PROJECT 2 - PORTFOLIO OPTIMIZATION

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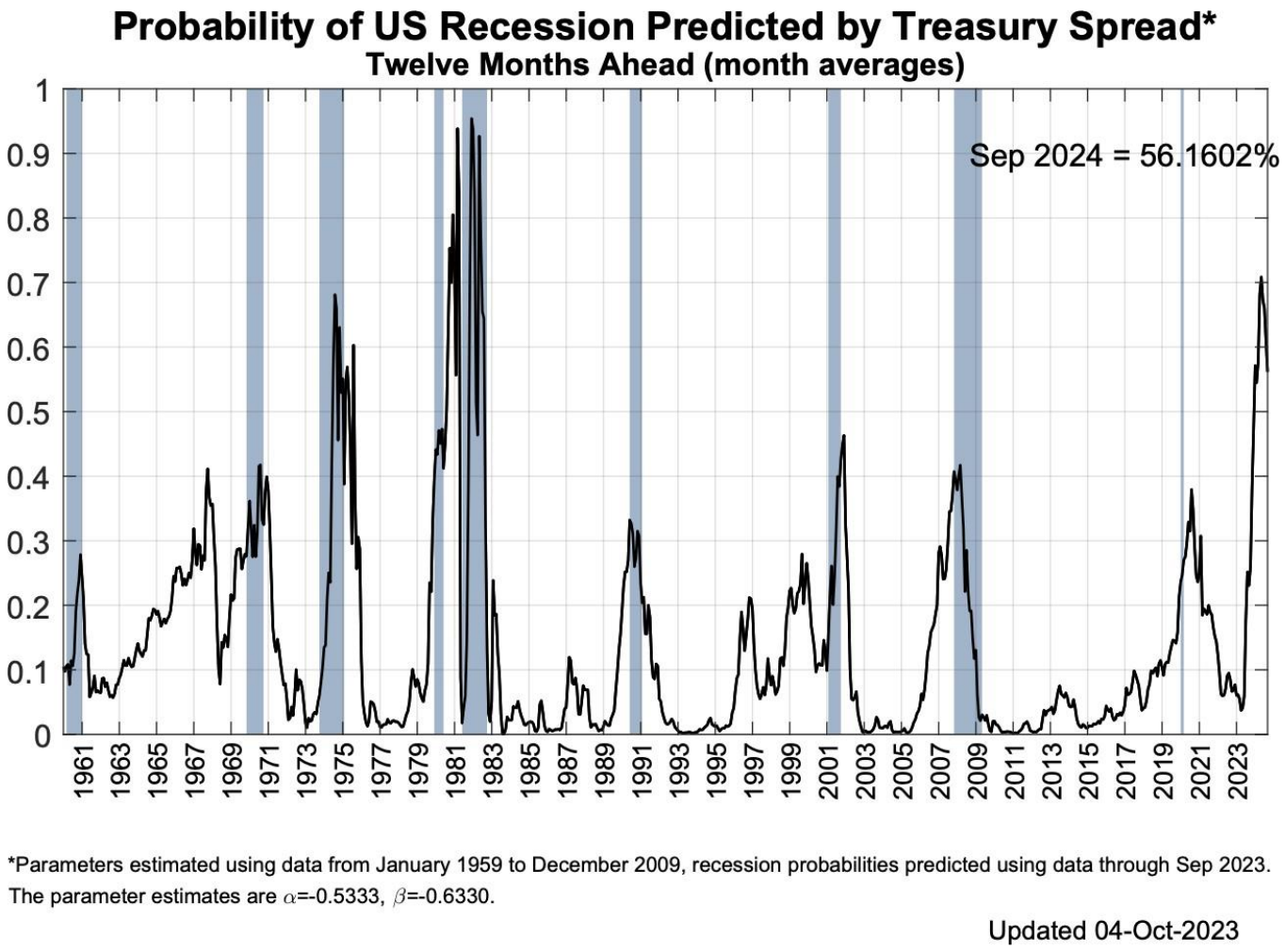
(I) INTRODUCTION/DELIVERABLE 1:

(A) ECONOMIC OULOOK

Recession or Soft Landing:

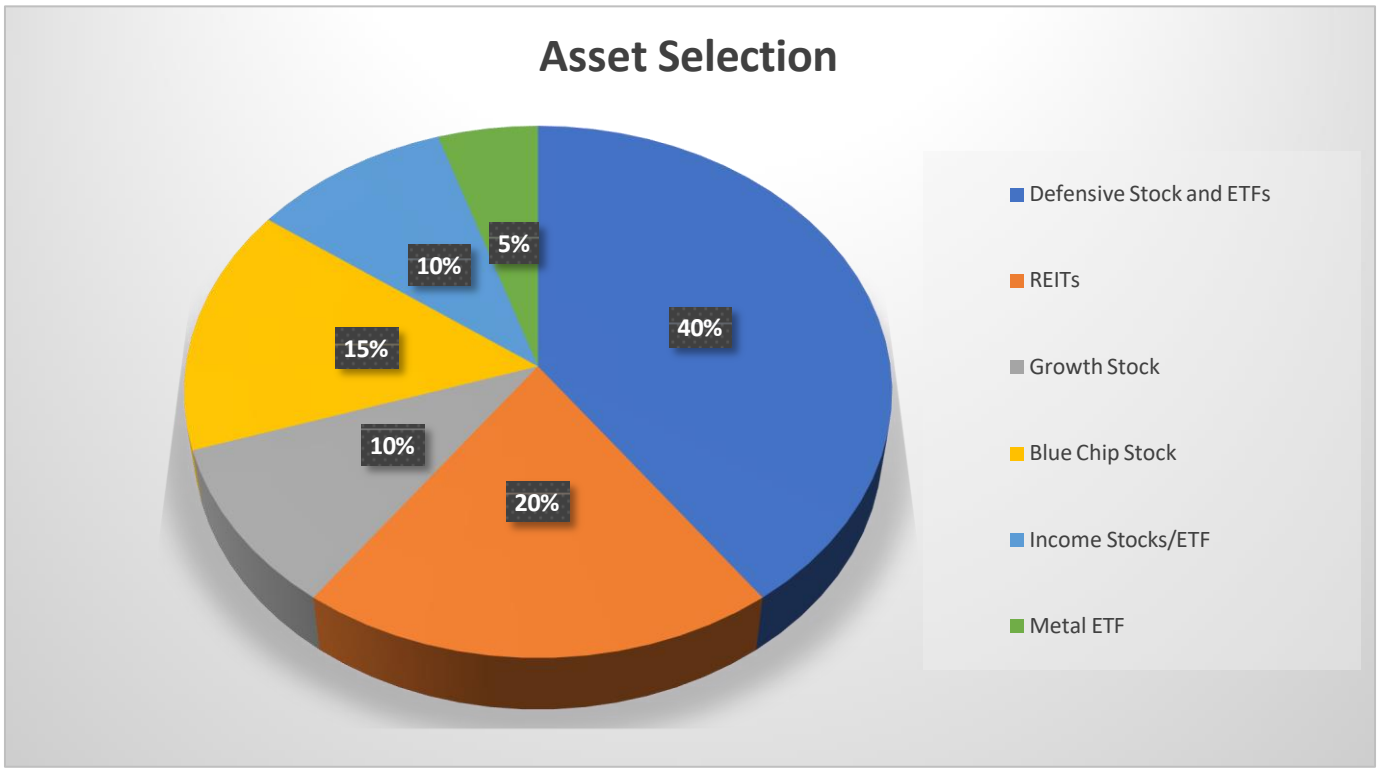
- Despite growing investor confidence in the U.S. economy's ability to sidestep a recession and achieve a soft landing, where inflation is under control and the economy continues to grow steadily, there are persistent threats to this optimistic scenario. Inflation has consistently trended downward for several months, but interest rates have reached their highest point in two decades. Furthermore, certain economic indicators indicate that the economy may not be entirely in the clear regarding a potential recession.
- The New York Fed's recession probability indicator still suggests a 56% likelihood of a U.S. recession within the next year, although this has decreased from a 66% reading in August. Other dependable indicators are sounding cautionary alarms, such as uneven job data, the persistent inversion of the yield curve, and a lack of consensus among experts regarding whether a recession may have been merely delayed rather than completely averted.
- The Federal Reserve has persistently cautioned that its extended series of interest rate hikes will hinder economic growth, even though its latest economic projections no longer foresee a recession. Furthermore, while the risk of a recession may be diminishing, interest rates are expected to stay elevated for a more extended period, suggesting that investors should adopt a prudent approach to the market.
- Even if the U.S. ultimately manages to steer clear of a recession in 2023, the consequences of the Federal Reserve's aggressive monetary policy strategy over the past year and a half may only be beginning to manifest in a negative way for the economy.



