Gravitational-Wave Astronomy

Franceso Pannarale
francesco.pannarale@uniroma1.it

SAPIENZA
UNIVERSITÀ DI ROMA

Lecture 2: Main Noise Sources, Waveform Sources and Phenomenology

Syllabus

1. Introduction to the course, General Relativity, gravitational waves and interferometric detectors
2. Waveform sources and phenomenology
3. Gravitational-wave searches and the case of GW150914
4. Gravitational-wave inference and the case of GW150914
5. Multimessenger astronomy and the case of GW170817

Key Concepts in Lecture 1 (1/3)

- Starting point – **Strong Principle of Equivalence**: in an arbitrary gravitational field, at any given spacetime point, we can choose a locally inertial reference frame such that locally all physical laws take the form prescribed in absence of gravity by Special Relativity ($\Rightarrow \exists$ GWs)
- Write laws of physics using tensors and they will automatically be valid in all frames (not just locally).
- Einstein Equation: $R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = \frac{8\pi G}{c^4} T_{\alpha\beta}$
- Detecting spacetime curvature requires monitoring the relative motion of at least two freely-falling test bodies.

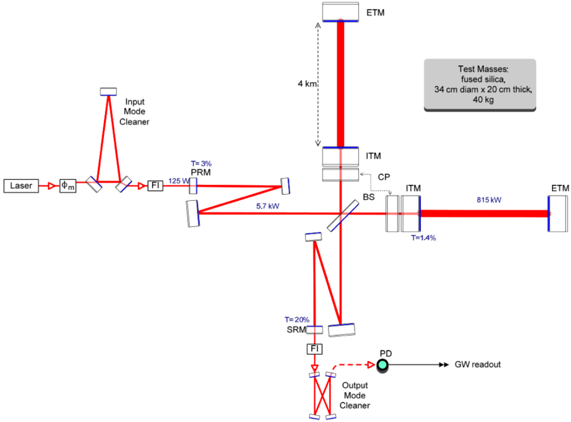
Key Concepts in Lecture 1 (2/3)

- Gravitational waves travel at the speed of light (c), are transverse to the propagation direction, have two independent polarisation states ($+$ and $\times$), carry energy away from a radiating system.

- As a gravitational wave passes through two bodies initially at rest, their coordinate positions (not tensors!) do not change, but the proper distance between them (tensor-related!) does.

Key Concepts in Lecture 1 (3/3)

The Michelson sensitivity $h = \delta\ell/\ell \sim 10^{-9}$ can be improved to $h = \delta\ell/\ell_{\mathrm{eff}} \sim 10^{-20}$ for a 1 W laser.



- Fabry–Perot cavities
- Power recycling (and 150 W)
- Signal recycling

Bibliography

- Einstein, *Relatività: esposizione divulgativa*, Bollati Boringhieri (1967)
- Born, *La sintesi einsteiniana* [Cap.6-7], Bollati Boringhieri (1969)
- https://www.ligo.caltech.edu/page/learn-more

Limits to Detector Sensitivity

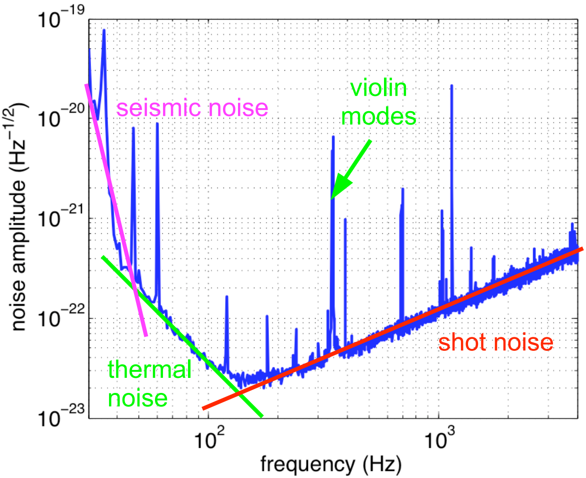
- Characterize sensitivity using the noise spectral density

$$\frac{1}{2}S_h(f) = \langle|\tilde{n}(f)|^2\rangle$$

or amplitude strain sensitivity $S_h^{1/2}(f)$.

