Probability and Statistics

5

Unit Overview

In this unit you will continue your study of probability. You will learn more about the language of probability, how to use the Addition Rule, how to calculate conditional probabilities, and the meaning of independence. You will investigate geometric probabilities, and you will also see how relevant probability concepts are to everyday scenarios.

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary

- elapsed
- summarize
- assume

Math Terms

- probability experiment
- outcome
- sample space
- event
- complement
- two-way frequency table
- intersection
- union
- mutually exclusive events
- conditional probability
- dependent events
- tree diagram
- independent events

ESSENTIAL QUESTIONS

? How does knowing that one event has happened change the probability of another event happening?

? How do such changes in probability influence the decisions we make?

EMBEDDED ASSESSMENTS

The two embedded assessments in this unit, following Activities 27 and 29, will allow you to demonstrate your ability to evaluate probabilities and your understanding of the relevance of these quantities.

Embedded Assessment 1:

Embedded Assessment 2:

Getting Ready

Answer each item. Show your work.

1. Express the following percentages as decimals.
- **a.** 43%
- **b.** 8%
- **c.** 89.2%

2. Round the following decimals to three decimal places.
- **a.** 0.78326
- **b.** 0.78372
- **c.** 0.78354
- **d.** 0.78349
- **e.** 0.78021
- **f.** 0.70041
- **g.** 0.78968
- **h.** 0.79968

3. Express the following fractions in simplest form. (Do not use a calculator.)
- **a.** $\frac{3}{6}$
- **b.** $\frac{14}{35}$
- **c.** $\frac{24,000}{36,000}$

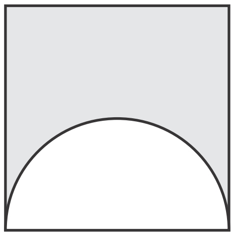
4. Evaluate the following. Express your answers as fractions in simplest form. (Do not use a calculator.)
- **a.** $\frac{3}{7}+\frac{2}{7}$
- **b.** $\frac{1}{6}+\frac{2}{3}$
- **c.** $\frac{5}{14}+\frac{1}{2}$
- **d.** $\frac{3}{8}+\frac{9}{10}$

5. Evaluate the following. Express your answers as fractions in simplest form. (Do not use a calculator.)
- **a.** $\frac{2}{3}\cdot\frac{4}{5}$
- **b.** $\frac{2}{21}\cdot\frac{21}{37}$
- **c.** $\frac{2}{3}\cdot\frac{6}{7}$

6. Evaluate the following. Express your answers as fractions in simplest form. (Do not use a calculator.)
- **a.** $\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)+\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)$
- **b.** $\left(\frac{1}{3}\right)\left(\frac{5}{8}\right)+\left(\frac{2}{3}\right)\left(\frac{3}{8}\right)$

7. There are 36 students in a class, and 20 of them are girls. If a student is selected at random from the class, what is the probability that the selected student is a girl?

8. The length of the diameter of the semicircle shown is 4 units. The diameter of the semicircle is also a side of the square. Find the exact fraction of the figure that is shaded. Then write this value as a decimal to the nearest hundredth.



Sample Spaces

SpringBoard Superstar and More

Lesson 25-1 Probability of a Single Event

My Notes

Learning Targets:

- Understand probability in real-world situations.
- Represent sample spaces as lists.
- Calculate the probability of a single event.

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Marking the Text, Visualization, Think-Pair-Share, Debriefing

1. Suppose that you have a cube with faces numbered 1 through 6. You will roll the cube 60 times.

a. On roughly how many of the rolls would you expect to get a result that is an even number?

b. On roughly how many of the rolls would you expect to get a 5?

c. On roughly how many of the rolls would you expect to get a number that is 5 or more?

DISCUSSION GROUP TIPS

As you share ideas in your group, ask your group members or your teacher for clarification of any language, terms, or concepts that you do not understand.

Rolling the cube is an example of a ***probability experiment***. A result of a probability experiment is called an ***outcome***. For the cube, the possible outcomes are 1, 2, 3, 4, 5, and 6.

The set of all possible outcomes is called the ***sample space***. Here the sample space is {1, 2, 3, 4, 5, 6}.

Getting an even number when you roll the cube is an example of an ***event***. To get an even number you have to roll a 2, a 4, or a 6. There are 3 outcomes that are even, out of a total of 6 possible outcomes. In the long run, $\frac{3}{6}$, or $\frac{1}{2}$, of the rolls of the cube will result in even numbers.

To generalize, if all the outcomes in the sample space are equally likely, then the probability of an event is given by

$$P(\text{event}) = \frac{\text{number of outcomes in the event}}{\text{number of outcomes in the sample space}}.$$

MATH TERMS

A **probability experiment** is the process of observing an outcome when the outcome is not known prior to the experiment.

An **outcome** is the result of a probability experiment.

The **sample space** is the set of all possible outcomes.

An **event** is any outcome or group of outcomes from a probability experiment.

The *probability* of an outcome tells you what part of the time you would expect the outcome to occur.

My Notes

WRITING MATH

Probability Notation

The *probability* of an outcome is written in symbols as *P*(outcome), where the *P* stands for probability.

For example, the probability that the outcome is an even number can be written as *P*(roll results in an even number), or simply as *P*(even).

When we roll the cube, as described in Item 1, we expect to get an even number about half of the time. Another way of saying this is that, when you roll the cube, the probability of getting an even number is $\frac{1}{2}$. To write this using probability notation,

$$P(\text{even}) = \frac{1}{2}.$$

2. a. When you roll the cube, what is the probability that you get a 5? Write your answer using probability notation.

b. When you roll the cube, what is the probability that you get a number that is 5 or more? Write your answer using probability notation.

3. A regular solid is constructed with four faces. The faces are numbered 1 through 4, and the four outcomes are equally likely. We will roll this object once.

a. What is the sample space for this probability experiment?

b. Find the probability that the roll results in the following. Use probability notation in your answers.

i. a 2

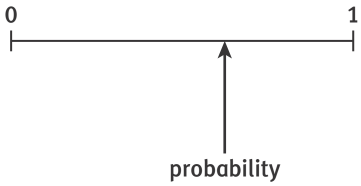
iii. a number greater than 4

ii. a number that is at least 2

iv. a number greater than 0

If an event can never happen, then there are no outcomes in the event, and so the probability of the event is 0. If an event is sure to happen, then every element of the sample space is in the event, and so the probability of the event is 1.

The probability of any event is a number between 0 and 1. How large the probability is tells us how likely the event is to occur.



My Notes

MATH TIP

The probability of an outcome is a number between 0 and 1, inclusive. It can be written as a fraction, a decimal, or a percent.

Check Your Understanding

4. In probability, what is the word or phrase for the set of all possible outcomes?

5. How is the probability that an event does not happen related to the probability that the event happens?

6. What is the least possible value for a probability?

7. What is the greatest possible value for a probability?

8. If a student is chosen at random from your class today, what is the probability that the student selected is male?

My Notes

The contestants for the SpringBoard Superstar contest are as follows. (The genders and ages of the contestants are in parentheses.)

Aaron Kelly (Male, 17)	Lacey Brown (Female, 24)
Andrew Garcia (Male, 24)	Lee DeWyze (Male, 24)
Casey James (Male, 27)	Michael Lynch (Male, 26)
Crystal Bowersox (Female, 24)	Paige Miles (Female, 24)
Didi Benami (Female, 23)	Siobhan Magnus (Female, 19)
Katie Stevens (Female, 17)	Tim Urban (Male, 20)

9. Select one of the twelve contestants at random.

a. What is the sample space for this probability experiment?

b. Find the probability that the selected contestant:

i. is Casey James.

ii. is female.

iii. has a last name beginning with Z.

iv. is under 30 years old.

v. has a last name beginning with B.

vi. does not have a last name beginning with B.

c. What do you get when you add the answers to parts b(v) and b(vi)? Will the probability that an event happens and an event does not happen always add to 1? Explain.

To generalize, when we add the probability that an event happens to the probability that it does not happen, we always get 1.

From this we can conclude that to find the probability that an event does *not* happen, we can subtract from 1 the probability that it *does* happen. This is called the ***complement*** of an event.

(probability that an event does *not* happen) = 1 − (probability that the event *does* happen)

My Notes

MATH TERMS

The **complement** of an event includes all possible outcomes of a probability experiment that are not outcomes of the event.

ACADEMIC VOCABULARY

The word ***assume*** means to consider a statement or fact as true. Often in mathematics, when first attempting a problem you list the assumptions, the information that you will consider as true to help solve the problem. Note the word *conjecture* is a synonym for *assume* when used as a verb.

10. Make sense of problems. Jenna and James were recently married, and are planning on having three children. They are wondering how many girls and how many boys they will have in their family. ***Assume*** that they are equally likely to have a girl as a boy.

a. Three of the possible outcomes are shown in the organized list. There are 8 possible outcomes. List the other five outcomes.

GGG

GGB

GBG

b. Find the probability that in their family they will have:

i. three girls.

ii. exactly two girls.

iii. at least two girls.

c. What is the probability that they will *not* have exactly two girls in the family?