(B) INVESTMENT STRATEGY:

(i) Asset Selection:

Diversifying across different asset classes, sectors, and industries can reduce the correlation between investments. This diversification spreads risk and helps ensure that one poor-performing asset doesn't unduly impact the entire portfolio. In this paper, we have considered 20 risky assets across different asset classes and industries and managed to construct a bullet proof portfolio to minimize the impact of market volatility and economic downturns on our investments as we predict a possible recession with our investment horizon. We tried to structure our portfolio to include mix of assets, defensive and cyclical which will allow the portfolio to adapt to changing economic cycles, providing stability during downturns and growth potential during upswings.



- **Defensive Stock and ETF's**
  Defensive stocks are often associated with industries that provide essential goods and services, such as healthcare, utilities, and consumer staples. People continue to use these products and services regardless of economic conditions. This stability contributes to consistent revenue and earnings for these companies.

  We have considered following Defensive stocks and ETFs in our Portfolio.
  - American International Group (AIG)
  - Public Service Enterprise Group (PSEG)
  - Verizon Communications (VZ)
  - The Kraft Heinz Company (KHC)
  - Walmart (WMT)
  - Vanguard Health Care ETF(VHT)
  - Caterpillar Inc. (CAT)

- **REITs**
  REITs are required by law to distribute at least 90% of their taxable income to shareholders in the form of dividends. This consistent income stream can be attractive to investors seeking regular cash flow, especially during economic downturns when income from other investments may be less reliable. REITs often hold portfolios of income-producing properties across different sectors, such as residential, commercial, industrial, or healthcare real estate.

  We have considered following REITs in our Portfolio.
  - Vanguard Real Estate ETF
  - iShares Global REIT ETF
  - iShares U.S. Insurance ETF
  - iShares U.S. Real Estate ETF
  - Simon Property Group

- **Growth Stock**
  While they are typically associated with economic expansion and bull markets, growth stocks can still be attractive investments during a recession as Diversifying the investment portfolio during a recession is important. Growth stocks can add diversification, as their performance may not be as closely correlated with traditional defensive assets.

  We have considered following Growth Stocks in our Portfolio.
  - Apple (AAPL)
  - Nvidia Corporation (NVDA)
  - Proctor & Gamble (PG)

- **Blue Chip Stock**
  Blue chip stocks, which are shares in well-established, large-cap companies with a history of stable performance and a reputation for reliability, are often considered a safe haven during a recession. Historical data often show that blue

chip stocks have performed relatively well during previous recessions, with milder declines compared to more volatile stocks.

We have considered following blue-chip stocks in our Portfolio.

- Tesla Inc (TSLA)
- Amazon (AMZN)

- **Income Stock/ETF**
  High dividend-paying stocks in defensive sectors, such as utilities, healthcare, and consumer staples, tend to be less affected by economic downturns. These sectors often maintain demand for their products or services regardless of economic conditions. High dividend-paying stocks can be attractive investments during a recession because they offer the potential for a steady income stream even in challenging economic times.

