3.1 I. A symmetric matrix $K \in R^{d\times d}$ is positive semidefinite $\Rightarrow \forall x \in R^d,\ x^T K x \geq 0$

By properties of symmetric matrix, we have

$K = Q A Q^T$ (Q is orthogonal matrix, A is diagonal matrix with entries to be eigenvalues of K)

$\forall x \in R^d$, $x^T K x = x^T Q A Q^T x \quad \cdots ①$

let $P = x^T Q$, then $① = P A P^T = P \begin{bmatrix} \lambda_1 & \cdots & \cdots \\ \vdots & \lambda_2 & \vdots \\ \vdots & \cdots & \lambda_d \end{bmatrix} P^T = \sum_{i=1}^{d} \lambda_i P_i^2 \quad \cdots ②$

And from the definition of positive semidefinite, we have $\lambda_i \geq 0$, and we also know $P_i^2 \geq 0$, then we know $② \geq 0 \Rightarrow x^T K x \geq 0$, proof done

II. $\forall x \in R^d,\ x^T K x \geq 0 \Rightarrow$ A symmetric matrix $K \in R^{d\times d}$ is positive semidefinite

Above is equal to prove its contrapositive, which is

$\forall$ symmetric matrix $K \in R^{d\times d}$ is not positive semidefinite $\Rightarrow \exists x \in R^d,\ x^T K x < 0$

Then we have $\lambda < 0$ such that $\exists x \in R^d,\ \lambda x = Kx \quad (x \neq \vec{0}) \cdots ①$

$① \Rightarrow x^T \lambda x = x^T K x \Rightarrow \lambda (x^T x) = x^T K x$

Since $\lambda < 0$, $x^T x > 0$, then we have $x^T K x = \lambda (x^T x) < 0$

Contrapositive is true, then II. is true, proof done.