My Notes

Check Your Understanding

11. Iris rolled a five-sided number cube numbered 1 through 5. Find the probability that a roll will result in each of the following. Write your answer using probability notation.

a. $P(\text{odd})$
b. $P(\text{not odd})$
c. $P(4)$
d. $P(\text{not } 4)$

12. Fiona's dog Sunshine is expecting a litter of four pups. It is equally likely that the pups will be male or female. What is the probability that the litter will contain two males and two females? Make an organized list to determine the probability.

LESSON 25-1 PRACTICE

13. Rafael has a bag of marbles. There are 5 green marbles, 6 red marbles, and 4 white marbles. Rafael picks a marble out of the bag without looking. What is the probability of each of the following?

a. $P(\text{green})$
b. $P(\text{not green})$
c. $P(\text{white})$
d. $P(\text{not white})$
e. $P(\text{red})$
f. $P(\text{not red})$

14. **Reason quantitatively.** The letters of a ten-letter word were placed in a box. Letters are chosen at random. Given that the probability of choosing a consonant is $\frac{3}{5}$, what is the probability of choosing a vowel? Explain.

15. Liam is tossing three coins in a game. He wants to determine the probability of tossing two heads.

a. Make an organized list.
b. What is the probability of tossing two heads? Explain.

Lesson 25-2
Geometric Probability

My Notes

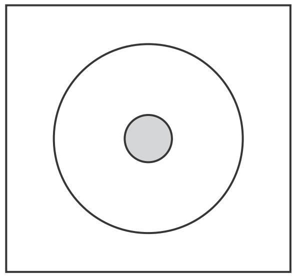
Learning Targets:

- Discover ways probability is used in real-life situations.
- Determine the probability of an event involving area.
- Use a linear model to determine probability involving elapsed time.

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Levels of Questions, Visualization, Discussion Groups, Debriefing

During the SpringBoard Superstar contest, Crystal challenges Paige to a game of darts. The dartboard is 15 inches by 15 inches, and the target is a large circle with a radius of 6 inches. The smaller, shaded circle has a radius of 2 inches, and is worth the most points.

1. Crystal and Paige each throw a dart. Both darts land on the board shown.



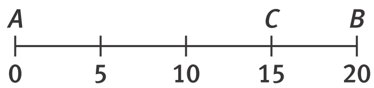
a. What is the probability that Crystal's dart landed within the large circle?

b. What is the probability that Paige's dart landed in the shaded region?

c. What is the probability that a dart landed in the outer ring of the circular target?

The previous example is an example of an area model for probability. Now let's look at a linear model.

2. Model with mathematics. Mr. Torres catches a bus each morning for work. The bus runs every 20 minutes. If he arrives at his bus stop at a random time, what is the probability that he will have to wait 5 minutes or more? Assume the bus stops for an insignificant amount of time. This number line represents ***elapsed*** time. Point B is when the next bus will arrive.



ACADEMIC VOCABULARY

The word ***elapse*** means to pass or go by. It is often used specifically in reference to an amount of time. In this case, the number 10 on the number line means that 10 minutes have gone by, or elapsed, since Mr. Torres arrived at his bus stop.

a. If Mr. Torres arrives at the bus stop between points C and B, how long will he have to wait for the next bus?

b. What happens if Mr. Torres arrives at any time between points A and C?

c. What is the probability that Mr. Torres will have to wait 5 minutes or more? Show your work.

My Notes

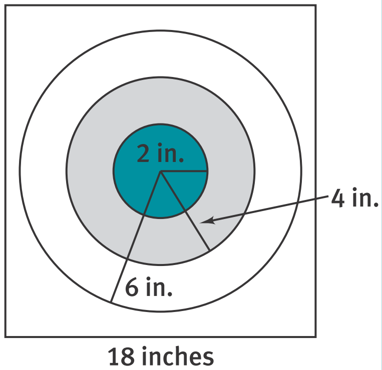
MATH TIP

Circles are concentric if they have the same center point.

Check Your Understanding

3. Game designers use probabilities to determine how points and prizes are awarded. How might understanding probabilities help both game designers and contestants?

4. Maggie wants to play darts at the school fair. The target is on a square dartboard that is 18 in. on a side. Maggie will score points if her dart lands in any of the concentric circles. She will score 25 points if her dart lands in the bull's-eye in the center. She will receive 15 points if her dart lands in the gray region, 10 points if her dart lands in the outer ring of the circles, and 0 points if her dart lands outside the circles on the dartboard. Write each probability as a decimal to the nearest ten thousandth and as a percent.



a. What is the probability that Maggie will score at least 15 points if her dart lands on the board?

b. What is the probability she will score exactly 15 points if she throws one dart that lands on the board? Explain your work.

5. Mrs. Ciaccio takes a bus every Sunday to visit her mother in the next town. The bus leaves every 15 minutes. If she arrives at the bus stop at a random time, what is the probability that she will have to wait 10 minutes or more?

POINT OF INTEGRATION

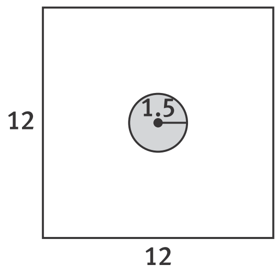
Geometry and Probability

The term *geometric probability* clearly identifies the two branches of mathematics being integrated. A probability is calculated by dividing the area of the desired outcome by the total area. This will always yield a result between 0 and 1. Generally, it is assumed that the outcome will be within the total area being considered.

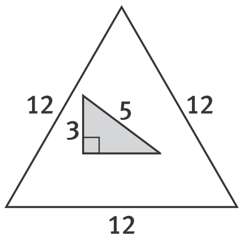
LESSON 25-2 PRACTICE

6. The members of the Math Club are designing dartboards. They experimented with different shapes. Suppose a dart lands on the board. For which dartboard is the probability of hitting the bull's-eye the least? Note that the dartboards are not drawn to scale.

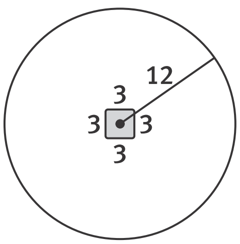
A.



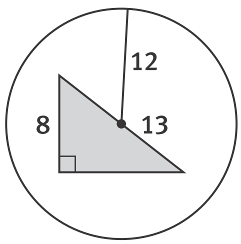
B.



C.



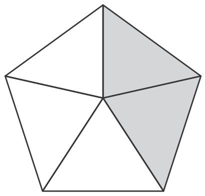
D.

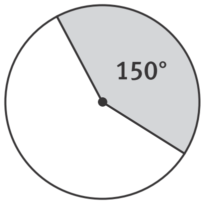


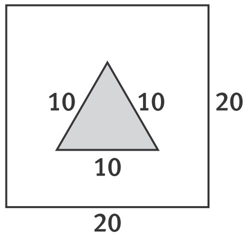
My Notes

7. **Attend to precision.** Students will not play a dartboard game if it is too difficult to win a prize. Design a dartboard with the probability that a dart that lands on the board has about a 15% chance of landing in the bull's-eye.

8. Suppose darts are thrown at random and land on each dartboard. Find the probability that a dart lands in the shaded region. Write your answer as a percent.







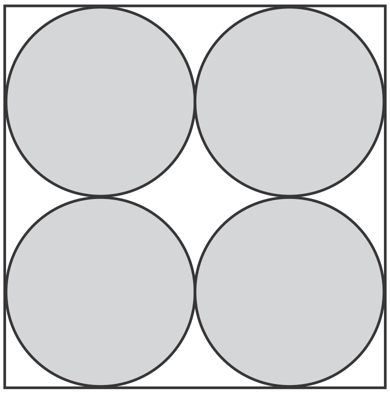
MATH TIP

In Item 8b, use the fact that the circle has 360° to find the shaded area, or *sector*, of the circle. You will learn more about sectors of circles in Unit 6.

9. Liam made a 3 × 3 target on a 10 × 10 dartboard.
   a. What is the probability that a dart that hits the board will hit Liam's target?
   b. **Critique the reasoning of others.** Olivia made a 6 × 6 target on a 10 × 10 dartboard. She said that the probability of a dart landing in her target area would be twice as great as the probability that a dart will land in Liam's target area. Do you agree? Explain.

10. Amelia plants an 8 ft by 6 ft garden. She plants roses in a 2 ft by 3 ft section of the garden. If a robin randomly lands in the garden, what is the probability that the robin will land among the rosebushes? What is the probability that the robin will not land among the rosebushes?

11. **Look for and make use of structure.** How would you find the probability of landing in the shaded region without knowing the length of the radii of the circles or the dimensions of the square? Show your work.



My Notes

Learning Targets:

- Understand probability in real-world situations.
- Describe events as subsets of a sample space using the characteristics of the outcomes.
- Represent sample spaces as tables of outcomes and as two-way frequency tables.
- Calculate the probability of events involving "and" and "or."

