Churn Management

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Contents

```r
library(bit64)
library(data.table)
library(ggplot2)
library(broom)
library(knitr)
library(dplyr)
library(kableExtra)
```

1 Overview

Cell2Cell, a wireless telecommunications company (with its name altered for confidentiality), is working to reduce customer churn. The objective of this project is to create a model that predicts customer churn at Cell2Cell and to leverage insights from the model to design a targeted incentive strategy aimed at decreasing churn rates.

In the assignment, you will address these key issues:

1. Is customer churn at Cell2Cell predictable from the customer information that Cell2Cell maintains?
2. What factors drive customer churn? Which factors are particularly important?
3. What incentives should Cell2Cell offer to prevent customer churn?
4. What is the economic value of a proposed targeted plan to prevent churn, and how does this value differ across customer segments? Compare the economic value to an incentive with a cost of $100 and another incentive with a cost of $175. Which customers segments should receive the incentive? Does the answer depend on the success probability?

Note that, in what follows, the key steps you need to take are highlighted in *italic*.

2 Data

All data are contained in the file `Cell2Cell.RData`, which is posted on Canvas.

```r
data_folder    = "./Data"
Cell2Cell = "Cell2Cell.RData"
load(paste0(data_folder, "/", Cell2Cell))
```

Please consult the file `Cell2Cell-Database-Documentation.xlsx` for a description of the data and some summary statistics. Note that *calibration sample* is an alternative term for *training* or *estimation* sample.

Report the churn rate in the calibration sample and in the validation sample and compare the two.

```r
# str(cell2cell_DT)

calibration_sample <- subset(cell2cell_DT, calibrat == 1)
validation_sample <- subset(cell2cell_DT, calibrat == 0)

# Calculate churn rate in the calibration sample
calibration_churn_rate <- mean(calibration_sample$churn)
cat("Churn Rate in Calibration Sample:", calibration_churn_rate, "\n")
```

```
Churn Rate in Calibration Sample: 0.5
```

```r
# Calculate churn rate in the validation sample
validation_churn_rate <- mean(validation_sample$churn)
cat("Churn Rate in Validation Sample:", validation_churn_rate, "\n")
```

```
Churn Rate in Validation Sample: 0.01961542
```

```r
# Compare churn rates
if (calibration_churn_rate > validation_churn_rate) {
  cat("The churn rate is higher in the calibration sample.\n")
} else if (calibration_churn_rate < validation_churn_rate) {
  cat("The churn rate is higher in the validation sample.\n")
} else {
  cat("The churn rates are the same in both samples.\n")
}
```

```
The churn rate is higher in the calibration sample.
```

You can see that the calibration sample was selected using *oversampling*. The purpose of oversampling was to obtain more precise estimates (lower standard errors) when estimating a logistic regression model. The validation sample, on the other hand, was not created using oversampling and represents the *true churn rate* in the data.

As you can see, some variables have missing values, which—as you know by now—is common and of no concern (unless the missing values indicate some *systematic* flaws or bias in how the data were constructed). Most estimation methods in R will automatically delete rows with missing values before estimating the model. However, the `predict` methods will yield `NA` values if a row in the data used for prediction contains missing values. Hence, in a situation where you don't need to keep the full data I recommend to remove any observations with missing values before you conduct the main analysis.

Perform this data-cleaning step.

```r
# Remove rows with missing values from the data
cell2cell_clean <- na.omit(cell2cell_DT)

# Verify that missing values have been removed
print(sprintf("Original dataset had %d rows", nrow(cell2cell_DT)))
```

```
[1] "Original dataset had 71047 rows"
```

```r
print(sprintf("Cleaned dataset has %d rows", nrow(cell2cell_clean)))
```

```
[1] "Cleaned dataset has 69309 rows"
```

```r
print(sprintf("Removed %d rows with missing values", nrow(cell2cell_DT) - nrow(cell2cell_clean)))
```

```
[1] "Removed 1738 rows with missing values"
```

3 Model estimation

Estimate a logit model to predict the conditional churn probability.

```r
# Fit the logistic regression model
fit <- glm(churn ~ .,
           data = cell2cell_clean[, .SD, .SDcols = !c("customer", "calibrat")],
           family = binomial())
```

You can inspect the regression output using methods you already used, such as `summary`. Having said this, especially when you have a large number of inputs, it can be convenient to store the regression estimates in a table. A simple way to do this is to install the broom package that has the purpose of cleaning up messy R output.

Using the `tidy` function in the `broom` package it is trivial to capture the regression output in the form of a data.table:

```r
# Tidy the model output and convert to data.table
results_DT <- as.data.table(tidy(fit))

# Display the regression results with 5 decimal places
kable(results_DT, digits = 5)
```

term	estimate	std.error	statistic	p.value
(Intercept)	-0.82035	0.07826	-10.48202	0.00000
revenue	0.00160	0.00066	2.42628	0.01525
mou	-0.00028	0.00004	-6.57237	0.00000
recchrge	-0.00263	0.00074	-3.55962	0.00037
directas	0.00023	0.00477	0.04781	0.96187
overage	0.00087	0.00023	3.73561	0.00019
roam	0.00281	0.00120	2.34317	0.01912
changem	-0.00057	0.00005	-12.57609	0.00000
changer	0.00290	0.00030	9.62096	0.00000
dropvce	0.00801	0.00698	1.14770	0.25109
blckvce	0.00345	0.00692	0.49930	0.61757
unansvce	0.00082	0.00037	2.23354	0.02551
custcare	-0.00650	0.00230	-2.82186	0.00477
threeway	-0.03574	0.00969	-3.68748	0.00023
mourec	0.00016	0.00011	1.39133	0.16413
outcalls	0.00056	0.00049	1.14705	0.25136
incalls	-0.00178	0.00090	-1.97778	0.04795
peakvce	-0.00063	0.00018	-3.41461	0.00064
opeakvce	-0.00018	0.00022	-0.82605	0.40878
dropblk	-0.00015	0.00683	-0.02230	0.98221
callfwdv	-0.01786	0.02199	-0.81229	0.41663
callwait	-0.00008	0.00258	-0.03224	0.97428
months	-0.02227	0.00168	-13.24110	0.00000
uniqsubs	0.17910	0.01576	11.36467	0.00000
actvsubs	-0.18508	0.02251	-8.22308	0.00000
phones	0.04958	0.01494	3.31785	0.00091
models	0.01377	0.02321	0.59341	0.55290
eqpdays	0.00142	0.00006	23.00599	0.00000
age1	-0.00409	0.00072	-5.69500	0.00000
age2	-0.00093	0.00056	-1.65423	0.09808
children	0.10973	0.02330	4.71042	0.00000
credita	-0.17827	0.02964	-6.01401	0.00000
creditaa	-0.39057	0.02924	-13.35695	0.00000