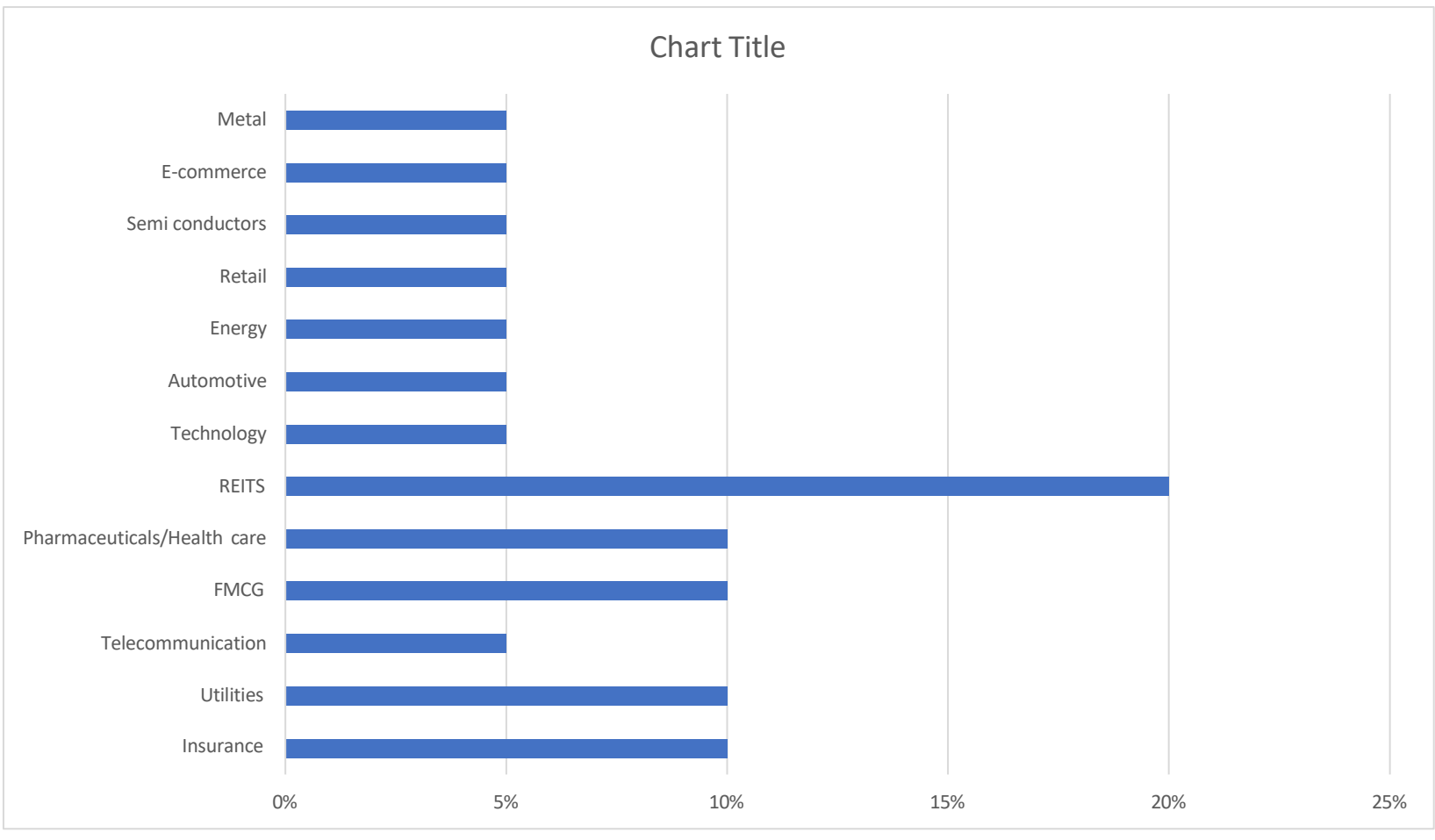
  We have considered following Income Stock/ETF in our Portfolio.
  - Chevron Corporation (CVX)
  - Pfizer Inc (PFE)

- **Metal ETF**
  Investing in metal exchange-traded funds (ETFs) during a recession can be a strategic move to diversify your portfolio, particularly if these ETFs exhibit a negative correlation with traditional asset classes like stocks. Metals, as tangible assets, often act as a hedge against economic uncertainty.

  We have considered following Metal ETF in our Portfolio.
  - Invesco DB Base Metals Fund (DBB)

- **Industry Exposure**
  Diversifying industry exposure during a recession is a strategic approach to managing risk and protecting your investment portfolio. It involves spreading your investments across different industries and sectors to reduce the impact of economic downturns on your holdings.

  We have diversified our portfolio with the following industry based on our above strategy for asset selection.



While defensive sectors are relatively stable during recessions, some cyclical sectors can offer growth potential during economic recoveries. Hence, we have tried to diversify our exposure accordingly.

- **Asset selection within same industry:**
  While choosing stock /ETF within same industry we have following the following rule-based approach while conducting Peer Analysis for market multiples:
  - **Low P/E**

- **Low Expense Ratio**

(C) INVESTMENT HORIZON: Our investment Horizon is 3 years.

(D) DATA: We downloaded the recent 5-year monthly price (Total Return Index (Gross Dividends)) from 10/30/2018-11/01/2023 for all our assets from Bloomberg to calculate the holding period returns.

(E) Risk Free rate: We have considered the 3-Month Treasury Bill Secondary Market Rate (Annualized) obtained from FRED website for our calculations.

(II) DELIVERABLE 2:

We calculated the mean monthly holding period returns and annualized the same by multiplying by 12 as under:

The formula for calculating these returns is as follows:

Rate of Return % = [(Current Value - Initial Value) / Initial Value] X 100 (Since our Price is inclusive of Dividends)

	AIG	PEG	VZ	KHC	WMT	VHT	CAT	VNQ	REET	IAK	IYR	SPG	AAPL	NVDA	TSLA	AMZN	PG	DBB	PFE	CVX
Monthly Average return (a)	1.39%	0.74%	-0.28%	0.11%	1.16%	0.63%	1.45%	0.30%	0.14%	1.04%	0.29%	0.44%	2.72%	4.91%	5.63%	1.19%	1.12%	0.29%	0.12%	1.16%
Average Annualized return (a) * 12	16.67%	8.87%	-3.34%	1.38%	13.86%	7.61%	17.41%	3.60%	1.66%	12.51%	3.54%	5.27%	32.58%	58.96%	67.60%	14.32%	13.45%	3.53%	1.38%	13.95%

(III) DELIVERABLE 3:

We used data Analysis function in excel to estimate our correlation matrix as under:

Correlation Matrix																			
	AIG	***PEG***	***VZ***	***KHC***	***WMT***	***VHT***	***CAT***	***VNQ***	***REET***	***IAK***	***IYR***	***SPG***	***AAPL***	***NVDA***	***TSLA***	***AMZN***	***PG***	***DBB***	***PFE***	***CVX***
AIG	1																			
PEG	0.411973273	1																		
VZ	0.198054434	0.523113114	1																	
KHC	0.322283669	0.205396513	0.176405807	1											**Very Low Correlation**					
WMT	0.214990114	0.471656243	0.196563203	0.270048497	1										**-Ve Correlation**					
VHT	0.418573762	0.515627637	0.431994814	0.43357335	0.558078046	1														
CAT	0.517243871	0.313145536	0.20523112	0.405766826	0.457662847	0.575218944	1													
VNQ	0.601197884	0.6424457	0.357941962	0.298833235	0.461123006	0.731993004	0.521006435	1												
REET	0.66051472	0.601962939	0.332270026	0.303891191	0.406637503	0.701764597	0.537883192	0.978545991	1											
IAK	0.8725666	0.479419423	0.376130873	0.361140427	0.35508368	0.586557482	0.634754931	0.694281994	0.741460162	1										
IYR	0.595686295	0.667137409	0.362531263	0.293663576	0.46818374	0.733064835	0.514873361	0.997681068	0.973580033	0.692177195	1									
SPG	0.690740182	0.334486335	0.205315411	0.285360243	0.195095967	0.529749095	0.536016185	0.734243243	0.797335435	0.735325062	0.720750931	1								
AAPL	0.172583482	0.368939266	0.292349439	0.339979882	0.489482324	0.640553695	0.406056428	0.619338881	0.598454912	0.398490406	0.623430879	0.427824282	1							
NVDA	0.078946381	0.134397266	0.176346536	0.095424278	0.305981275	0.420782184	0.388795778	0.446225978	0.432875098	0.162208585	0.435893993	0.318920693	0.649055262	1						
TSLA	0.047996254	0.106173965	0.076164132	0.152073483	0.304858554	0.394468657	0.237586855	0.416702804	0.417852427	0.222348341	0.414612632	0.351471529	0.690477937	0.533166133	1					
AMZN	0.053357342	0.299237511	0.246944483	0.229534719	0.346962347	0.455217268	0.201062009	0.52301401	0.465482344	0.197745019	0.523571374	0.294757658	0.726472714	0.638247881	0.603344964	1				
PG	0.250657502	0.637171136	0.431470144	0.410034076	0.545337392	0.492541554	0.310987438	0.493903654	0.453094967	0.377408916	0.512013905	0.27141514	0.456588168	0.082040626	0.045934504	0.127124883	1			
DBB	-0.048451234	-0.001651646	-0.057117455	-0.094438002	-0.129607545	-0.239993833	-0.112592785	-0.173745995	-0.120708162	0.000994017	-0.170457133	-0.111484624	-0.15247877	-0.150252791	-0.071291243	-0.321827735	0.040110488	1		
PFE	0.160641334	0.457316189	0.299051492	0.226833759	0.302429704	0.595777552	0.27012067	0.352210013	0.341679856	0.240668885	0.361045139	0.211290237	0.319516174	0.235937537	0.14858001	0.16382622	0.351926495	-0.077031754	1	
CVX	0.600007882	0.297611325	0.296370664	0.426945591	0.28096572	0.584888648	0.607409789	0.532971906	0.578350346	0.671288611	0.520455482	0.685899598	0.342718083	0.204007305	0.219411824	0.202803515	0.234888033	-0.224575335	0.325909578	1

- We have used above matrix to make decisions about our portfolio diversification. Low or negative correlations is used an indicator of potential diversification opportunities, as they suggest that movements in one asset may not strongly affect others.

- We observed the following from the above correlation Matrix:
  - DBB demonstrates generally low correlations with the majority of the listed ETFs and stocks. This suggests that the price movements of DBB are relatively independent of the other assets.
  - For example, the correlation coefficients between DBB and many of the stocks in the list are near zero or slightly negative, indicating minimal linear relationships.

- There are a few weak negative correlations observed between DBB and some assets. For instance, DBB exhibits a modest negative correlation with PG (-0.0770) and KHC (-0.0944). This suggests that when the prices of base metals represented by DBB tend to move in one direction, the prices of these consumer goods stocks may move in the opposite direction, albeit weakly.
- AIG and IAK have a very high positive correlation of 0.8726, suggesting that the stock prices of these two companies tend to move closely in the same direction.
- VNQ and REET also exhibit a strong positive correlation of 0.9785, indicating a tight relationship in their stock price movements.
- There are several sectors within the correlation matrix that show strong positive correlations. For example, IYR, SPG, and VNQ are all related to real estate, which is reflected in their high positive correlations.
- AAPL, NVDA, and TSLA, being tech companies, also exhibit some positive correlations among themselves.

(IV) DELIVERABLE 4:

We used **cov-matrix.xlam** application in excel to calculate our sample variance-covariance matrix as under, which will help us further in the calculation of Minimum Variance Efficient Portfolio. Please note this is the annualize Sample Variance-Covariance matrix which was derived by multiplying the whole table by 12.

