3.2.1 $\forall x$, let $\phi(x) = \sqrt{2}$, then we have $k(x,y) = \langle \phi(x), \phi(y) \rangle = \sqrt{2} \cdot \sqrt{2} = 2$

2. for all $f: R^d \to R$, we can construct $W = [f(x^{(1)}), f(x^{(2)}), \ldots, f(x^{(n)})]$
such that $K = W^T W$, and $\forall y \in R^d$, we have $Y^T K Y = Y^T W^T W Y = (WY)^2 \geq 0$
From what we have from 3.1.1 we show that K is positive semedefinite, then $k(x,y) = f(x) \cdot f(y)$ is a kernel for all $f: R^d \to R$

3. $k_1(x,y) = \langle \phi_1(x), \phi_1(y) \rangle \quad k_2(x,y) = \langle \phi_2(x), \phi_2(y) \rangle$

$$a k_1(x,y) + b k_2(x,y) = \langle \sqrt{a}\phi_1(x), \sqrt{a}\phi_1(y) \rangle + \langle \sqrt{b}\phi_2(x), \sqrt{b}\phi_2(y) \rangle$$

$$= \langle [\sqrt{a}\phi_1(x), \sqrt{b}\phi_2(x)], [\sqrt{a}\phi_1(y), \sqrt{b}\phi_2(y)] \rangle$$

(properties of concatenation of feature maps)

Then we can let $\phi(x) = [\sqrt{a}\phi_1(x), \sqrt{b}\phi_2(x)]$

$$\Rightarrow k(x,y) = \cancel{a k_1(x,y) + b k_2(x,y)} \langle \phi(x), \phi(y) \rangle$$

$$= \langle [\sqrt{a}\phi_1(x), \sqrt{b}\phi_2(x)], [\sqrt{a}\phi_1(y), \sqrt{b}\phi_2(y)] \rangle$$

$$= a \cdot k_1(x,y) + b\, k_2(x,y)$$ is a kernel