in this assignment: theoretical part and practical part.
- The practical part that is solved with Python should be submitted as an ipynb file (Jupyter Notebook or Google Colab), where every function is written in a separate block.
- The theoretical part can either be written by hand and scanned, or typed with LaTeX in the text / markdown area of ipynb file.
- Submission of both parts should be done via Microsoft Teams by one of the group members.
- Submission should be named as: Ex06_matriculationnumber1_matriculationnumber2.zip

The submission should contain the following files:

- file "README" that contains an information on all team members:
  name, matriculation number, and email address.
- code files
- file "answers.pdf" which contains answers to the questions appearing in the exercise sheet. *Note: If you use ipynb file, you don't have to submit "answers.pdf". You can embed your scanned copy or write your answers in the text / markdown area.*

1 (1.5P) Steps in Kalman Filter implementation

Based on your understanding in Kalman Filter, formulate / list the general steps required to implement Kalman Filter. *Hint: there can be about 6 steps.*

2 Implementation of 2-D Kalman Filter

Allowed Python libraries: numpy, math, and matplotlib.pyplot. Matlab libraries would be the equivalent ones.

Problem Formulation

In this exercise, we will implement a Kalman filter for a travelling object. A well calibrated and reliable droid C3PO launched a ball towards the sky at angle $\alpha = 45°$ with initial velocity v_0 of 20 m/sec. His partner R2D2 equipped with a LIDAR sensor measured regularly the position p from the starting point and velocity v of the ball in both x and y directions (x is parallel to the ground, while y is perpendicular to the ground). In their location, the atmospheric resistance is negligible.

We can express the kinematic equations for this problem in discrete form as follows:

$$p_{x_k} = p_{x_{k-1}} + v_{x_{k-1}}\Delta t \tag{1}$$

$$v_{x_k} = v_{x_{k-1}} \tag{2}$$

$$p_{y_k} = p_{y_{k-1}} + v_{y_{k-1}}\Delta t + \frac{1}{2}a_y(\Delta t)^2 \tag{3}$$

$$v_{y_k} = v_{y_{k-1}} + a_y\Delta t \tag{4}$$

2.1 (1P) Function to generate position and velocity

Implement a function to generate p_{x_k}, p_{y_k}, v_{x_k}, and v_{y_k} with inputs a_y, Δt, $p_{x_{k-1}}$, $p_{y_{k-1}}$, $v_{x_{k-1}}$, and $v_{y_{k-1}}$.

2.2 (0.5P) Function to add noise

Implement a function that adds random noise following a normal distribution $\mathcal{N}(0, \sigma^2)$ to the output of task 2.1.

2.3 (5P) Prediction with Kalman Filter

Since the sensor used was not perfect, the measurements of position and velocity contained some additive random errors. For simplicity, we assume that the errors η for both position and velocity follow the same distribution, i.e. $\eta \sim \mathcal{N}(0, \sigma_\eta^2)$.

Similarly, since it was not a perfect place, the real position and velocity of the travelling ball were affected by some additive perturbations that made the values differ from the model described in the Problem Formulation. For simplicity, we assume that all perturbations γ for both position and velocity follow the same distribution, i.e. $\gamma \sim \mathcal{N}(0, \sigma_\gamma^2)$.

Implement Kalman Filter to obtain the optimal estimation of position and velocity of the ball. Assume that all errors and perturbations are independent of each other, and use the following conditions:

- $a_y = -3.7 m/sec^2$
- $\Delta t = 0.01 sec$
- $\sigma_\eta^2 = 1.0$ (depending on the variable in which the noise is, the unit is either m^2 or m^2/sec^2)
- $\sigma_\gamma^2 = 0.01$ (depending on the variable in which the perturbation is, the unit is either m^2 or m^2/sec^2)
- N = 760 iterations (i.e. the total number of time steps)
- Initial position $p = (0, 0)$ m
- Additionally, the initial velocity v_0 can be decomposed into the following:

$$v_{x_0} = v_0 \cos\left(\frac{\pi\alpha}{180^\circ}\right) \; m/sec \tag{5}$$

$$v_{y_0} = v_0 \sin\left(\frac{\pi\alpha}{180^\circ}\right) \; m/sec \tag{6}$$

Plot the ground truth (i.e. model added with perturbations), the measurements, and optimal estimation (i.e. the output of Kalman Filter) against time in the same graph. The plots should be properly labelled and titled. *Hint: You should generate 4 plots for the variable position and velocity, i.e. p_x, p_y, v_x, and v_y, and properly labelled axis includes the title and its unit, e.g. time (sec).*

2.4 (1P) Behaviour of Kalman Filter

Provide 2 new sets of plots by varying the parameters σ_γ^2 and σ_η^2, and argue why you have selected such values. Additionally, explain the results you obtained.

2.5 (1P) Kalman Gain

Plot the diagonal elements of Kalman gain matrix against time for the three sets of data you have obtained from task 2.3 and 2.4. The plots should be properly labelled and titled. What do you observe? Explain your findings. *Hint: Similarly to task 2.3, you should generate 4 plots.*