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Marking the Text, Visualization, Think-Pair-Share, Debriefing

Consider probabilities that involve the words "and" and "or." Look again at the SpringBoard Superstar contestants.

Aaron Kelly (Male, 17)	Lacey Brown (Female, 24)
Andrew Garcia (Male, 24)	Lee DeWyze (Male, 24)
Casey James (Male, 27)	Michael Lynch (Male, 26)
Crystal Bowersox (Female, 24)	Paige Miles (Female, 24)
Didi Benami (Female, 23)	Siobhan Magnus (Female, 19)
Katie Stevens (Female, 17)	Tim Urban (Male, 20)

A contestant will be chosen at random.

1. Find the probability that the contestant will be female *and* under 21.
   a. How many of the contestants are females who are under 21?

   b. How many contestants are there in total?

   c. What is the probability that the randomly chosen contestant will be female *and* under 21?

MATH TIP

In probability, "and" means the event has the characteristics of *both* possibilities.

2. Now find the probability that the contestant will be female *or* under 21.
   a. Refer to the list of contestants. Put a check mark next to any contestant who is either female or under 21, or both female and under 21. In probability, "or" always includes the possibility of "one or both."

   b. How many contestants are female *or* under 21, or both female *and* under 21?

   c. What is the probability that the randomly chosen contestant will be female *or* under 21?

MATH TIP

The probability of "A or B" includes the probability of just A, just B, or both A and B.

Lesson 25-3
Events Involving "And" and "Or"

My Notes

POINT OF INTEGRATION

Probability and Algebra
The words "or" and "and" used in probability are also used in algebra to describe solutions to compound inequalities. For example, the compound inequality $x > 3$ or $x < 0$ represents values that will meet either of the two conditions, while the inequality $0 < x < 3$ represents values that must be both greater than 0 and less than 3.

Very often, we show sample spaces using tables rather than lists.

3. Kathy likes Honey Crisp breakfast cereal. Each box of Honey Crisp contains a plastic animal—a monkey, an elephant, a caribou, or a bear—and each of these four animals is equally likely to be found in any given box. Kathy's younger brother, Brian, prefers Korn Invaders breakfast cereal. Each box of Korn Invaders contains either Hero Comic 1, Hero Comic 2, or Hero Comic 3, and each of the comics is equally likely to be found in any given box.

Tomorrow, Kathy will open a new box of Honey Crisp and Brian will open a new box of Korn Invaders. They will find out what prizes they have.

The table representing the sample space for this probability experiment is given.

	Hero Comic 1	Hero Comic 2	Hero Comic 3
Monkey			
Elephant			
Caribou			
Bear			

a. Using the eraser end of your pencil, locate the cell in the table that represents getting an elephant *and* Hero Comic 2.

b. Which cells of the table represent getting a caribou?

c. Which cells of the table represent getting Hero Comic 3?

d. How many outcomes are there in the sample space for this probability experiment?

e. Locate, again, the cell in the table that represents getting an elephant and Hero Comic 2. What is the probability of getting an elephant and Hero Comic 2?

f. Now find the probability of getting an elephant *or* Hero Comic 2.

Write stars in all the cells of the table that represent getting an elephant or Hero Comic 2. (Note: You are writing a star in any cell that represents getting an elephant, any cell that represents getting Hero Comic 2, and any cell that represents getting both. You should be writing 6 stars in total.)

What is the probability that they get an elephant *or* Hero Comic 2?

My Notes

Check Your Understanding

4. If a student is chosen at random from your class, what is the probability that the student has a:

a. first name beginning with a letter in the first half of the alphabet?

b. first name beginning with a letter in the first half of the alphabet and is female?

c. first name beginning with a letter in the first half of the alphabet or is female?

MATH TERMS

A **two-way frequency table** shows data that pertain to two different categories.

Sometimes, a sample space is represented using a table showing the numbers of occurrences of different outcomes.

5. A store has 48 different phones available with or without caller ID, and with or without a speakerphone. The numbers of phones with and without these features are shown in the ***two-way frequency table*** below.

	Speakerphone	No Speakerphone	Total
Caller ID	18	2	20
No Caller ID	16	12	28
Total	34	14	48

a. How many of the phones have caller ID?

b. What is the total number of phones that the store has available?

c. If a phone is chosen at random, what is the probability that it has caller ID? Using a calculator, give your answer as a decimal rounded to three decimal places.

d. A phone is chosen at random from the ones available at the store. To three decimal places, find the probability that the phone has a:

i. speakerphone.

ii. speakerphone and caller ID.

iii. speakerphone but does not have caller ID.

iv. speakerphone or has caller ID.

My Notes

Check Your Understanding

6. **Attend to precision.** Students who take an AP Calculus exam can take either the AP Calculus AB exam or the more advanced AP Calculus BC exam. On each exam the highest possible grade is a 5 and the lowest is a 1. No student takes both exams. The approximate results (in thousands) for 2011 are ***summarized*** in the table below. (For example, the table tells us that 39,000 students took the AB exam and got a 5.)

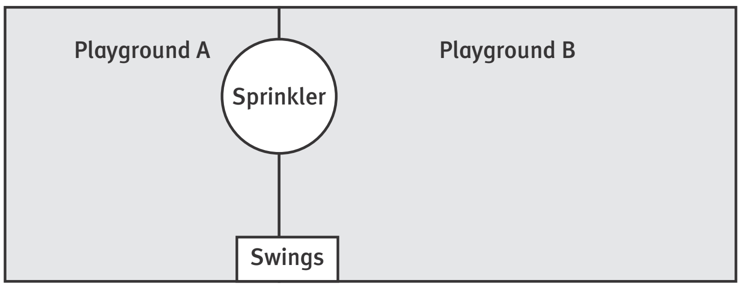
	5	4	3	2	1	Total
AB	39	32	35	23	65	194
BC	28	10	12	4	8	
Total						

a. Complete the table by writing the row and column totals in the empty cells.
b. Suppose that one of these 256,000 students is chosen at random. Using a calculator, and giving your answers rounded to three decimal places, find the probability that the student:
  i. took the BC exam.
  ii. got a 5.
  iii. did not get a 5.
  iv. took the BC exam and got a 5.
  v. took the BC exam or got a 5.

ACADEMIC VOCABULARY

The word ***summarize*** means to create a brief statement (called a summary) to review the important points of something. In this case, the important points are captured in a table, and are the numerical data for the number of students that received a certain score on a particular AP calculus exam.

The Superstar contestants had a lunch break at Pleasant Park. The park is a rectangle with two playgrounds. The sprinkler region is divided equally between the two playgrounds. Tim lost his room key somewhere inside Pleasant Park. He spent time in all regions of the park.



7. The table shows areas of different regions in the park. For instance, the total area of Playground A is 576 square yards. A part of the swings region is in Playground A, and the area of that part is 25 square yards. Complete the table to show the areas, in square yards, of the different regions of the park.

POINT OF INTEGRATION

Probability and Geometry
Geometrically, "A and B" is represented by the area overlapped by two regions A and B, while "A or B" is represented by just region A, just region B, and the area where the two regions overlap.

	Sprinkler Region	Swings Region	Not Sprinkler or Swings	Total Area
Area in Playground A		25 yd^2		576 yd^2
Area in Playground B				960 yd^2
Area in Both Playgrounds	78.5 yd^2	75 yd^2		1536 yd^2

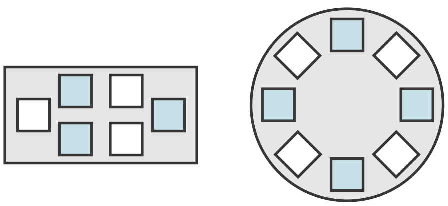
8. a. What is the probability that Tim lost his key in the swings *or* in the sprinkler region? Round to the nearest tenth of a percent.

b. What is the probability that Tim's key is in Playground A *and* in the swings region? Round to the nearest tenth of a percent.

c. What is the probability that Tim's key is in Playground B and *not* in the swings *or* in the sprinkler region? Round to the nearest tenth of a percent.

Check Your Understanding

9. A party room has two tables. One table is rectangular with a length of 6 feet and a width of 3 feet. The other table is circular with a diameter of 8 feet. Blue and white placemats are placed around the tables in an alternating pattern. Each placemat is a square with side length of 1 foot. A sticker is hidden somewhere on one of the tables under the tablecloth.



a. What is the probability that the sticker is hidden under a white placemat *and* on the circular table? Round to the nearest thousandth.

b. What is the probability that the sticker is hidden under a blue placemat *or* on the rectangular table? Round to the nearest thousandth.

My Notes

LESSON 25-3 PRACTICE

10. Suppose that you are going to roll a red cube with faces numbered 1 through 6 and a blue cube with faces numbered 1 through 6.

a. Draw a table to represent the sample space for this probability experiment.

b. Giving your answers as fractions in their lowest terms, find the probability that:

i. the red cube shows a 4 and the blue cube shows a 6.

ii. the red cube shows a 4 *or* the blue cube shows a 6.

iii. it is *not* the case that "the red cube shows a 4 or the blue cube shows a 6."

iv. the sum of the numbers on the two cubes is 9.

v. the sum of the numbers on the two cubes is more than 8.