- Let's look at some of the sources of noise that limit the sensitivity
  1. Quantum noise: shot noise and radiation pressure.
  2. Thermal noise
  3. Seismic noise

LIGO-Hanford Detector Noise Spectrum, 2007



1. Quantum Noise: Shot Noise

Shot noise is simply the photon counting noise we met before.

$$\frac{\Delta\ell}{\ell} \sim \frac{\lambda_{\rm laser}}{\ell N_{\rm photons}^{1/2}} = \frac{1}{\ell}\sqrt{\frac{h_{\rm Planck}\, c\, \lambda_{\rm laser}}{P\,\mathcal{T}}}$$

Detailed calculation for a Michelson interferometer yields:

$$S_{\rm shot}^{1/2}(f) = \frac{\lambda_{\rm laser}}{4\pi L}\left(\frac{2\hbar\omega_{\rm laser}}{P}\right)^{1/2}$$

1. Frequency independent (but for a realistic Fabry – Perot interferometer need to account for detector response).
2. Reduced by higher power.

1. Quantum Noise: Radiation Pressure

We cannot increase laser power indefinitely.

- Photons bouncing off mirrors impart momentum, moving the mirrors stochastically.
- More power $\longrightarrow$ more photons $\longrightarrow$ more **radiation pressure noise**.

Force due to wave with power P ($F = 2P/c$) fluctuates over time: the power spectrum of the fractional relative length change from the sum of the 2 arms is

$$S_{\mathrm{rad}}^{1/2}(f) = \frac{4\sqrt{2\hbar\omega_{\mathrm{laser}} P}}{MLc(2\pi f)^2}$$

1. Frequency dependent
2. Reduced by lower power

1. Quantum Noise: the Standard Quantum Limit

- At any given frequency f, $S_{\text{shot}} + S_{\text{rad}}$ is minimized by choosing the power P such that $S_{\text{shot}} = S_{\text{rad}}$. The total noise is then

$$S_{\text{SQL}}^{1/2} = \frac{1}{2\pi f L}\sqrt{\frac{8\hbar}{M}}.$$

- This minimum noise level is called the **standard quantum limit**.

- We have a km-scale machine that “sees” the consequences of Heisenberg’s uncertainty principle.

2. Seismic Noise

- Seismic noise is due to shaking of the ground, due to earthquakes, weather, human activity, etc. It affects the interferometer by shaking the optical components.
- If the mirrors were resting on the ground, it would be

$$S_{\rm seis}^{1/2} \sim 10^{-12}\mathrm{Hz}^{-1/2}\left(\frac{10\mathrm{Hz}}{f}\right)^2 \quad \text{for } f > 10\mathrm{Hz}$$

- This is many orders of magnitude larger than the gravitational wave signal we are trying to measure.
- To suppress the effect of seismic motion, the mirrors are suspended as pendula.

2. Seismic Noise

- Suspended mirrors are simple harmonic oscillators.
- If X is the position of the pivot point, and x is the position of the mirror and ℓ is the length of the pendulum, the two are related by

$$\tilde{x}(f) = A(f)\tilde{X}(f) \quad \text{where} \quad A(f) = \frac{1}{1 - (f/f_{\text{pend}})^2}$$

$$\text{and} \quad f_{\text{pend}} = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$$

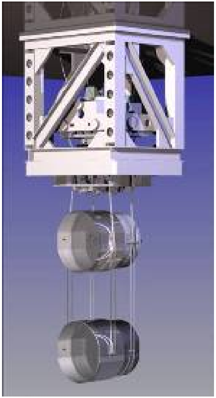
- For $f > f_{\text{pend}}$, $\tilde{x}(f) \approx -(f_{\text{pend}}/f)^2\,\tilde{X}(f)$, so seismic noise is suppressed.

2. Seismic Noise

Increase isolation from ground by "stacking." E.g., Advanced LIGO/Virgo use a quadruple/heptaple stage pendulum.

They also use **active isolation**: sensors monitor motion of suspension system; control motors that push back on system to cancel ground motion.

Can get required sensitivity, but not easy, especially at low frequencies.



Advanced LIGO suspension.

3. Thermal Noise

Equipartition Theorem: each degree of freedom of a system in thermodynamic equilibrium at temperature T should have an energy with expectation value $\frac{1}{2}k_\mathrm{B}T$.

Main physical manifestations of thermal noise:

- Random motion of atoms in mirrors.
- Vibration in wire suspensions of mirrors ("violin modes").
- Swinging of the mirror pendula.