term	estimate	std.error	statistic	p.value
prizmrur	0.12196	0.04067	2.99893	0.00271
prizmub	-0.03453	0.02019	-1.71005	0.08726
prizmtwn	0.04801	0.02580	1.86104	0.06274
refurb	0.25268	0.02617	9.65444	0.00000
webcap	-0.13360	0.03013	-4.43361	0.00001
truck	0.03195	0.02968	1.07643	0.28173
rv	0.01306	0.03957	0.33005	0.74136
occprof	-0.01636	0.02689	-0.60840	0.54292
occcler	0.08278	0.06123	1.35196	0.17639
occcrft	-0.02871	0.05222	-0.54977	0.58248
occstud	0.07535	0.09816	0.76766	0.44269
occhmkr	0.11401	0.14800	0.77031	0.44112
occret	-0.05320	0.07590	-0.70091	0.48336
occself	-0.01679	0.06743	-0.24896	0.80339
ownrent	0.04038	0.03541	1.14025	0.25418
marryun	0.09073	0.02811	3.22759	0.00125
marryyes	0.05576	0.02686	2.07578	0.03791
mailord	-0.04646	0.07135	-0.65115	0.51495
mailres	-0.09220	0.07162	-1.28725	0.19801
mailflag	0.01346	0.07106	0.18935	0.84982
travel	0.00532	0.03914	0.13598	0.89184
pcown	0.02450	0.02556	0.95851	0.33781
creditcd	0.08633	0.03585	2.40792	0.01604
retcalls	0.18640	0.14799	1.25950	0.20785
retaccpt	-0.17696	0.08357	-2.11759	0.03421
newcelly	-0.03518	0.02261	-1.55586	0.11974
newcelln	0.02663	0.02596	1.02601	0.30489
refer	-0.08079	0.03543	-2.28011	0.02260
incmiss	-0.07086	0.04938	-1.43484	0.15133
income	-0.00712	0.00499	-1.42673	0.15366
mcycle	0.09062	0.07317	1.23859	0.21550
setprcm	-0.09003	0.03317	-2.71375	0.00665
setprc	0.00047	0.00023	1.99417	0.04613
retcall	0.64385	0.15547	4.14117	0.00003

For `kable` to work, you need to load the `knitr` library.

4 Prediction: Accounting for oversampling

The idea of oversampling is as follows. If the response rate in the data is small, there is a strong imbalance between observations with a response of $Y = 1$ and a response of $Y = 0$. As a consequence, estimating the model is difficult and the estimates will be imprecise, i.e. they will have large standard errors.

The solution: Create a training sample with one half of observations randomly chosen from the original data with response $Y = 1$, and the other half randomly chosen from the original data with response $Y = 0$. Now estimation is easier and the standard errors will be smaller.

However, when applied to logistic regression, oversampling will result in an inconsistent estimate of the intercept (constant) term, although all other estimates will be consistent. Hence, if we do not de-bias (adjust) the intercept, the predicted probabilities will be too large, reflecting the artificial response rate of $\frac{1}{2}$ in the over-sampled training data.

In order to de-bias the scale of the predicted response (in this example: churn) in the validation sample we need to supply an *offset variable* to the logistic regression model. An offset is a known number that is added to the right-hand side of the regression when estimating the model, and adding the offset will correspondingly change the estimate of the intercept. The offset takes the form:

$$\text{offset} = [\log(\bar{p}_t) - \log(1 - \bar{p}_t)] - [\log(\bar{p}_v) - \log(1 - \bar{p}_v)]$$

Here, $\bar{p}_t$ is the average response rate in the training sample and $\bar{p}_v$ is the average response rate in the validation sample. Note that the offset is positive (given that $\bar{p}_t > \bar{p}_v$), so that including the offset term when estimating the model accounts for the fact that the training sample has a higher share of $Y = 1$ relative to the validation sample.

Conversely, when we predict the response rate in the validation sample, we set the offset variable to 0.

Why does this work? — Conceptually, logistic regression is a regression model for the log-odds of the response (outcome) probability,

$$\log\left(\frac{p}{1-p}\right) = \log(p) - \log(1-p) = \beta_0 + \beta_1 X_1 + \beta_2 X_1 + \ldots$$

When we add the offset variable to the right hand side of the regression model the estimation algorithm will "incorporate" the offset in the intercept, β_0. The effect of setting the offset to 0 (when applying the model to the validation sample) is equivalent to subtracting the offset from the intercept. Subtracting the offset amounts to:

(i) Subtracting $\log(\bar{p}_t) - \log(1 - \bar{p}_t)$, the log-odds of the artificial response rate in the training sample, and

(ii) Adding $\log(\bar{p}_v) - \log(1 - \bar{p}_v)$, the log-odds in the validation sample that reflects the true log-odds in the data.

This process de-biases the predicted response, i.e. restores the correct response level in the validation sample.

Note: Never use over-sampling to create the validation sample, otherwise the offset variable approach will not work.