11. Aslan's store has the following tablet computers in stock.

	Tablet Classic	Tablet Mini	Total
16 GB	75	95	
32 GB	42	63	
Total			

a. Complete the table by writing the row and column totals in the empty cells.

b. If a tablet is chosen at random, to the nearest thousandth, what is the probability it has 32 GB?

c. If a tablet is chosen at random, to the nearest thousandth, what is the probability it is a Tablet Mini with 16 GB?

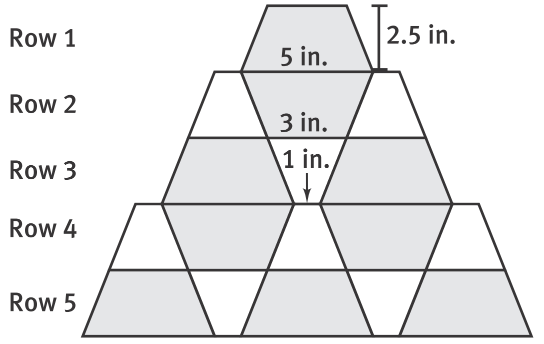
d. If a tablet is chosen at random, to the nearest thousandth, what is the probability it is not a Tablet Mini?

e. If a tablet is chosen at random, to the nearest thousandth, what is the probability it is a Tablet Classic?

f. If a tablet is chosen at random, to the nearest thousandth, what is the probability it is a Tablet Classic or a Tablet Mini?

My Notes

12. Model with mathematics. A target is made of gray and white trapezoids. Each gray and white trapezoid has a height of 2.5 inches. Each gray trapezoid has base lengths of 3 inches and 5 inches. Each white trapezoid has base lengths of 1 inch and 3 inches. Darts are thrown randomly at the board as part of a game. Assume all darts hit the board.



- **a.** What is the probability that a dart lands in Row 1 *and* in a gray trapezoid?
- **b.** What is the probability that a dart lands in Row 1 *or* in a gray trapezoid?
- **c.** What is the probability that a dart lands in Row 1 *and* in a white trapezoid?
- **d.** What is the probability that a dart lands in Row 1 *or* in a white trapezoid?
- **e.** What is the probability that a dart lands in Row 2 *or* in Row 3?
- **f.** What is the probability that a dart lands in Row 4 *and* in a white trapezoid?
- **g.** Assuming a dart is thrown at the board and lands on the board, describe an event that could have a probability of $\frac{1}{2}$.

ACTIVITY 25 PRACTICE

Answer each item. Show your work.

Lesson 25-1

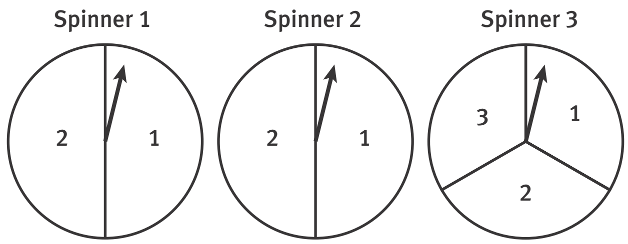
1. A garden center has 20 plants in its "Special Offer" section. Information about these plants is given in the table below.

Plant Number	Type of Plant	Color of Flowers
1	Hyacinth	Pink
2	Hyacinth	White
3	Hyacinth	Blue
4	Hyacinth	Blue
5	Tulip	Pink
6	Tulip	White
7	Tulip	Pink
8	Pansy	Blue
9	Pansy	White
10	Pansy	Pink
11	Pansy	Pink
12	Zinnia	Pink
13	Zinnia	White
14	Zinnia	White
15	Daisy	White
16	Daisy	White
17	Daisy	Pink
18	Daisy	Blue
19	Orchid	Pink
20	Orchid	White

A customer picks a plant at random from the "Special Offer" section. Find the probability that the plant:

a. is a hyacinth.
b. has flowers that are blue.
c. does not have flowers that are blue.
d. is a rose.
e. has flowers that are pink, white, or blue.

2. Each of the three spinners will be spun once. All outcomes are equally likely.



a. Complete the list of all the possible outcomes for the three spinners.

1 1 1
1 1 2
1 1 3
1 2 1
…

b. What is the probability that the sum of the scores for the three spinners will be more than 5?

3. A recently married couple are planning to have four children. They are equally as likely to have a boy as a girl. What is the probability that they will have three boys and one girl?

A. $\frac{1}{4}$

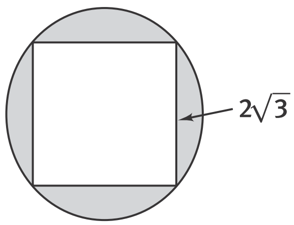
B. $\frac{3}{8}$

C. $\frac{3}{4}$

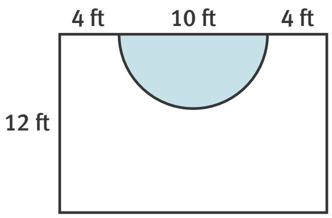
D. $\frac{5}{8}$

Lesson 25-2

4. The shaded region is outside of a square with side length of $2\sqrt{3}$ units on a circular dartboard. Estimate the probability that a dart that hits the board will hit the shaded region. Show your work.



5. In a game at a school fundraiser, players throw coins into the target shown. The shaded region inside the rectangle is a half circle and represents the prize zone. Assume that a coin that lands on the target is equally likely to land anywhere on the target. What is the probability of the coin landing in the prize zone? Round to the nearest thousandth.



Lesson 25-3

6. Refer to the information given in Item 1. A customer picks a plant at random from the "Special Offer" section. Find the probability that the plant is:

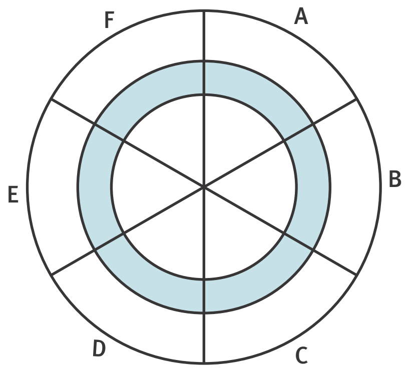
a. a daisy with white flowers.
b. not a daisy with white flowers.
c. a daisy or has white flowers.
d. a tulip with pink flowers or a zinnia with pink flowers.

7. A high school current affairs club has members in all four grades (9, 10, 11, and 12) and of both genders. The numbers of students in the various grade/gender categories are given in the table below.

Grade	Gender		Total
	Male	Female	
9	8	6	
10	4	2	
11	3	7	
12	9	9	
Total			

a. Complete the table by entering all the totals in the empty cells.
b. A student from the club is chosen at random to make an announcement in assembly. Find the probability that the student is:
- **i.** in the 9th grade.
- **ii.** female.
- **iii.** an 11th-grade girl.
- **iv.** in the 11th grade or is a girl.

8. A circular dartboard is divided into six equal-sized sectors labeled *A* through *F*. The diameter of the circular dartboard is 21 inches. The smallest circle on the dartboard has a diameter of 11 inches. The next-largest circle has a diameter of 15 inches. Darts are thrown randomly at the board.



a. What is the probability that a dart that lands on the board lands in Sector A *or* in Sector E?
b. What is the probability that a dart that lands on the board lands in Sector B *and* in the white outer ring?
c. What is the probability that a dart that lands on the board lands in Sector C *or* inside the smallest circle?

MATHEMATICAL PRACTICES
Reason Abstractly and Quantitatively

9. Suppose that you're going to pick a student at random from your school. Put the following three events in order by how likely they are, from the most likely event to the least likely. Write a few sentences explaining why you ordered them the way you did.

1. The student plays basketball.
2. The student plays basketball and takes a music class.
3. The student plays basketball or takes a music class.

Venn Diagrams and Probability Notation

ACTIVITY 26

Annabel High School

Lesson 26-1 Using a Venn Diagram to Represent a Sample Space

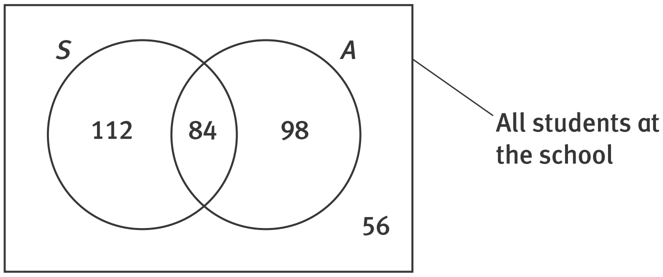
Learning Targets:

- Use Venn diagrams to represent events.
- Translate Venn diagrams of counts into Venn diagrams of probabilities.

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Close Reading, Create Representations, Visualization, Think-Pair-Share, Debriefing

You have represented sample spaces using lists and tables. You can also use Venn diagrams to represent sample spaces.