Induces random "jittering" of mirror positions:

- $m = 10\,\mathrm{kg}$ mirror suspended from an $\ell = 1\,\mathrm{m}$ wire:

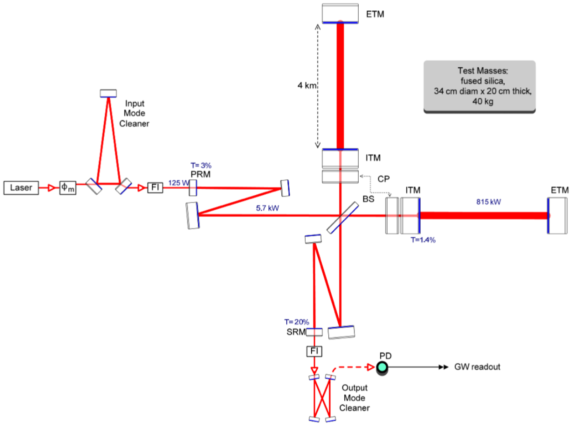
$$\Delta L_\mathrm{rms} \simeq \sqrt{\frac{k_B T \ell}{mg}} \simeq 6 \times 10^{-12} m \quad [\text{Increase } m]\,.$$

3. Thermal Noise

Can minimize impact by concentrating thermal noise power at resonant frequency f_0. In the pendulum example, this is equivalent to minimizing the coefficient in the dissipative force $-\beta v$.

- Suspend mirrors as pendulums (low β, resonant frequency $\sim 1Hz$ – well below observation band).
- Use high-quality-factor wires for suspensions, so vibrational motion of "violin modes" is concentrated in narrow frequency range.
- Minimize the thermal noise from mirror internal vibrations by making them from material with very low dissipation at acoustic frequencies at room temperature (fused silica).

Gravitational-Wave Detector



- Fabry–Perot cavities
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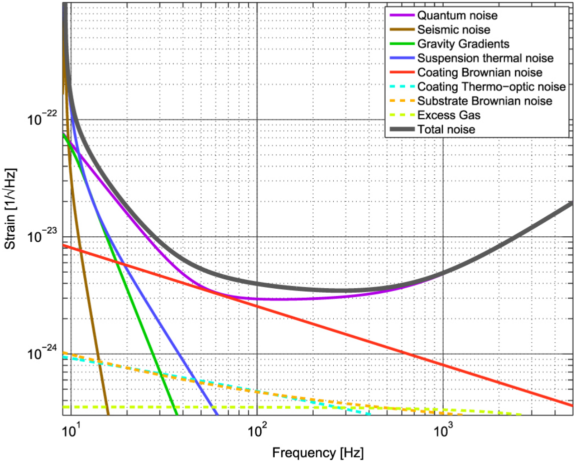
Noise Budget Example

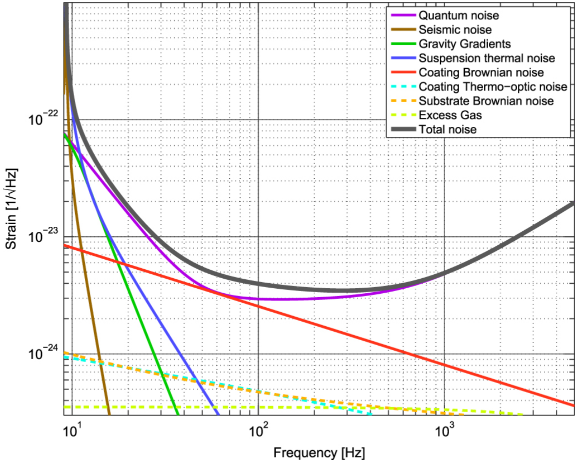


Quantum noise: Shot noise and radiation pressure

Seismic noise: Limits low frequency sensitivity

[Try out the Gravitational Wave Interferometer Noise Calculator]

Noise Budget Example



Thermal noise: Coating Brownian noise: dissipation in mirror coatings

Suspension thermal noise: loss in fibres used to suspend mirrors

[Try out the Gravitational Wave Interferometer Noise Calculator]

Key Components

- Vacuum
- Laser source: Maximum power 200 W.
- Input mode cleaner: stabilize beam position and only let through the desired modes.
- Core optics: transmission, smoothness
- Suspensions
- Active seismic isolation
- Scattered light control
- Global sensing and control
- Physical environment monitoring: $\mathcal{O}(10^6)$ auxiliary channels
- Calibration