Create an `offset_var` variable and add it to the data set. Then re-estimate the logistic regression. To tell `glm` that you want to use `offset_var`, you need to use a formula of the form:

```
y ~ offset(offset_var) + <all other variables>
```

```
# Oversampling Bias: Helps in training but inflates the response rate, requiring adjustment.
# Offset Variable: Corrects the intercept to align predictions with the true response rate.
# Validation Data: Always use non-oversampled data for evaluation and prediction.

# Separate calibration and validation samples
calibration_sample <- subset(cell2cell_clean, calibrat == 1)
validation_sample <- subset(cell2cell_clean, calibrat == 0)

# Calculate the average churn rates in each sample
```

```r
pt <- mean(calibration_sample$churn)
pv <- mean(validation_sample$churn)

# Calculate the offset variable
offset_var <- (log(pt) - log(1 - pt)) - (log(pv) - log(1 - pv))

# Add the offset variable to the calibration sample
calibration_sample$offset_var <- offset_var
```

```r
# Fit the logistic regression model with the offset
fit_with_offset <- glm(churn ~ . + offset(offset_var),
                       data = calibration_sample[, .SD, .SDcols = !c("customer", "calibrat")],
                       family = binomial)

# View the summary of the model
summary(fit_with_offset)
```

```
Call:
glm(formula = churn ~ . + offset(offset_var), family = binomial,
    data = calibration_sample[, .SD, .SDcols = !c("customer",
        "calibrat")])

Coefficients: (1 not defined because of singularities)
              Estimate Std. Error z value Pr(>|z|)
(Intercept) -3.771e+00  9.527e-02 -39.583  < 2e-16 ***
revenue      1.963e-03  7.981e-04   2.460 0.013888 *
mou         -2.809e-04  4.965e-05  -5.657 1.54e-08 ***
recchrge    -3.123e-03  8.888e-04  -3.513 0.000443 ***
directas    -1.196e-03  5.939e-03  -0.201 0.840425
overage      7.602e-04  2.804e-04   2.711 0.006704 **
roam         7.091e-03  2.064e-03   3.436 0.000589 ***
changem     -4.919e-04  5.351e-05  -9.194  < 2e-16 ***
changer      2.303e-03  3.687e-04   6.247 4.20e-10 ***
dropvce      1.134e-02  7.254e-03   1.563 0.118044
blckvce      6.403e-03  7.157e-03   0.895 0.371044
unansvce     9.215e-04  4.478e-04   2.058 0.039609 *
custcare    -5.951e-03  2.553e-03  -2.331 0.019738 *
threeway    -3.029e-02  1.125e-02  -2.691 0.007122 **
mourec       1.339e-04  1.316e-04   1.018 0.308816
outcalls     1.119e-03  5.906e-04   1.894 0.058164 .
incalls     -3.107e-03  1.058e-03  -2.937 0.003314 **
peakvce     -6.696e-04  2.190e-04  -3.058 0.002229 **
opeakvce    -2.080e-04  2.657e-04  -0.783 0.433722
dropblk     -3.115e-03  7.039e-03  -0.442 0.658135
callfwdv    -2.643e-03  2.315e-02  -0.114 0.909128
callwait     2.085e-03  3.141e-03   0.664 0.506825
months      -2.128e-02  1.998e-03 -10.652  < 2e-16 ***
uniqsubs     1.844e-01  1.999e-02   9.225  < 2e-16 ***
actvsubs    -2.057e-01  2.791e-02  -7.372 1.68e-13 ***
phones       4.866e-02  1.817e-02   2.678 0.007398 **
models       1.380e-02  2.787e-02   0.495 0.620596
eqpdays      1.442e-03  7.466e-05  19.309  < 2e-16 ***
age1        -3.303e-03  8.723e-04  -3.787 0.000152 ***
```

```
age2        -1.168e-03  6.800e-04  -1.718 0.085778 .
children     9.455e-02  2.815e-02   3.359 0.000782 ***
credita     -1.781e-01  3.550e-02  -5.016 5.28e-07 ***
creditaa    -3.626e-01  3.458e-02 -10.488  < 2e-16 ***
prizmrur     6.649e-02  4.956e-02   1.342 0.179746
prizmub     -3.963e-02  2.441e-02  -1.624 0.104400
prizmtwn     4.622e-02  3.145e-02   1.470 0.141602
refurb       2.340e-01  3.196e-02   7.323 2.42e-13 ***
webcap      -1.561e-01  3.756e-02  -4.157 3.23e-05 ***
truck        2.689e-02  3.600e-02   0.747 0.455077
rv           1.186e-02  4.801e-02   0.247 0.804909
occprof     -1.987e-02  3.250e-02  -0.611 0.540996
occcler      3.949e-02  7.491e-02   0.527 0.598053
occcrft     -2.013e-02  6.290e-02  -0.320 0.748897
occstud      1.200e-01  1.219e-01   0.984 0.324916
occhmkr      2.559e-01  1.901e-01   1.346 0.178266
occret      -3.993e-02  9.055e-02  -0.441 0.659244
occself     -7.057e-02  8.059e-02  -0.876 0.381215
ownrent      2.554e-03  4.272e-02   0.060 0.952328
marryun      1.088e-01  3.403e-02   3.198 0.001385 **
marryyes     5.570e-02  3.249e-02   1.714 0.086444 .
mailord      7.687e-04  8.565e-02   0.009 0.992840
mailres     -1.297e-01  8.604e-02  -1.508 0.131672
mailflag    -4.818e-02  8.445e-02  -0.571 0.568303
travel      -5.320e-04  4.732e-02  -0.011 0.991030
pcown        3.418e-02  3.096e-02   1.104 0.269633
creditcd     4.202e-02  4.371e-02   0.961 0.336435
retcalls     1.203e-02  1.837e-01   0.066 0.947760
retaccpt    -1.279e-01  1.076e-01  -1.188 0.234909
newcelly    -7.053e-02  2.727e-02  -2.586 0.009708 **
newcelln    -5.084e-03  3.153e-02  -0.161 0.871883
refer       -5.003e-02  4.214e-02  -1.187 0.235215
incmiss     -9.151e-02  6.006e-02  -1.524 0.127615
income      -1.324e-02  6.035e-03  -2.195 0.028177 *
mcycle       1.223e-01  8.898e-02   1.374 0.169302
setprcm     -9.632e-02  4.051e-02  -2.377 0.017431 *
setprc       6.203e-04  2.827e-04   2.194 0.028222 *
retcall      7.937e-01  1.946e-01   4.079 4.52e-05 ***
offset_var          NA         NA      NA       NA
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 53983  on 38940  degrees of freedom
Residual deviance: 52300  on 38874  degrees of freedom
AIC: 52434

Number of Fisher Scoring iterations: 4
```

Where you place `offset()` on the right-hand side of the formula is irrelevant.

Before predicting the response rate in the validation sample set the offset to 0. Then, when you invoke the `predict` function, supply the data with the offset set to 0 using the `newdata` option.

```r
# Add offset_var as 0 in the validation sample
validation_sample$offset_var <- 0

# Predict churn probabilities in the validation sample with offset set to 0
validation_sample$predicted_churn <- predict(
  fit_with_offset,
  newdata = validation_sample[, .SD, .SDcols = !c("customer","calibrat")],
  type = "response")
```

```
Warning in predict.lm(object, newdata, se.fit, scale = 1, type = if (type == :
prediction from rank-deficient fit; attr(*, "non-estim") has doubtful cases
```

Compare the average predicted response with the average observed response rate in the validation sample.

```r
# Calculate the average predicted churn rate
average_predicted_churn <- mean(validation_sample$predicted_churn)

# Calculate the observed churn rate in the validation sample
average_observed_churn <- mean(validation_sample$churn)

# Display the results
cat("Average Predicted Churn Rate in Validation Sample:", average_predicted_churn, "\n")
```

```
Average Predicted Churn Rate in Validation Sample: 0.01943244
```

```r
cat("Average Observed Churn Rate in Validation Sample:", average_observed_churn, "\n")
```

```
Average Observed Churn Rate in Validation Sample: 0.01929663
```

```r
cat("Average of predicted churn rate and the actual churn rate are close after de-biasing with the offs
```

```
Average of predicted churn rate and the actual churn rate are close after de-biasing with the offset va
```