Annualised Sample Variance/Covariance Matrix																				
	AIG	***PEG***	***VZ***	***KHC***	***WMT***	***VHT***	***CAT***	***VNQ***	***REET***	***IAK***	***IYR***	***SPG***	***AAPL***	***NVDA***	***TSLA***	***AMZN***	***PG***	***DBB***	***PFE***	***CVX***
AIG	0.00986	0.00235	0.00097	0.00293	0.00108	0.00187	0.00467	0.00349	0.00389	0.00508	0.00344	0.00815	0.00149	0.00113	0.00105	0.00051	0.00126	-0.00028	0.00126	0.00574
PEG	0.00235	0.00330	0.00147	0.00108	0.00138	0.00134	0.00164	0.00216	0.00205	0.00161	0.00223	0.00228	0.00184	0.00111	0.00134	0.00164	0.00185	-0.00001	0.00207	0.00165
VZ	0.00097	0.00147	0.00241	0.00079	0.00049	0.00096	0.00092	0.00103	0.00097	0.00108	0.00104	0.00120	0.00125	0.00125	0.00082	0.00116	0.00107	-0.00017	0.00116	0.00140
KHC	0.00293	0.00108	0.00079	0.00840	0.00126	0.00179	0.00338	0.00160	0.00165	0.00194	0.00157	0.00311	0.00271	0.00126	0.00306	0.00201	0.00190	-0.00051	0.00164	0.00377
WMT	0.00108	0.00138	0.00049	0.00126	0.00258	0.00128	0.00211	0.00137	0.00122	0.00106	0.00138	0.00118	0.00216	0.00224	0.00340	0.00168	0.00140	-0.00039	0.00121	0.00137
VHT	0.00187	0.00134	0.00096	0.00179	0.00128	0.00203	0.00236	0.00193	0.00187	0.00155	0.00192	0.00284	0.00251	0.00274	0.00391	0.00196	0.00113	-0.00064	0.00212	0.00254
CAT	0.00467	0.00164	0.00092	0.00338	0.00211	0.00236	0.00827	0.00277	0.00290	0.00338	0.00273	0.00579	0.00321	0.00510	0.00475	0.00174	0.00143	-0.00060	0.00193	0.00532
VNQ	0.00349	0.00216	0.00103	0.00160	0.00137	0.00193	0.00277	0.00342	0.00339	0.00238	0.00339	0.00510	0.00314	0.00376	0.00535	0.00292	0.00146	-0.00060	0.00162	0.00300
REET	0.00389	0.00205	0.00097	0.00165	0.00122	0.00187	0.00290	0.00339	0.00351	0.00257	0.00336	0.00562	0.00308	0.00370	0.00544	0.00263	0.00136	-0.00042	0.00159	0.00330
IAK	0.00508	0.00161	0.00108	0.00194	0.00106	0.00155	0.00338	0.00238	0.00257	0.00343	0.00236	0.00512	0.00203	0.00137	0.00286	0.00111	0.00112	0.00000	0.00111	0.00379
IYR	0.00344	0.00223	0.00104	0.00157	0.00138	0.00192	0.00273	0.00339	0.00336	0.00236	0.00339	0.00499	0.00315	0.00366	0.00531	0.00291	0.00151	-0.00058	0.00166	0.00292
SPG	0.00815	0.00228	0.00120	0.00311	0.00118	0.00284	0.00579	0.00510	0.00562	0.00512	0.00499	0.01413	0.00441	0.00547	0.00918	0.00334	0.00163	-0.00078	0.00198	0.00786
AAPL	0.00149	0.00184	0.00125	0.00271	0.00216	0.00251	0.00321	0.00314	0.00308	0.00203	0.00315	0.00441	0.00754	0.00812	0.01317	0.00602	0.00201	-0.00078	0.00218	0.00287
NVDA	0.00113	0.00111	0.00125	0.00126	0.00224	0.00274	0.00510	0.00376	0.00370	0.00137	0.00366	0.00547	0.00812	0.02078	0.01689	0.00878	0.00060	-0.00128	0.00268	0.00283
TSLA	0.00105	0.00134	0.00082	0.00306	0.00340	0.00391	0.00475	0.00535	0.00544	0.00286	0.00531	0.00918	0.01317	0.01689	0.04829	0.01265	0.00051	-0.00092	0.00257	0.00465
AMZN	0.00051	0.00164	0.00116	0.00201	0.00168	0.00196	0.00174	0.00292	0.00263	0.00111	0.00291	0.00334	0.00602	0.00878	0.01265	0.00910	0.00061	-0.00181	0.00123	0.00186
PG	0.00126	0.00185	0.00107	0.00190	0.00140	0.00113	0.00143	0.00146	0.00136	0.00112	0.00151	0.00163	0.00201	0.00060	0.00051	0.00061	0.00257	0.00012	0.00140	0.00115
DBB	-0.00028	-0.00001	-0.00017	-0.00051	-0.00039	-0.00064	-0.00060	-0.00060	-0.00042	0.00000	-0.00058	-0.00078	-0.00078	-0.00128	-0.00092	-0.00181	0.00012	0.00347	-0.00036	-0.00128
PFE	0.00126	0.00207	0.00116	0.00164	0.00121	0.00212	0.00193	0.00162	0.00159	0.00111	0.00166	0.00198	0.00218	0.00268	0.00257	0.00123	0.00140	-0.00036	0.00620	0.00247
CVX	0.00574	0.00165	0.00140	0.00377	0.00137	0.00254	0.00532	0.00300	0.00330	0.00379	0.00292	0.00786	0.00287	0.00283	0.00465	0.00186	0.00115	-0.00128	0.00247	0.00929

- We have used above matrix to construct our MVE Portfolios.
- The diagonal elements of the matrix (e.g., 0.00986 for AIG, 0.00330 for PEG) represent the annualized sample variances for each asset. These values reflect the dispersion of returns for each individual asset over the given time period.
- A higher variance indicates greater price volatility and risk associated with the asset. For example, AIG exhibits relatively high variance, implying that its stock price experiences larger fluctuations.

- The off-diagonal elements of the matrix (e.g., 0.00235 for AIG and PEG, 0.00097 for AIG and VZ) represent the annualized sample covariances between pairs of assets. These values quantify how two assets move in relation to each other.
- A positive covariance suggests that the returns of the two assets tend to move in the same direction. A higher positive covariance indicates a stronger positive relationship. For example, AIG and PEG have a positive covariance, implying that their returns are positively correlated.
- A negative covariance suggests that the returns of the two assets tend to move in opposite directions. A higher negative covariance indicates a stronger negative relationship. For example, AIG and PG have a negative covariance, suggesting that their returns move in opposite directions.

(V) DELIVERABLE 5 &7:

- Modern Portfolio Theory uses an optimization process to determine the portfolio that lies on the efficient frontier and suits the investor's risk-return preferences.
- The optimizer will select the portfolio that provides the highest return for a given level of risk or the lowest risk for a specified level of return. The optimization process takes into account the Variance-Covariance Matrix, the expected returns, and the investor's risk tolerance.
- The optimization process calculates the weights of each asset in the portfolio. These weights represent the allocation of each asset within the portfolio. The weights are determined based on the Variance-Covariance Matrix and the desired portfolio characteristics (e.g., maximum return for a given risk level).
- The analysis of Minimum Variance Efficient (MVE) portfolios involves several key steps and Excel formulas. The first Excel formula used, **MMULT(MMULT(TRANSPOSE(matrix of stock weights),covariance matrix),matrix of stock weights)**, is crucial for calculating the **portfolio variance**. This formula employs matrix operations to consider both the weights of individual assets and their respective covariances, resulting in an accurate measure of the overall portfolio risk. The second formula, **SQRT(Variance)**, computes the **standard deviation**, which provides a more intuitive understanding of risk compared to variance. It quantifies how spread out the returns of the portfolio are. The third formula, **MMULT(TRANSPOSE(matrix of stock weights), covariance matrix)**, is utilized to determine the **expected return** of the portfolio.
- We calculated MVE under following scenarios:
  Scenario 1- No Short Selling
  Scenario 2- Long and Short Selling
- To find the optimal portfolio weights, the Solver function in MS Excel was employed as under:

  Scenario 1 (Long only)
  - **Objective Function:** Minimize Variance of the Portfolio
  - **Constraints:**
    Sum of weights= 1
    All individual weights are non-negative.
    Weights of individua asset >=0.01 and <=1 (to make sure that we take a position in all the 20 assets to benefit from diversification)
  - Furthermore, an additional constraint was introduced, adjusting the expected return by 1% in each iteration. This rigorous approach was repeated 10 times to generate a set of MVE portfolios, each offering a unique risk-return profile and constraints.