At Annabel High School, some students take Spanish and some do not. Some students take an art class, and some do not. Let *S* be the set of students who take Spanish, and let *A* be the set of students who take an art class. The numbers of students in these classes are shown in the Venn diagram.



As you answer the following items, remember to use complete sentences and words such as *and, or, since, for example, therefore, because of, by the*, to make connections between your thoughts.

1. a. Look at the number "112" in the diagram. This number is inside the circle for *S* and outside the circle for *A*. It tells us that there are 112 students who take Spanish but do not take an art class. What does the number "98" tell us?

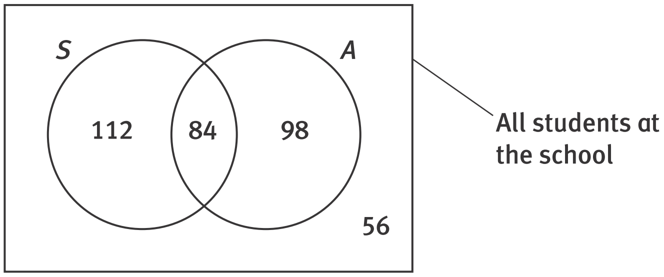
b. What does the number "84" tell us?

My Notes

DISCUSSION GROUP TIPS

If you do not understand something in group discussions, ask for help or raise your hand for help. Describe your questions as clearly as possible, using synonyms or other words when you do not know the precise words to use.

My Notes



DISCUSSION GROUP TIPS

As you listen to the group discussion, take notes to aid comprehension and to help you describe your own ideas to others in your group. Ask questions to clarify ideas and to gain further understanding of key concepts

c. What does the number “56” tell us?

d. What is the total number of students who take Spanish?

e. What is the total number of students who take an art class?

f. How many students are there at the school?

g. How many students at this school take Spanish or an art class?

h. How many students do not take an art class?

My Notes

2. **Reason quantitatively.** Now we can talk about probabilities.
   a. If a student is selected at random from the school, what is the probability that the selected student takes Spanish? Explain.

   b. A student is selected at random. Find the probability that the student selected:

      i. takes an art class.

      ii. takes Spanish *and* an art class.

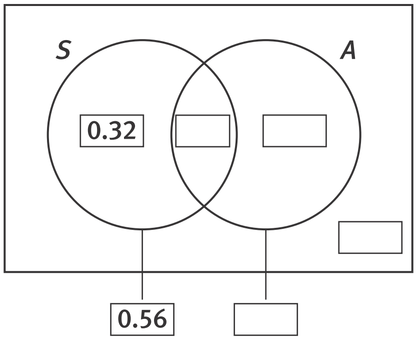
      iii. takes an art class but does not take Spanish.

      iv. takes Spanish *or* an art class.

      v. takes neither Spanish nor an art class.

My Notes

3. Model with mathematics. You can write the probabilities in the Venn diagram, rather than the counts. Suppose a student is selected at random from Annabel High School. Let *S* be the event that the student takes Spanish and let A be the event that the student takes an art class.

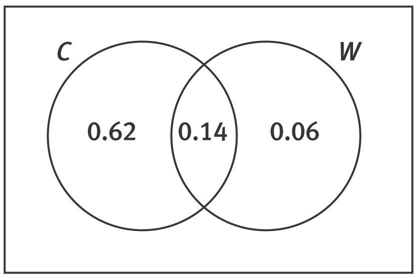


a. The number 0.32 in set *S* tells you the probability that the selected student takes Spanish and does not take an art class. How do you calculate this probability?

b. What does the "0.56" written in the box connected to the circle for event *S* tells us?

c. Write the appropriate probabilities in all the remaining empty boxes in the Venn diagram.

4. The most popular color for vehicles is white. Suppose that a vehicle is selected at random from a city, and let *C* be the event that the vehicle is a car and *W* be the event that the vehicle is white. Some of the relevant probabilities are shown in the Venn diagram below.



a. What two probabilities do you add to find the probability that the selected vehicle is a car? Find the probability.

My Notes

b. Find the probability that the selected vehicle is:

i. white.

ii. a white car.

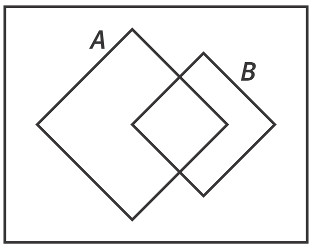
iii. a car that is not white.

iv. a car or is white.

c. What region in the Venn diagram represents the event that the vehicle selected is neither white nor a car? Find the probability of this event.

In Lesson 25-2, you found geometric probabilities. You can use Venn diagrams to represent the areas shown in geometric diagrams in order to find probabilities.

Some students at Annabel High School developed a game based on a board like the one below. The game board measures 5 in. by 10 in. Each side of Square *A* is 4 in. Each side of Square *B* is 3 in. The overlapping square has side 2 in.

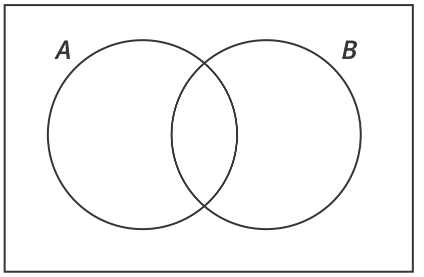


My Notes

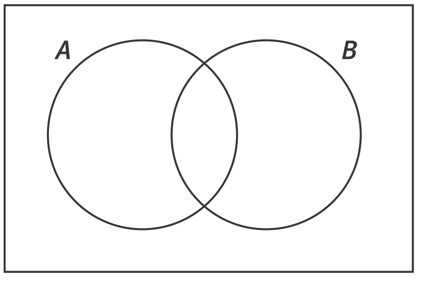
MATH TIP

Find the area of the overlapping square first, and then work outward to accurately find the value of each region.

5. In the Venn diagram, place the number of square inches for the area of each region of the diagram above. Be sure to include the area outside the rectangle not included in the squares.



6. Suppose a dime is tossed at random on the game board. Any time a coin is on a line, it is not counted and is tossed again. In the Venn diagram, write the probability that the coin lands in each region, assuming the coin is counted.



7. Use the probability diagram in Item 6 to find each probability.

a. What is the probability that the coin lands in *A*?

b. What is the probability that the coin does not land in *A*?

c. What is the probability that the coin lands in *A* or *B*?

MATH TIP

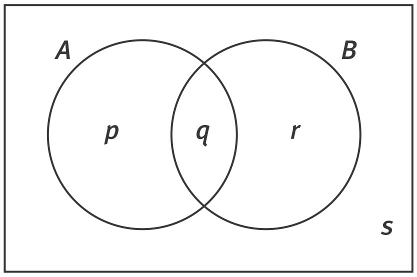
The three given probabilities and the required probability added together must be 1, since the four probabilities cover all probabilities.

Check Your Understanding

8. At the beginning of this activity you were given a Venn diagram with the numbers (counts) of students who take the various combinations of Spanish (or not) and an art class (or not). From the counts you were given, how did you calculate the following?

a. the total number of students in the school
b. the probability that a randomly selected student takes Spanish
c. the probability that a randomly selected student takes an art class but does not take Spanish

9. Suppose that a student will be selected at random from Annabel High School. Let A be the event that the student takes an art class and let B be the event that the student plays basketball. The quantities p, q, r, and s given in the Venn diagram represent probabilities.

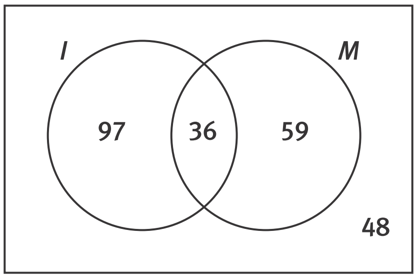


a. How would you calculate the probability that the student:
- **i.** plays basketball?
- **ii.** takes an art class and plays basketball?
- **iii.** plays basketball but does not take an art class?
- **iv.** takes an art class or plays basketball?
- **v.** does not take an art class?

b. If you knew the values of p, q, and r, how would you find the value of s? What does the probability s represent?

LESSON 26-1 PRACTICE

10. At Kennedy High School, some students take Italian and some do not. Some students take music and some do not. Let I be the set of students who take Italian, and let M be the set of students who take music.



a. What does the number "36" in the Venn diagram tell us?
b. What does the number "59" tell us?
c. What does the number "48" tell us?
d. How many students take Italian or music?
e. How many students do not take Italian?

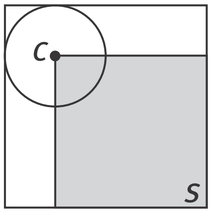
My Notes

11. If a student is selected at random from Kennedy High School, to the nearest thousandth, what is the probability that the selected student:

a. takes Italian?
b. takes Italian or music?
c. does not take Italian and does not take music?
d. only takes music?

12. Make sense of problems and persevere in solving them. Suppose that a student will be selected at random from the High School of Arts and Sciences. Let *P* be the event that the student takes physics and let *D* be the event that the student takes a dance class. The probability that the student takes physics is 0.28, while the probability that the student takes a dance class is 0.45. The probability that the student takes physics and a dance class is 0.07. What is the probability that the student takes a dance class but not a physics class?