5 Predictive power: Lift

We evaluate the predictive fit of the logistic regression model using a lift table and lift chart. To develop reusable code, we develop a function that returns a lift table. The function (call it `liftTable`) will need to take the following inputs:

- Predicted outcome or score
- Observed outcome
- Number of segments to be created based on the score

`liftTable` will return a data.table that contains:

- An index (`score_group`) for each segment that was created based on the score
- The average score value (predicted outcome) in the `score_group`
- The average observed outcome in the `score_group`
- The lift factor

To code the `liftTable` command, I recommend to use the `cut_number` function in the ggplot2 package. `cut_number` takes a variable `x` and creates `n` groups with an approximately equal number of observations in each group. Observations are assigned to the groups based on their ranking along the variable `x`. The format is:

```
cut_number(x, n = <no. of groups>)
```

To illustrate, we draw 10,000 random numbers from a uniform distribution on $[0, 5]$. `cut_number` assigns each number to one of five (because we set `n = 5`) groups.

```r
set.seed(123)
DT = data.table(x = runif(10000, min = 0, max = 5))
DT[, group    := cut_number(x, n = 5)]
DT[, group_no := as.integer(group)]
```

```r
head(DT)
```

```
           x        group group_no
       <num>       <fctr>    <int>
1: 1.4378876     (1,2.01]        2
2: 3.9415257  (2.98,3.98]        4
3: 2.0448846  (2.01,2.98]        3
4: 4.4150870     (3.98,5]        5
5: 4.7023364     (3.98,5]        5
6: 0.2277825 [0.000327,1]        1
```

```r
table(DT$group)
```

```

[0.000327,1]      (1,2.01]  (2.01,2.98]  (2.98,3.98]     (3.98,5] 
        2000          2000         2000         2000         2000 
```

As expected, because `x` is uniformly distributed on $[0, 5]$, the five groups created by `cut_number` correspond almost exactly to a $[k, k + 1]$ interval ($k = 0, 1, \dots, 4$), and each of these intervals contains exactly 20 percent of all observations based on the rank of the `x` values. The `group` variable that we created is a factor that we converted to an integer score.

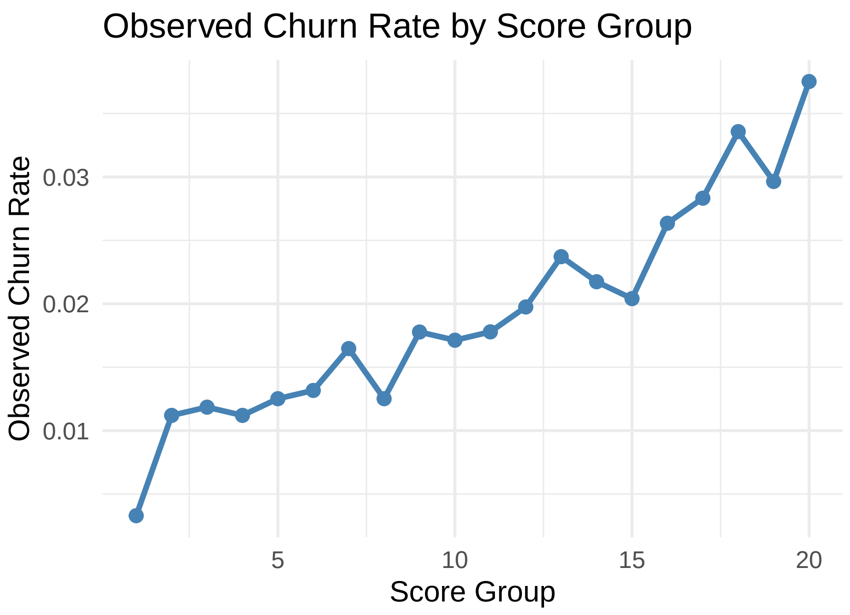
Calculate a lift table for 20 segments. Inspect the lift table. Then provide two charts. First, plot the `score_group` *segments on the x-axis versus the observed churn rate on the y-axis. Second, plot the segments versus the lift factor, and add a horizontal line at* $y = 100$. How to do this in ggplot2 is explained in the ggplot2 guide (look for the `yintercept` option).

```r
liftTable <- function(predicted, observed, segments = 20) {
  # Combine inputs into a data.table
  DT <- data.table(predicted = predicted, observed = observed)

  # Create score groups using cut_number
  DT[, score_group := as.integer(cut_number(predicted, n = segments))]

  # Calculate lift table metrics
  lift_table <- DT[, .(
    avg_score = mean(predicted),
    avg_observed = mean(observed),
    lift_factor = mean(observed) / mean(DT$observed) * 100
  ), by = score_group]

  return(lift_table)
}

# Apply the liftTable function
lift_table <- liftTable(
  predicted = validation_sample$predicted_churn,
  observed = validation_sample$churn,
  segments = 20
)

# Inspect the lift table
print(lift_table)
```