NO SHORTING												
Ticker Name Name	**Avg Annualised Return**	**Weights MVE**	**Weights MVE +1%**	**Weights MVE +2%**	**Weights MVE +3%**	**Weights MVE +4%**	**Weights MVE +5%**	**Weights MVE +6%**	**Weights MVE +7%**	**Weights MVE +8%**	**Weights MVE +9%**	**Weights MVE +10%**
AIG	14.38%	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
PEG	8.87%	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
VZ	-3.34%	0.21	0.15	0.11	0.09	0.07	0.05	0.03	0.01	0.01	0.01	0.01
KHC	1.38%	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
WMT	13.86%	0.16	0.18	0.19	0.19	0.20	0.20	0.20	0.20	0.20	0.18	0.17
VHT	7.61%	0.14	0.14	0.13	0.12	0.11	0.10	0.08	0.05	0.01	0.01	0.01
CAT	17.41%	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
VNQ	3.60%	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
REET	1.66%	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
IAK	12.51%	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.04	0.05	0.05	0.04
IYR	3.54%	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
SPG	5.27%	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
AAPL	32.58%	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
NVDA	58.96%	0.01	0.01	0.02	0.03	0.04	0.05	0.07	0.08	0.09	0.11	0.13
TSLA	67.60%	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
AMZN	14.32%	0.02	0.03	0.03	0.03	0.02	0.01	0.01	0.01	0.01	0.01	0.01
PG	13.45%	0.01	0.04	0.07	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.23
DBB	3.53%	0.33	0.33	0.32	0.32	0.32	0.32	0.31	0.31	0.30	0.29	0.28
PFE	1.38%	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
CVX	13.95%	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
SUM OF WEIGHTS		1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
NO SHORTING												
Variance of the portfolio		0.08917%	0.09025%	0.09298%	0.09656%	0.10080%	0.10571%	0.11123%	0.11739%	0.12438%	0.13268%	0.14254 %
Std Deviation of the portfolio		2.99%	3.00%	3.05%	3.11%	3.17%	3.25%	3.34%	3.43%	3.53%	3.64%	3.78%
Expected return of the Portfolio		6.57%	7.57%	8.57%	9.57%	10.57%	11.57%	12.57%	13.57%	14.57%	15.57%	16.57%

- **We used the following formula in the calculation of Variance(p), SD(p) and E(rp)**

$$\min_{\omega} \sigma_{pf}^2$$

subject to:

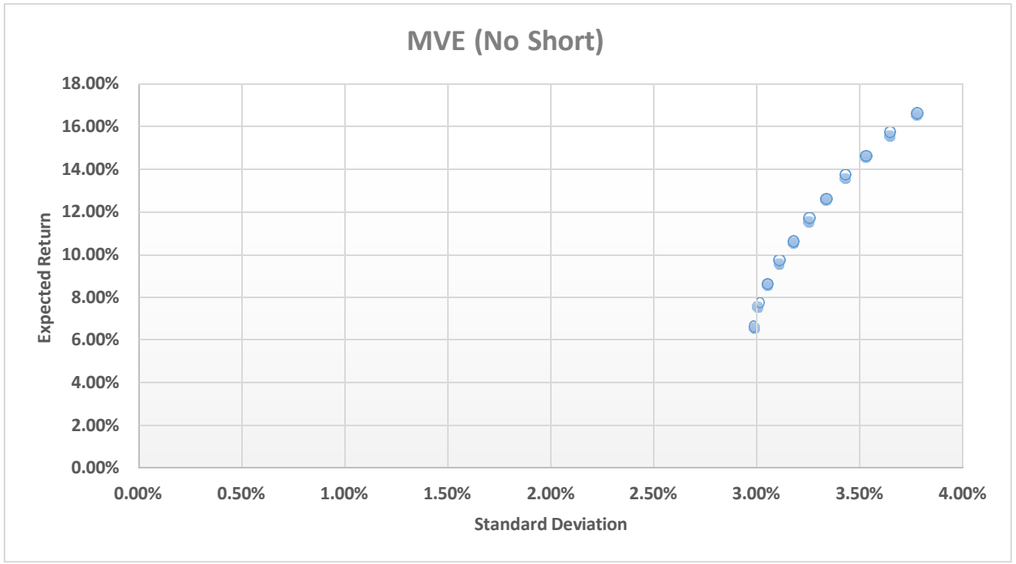
$$\sigma_{pf}^2 = \omega \Omega \omega'$$

$$\sigma_{pf} = \sqrt{\sigma_{pf}^2}$$

$$\sum_{i=1}^{N} w_i = 1$$

$$E[r_{pf}] = R\omega'$$

- The efficient frontier is a graph that illustrates all possible portfolios (in the table above) that can be constructed from the available assets. These portfolios are combinations of the assets, and they vary in terms of expected return and risk (standard deviation). The efficient frontier is derived from the Variance-Covariance Matrix and the expected returns.

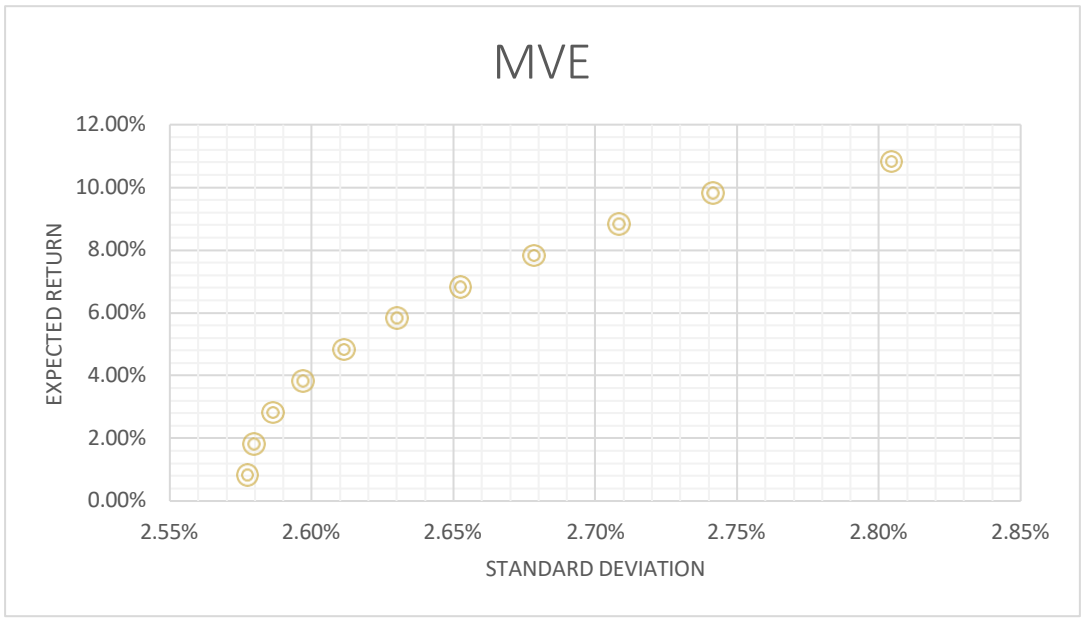


- The above efficient frontier is a graphical representation of portfolios that maximize returns for the risk assumed, showing the benefit of diversification.
- It is the set of efficient portfolios. It is the upper portion of the minimum variance frontier starting at the minimum variance portfolio.

