4. $k(x,y) = \dfrac{k_1(x,y)}{\sqrt{k_1(x,x)}\sqrt{k_1(y,y)}} = \dfrac{<\phi_1(x), \phi_1(y)>}{\sqrt{<\phi_1(x), \phi_1(x)>}\sqrt{<\phi_1(y), \phi_1(y)>}}$

$$= \frac{<\phi_1(x), \phi_1(y)>}{\sqrt{\phi_1(x_1)^2 + \cdots + \phi_1(x_d)^2}\sqrt{\phi_1(y_1)^2 + \cdots \phi_1(y_d)^2}}$$

$$= \frac{<\phi_1(x), \phi_1(y)>}{||\phi_1(x)|| \cdot ||\phi_1(y)||}$$

then we can let $\phi(x) = \dfrac{\phi_1(x)}{||\phi_1(x)||}$, we have

$$<\phi(x), \phi(y)> = \frac{<\phi_1(x), \phi_1(y)>}{||\phi_1(x)|| \cdot ||\phi_1(y)||}$$

$\Rightarrow k(x,y) = \dfrac{<\phi_1(x), \phi_1(y)>}{||\phi_1(x)|| \cdot ||\phi_1(y)||} = <\phi(x), \phi(y)>$ is a kernel