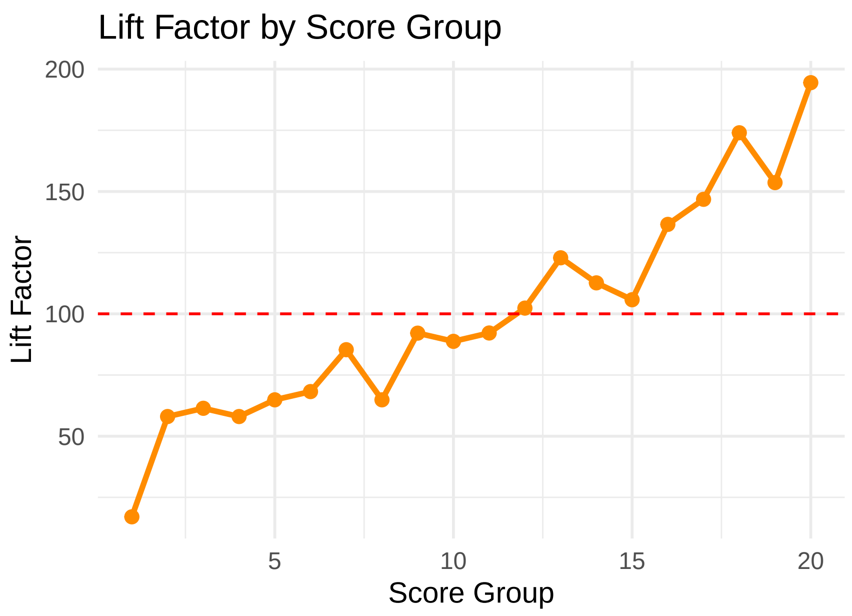
```
    score_group   avg_score avg_observed lift_factor
          <int>       <num>        <num>       <num>
 1:           1 0.007345874  0.003291639    17.05811
 2:           2 0.009751282  0.011198946    58.03577
 3:           3 0.011117084  0.011849901    61.40918
 4:           9 0.016544030  0.017774852    92.11377
 5:          10 0.017328919  0.017127800    88.76058
 6:           5 0.013289831  0.012516469    64.86350
 7:          20 0.047617454  0.037524687   194.46241
 8:           7 0.014985665  0.016469038    85.34672
 9:          17 0.025339449  0.028326746   146.79635
10:          16 0.023520373  0.026350461   136.55474
11:          19 0.031557441  0.029644269   153.62409
12:           4 0.012293521  0.011198946    58.03577
13:          14 0.020985705  0.021739130   112.65766
14:           6 0.014176228  0.013166557    68.23242
15:          12 0.019027414  0.019749835   102.34864
16:           8 0.015772776  0.012516469    64.86350
17:          13 0.019964653  0.023715415   122.89927
18:          11 0.018149515  0.017786561    92.17445
19:          15 0.022128569  0.020408163   105.76026
20:          18 0.027746205  0.033574720   173.99268
    score_group   avg_score avg_observed lift_factor
```

```r
#  Churn Rate vs. Score Groups
# Higher score groups (with higher predicted churn probabilities) should have higher
# observed churn rates.
```

```r
ggplot(lift_table, aes(x = score_group, y = avg_observed)) +
  geom_line(color = "steelblue", linewidth = 1) +  # Use `linewidth` for line width
  geom_point(color = "steelblue", size = 2) +  # Points on the line
  labs(
    title = "Observed Churn Rate by Score Group",
    x = "Score Group",
    y = "Observed Churn Rate"
  ) +
  theme_minimal()
```



```r
# Lift Factor vs. Score Groups
# The lift factor should be highest for the top score groups (high-risk churners).
# The horizontal line at y = 100 represents the baseline (random guessing or avg churn rate).
# It hits close to the mid segment group reflects that the churn predictions align
# with the reality (observation) of churn that is expected per each segment.
ggplot(lift_table, aes(x = score_group, y = lift_factor)) +
  geom_line(color = "darkorange", linewidth = 1) +  # Use `linewidth` for line width
  geom_point(color = "darkorange", size = 2) +  # Points on the line
  geom_hline(yintercept = 100, color = "red", linetype = "dashed") +  # Horizontal reference line
  labs(
    title = "Lift Factor by Score Group",
    x = "Score Group",
    y = "Lift Factor"
  ) +
  theme_minimal()
```


Lift Factor by Score Group
Lift Factor
Score Group
200
150
100
50
5
10
15
20

6 Why do customers churn? — Effect sizes

We would like to understand *why* customers churn, which can help us to propose incentives to prevent customer churn

To this end, construct a table that contains comparable effect sizes (changes in the churn probability) for all independent variables, as we discussed in class.

Here are a few more details on the steps needed to create this table:

1. Because logistic regression coefficients are not directly interpretable, we estimate a linear probability model of customer churn. In a linear probability model we regress the $Y = 0, 1$ output on all the customer features. The estimated coefficients can be interpreted as differences in $\Pr\{Y = 1|X_1, X_2, ...\}$ for a one-unit difference in one of the features, X_k. Note: **The *offset variable* should not be included in the linear probability model as it is specific to logistic regression.**
2. Note that our analysis is based on a *comparison* of the effect sizes of the different variables. However, because the variables have different scales, the effect sizes are not directly comparable. For example, `revenue` (mean monthly revenue) and `mou` (mean monthly minutes use) have different means and standard deviations, and hence the effects of increasing `revenue` and `mou` by one unit on the churn probabilities are not comparable without taking the scale differences into account.
3. To solve this problem we **standardize** the independent variables in the data. To standardize, we divide the values of each independent variable by its standard deviation, except if the variable is a 0/1 dummy. Once standardized, all variables except the dummies will have a standard deviation of 1, and a one unit difference corresponds to a one standard deviation difference in the original, non-standardized variable. Here's a function, `standardize_columns`, that takes a column `x` as input and returns the standardized values of the column:

```r
standardize_columns <- function(x) {

   # Check if the column is a dummy variable
   elements = unique(x)
   if (length(elements) == 2L & all(elements %in% c(0L,1L))) {
      is_dummy = TRUE
   } else {
      is_dummy = FALSE
   }

   # If not a dummy, divide values in x by its standard deviation
   if (is_dummy == FALSE) x = x/sd(x, na.rm = TRUE)

   return(x)
}
```

The first part of the function checks that the input `x` has exactly two elements and that these elements are the integers 0 and 1. Note that in R, numbers are represented as floating point numbers by default. However, adding `L` after the numbers tells R to represent the number as an integer.

```r
class(1)
```

```
[1] "numeric"
```

```r
class(1L)
```

```
[1] "integer"
```

```r
DT_lin_prob = cell2cell_clean[calibrat == 1]

# Create a vector that contains the names of all inputs (covariates)
# remove customer, calibrat, churn columns, retcall
all_columns    = names(DT_lin_prob)
```

```r
input_columns = all_columns[-c(1:3, length(all_columns))]

# Standardize all input columns
DT_lin_prob[, (input_columns) := lapply(.SD, standardize_columns), .SDcols = input_columns]

library(tidyverse)

Dv_lin_prob = cell2cell_clean[calibrat == 0]

# Create a vector that contains the names of all inputs (covariates)
all_columns   = names(Dv_lin_prob)
input_columns = all_columns[-c(1:3, length(all_columns))]

# Standardize all input columns
Dv_lin_prob[, (input_columns) := lapply(.SD, standardize_columns), .SDcols = input_columns]

# Calculate average churn probabilities in training and validation samples
avg_churn_train <- mean(DT_lin_prob$churn)
avg_churn_valid <- mean(Dv_lin_prob$churn)

# Specify the formula excluding the first three columns
independent_vars = names(DT_lin_prob)[-(1:3)]
formula <- as.formula(paste("churn ~", paste(independent_vars, collapse = " + ")))

# Fit the lpm model
lm_model <- lm(formula, data = cell2cell_clean, family = "binomial")
```

```
Warning: In lm.fit(x, y, offset = offset, singular.ok = singular.ok, ...) :
 extra argument 'family' will be disregarded
```

```r
# Tidy the linear probability model output
tidy_model <- tidy(lm_model)

# Add effect_size column
tidy_model <- tidy_model %>%
  mutate(effect_size = estimate * (100 * avg_churn_valid / avg_churn_train))

# Sort by absolute effect_size and print the table
tidy_model <- tidy_model %>% arrange(desc(abs(effect_size)))
kable(tidy_model)
```

