# Lecture 11-2

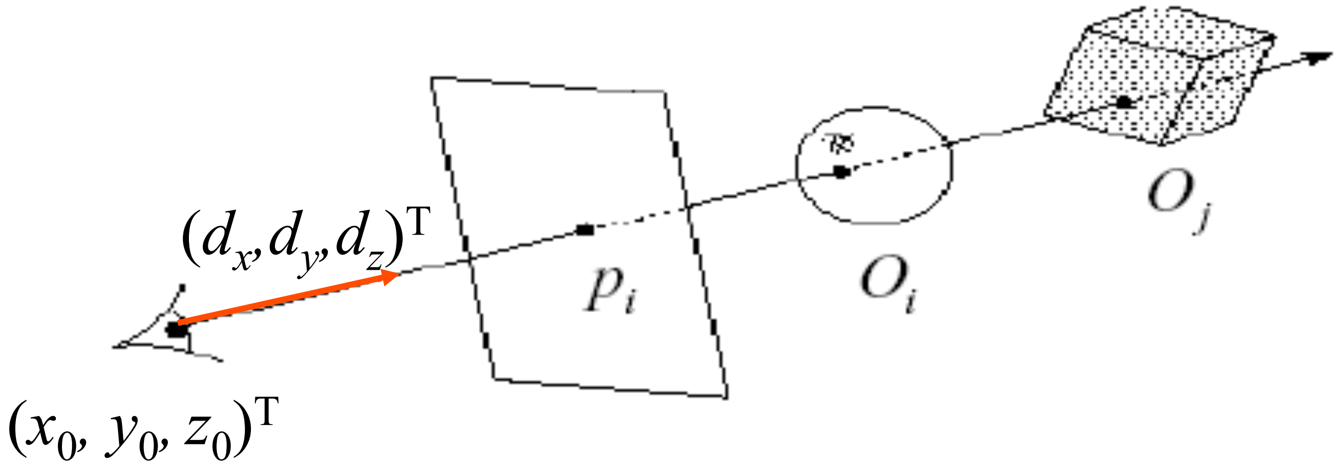
# Ray-Object Intersections

Lecture outline:

1. Basic Concept: Ray-Object Intersection
2. Ray-intersections with different basic objects:
   - Sphere
   - Quadrilateral
   - Disk
   - Cylinder
   - Cone

# Mathematics

- The heart of any ray tracer, or ray casting for hidden surface removal, is the intersection routines.
- Each kind of primitive has different properties, so we have different intersection equations.



# Parametric Ray Equation

- Let
  - the COP be $\mathbf{P}_0 = ( x_0 , y_0 , z_0 )^T$ and
  - the viewing direction be $\mathbf{D} = ( d_x , d_y , d_z )^T$
- Any point P lying on the eye ray is given by:

$$\mathbf{P} = \mathbf{P}_0 + \mathbf{D}\, t$$

- Or writing each coordinate separately:

$$x = x_0 + d_x\, t$$
$$y = y_0 + d_y\, t$$
$$z = z_0 + d_z\, t$$

# Ray Parameterization

- The parametric ray equation is given by:

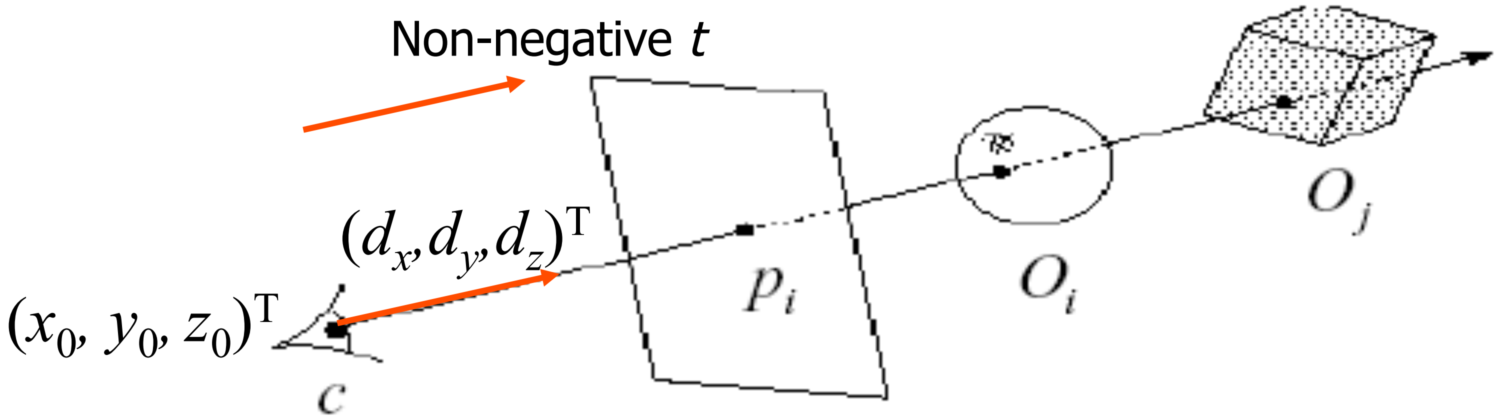
$$\mathbf{P} = \mathbf{P}_0 + \mathbf{D}\, t$$