Scenario 2 (Long and Short)

- **Objective Function:** Minimize Variance of the Portfolio
- **Constraints:**
  Sum of weights= 1
- Furthermore, an additional constraint was introduced, adjusting the expected return by 1% in each iteration. This rigorous approach was repeated 10 times to generate a set of MVE portfolios, each offering a unique risk-return profile and constraints.

SHORTING												
Ticker Name Name	**Avg Annualised Return**	**Weights MVE**	**Weights MVE +1%**	**Weights MVE +2%**	**Weights MVE +3%**	**Weights MVE +4%**	**Weights MVE +5%**	**Weights MVE +6%**	**Weights MVE +7%**	**Weights MVE +8%**	**Weights MVE +9%**	**Weights MVE +10%**
AIG	14.38%	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.05
PEG	8.87%	-0.16	-0.15	-0.13	-0.12	-0.11	-0.10	-0.08	-0.07	-0.06	-0.04	-0.09
VZ	-3.34%	0.16	0.14	0.13	0.11	0.10	0.09	0.07	0.06	0.04	0.03	0.01
KHC	1.38%	-0.02	-0.02	-0.02	-0.03	-0.03	-0.03	-0.04	-0.04	-0.04	-0.05	-0.05
WMT	13.86%	0.16	0.16	0.15	0.15	0.14	0.13	0.12	0.12	0.11	0.10	0.11
VHT	7.61%	0.36	0.37	0.37	0.38	0.38	0.39	0.39	0.40	0.41	0.41	0.37
CAT	17.41%	-0.02	-0.02	-0.03	-0.03	-0.03	-0.04	-0.04	-0.05	-0.05	-0.05	-0.05
VNQ	3.60%	-0.16	-0.14	-0.06	0.04	0.14	0.24	0.34	0.44	0.55	0.65	-0.15
REET	1.66%	0.40	0.37	0.33	0.29	0.25	0.22	0.18	0.14	0.10	0.06	0.06
IAK	12.51%	0.02	0.04	0.05	0.07	0.09	0.10	0.12	0.14	0.16	0.17	0.17
IYR	3.54%	-0.16	-0.16	-0.23	-0.32	-0.41	-0.50	-0.59	-0.67	-0.76	-0.85	-0.04
SPG	5.27%	-0.06	-0.06	-0.06	-0.06	-0.07	-0.07	-0.07	-0.07	-0.08	-0.08	-0.10
AAPL	32.58%	-0.18	-0.18	-0.18	-0.18	-0.18	-0.18	-0.17	-0.17	-0.17	-0.17	-0.17
NVDA	58.96%	-0.01	0.00	0.00	0.01	0.01	0.02	0.03	0.03	0.04	0.05	0.05
TSLA	67.60%	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00
AMZN	14.32%	0.18	0.18	0.18	0.17	0.17	0.17	0.16	0.16	0.16	0.16	0.16
PG	13.45%	0.16	0.18	0.19	0.20	0.22	0.23	0.25	0.26	0.28	0.29	0.31
DBB	3.53%	0.31	0.31	0.31	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.31
PFE	1.38%	-0.01	-0.01	-0.02	-0.02	-0.02	-0.03	-0.03	-0.04	-0.04	-0.04	-0.04
CVX	13.95%	0.02	0.02	0.03	0.03	0.04	0.04	0.04	0.05	0.05	0.06	0.08
SUM OF WEIGHTS		1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
NO SHORTING		**SHORTING**										
Varience of the portfolio		0.06644%	0.06656%	0.06691%	0.06746%	0.06822%	0.06919%	0.07037%	0.07176%	0.07336%	0.07517%	0.07867%
Std Deviation of the portfolio		2.58%	2.58%	2.59%	2.60%	2.61%	2.63%	2.65%	2.68%	2.71%	2.74%	2.80%
Expected return of the Portfolio		0.82%	1.82%	2.82%	3.82%	4.82%	5.82%	6.82%	7.82%	8.82%	9.82%	10.82%



As can be inferred from the above, short selling increases the variance and overall risk of a portfolio. With the ability to short sell, the efficient frontier may extend beyond the typical upper boundary of the long-only efficient frontier. This means that portfolios on the efficient frontier can have negative weights (short positions) in certain assets. Short selling can help further optimize the risk-return trade-off.

(VI) DELIVERABLE 6

- The Tangency Portfolio is the point on the efficient frontier where the CML is tangent, or just touches, the efficient frontier. In other words, it's the portfolio that offers the highest risk-adjusted return, taking into account the risk-free rate. We identify tangency ratio by maximizing the sharpe ratio.

- **Risk Free Rate** – 1.8137% (This is the annualized risk free 3-month treasury bill rates that we have downloaded for the same five-year period From the Federal Reserve website.) (https://fred.stlouisfed.org/series/TB3MS)

- **Sharpe Ratio:** The Sharpe Ratio is the difference between the risk-free return and the return of an investment divided by the investment's standard deviation.

$$\left[\text{Sharpe ratio} = \frac{E[r_{pf}] - r_f}{\sigma_{pf}}\right]$$

Where,
E [rpf] = the Expected return of the portfolio
rf = the risk-free rate
σpf = the standard deviation of the portfolio

The Sharpe ratio is a measure of risk-adjusted return. It describes how much excess return you receive for the volatility of holding a riskier asset.

Here is a dummy portfolio with equal weights allowed to each stock.

	Weights
AIG	5.0%
PEG	5.0%
VZ	5.0%
KHC	5.0%
WMT	5.0%
VHT	5.0%
CAT	5.0%
VNQ	5.0%
REET	5.0%
IAK	5.0%
IYR	5.0%
SPG	5.0%

AAPL	5.0%
NVDA	5.0%
TSLA	5.0%
AMZN	5.0%
PG	5.0%
DBB	5.0%
PFE	5.0%
CVX	5.0%

We find the expected return of the portfolio, the standard deviation and Sharpe ratio to be as follows.