term	estimate	std.error	statistic	p.value	effect_size
(Intercept)	0.3155318	0.0154054	20.4819562	0.0000000	1.2222911
retcall	0.1428622	0.0334030	4.2769308	0.0000190	0.5534125
creditaa	-0.0746562	0.0054187	-13.7775804	0.0000000	-0.2891994
refurb	0.0497153	0.0052670	9.4389822	0.0000000	0.1925846
retaccpt	-0.0454461	0.0180450	-2.5184894	0.0117882	-0.1760470
retcalls	0.0436851	0.0318359	1.3721972	0.1700065	0.1692253
credita	-0.0374406	0.0057344	-6.5290790	0.0000000	-0.1450354
webcap	-0.0325533	0.0062916	-5.1741180	0.0000002	-0.1261035
actvsubs	-0.0260920	0.0042041	-6.2063671	0.0000000	-0.1010739
occhmkr	0.0255888	0.0300500	0.8515430	0.3944707	0.0991247
prizmrur	0.0251413	0.0082316	3.0542525	0.0022571	0.0973913

term	estimate	std.error	statistic	p.value	effect_size
children	0.0219170	0.0045942	4.7705388	0.0000018	0.0849008
uniqsubs	0.0216580	0.0023639	9.1619086	0.0000000	0.0838977
marryun	0.0184553	0.0055491	3.3258146	0.0008821	0.0714911
mailres	-0.0182875	0.0139969	-1.3065363	0.1913746	-0.0708411
mcycle	0.0179938	0.0147568	1.2193553	0.2227135	0.0697033
setprcm	-0.0179207	0.0065143	-2.7509916	0.0059431	-0.0694203
refer	-0.0164161	0.0065690	-2.4990389	0.0124554	-0.0635920
creditcd	0.0162241	0.0069966	2.3188679	0.0204051	0.0628480
occcler	0.0159186	0.0122084	1.3038995	0.1922722	0.0616646
occstud	0.0150096	0.0195761	0.7667340	0.4432423	0.0581436
incmiss	-0.0137615	0.0097467	-1.4119094	0.1579811	-0.0533085
marryyes	0.0109617	0.0052656	2.0817593	0.0373681	0.0424629
occret	-0.0107438	0.0146348	-0.7341284	0.4628730	-0.0416190
prizmtwn	0.0099962	0.0051531	1.9398633	0.0524004	0.0387229
phones	0.0091587	0.0028875	3.1718725	0.0015153	0.0354785
mailord	-0.0086711	0.0139311	-0.6224300	0.5336612	-0.0335897
ownrent	0.0080209	0.0069890	1.1476508	0.2511167	0.0310710
newcelly	-0.0068243	0.0044463	-1.5348272	0.1248309	-0.0264356
newcelln	0.0067779	0.0051506	1.3159517	0.1881946	0.0262560
prizmub	-0.0065886	0.0039796	-1.6555940	0.0978086	-0.0255224
truck	0.0061767	0.0058655	1.0530690	0.2923130	0.0239271
occcrft	-0.0058655	0.0102090	-0.5745377	0.5656058	-0.0227213
threeway	-0.0056046	0.0015879	-3.5296127	0.0004164	-0.0217107
pcown	0.0046355	0.0050451	0.9188126	0.3581968	0.0179568
months	-0.0039985	0.0003112	-12.8490599	0.0000000	-0.0154893
rv	0.0032359	0.0078461	0.4124217	0.6800316	0.0125350
models	0.0030603	0.0044645	0.6854850	0.4930402	0.0118550
occprof	-0.0028973	0.0052951	-0.5471700	0.5842637	-0.0112236
callfwdv	-0.0023206	0.0030577	-0.7589262	0.4478993	-0.0089893
occself	-0.0022278	0.0131317	-0.1696487	0.8652869	-0.0086298
mailflag	0.0021229	0.0141718	0.1497977	0.8809247	0.0082236
dropvce	0.0015669	0.0013081	1.1978461	0.2309810	0.0060696
income	-0.0013005	0.0009818	-1.3246965	0.1852762	-0.0050380
travel	0.0011499	0.0077021	0.1493014	0.8813163	0.0044545
custcare	-0.0009738	0.0003752	-2.5958113	0.0094388	-0.0037724
age1	-0.0008056	0.0001416	-5.6910243	0.0000000	-0.0031208
blckvce	0.0006473	0.0012956	0.4995951	0.6173618	0.0025074
recchrge	-0.0005385	0.0001443	-3.7308807	0.0001910	-0.0020861
roam	0.0005263	0.0002299	2.2892024	0.0220706	0.0020389
changer	0.0005231	0.0000555	9.4178059	0.0000000	0.0020265
revenue	0.0003348	0.0001279	2.6177138	0.0088540	0.0012970
eqpdays	0.0002750	0.0000118	23.3543418	0.0000000	0.0010653
incalls	-0.0002686	0.0001654	-1.6240220	0.1043757	-0.0010404
callwait	0.0001829	0.0004725	0.3871468	0.6986487	0.0007086
age2	-0.0001770	0.0001100	-1.6091271	0.1075931	-0.0006856
overage	0.0001579	0.0000451	3.4999792	0.0004656	0.0006116
unansvce	0.0001527	0.0000705	2.1647194	0.0304126	0.0005914
peakvce	-0.0001153	0.0000345	-3.3417639	0.0008329	-0.0004468
outcalls	0.0001038	0.0000917	1.1319893	0.2576429	0.0004021
changem	-0.0001028	0.0000085	-12.0987909	0.0000000	-0.0003981
setprc	0.0000899	0.0000454	1.9797035	0.0477408	0.0003482
mou	-0.0000508	0.0000080	-6.3614191	0.0000000	-0.0001970

term	estimate	std.error	statistic	p.value	effect_size
opeakvce	-0.0000471	0.0000416	-1.1318473	0.2577025	-0.0001824
dropblk	-0.0000304	0.0012767	-0.0238423	0.9809785	-0.0001179
mourec	0.0000274	0.0000212	1.2945619	0.1954757	0.0001061
directas	0.0000122	0.0008945	0.0136384	0.9891185	0.0000473