- Points along the line of sight is parametrized by *t*:

    t = 0 , at COP (eye/viewpoint)

    t < 0 , behind COP

    t > 0 , in front of COP



# Mathematics

- Given an implicit surface (i.e., spheres and other quadrics defined by an implicit equation)

$$F(x, y, z) = 0$$

- In the followings, all surface equations are assumed to be in the object space coordinate system. Therefore, we need to transform the ray before testing for intersection.
- Let's start with a simple 2D example: a ray in 2D and a circle in 2D… what are their equations?

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# (1) Intersecting Spheres

- The (implicit) equation of a unit sphere is given by:

$$x^2 + y^2 + z^2 = 1$$

- Assuming a unit sphere (radius is equal to one and center at origin). Substituting the parametric ray equation yields the following:

$$(d_x{}^2 + d_y{}^2 + d_z{}^2)\,t^2 + 2(d_x x_0 + d_y y_0 + d_z z_0)\,t + (x_0{}^2 + y_0{}^2 + z_0{}^2) - 1 = 0$$

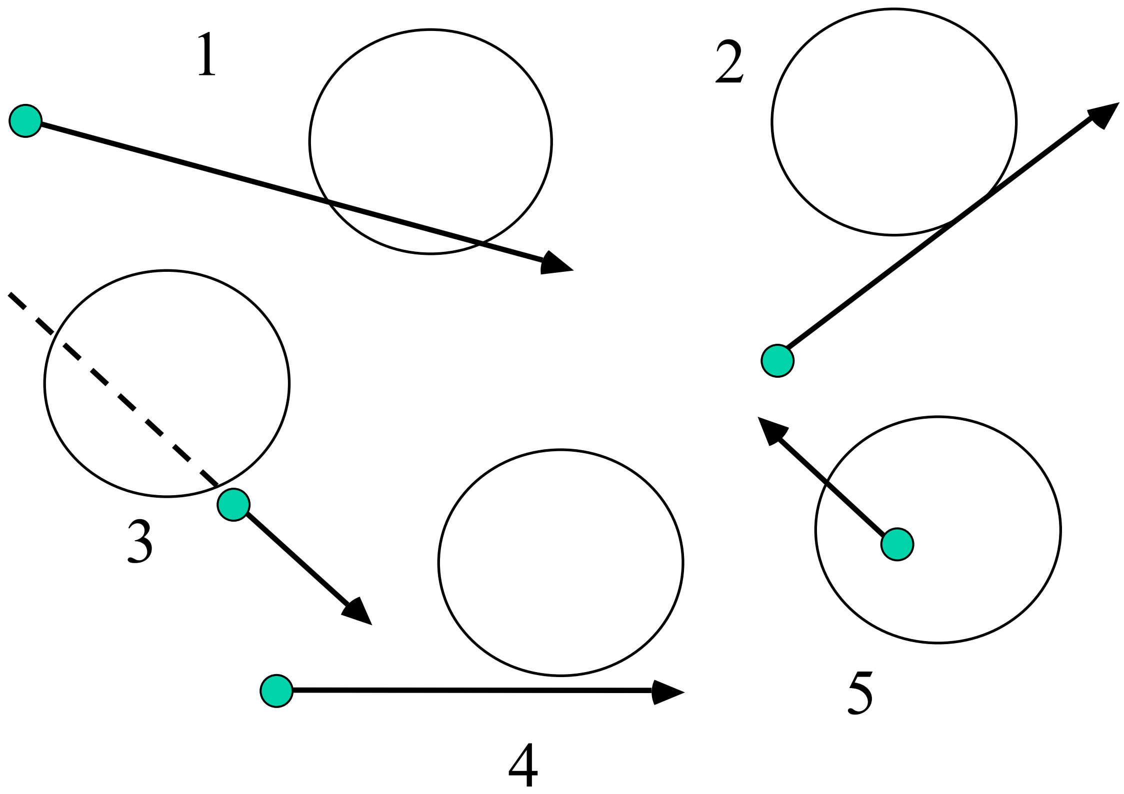
which is a quadratic equation in *t*.

