1) $N \times N$ cosine transform Matrix $C = \{c(k,n)\}$

1.1

$$C(k,n) = \begin{cases} \frac{1}{\sqrt{N}} & k=0, \quad 0 \le n \le N-1 \\ \sqrt{\frac{2}{N}} \cos \frac{\pi(2n+1)k}{2N}, & 1 \le k \le N-1 \\ & 0 \le n \le N-1 \end{cases}$$

DCT matrix for $N=4$

$C(4,4)$:

for $k=0$

for $n=0$, $C(0,0) = \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$

$n=1$, $c(0,1) = \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$

$n=2$, $c(0,2) = \frac{1}{2}$

$n=3$, $c(0,3) = \frac{1}{2}$

for $k=1$

$n=0$ $\quad c(1,0) = \sqrt{\frac{2}{N}} \cos\left(\frac{\pi(2n+1)k}{2N}\right) = \frac{1}{\sqrt{2}} \cos\left(\frac{\pi}{8}\right)$

$n=1$ $\quad c(1,1) = \frac{1}{\sqrt{2}} \cos\left(\frac{3\pi}{8}\right)$

$n=2$ $\quad c(1,2) = \frac{1}{\sqrt{2}} \cos\left(\frac{5\pi}{8}\right)$

$n=3$ $\quad c(1,3) = \frac{1}{\sqrt{2}} \cos\left(\frac{7\pi}{8}\right)$

for $k=2$

$n=0$ $\quad c(2,0) = \sqrt{\frac{2}{N}} \cos\left(\frac{\pi(1)2}{8}\right) = \frac{1}{\sqrt{2}} \cos\left(\frac{\pi}{4}\right)$

$n=1$ $\quad c(2,1) = \frac{1}{\sqrt{2}} \cos\left(\frac{6\pi}{8}\right)$

$n=2$ $\quad c(2,2) = \frac{1}{\sqrt{2}} \cos\left(\frac{10\pi}{8}\right)$

$n=3$ $\quad c(2,3) = \frac{1}{\sqrt{2}} \cos\left(\frac{14\pi}{8}\right)$

for $K=3$

$n=0 \quad C(3,0) = \sqrt{\frac{2}{N}} \cos\left(\frac{\pi(2n+1)k}{2N}\right) = \frac{1}{\sqrt{2}} \cos\left(\frac{3\pi}{8}\right)$

$n=1 \quad C(3,1) = \frac{1}{\sqrt{2}} \cos\left(\frac{9\pi}{8}\right)$

$n=2 \quad C(3,2) = \frac{1}{\sqrt{2}} \cos\left(\frac{15\pi}{8}\right)$

$n=3 \quad C(3,3) = \frac{1}{\sqrt{2}} \cos\left(\frac{21\pi}{8}\right)$

DCT matrix $= C(\cancel{k},n) =$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \cos\frac{\pi}{8} & \cos\frac{3\pi}{8} & \cos\frac{5\pi}{8} & \cos\frac{7\pi}{8} \\ \cos\frac{2\pi}{8} & \cos\frac{6\pi}{8} & \cos\frac{10\pi}{8} & \cos\frac{14\pi}{8} \\ \cos\frac{3\pi}{8} & \cos\frac{9\pi}{8} & \cos\frac{15\pi}{8} & \cos\frac{21\pi}{8} \end{bmatrix}$$

1.2 DCT of a given signal

1-D signal : $(2, -1, 0, 1)^T$

DCT transformed signal using 4x4 DCT matrix derived in part 1.1

transformed signal = input signal × DCT matrix

$$= \begin{bmatrix} 2 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \cos\frac{\pi}{8} & \cos\frac{3\pi}{8} & \cos\frac{5\pi}{8} & \cos\frac{7\pi}{8} \\ \cos\frac{2\pi}{8} & \cos\frac{6\pi}{8} & \cos\frac{10\pi}{8} & \cos\frac{14\pi}{8} \\ \cos\frac{3\pi}{8} & \cos\frac{9\pi}{8} & \cos\frac{15\pi}{8} & \cos\frac{21\pi}{8} \end{bmatrix}$$

$$= \left[\sqrt{2} - \cos\frac{\pi}{8} + \cos\frac{3\pi}{8},\ \sqrt{2} - \cos\frac{3\pi}{8} + \cos\frac{9\pi}{8},\ \sqrt{2} - \cos\frac{5\pi}{8} + \cos\frac{15\pi}{8},\ \sqrt{2} - \cos\frac{7\pi}{8} + \cos\frac{21\pi}{8}\right]$$

$$= \left[\sqrt{2}, \sqrt{2}, \sqrt{2}, \sqrt{2}\right]$$ → (Is it correct?)

1.3 Inverse DCT of a given signal.

for inverse DCT the DCT coefficient matrix obtained in part one can be used.

Inverse DCT coefficient matrix would be transpose of DCT coefficient matrix $C(K, N)$.