Sum of weights	100.0%
Portfolio Expected Return	14.74%
Std Dev of Portfolio	18.01%
Sharpe Ratio	0.71764

We want to maximize this Sharpe ratio. We use Excel to solve the following optimization problem:

$$\max_{\omega} \frac{E[r_{pf}]-r_f}{\sigma_{pf}}$$

subject to:

$$\sum_{i=1}^{N} w_i = 1$$

No

- **No Shorting**

By running solver and by choosing not to short shorts, we get the following values.

	Weights
AIG	0.05
PEG	0.01
VZ	0.01
KHC	0.01
WMT	0.01
VHT	0.01
CAT	0.01
VNQ	0.01
REET	0.01
IAK	0.01
IYR	0.01
SPG	0.01
AAPL	0.01
NVDA	0.27
TSLA	0.06
AMZN	0.01
PG	0.32
DBB	0.14
PFE	0.01
CVX	0.01

Sum of weights	100%
Portfolio Expected Return	27%
Std Dev of Portfolio	20%
Sharpe Ratio	1.26755

As u can see the Sharpe ratio has increased and the weights have been allotted to each stock in the portfolio in a way that maximizes our Sharpe ratio.

In summary, a tangency portfolio with an expected return of 27% and a standard deviation of 20% represents a balanced investment strategy. It suggests that the portfolio has been carefully constructed to achieve a relatively high return for a moderate level of risk.

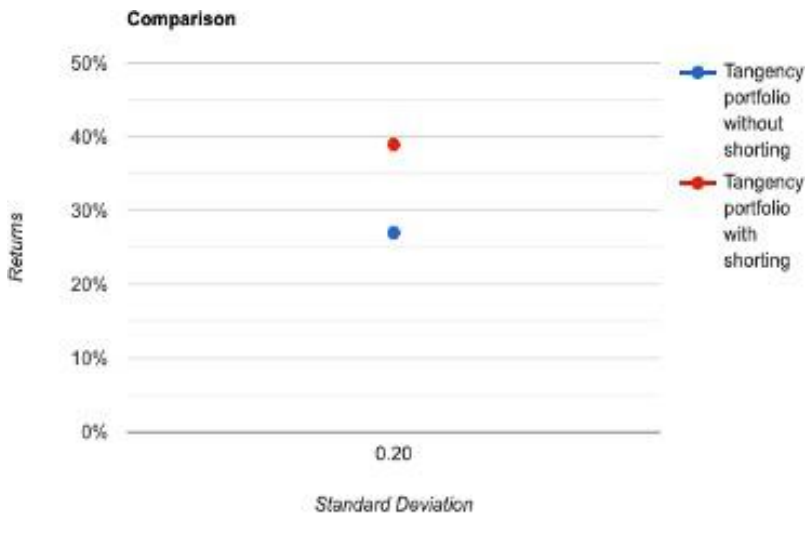
- **Shorting**

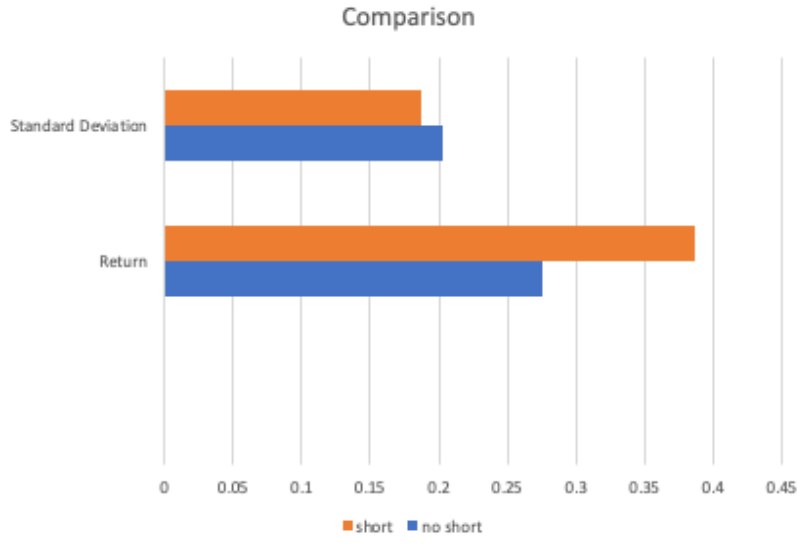
We repeated the same steps here but chose to allow shorting hence the negative weights.

	Weights
AIG	0.20
PEG	0.20
VZ	-0.04
KHC	-0.08
WMT	0.20
VHT	0.20
CAT	-0.02
VNQ	0.20
REET	-0.98
IAK	0.20
IYR	0.20
SPG	-0.17
AAPL	0.20
NVDA	0.20
TSLA	0.07
AMZN	-0.09
PG	0.20
DBB	0.20
PFE	-0.10
CVX	0.20

Sum of weights	100%
Portfolio Expected Return	39%
Std Dev of Portfolio	19%
Sharpe Ratio	1.96556

- **Comparison – With and without shorting**





As you can see, shorting a few stocks in the portfolio can potentially increase returns for the same level of risk. Choosing to short stocks in a portfolio can increase returns for several reasons:

- **Profit from Declining Prices:** Shorting allows investors to profit from stocks that are expected to decrease in value. When the price falls, short sellers buy the stock back at a lower price, making a profit from the difference. This additional source of profit can enhance overall portfolio returns.
- **Diversification of Strategies:** Allowing shorting enables investors to diversify their investment strategies. Long positions (buying stocks) and short positions (selling borrowed stocks) can be strategically balanced to take advantage of both upward and downward market movements. This flexibility can lead to higher returns in various market conditions.
- **Hedging Against Market Downturns:** Shorting specific stocks can act as a hedge against market downturns. If the overall market is expected to decline, shorting certain stocks can offset losses in other parts of the portfolio, helping to preserve capital and potentially increase overall returns.
- **Active Management:** Allowing shorting provides active portfolio managers with more tools to manage their portfolios dynamically. They can capitalize on both positive and negative market movements, making strategic shorting decisions to optimize returns and manage risk actively.

(VII) CONCLUSION:

In this project, we explored portfolio optimization techniques based on modern portfolio theory. Specifically, we looked at how to construct minimum variance efficient frontiers and identify the tangency portfolio that provides the highest possible return for a given level of risk.

Our analysis showed that by carefully selecting the weights of different assets in a portfolio, it is possible to minimize the risk for a target level of expected return. The minimum variance efficient frontier plots the portfolio with the lowest possible risk at each target return level. The tangency portfolio identifies the point where the efficient frontier is tangent to the capital allocation line, representing the theoretically optimal risk-return tradeoff.