# Intersecting Spheres

- Solving the quadratic equation in $t$ gives the solution.
- Ray misses the sphere if the discriminant is negative.
- If the discriminant is non-negative, the smallest positive $t$ is taken.
- Else, the intersection point is given by:

$$x = x_0 + d_x t_1$$
$$y = y_0 + d_y t_1$$
$$z = z_0 + d_z t_1$$

# How about Possible cases? Sphere-Ray Intersection




1. Ray intersects sphere twice with $t>0$
2. Ray tangent to sphere
3. Ray intersects sphere with $t<0$
4. Ray does not intersect sphere
5. Ray originates inside sphere

# (2) Intersecting (planar) Quadrilaterals

- Solving a ray-plane equation determines if the ray hits the polygon plane. It is followed by an extent check to see if the ray hits the polygon.

- Let's write the ray equation as:

$$\mathbf{P} = \mathbf{P}_0 + \mathbf{D}\, t$$

which defines a ray with

$\mathbf{P}_0 = ( x_0 , y_0 , z_0 )^T$ is the ray's origin

$\mathbf{D} = ( d_x , d_y , d_z )^T$ is the ray's direction

# Intersecting Quadrilaterals

- Define the plane in terms of [*A B C D* ] as:

$$[\ (x,y,z) - (x_0,y_0,z_0)\ ] \text{ dot } [N_x,N_y,N_z] = 0$$

$$\Rightarrow \quad A\,x + B\,y + C\,z + D = 0$$

- Note: the unit vector normal of the plane is defined by:

$$\boldsymbol{P}_{normal} = \boldsymbol{P}_n = [A\ B\ C\ ]^{T}$$

# Intersecting Quadrilaterals

- Substituting the ray equation into the plane equation yields:

$$A\,(x_0 + d_x\,t) + B\,(y_0 + d_y\,t) + C\,(z_0 + d_z\,t) + D = 0$$

- Solving for $t$

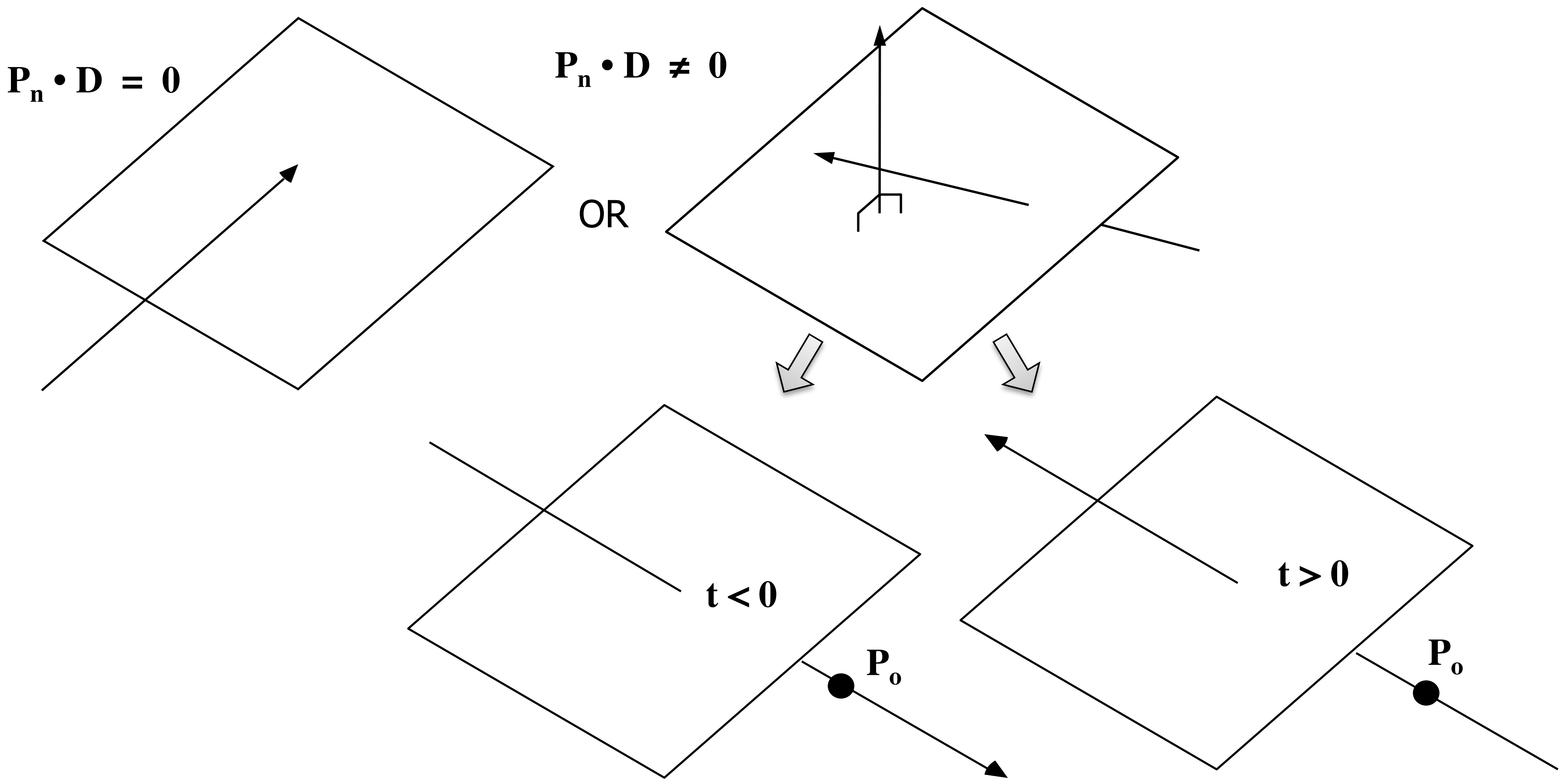
$$t = \frac{-(Ax_0 + By_0 + Cz_0 + D)}{Ad_x + Bd_y + Cd_z}$$