4. In order to create a table that captures the linear probability model estimates, use the `tidy` function. Add a column, e.g. `effect_size`, that scales the estimates by the factor

$$100 \cdot \frac{\bar{p}_v}{\bar{p}_t}$$

   This scales the effect sizes to the correct magnitude of the churn probabilities in the validation sample and puts the effects on a 0-100% scale. Sort the variables according to the magnitude of the effect sizes, and print the results table using `kable`.
5. Inspect the results. Identify some factors that are strongly associated with churn. If actionable, propose an incentive that can be targeted to the customers to prevent churn.

```
sorted_model <- tidy_model %>% arrange(desc(effect_size))
top_5 <- head(sorted_model, 5)
bottom_5 <- tail(sorted_model, 5)

print(top_5)
```

```
# A tibble: 5 x 6
  term        estimate std.error statistic  p.value effect_size
  <chr>          <dbl>     <dbl>     <dbl>    <dbl>       <dbl>
1 (Intercept)   0.316    0.0154     20.5   5.89e-93      1.22
2 retcall       0.143    0.0334      4.28  1.90e- 5      0.553
3 refurb        0.0497   0.00527     9.44  3.88e-21      0.193
4 retcalls      0.0437   0.0318      1.37  1.70e- 1      0.169
5 occhmkr       0.0256   0.0300      0.852 3.94e- 1      0.0991
```

```
print(bottom_5)
```

```
# A tibble: 5 x 6
  term     estimate std.error statistic  p.value effect_size
  <chr>       <dbl>     <dbl>     <dbl>    <dbl>       <dbl>
1 actvsubs  -0.0261   0.00420     -6.21 5.45e-10      -0.101
2 webcap    -0.0326   0.00629     -5.17 2.30e- 7      -0.126
3 credita   -0.0374   0.00573     -6.53 6.66e-11      -0.145
4 retaccpt  -0.0454   0.0180      -2.52 1.18e- 2      -0.176
5 creditaa  -0.0747   0.00542    -13.8  3.96e-43      -0.289
```

Factors that are strongly associated with churn. Strong positively churn 1. retcall (0.55341249) : Customers with retention calls are more likely to churn, possibly due to dissatisfaction with previous interactions or unresolved issues. Highly significant (p=1.89e-05) Incentive : - Personalized Service Recovery: Provide dedicated, high-quality customer service follow-ups for customers who have made a retention call, ensuring their issues are fully resolved. - Offer discounts or credits (e.g., one-time $50 credit) for the inconvenience.

2. refurb (0.19258463) : Customers with refurbished devices are more likely to churn, possibly due to dissatisfaction with older hardware. Highly significant(p=3.87e10-21) Incentive :

- Upgrade Program : Create trade-in programs allowing refurbished users to easily swap for newer models at minimal cost.

3. retcalls frequency (0.16922533) : Number of calls previously made to retention team, possibly due to dissatisfaction with the too frequent callings or unresolved issues. Incentive :

- Implement a proactive monitoring system to identify frequent retention team callers and address their issues before they escalate.
- Provide special discounts or perks (e.g., reduced fees for 3 months) to frequent retention callers to demonstrate appreciation and regain trust.

4. occhmkr (0.09912475) : Occupation - homemaker are likely to churn, potentially due to higher demands for quality of product or services. Incentive :

- Introduce a family-oriented subscription plans with better value for households.
- Offer special perks for homemakers.

Strong negatively churn (unlikely to churn) 1. creditaa (-0.2891994) : Customers with excellent credit ratings ‘aa’ are less likely to churn, possibly indicating higher satisfaction or loyalty. Incentive : - Offer loyalty programs, such as discounts on premium services or free upgrades, for customers who meet certain payment and credit score thresholds.

2. retaccpt (-0.1760470) : Customers who accept retention offers are less likely to churn, highlighting the effectiveness of these interventions. Incentive :

- Post retention engagement : Provide incentives for continued loyalty, such as bonus points or exclusive access to new features.

3. credita (-0.1450354) : Customers with excellent credit ratings ‘a’ are less likely to churn. Similar to creditaa. Incentive :

- Send message to encourage them to earn more points to move to the ‘aa’ tier.

7 Economics of churn management

Next, we would like to predict the value of a proposed churn management program in order to assess the maximum amount that we would spend to prevent a customer from churning for one year.

Perform this prediction, under the following assumptions:

1. We consider a planning horizon of 4 years (the current year and three additional years), and an annual discount rate of 8 percent.
2. Predict the churn management value for 20 groups, but keep in mind that it is good practice to make sure the code works for an arbitrary number of groups in case we wish to modify that in the future. Predict the program value for these 20 customer segments based on the predicted churn rate. Note that we create these segments based on the validation sample data. We predict current and future customer profits at the year-level. Hence, we also need to convert both the monthly churn rate and the revenue data to the year-level.
3. Assume that the churn management program has a success probability `gamma` (γ) and compare the results for $\gamma = 0.25$ and $\gamma = 0.5$.

Hint: It is easy to make little mistakes in the lifetime value predictions. Hence, be very clear about what your code is supposed to achieve, and check that every step is correct.

```r
# Program Value
# Gamma = 0.25 : Adjusted LTV for success probability 25%.
# Gamma = 0.50 : Adjusted LTV for success probability 50%.

# Parameters
discount_rate <- 0.08
gamma_values <- c(0.25, 0.5)
groups <- 20
horizon <- 4  # 4 years

# Convert monthly churn rate and revenue to yearly
validation_data <- validation_sample
validation_data[, churn_rate_yearly := 1 - (1 - predicted_churn)^12]
validation_data[, revenue_yearly := revenue * 12]

# Group data into segments
# The segment is in the score_group column
validation_data[, score_group := as.integer(cut_number(predicted_churn, n = groups))]

# Function to calculate LTV
calculate_ltv <- function(revenue, churn_rate, discount_rate, horizon) {
  ltv <- 0
  for (t in 0:(horizon - 1)) {
    ltv <- ltv + (revenue * (1 - churn_rate)^t) / ((1 + discount_rate)^t)
  }
  return(ltv)
}
```

```r
# Calculate metrics for each group
group_summary <- validation_data[, .(
  num_customers = .N,  # Count the number of rows (customers) in each group
  avg_churn_rate = mean(churn_rate_yearly),
  avg_revenue = mean(revenue_yearly),
  avg_ltv = calculate_ltv(mean(revenue_yearly), mean(churn_rate_yearly),
                          discount_rate, horizon)
  ), by = score_group]
```

```r
# Add program values for different success probabilities
# If gamma = 0.25, the program is expected to succeed for 25% of the customers it
# targets,so only 25% of the LTV is realized.
# Program value shows the expected realized benefit
group_summary[, program_value_gamma_0.25 := 0.25 * avg_ltv]
group_summary[, program_value_gamma_0.5 := 0.5 * avg_ltv]

# Add max_spend per each program to the table
group_summary[, max_spend_gamma_0.25 := program_value_gamma_0.25 / horizon]
group_summary[, max_spend_gamma_0.5 := program_value_gamma_0.5 / horizon]
group_summary <- group_summary[order(score_group)]

group_summary[, total_program_value_gamma_0.5 := max_spend_gamma_0.5 * num_customers]
group_summary[, total_program_value_gamma_0.25 := max_spend_gamma_0.25 * num_customers]

# View the updated table
kable(group_summary[, 1:7], digits = 2) %>%
  kable_styling(font_size = 8)
```

