Lecture "Digital Signal Processing"

Prof. Dr. Dietrich Klakow, Summer Term 2021

Assignment 5

Submission deadline: 24 May 2021, 23:59

Submission Instructions:

You have one week to solve the assignments.
The code should be well structured and commented. Do not use any Matlab-Toolbox or Python external libraries if it is not mentioned that you can use them.

- You are allowed and encouraged to hand in your solutions in a group of two students.
- There are two parts in this assignment: theoretical part and practical part.
- The practical part that is solved with Python should be submitted as an ipynb file (Jupyter Notebook or Google Colab), where every function is written in a separate block.
- The theoretical part can either be written by hand and scanned, or typed with LaTeX in the text / markdown area of ipynb file.
- Submission of both parts should be done via Microsoft Teams by one of the group members.
- Submission should be named as: Ex02_matriculationnumber1_matriculationnumber2.zip

The submission should contain the following files:

- file "README" that contains an information on all team members:
  name, matriculation number, and email address.
- code files
- file "answers.pdf" which contains answers to the questions appearing in the exercise sheet. *Note: If you use ipynb file, you don't have to submit "answers.pdf". You can embed your scanned copy or write your answers in the text / markdown area.*

1 (5P)Exercise

1.1 (3P) Subtask

Consider a random signal

$$X(t) = A\cos\left(2\pi f_0\left(t + \Theta\right)\right)$$

where A, Θ are independent random variables of finite variance, and Θ is uniformly distributed on the time interval $[0, P = 1/f_0]$. Is this signal stationary? Find its mean and autocorrelation functions.

1.2 (2 P) Subtask

Given that $\vec{y} = \mathbf{A}\vec{x} + \vec{e}$
Prove that minimizing trace of error covariance $Tr(\mathbf{E}[\vec{e}\vec{e}^T])$ is maximizing $\mathbf{P}(\vec{y}|\vec{x})$ assuming the data is in Gaussian Distribution, the data points are i.i.d. (independent and identically distributed).
Here $\mathbf{E}$ is the expectation and $\mathbf{P}$ is the (conditional) probability.

2 (5 P)Exercise

2.1 (2 P) Subtask

Here you will implement a sensitivity pattern of microphone array for following configuration. Folloing information are given:

1. Location of signal sources in 3D coordinate is $s_1 = Rcos(\theta)sin(\phi)$, $s_2 = Rsin(\theta)sin(\phi)$ and $s_3 = Rcos(\phi)$. By varying the ϕ and θ you can simulate many source from different direction.
2. The specific location of source which you want to listen is [0,1,1]
3. Sound wave is $sin(\omega t)$ and velocity is 330
4. All of your microphones are omnidirectional
5. Location of microphones $x = [[m1_x, m1_y, m1_z], [m2_x, m2_y, m2_z], [m3_x, m3_y, m3_z], ..]$

Write a function *directivity(x, θ, ϕ)* that calculate the power ratio (Total power:Single Power) and returns that value. Use $R = 1$.

2.2 (2 P) Subtask

Use *directivity(x, θ, ϕ)* to find the power ratio in the given specific direction for a (4 microphones are situated in four corners, $x = [[1, 0, 0], [0, 1, 0], [0, -1, 0], [-1, 0, 0]]$) and plot all the values in this 3D polar coordinate using all combination of θ and ϕ.
Note: You can convert the values from polar coordinate to Cartesian 3D coordinate and then plot in 3D.

2.3 (1 P) Subtask

Now do the same task in 2.2 for an **Octahedron** structure (includes two new coordinates $[0, 0, 1], [0, 0, -1]$)