13. A student developed the game board shown. The radius of circle *C* is 5 in. and each side of square *S* is 10 in. Students will toss a dime at random on the game board.



a. What is the length of a side of the outside square?
b. Make a Venn diagram showing the areas of each region. Find these areas to the nearest tenth of a square inch.
c. Any time the coin is on a line, it is not counted and it is tossed again. Make a second Venn diagram showing the probability that a dime (that counts) would land in each region identified in part b. Find these probabilities to the nearest hundredth.
d. What is the probability that a coin will land in circle *C* and square *S*?
e. What is the probability that a coin will land in circle *C* or square *S*?
f. What is the probability that a coin will not land in square *S*?
g. What is the probability that a coin will not land in either circle *C* or in square *S*?

My Notes

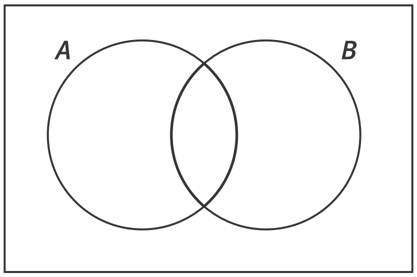
Learning Targets:

- Use Venn diagrams to represent "and," "or," and "not."
- Use set notation to describe events.

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Close Reading, Create Representations, Visualization, Think-Pair-Share, Debriefing

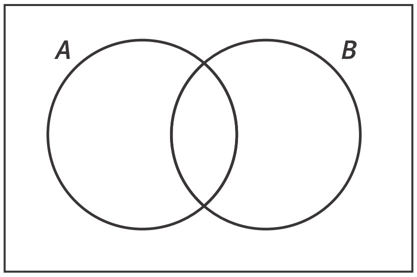
1. Now look at the mathematical notation for "and," "or," and "not." Suppose that we are selecting a student at random from Annabel High School, and let A be the event that the student takes an art class and let B be the event that the student plays basketball.

a. Shade the region representing the event that the selected student takes an art class *and* plays basketball.



This region is called the ***intersection*** of the sets A and B, and can be written as $A \cap B$ (read as "A intersect B"). The probability that the selected student takes an art class and plays basketball can be written as $P(A \cap B)$.

b. Shade the region representing the event that the selected student takes an art class *or* plays basketball.



This region is called the ***union*** of the sets A and B, and can be written as $A \cup B$ (read as "A union B"). The probability that the selected student takes an art class or plays basketball can be written as $P(A \cup B)$.

MATH TERMS

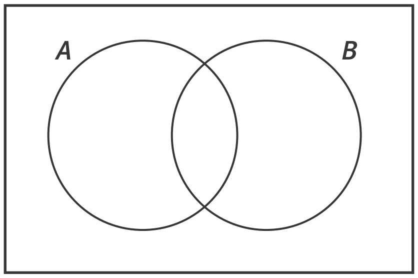
The set of elements that are in A and in B is referred to as the **intersection** of A and B, written $A \cap B$.

MATH TERMS

The set of elements that are in A or in B, or in both, is referred to as the **union** of A and B, and is written $A \cup B$.

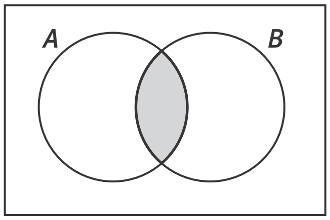
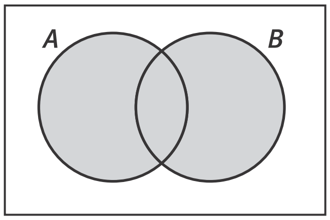
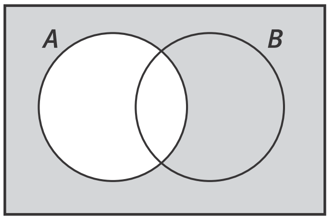
My Notes

c. Shade the region representing the event that the selected student does *not* take an art class.



This region is the ***complement*** of set A, and can be written as A^C (read as "A complement"). The probability that the selected student does not take an art class can be written as $P(A^C)$.

These ideas are summarized in the table below.

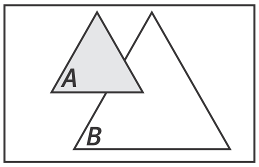
Shading	In Words	In Probability Notation
	The probability that a randomly selected student takes an art class *and* plays basketball	$P(A \cap B)$
	The probability that a randomly selected student takes an art class *or* plays basketball	$P(A \cup B)$
	The probability that a randomly selected student does *not* take an art class	$P(A^C)$

MATH TERMS

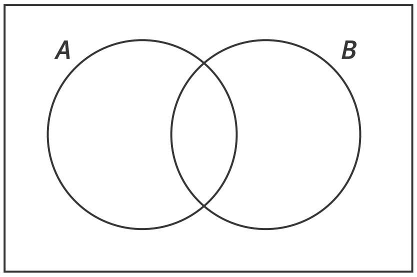
The set of elements that are not in A is referred to as the **complement** of A, and is written A^C.

My Notes

2. In the game board below, the area of triangle A is 3.90 in.^{2}, the area of triangle B is 10.83 in.^{2}, the area of the overlapping region is 0.43 in.^{2}, and the area of the rectangle is 40 in.^{2}.



a. **Model with mathematics.** Suppose a dime is tossed at random on the game board. Any time the coin is on a line, it is not counted and is tossed again. In the Venn diagram, write the probability that the coin lands in each region. Find these probabilities to the nearest hundredth.



b. Find $P(A \cup B)$.

c. Find $P(A \cap B)$.

d. Find $P(A^c)$.

e. Describe the event $A^c \cap B^c$ in words.

f. Find $P(A^c \cap B^c)$.

MATH TIP

It may be helpful to first make a Venn diagram showing the areas of each region and then make the one showing the probabilities.

My Notes

Check Your Understanding

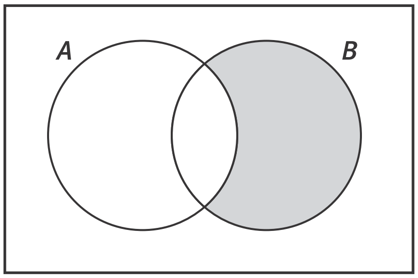
3. Suppose that a student is selected at random from Annabel High School. Let S be the event that the student plays soccer and let H be the event that the student is on the honor roll. Copy each diagram and shade the region for each event.
   a. Make a Venn diagram and shade the region that represents the event $S \cap H$.
   b. Describe the event $S \cap H$ in words: "$S \cap H$ is the event that the student . . ."
   c. Make a Venn diagram and shade the region that represents the event $S \cup H$.
   d. Describe the event $S \cup H$ in words.
   e. Make a Venn diagram and shade the region representing the event H^C.
   f. Describe the event H^C in words.

4. Consider picking a vehicle at random from a city. Denote the event that the vehicle is a sedan by S and the event that the vehicle is black by B. Write the following probabilities in probability notation. (Use the symbols $\cap$, $\cup$, and ^{C} where they apply.)
   a. the probability that the vehicle is black
   b. the probability that the vehicle is not black
   c. the probability that the vehicle is a black sedan
   d. the probability that the vehicle is either a sedan or is black
   e. the probability that the vehicle is a sedan and is not black
   f. the probability that the vehicle is a sedan or is not black

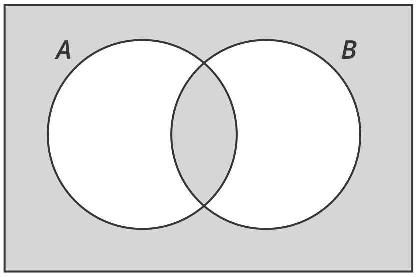
LESSON 26-2 PRACTICE

5. A student is selected at random from Annabel High School. Let A be the event that the student takes an art class and B be the event that the student plays basketball. Describe in words the event represented by each shaded region, and write the event in set notation.

a.

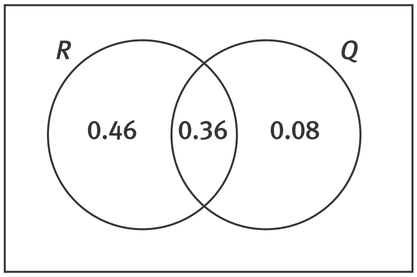


b.



My Notes

6. Suppose that a player will be selected at random from the roster of a local baseball team. Let the event that the player is a right-handed thrower be R and let the event that the player is a pitcher be Q. Some of the relevant probabilities are shown in the Venn diagram below.



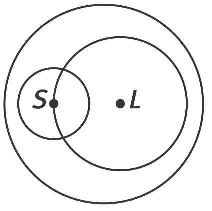
a. One probability has been omitted from the Venn diagram. Calculate this probability, and write the value in the appropriate place on the diagram.

b. **Reason abstractly and quantitatively.** Complete the table below by writing the appropriate words, numbers, or probability notation in the empty cells.