score_group	num_customers	avg_churn_rate	avg_revenue	avg_ltv	program_value_gamma_0.25	program_value_gamma_0.5
1	1519	0.08	1033.05	3279.97	819.99	1639.99
2	1518	0.11	845.28	2585.54	646.38	1292.77
3	1519	0.13	821.25	2460.57	615.14	1230.28
4	1518	0.14	764.15	2249.57	562.39	1124.79
5	1518	0.15	731.57	2122.20	530.55	1061.10
6	1519	0.16	690.73	1977.92	494.48	988.96
7	1518	0.17	685.86	1941.09	485.27	970.55
8	1518	0.17	660.22	1847.54	461.89	923.77
9	1519	0.18	636.39	1761.43	440.36	880.71
10	1518	0.19	637.45	1744.94	436.24	872.47
11	1518	0.20	664.12	1797.23	449.31	898.61
12	1519	0.21	626.87	1675.94	418.98	837.97
13	1518	0.21	623.76	1646.35	411.59	823.18
14	1518	0.22	640.54	1667.42	416.86	833.71
15	1519	0.24	648.65	1662.94	415.73	831.47
16	1518	0.25	645.78	1625.52	406.38	812.76
17	1518	0.27	625.23	1537.26	384.32	768.63
18	1519	0.29	637.67	1521.07	380.27	760.53
19	1518	0.32	696.42	1586.37	396.59	793.18
20	1519	0.42	856.16	1695.42	423.85	847.71

```r
kable(group_summary[, 8:ncol(group_summary)], digits = 2) %>%
  kable_styling(font_size = 8)
```

max_spend_gamma_0.25	max_spend_gamma_0.5	total_program_value_gamma_0.5	total_program_value_gamma_0.25
205.00	410.00	622784.5	311392.2
161.60	323.19	490605.7	245302.8
153.79	307.57	467200.5	233600.2
140.60	281.20	426856.8	213428.4
132.64	265.27	402686.9	201343.5
123.62	247.24	375558.0	187779.0
121.32	242.64	368321.9	184160.9
115.47	230.94	350571.3	175285.7
110.09	220.18	334450.8	167225.4

109.06	218.12	331102.7	165551.3
112.33	224.65	341024.3	170512.2
104.75	209.49	318219.0	159109.5
102.90	205.79	312395.8	156197.9
104.21	208.43	316393.7	158196.8
103.93	207.87	315749.8	157874.9
101.59	203.19	308442.4	154221.2
96.08	192.16	291695.6	145847.8
95.07	190.13	288813.0	144406.5
99.15	198.30	301013.5	150506.7
105.96	211.93	321917.8	160958.9

```r
# Select high-risk, high-value group to enter the program

# Step 1: Filter for high-risk groups (avg_churn_rate > 0.2)
high_risk_groups <- group_summary[avg_churn_rate > 0.2]

# Step 2: Prioritize by program value
prioritized_groups <- high_risk_groups[order(-program_value_gamma_0.5)]

kable(prioritized_groups[, 1:7], digits = 2) %>%
  kable_styling(font_size = 8)
```

score_group	num_customers	avg_churn_rate	avg_revenue	avg_ltv	program_value_gamma_0.25	program_value_gamma_0.5
20	1519	0.42	856.16	1695.42	423.85	847.71
12	1519	0.21	626.87	1675.94	418.98	837.97
14	1518	0.22	640.54	1667.42	416.86	833.71
15	1519	0.24	648.65	1662.94	415.73	831.47
13	1518	0.21	623.76	1646.35	411.59	823.18
16	1518	0.25	645.78	1625.52	406.38	812.76
19	1518	0.32	696.42	1586.37	396.59	793.18
17	1518	0.27	625.23	1537.26	384.32	768.63
18	1519	0.29	637.67	1521.07	380.27	760.53

```r
kable(prioritized_groups[, 8:ncol(prioritized_groups)], digits = 3) %>%
  kable_styling(font_size = 8)
```

max_spend_gamma_0.25	max_spend_gamma_0.5	total_program_value_gamma_0.5	total_program_value_gamma_0.25
105.964	211.927	321917.8	160958.9
104.746	209.492	318219.0	159109.5
104.214	208.428	316393.7	158196.8
103.933	207.867	315749.8	157874.9
102.897	205.794	312395.8	156197.9
101.595	203.190	308442.4	154221.2
99.148	198.296	301013.5	150506.7
96.079	192.158	291695.6	145847.8
95.067	190.134	288813.0	144406.5

```r
# Find the Aggregate total max spending for the targeted group


# Aggregate total max spend for gamma = 0.5
total_program_value_gamma_0.5 <- sum(prioritized_groups$total_program_value_gamma_0.5)
```

```r
# Aggregate total max spend for gamma = 0.25
total_program_value_gamma_0.25 <- sum(prioritized_groups$total_program_value_gamma_0.25)

cat("Total max spend represents is the cumulative justification for running \
    the churn management program across all selected groups. \n")
```

```
Total max spend represents is the cumulative justification for running
    the churn management program across all selected groups.
```

```r
cat("Total max spend for program (gamma = 0.5) \
    on selected group $",
    total_program_value_gamma_0.5, "per year. \n")
```

```
Total max spend for program (gamma = 0.5)
    on selected group $ 2774641 per year.
```

```r
cat("Total max spend for program (gamma = 0.25) \
    on selected group $",
    total_program_value_gamma_0.25, "per year. \n")
```

```
Total max spend for program (gamma = 0.25)
    on selected group $ 1387320 per year.
```

Analyzing Result :

Per segment

- For example, for score group 20, we can spend up to $211.93 per customer per year, assuming a 50% success probability (□= 0.5).
- For score group 20, we can spend up to $105.96 per customer per year, assuming a 25% success probability (□= 0.25).

Per total

- 'Total_program_value_gamma_0.5' column is indicating the total budget for each segment by taking into account of the number of customers.

Aggregate Sum on selected group

- Total max spend represents indicates the cumulative justification for running the churn management program across all selected groups. This is total budget we could allocate per each program.
- Total max spend for program (gamma = 0.5) across selected group: $ 2774641 per year. - Total max spend for program (gamma = 0.25) across selected group: $ 1387320 per year.

8 Summarize your main results

Please organize your main results along the four questions posed in the overview.