- In vector form, the equation becomes

$$t = \frac{-(\vec{P}_n \cdot P_0 + D)}{\vec{P}_n \cdot \vec{D}}$$

- The vector equation will have no solution if the dot product of $\boldsymbol{P}_n$ and $\boldsymbol{D}$ is zero (ray direction exactly perpendicular to plane normal).

# Possible Cases



# Intersecting Quadrilaterals

- Define

$$v_d = \mathbf{P_n} \bullet \mathbf{D}$$
$$v_0 = -(\mathbf{P_n} \bullet \mathbf{P_0} + D)$$

- Hence,

$$t = v_0 / v_d$$

- If t < 0, then the line defined by the ray intersects the plane behind the COP. Therefore, no intersection actually occurs.
- Else, the intersection point is given by:

$$\mathbf{P} = \mathbf{P}_0 + \mathbf{D}\,(v_0 / v_d)$$

# Intersecting Quadrilaterals

Final Step!!!

- Further extent check is required if the intersection point lies within the region bounded by the quadrilaterals

Two questions:

- Any fast way to do this?
- How about intersecting a triangle?

# (3) Intersecting a disk

- Intersecting circles is similar to intersecting quadrilaterals
- The extent check, after computing the intersection point, becomes one of using the circle equation

- Consider a circle lying on the z=0 plane. If the ray intersects the z=0 plane, it also intersects the circle if:

$$x^2 + y^2 - 1 \leq 0$$

# (4) Intersecting Cylinders

- Recall the parametric ray equation is:

$$x = x_0 + d_x t$$

$$y = y_0 + d_y t$$

$$z = z_0 + d_z t$$

- The equation for an infinite cylinder (along Z-axis) is:

$$x^2 + y^2 - 1 = 0$$

- Substituting the ray equation yields a quadratic equation in *t*:

$$(x_0 + d_x t)^2 + (y_0 + d_y t)^2 - 1 = 0$$

$$t^2 (d_x^2 + d_y^2) + 2(x_0 d_x + y_0 d_y) t + (x_0^2 + y_0^2) - 1 = 0$$

- An extent check is applied for a finite cylinder.

# (5) Intersecting Cones

- The implicit equation for a cone is

$$x^2 + y^2 - z^2 = 0$$

- Substituting the ray equation into the above yields a quadratic equation in $t$:

$$(x_0 + d_x t)^2 + (y_0 + d_y t)^2 - (z_0 + d_z t)^2 = 0$$

$$t^2(d_x^2 + d_y^2 - d_z^2) + 2(x_0 d_x + y_0 d_y - z_0 d_z)t + (x_0^2 + y_0^2 - z_0^2) = 0$$

- Compute the discriminant, and solve for $t$ if the discriminant is non-negative.