Probability in Words	Probability Notation	Value of Probability
The probability that the player is a right-handed thrower	$P(R)$	0.82
The probability that the player is a pitcher		
The probability that the player is a right-handed thrower and is a pitcher		
	$P(R \cup Q)$	
	$P(Q^c)$	
The probability that the player is a right-handed thrower and is not a pitcher		
	$P(R^C \cap Q^C)$	

My Notes

7. The circle with the two smaller circles represents a game board with an area of 144π in.^{2}. The area of the smaller circle, S, is 16π in.^{2}. The area of the larger circle, L, is 64π in.^{2}. The area of the overlap of S and L is 7.4π in.^{2}. Students will toss a dime at random on the game board.



a. If a coin lands on a line, it is not counted and it is tossed again. Make a Venn diagram showing the probability that a dime (that counts) would land in each of the regions. Find these probabilities to the nearest hundredth.

b. Find $P(S)$.

c. What is the set notation for the probability that the coin will land in a region that is not a part of circle S?

d. Find $P(S \cap L)$.

e. Describe $P(S \cap L^c)$ with respect to the game board and find $P(S \cap L^c)$.

f. Find $P(S \cup L)$.

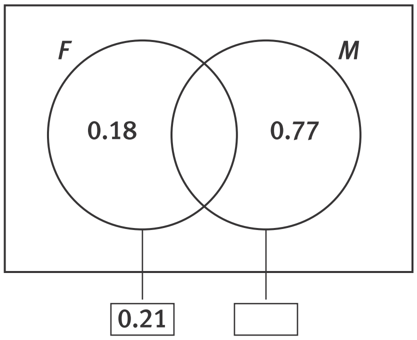
g. Describe $P(S \cup L^c)$ with respect to the game board and find $P(S \cup L^c)$.

ACTIVITY 26 PRACTICE

Answer each item. Show your work.

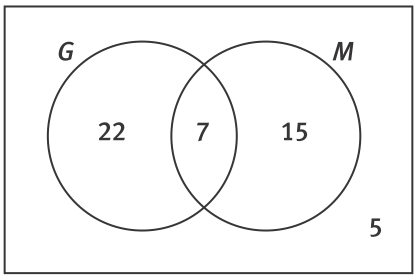
Lesson 26-1

1. Suppose that a student will be selected at random from Annabel High School. Let *F* be the event that the student plays football and let *M* be the event that the student takes a music class. Some of the probabilities of the events are shown in the Venn diagram.



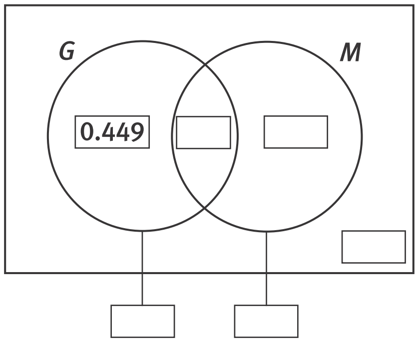
   a. Complete the diagram.
   b. What is the probability that the student will play football but will not take a music class?
   c. What is the probability that the student will take a music class?

2. A restaurant has a large fish tank. Some of the fish are goldfish, and some are not. Some of the fish are more than a year old, and some are not. Let the set of fish that are goldfish be *G* and let the set of fish that are more than a year old be *M*. The numbers of fish in various categories are shown below.



   a. How many fish are goldfish?
   b. How many fish are goldfish and are more than a year old?
   c. How many fish are goldfish or are more than a year old?
   d. How many fish are not goldfish and are not more than a year old?
   e. What is the total number of fish?
   f. How many fish are not goldfish?
   g. How many fish are less than or equal to a year old?
   h. What is the complement of set *G*? Describe the set in words.
   i. Compare the answers to parts e and f. Are they the same? Explain.

3. A fish will be selected at random from the tank. Complete the Venn diagram by writing the appropriate probabilities in the boxes. Give the probabilities as decimals to the nearest thousandth.



   a. Explain how you found *P*(goldfish less than or equal to a year old).
   b. What is the total of all the probabilities for this experiment? Explain.

Lesson 26-2

4. Refer to the Venn diagram you completed in Item 3. Describe each of the following probabilities in words. Then determine the value of the probability.

a. $P(G)$

b. $P(M)$

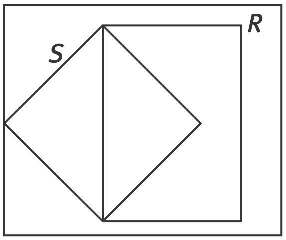
c. $P(G \cap M)$

d. $P(G \cup M)$

e. $P(G^C)$

f. What is the probability that the fish that is selected is more than a year old and is not a goldfish?

5. The large rectangle with square S and the smaller rectangle, R, represent a game board with an area of 30 in.^{2}. The area of square S is 9 in.^{2}. The area of the rectangle R is 12.73 in.^{2}. The area of the overlap of S and R is 4.5 in.^{2}. Students will toss a dime at random on the game board.



a. Any time the coin is on a line, it is not counted and it is tossed again. Make a Venn diagram showing the probability that a dime (that counts) lands in each region. Find these probabilities to the nearest hundredth.

b. What is the probability that the coin does not land in rectangle R?

c. What is the probability that the coin lands in rectangle R but not square S?

d. What is the probability that the coin lands outside both rectangle R and square S?

e. What is the probability that the coin lands outside rectangle R or in square S?

6. Refer to your Venn diagram from Item 5. Find each probability to the nearest hundredth.

a. $P(S)$

b. $P(S^c)$

c. $P(S \cap R)$

d. $P(S^c \cap R)$

e. $P(S \cup R)$

f. $P(S^c \cup R)$

MATHEMATICAL PRACTICES
Model with Mathematics

7. The following table shows the numbers of phones available with and without speakerphone and with and without caller ID in Joe's Phone Store.

	Speakerphone	No Speakerphone
Caller ID	18	2
No Caller ID	16	12

Draw a Venn diagram to represent this information. Label the circles C (for Caller ID) and S (for Speakerphone). Write the four numbers in the table in the appropriate places in the diagram.

Addition Rule and Mutually Exclusive Events

ACTIVITY 27

Hector Street

Lesson 27-1 Applying the Addition Rule

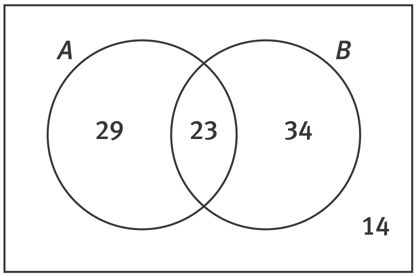
My Notes

Learning Targets:

- Learn the Addition Rule and understand why it applies.
- Use the Addition Rule to calculate probabilities.

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Think Aloud, Create Representations, Visualization, Think-Pair-Share, Debriefing

There are 100 houses on Hector Street. Some of the houses have attics, and some do not. Some of the houses have basements, and some do not. Let the set of houses that have attics be *A* and the set of houses that have basements be *B*. The numbers of houses with various combinations of attic and basement are shown.



1. How many of the houses on the street have attics?

2. How many of the houses on the street have basements?

3. **Reason quantitatively.** If a house is selected at random from the street, what is the probability that it has an attic?

4. If a house is selected at random from the street, what is the probability that it has a basement?

DISCUSSION GROUP TIPS

Use your group discussions to clarify the meaning of mathematical concepts and other language used to describe problem information. With your group or your teacher, review background information that will be useful in applying concepts.

My Notes

WRITING MATH

Set Notation

$A \cap B$ means that the outcome belongs to both A **and** B.

$A \cup B$ means that the outcome belongs to A **or** B.

^{C} means the outcome does **not** belong to the set.

5. If a house is selected at random from the street, what is the probability that it has an attic *and* a basement?

6. Write the answers to Items 3–5 using probability notation. Use set notation as necessary.

7. What is the probability that a randomly selected house from the street has an attic *or* a basement?

a. What is the value of $P(A \cup B)$?

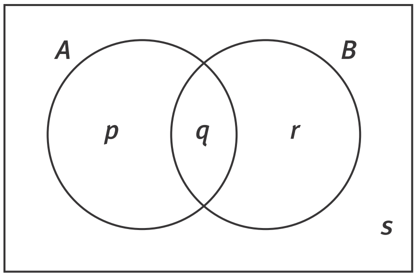
b. What is the value of $P(A) + P(B)$?

c. **Construct viable arguments.** Explain why $P(A) + P(B)$ is not equal to $P(A \cup B)$.

Complete the formula for the union of two intersecting sets.

$$P(A \cup B) = P(A) + P(B) - \_\_\_\_\_\_$$

Let's generalize. Suppose that various combinations of events A and B have the probabilities p, q, r, and s as shown in this Venn diagram.



To calculate $P(A \cup B)$, find $p + q + r$.

To calculate $P(A)$, find $p + q$, and to calculate $P(B)$, find $q + r$.

If we calculate $P(A) + P(B)$, we get $p + q + q + r$, which includes two q's.

In $P(A \cup B)$ we only want one q.

Lesson 27-1
Applying the Addition Rule

My Notes

When we calculate $P(A \cup B)$ using $P(A)$ and $P(B)$, we have to subtract q, which is $P(A \cap B)$. This leads to the following formula:

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

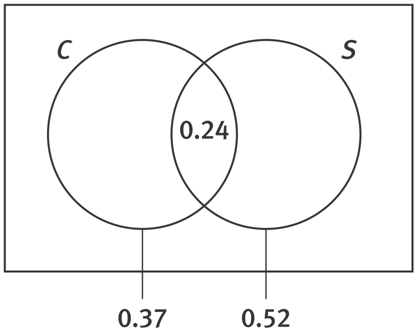
This formula is called the *Generalized Addition Rule*, and is often simply referred to as the Addition Rule.

MATH TIP

To find $P(A \text{ or } B)$, use the **Addition Rule:**

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Of the houses on Hector Street, 37% have central air conditioning, 52% have a security system, and 24% have both central air and a security system. Suppose that a house is selected at random from the street. Let C be the event that the house has central air and S be the event that the house has a security system. The given information is shown in the Venn diagram below.



8. What is the value of each of the following?

a. $P(C)$

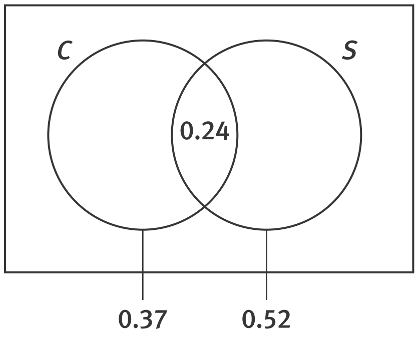
b. $P(S)$

c. $P(C \cap S)$

9. Make sense of problems. Now use the Addition Rule to find $P(C \cup S)$. Express in words the meaning of the probability you just found.

My Notes

10. Model with mathematics. Shade the region represented by $C \cup S$.

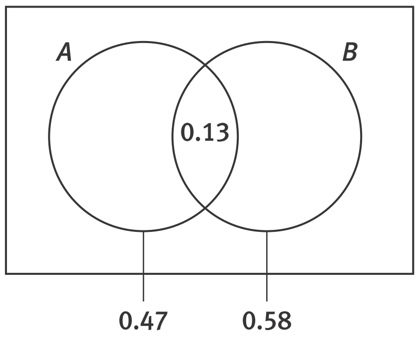


11. If a house is selected at random from this street, what is the probability that it has neither central air nor a security system?

Check Your Understanding

12. Write the Addition Rule.

13. Using the information given in the Venn diagram below, calculate $P(A \cup B)$.



14. What is the probability of the complement of $P(A \cup B)$?

My Notes

15. If a household is selected at random from Elm Street, the probability that the household has lagged pipes is 0.26 (lagged piping saves energy). The probability that the household recycles its refuse is 0.63. The probability that the household either has lagged pipes or recycles its refuse is 0.67.

a. Write the given information using probability notation. Let the event that a household has lagged pipes be L and the event that a household recycles most of its refuse be R.

b. Find the probability that a randomly selected household has lagged pipes *and* recycles its refuse. That is, find $P(L \cap R)$. Use the Addition Rule. Fill in the boxes with the given information and solve for $P(L \cap R)$.

$$P(L \cup R) = P(L) + P(R) - P(L \cap R)$$

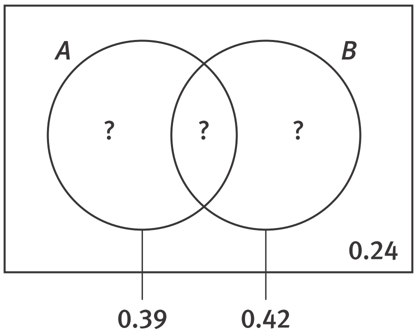
$$\boxed{} = \boxed{} + \boxed{} - \boxed{?}$$

c. What is the value of $P(L \cap R)$? Show your work.

Check Your Understanding

16. Reason quantitatively. For two intersecting sets, $P(C \cup D) = 0.85$. What is $P(C \cap D)$ if $P(C) = 0.7$ and $P(D) = 0.3$?

17. Calculate the three probabilities indicated by the question marks in the Venn diagram below.



My Notes

LESSON 27-1 PRACTICE

18. Dr. Zadok's Museum has a collection of cameras. If a camera is selected at random from the museum's collection, the probability that it is digital is 0.43 and the probability that it is a single lens reflex (SLR) camera is 0.51. The probability that the randomly selected camera is both digital and an SLR is 0.19.

a. Let the event that a camera is digital be D and the event that a camera is an SLR be S. Draw a Venn diagram to illustrate the information that you were given.

b. Suppose that a camera is selected at random from the museum's collection. Find the probability that it is

i. either digital or an SLR.

ii. digital but not an SLR.

iii. an SLR but is not digital.

iv. neither digital nor an SLR.

v. not digital.

19. Make sense of problems. In the collection of clocks at Dr. Zadok's Museum, 28% run on electrical power, 45% have alarms, and 54% either run on electrical power or have alarms. If a clock is selected at random, what is the probability that it runs on electrical power and has an alarm? Show your work.

20. Dr. Zadok's Museum has a cafeteria. If a customer is selected at random, the probability that the customer orders a beverage is 0.74, the probability that the customer orders a dessert is 0.66, and the probability that the customer orders neither a beverage nor a dessert is 0.07.

a. Draw a Venn diagram to represent this information.

b. You are given that the probability that a randomly selected customer orders neither a beverage nor a dessert is 0.07. What does this tell you about the probability that the customer orders either a beverage or a dessert?

c. Calculate the probability that a randomly selected customer orders a beverage and a dessert.

My Notes

Learning Targets:

- Learn the meaning of "mutually exclusive" events.
- Use Venn diagrams to represent mutually exclusive events.
- Use the Addition Rule to calculate the probability of mutually exclusive events.

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Marking the Text, Visualization, Think-Pair-Share, Debriefing

Mutually exclusive events are events that can't happen at the same time. They have no outcomes in common.

For example, suppose we have a cube with faces numbered 1 through 6. We will roll the cube once.

Let L = the event that the cube shows a number less than 3.

Let G = the event that the cube shows a number greater than 4.

On a roll of the cube, it is not possible that the result could be less than 3 and greater than 4. Therefore, L and G are mutually exclusive events.

MATH TERMS

Events are **mutually exclusive** if they have no outcomes in common.

1. **Reason abstractly.** A cube with faces numbered 1 through 6 will be rolled once.
   a. Let E be the event that the cube shows an even number and let F be the event that the cube shows a 5. Are E and F mutually exclusive events? Explain.

   b. Let E be the event that the cube shows an even number and let H be the event that the cube shows a number greater than 3. Are E and H mutually exclusive events? Explain.

My Notes

2. Sam selects one car at random, and notes the color of the car.

a. List five possible outcomes in this context.

b. Give two events (in this context) that are mutually exclusive.

Event A: The car is ________.

Event B: The car is ________.

Explain why your events are mutually exclusive.

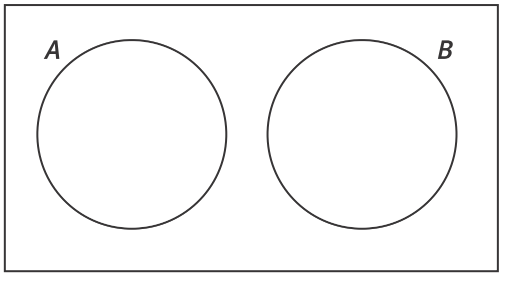
3. Suppose that when you randomly choose a car, you note both its color and its body style. Give two possible events that are <u>not</u> mutually exclusive.

Event A: The car is ________.

Event B: The car is ________.

Explain why your events are not mutually exclusive.

Since mutually exclusive events have no outcomes in common, they can be represented in a Venn diagram where the circles do not intersect.



4. If the events A and B are mutually exclusive, what is the value of $P(A \cap B)$?

Lesson 27-2
Adapting the Addition Rule for Mutually Exclusive Events

My Notes

5. The Addition Rule states that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Rewrite the Addition Rule for mutually exclusive events.

If the events A and B are mutually exclusive, then $P(A \cap B) = 0$. The Addition Rule for mutually exclusive events is as follows:

If the events A and B are mutually exclusive, then $P(A \cup B) = P(A) + P(B)$.

Check Your Understanding

6. Describe two events that are mutually exclusive.

7. Describe two events that are not mutually exclusive.

8. When a house is selected at random from Hector Street, the probability that it is a two-story house is 0.63 and the probability that it is a three-story house is 0.13. Let A be the event that a house has exactly two stories and let B be the event that a house has exactly three stories.

a. Are A and B mutually exclusive events? Explain.

b. Find the probability that a randomly selected house from Hector Street is a two- or three-story house.

9. Make sense of problems. When a student is selected at random from Annabel High School, the probability that the student plays soccer is 0.143 and the probability that the student takes a foreign language class is 0.682. Is it possible to use this information to find the probability that a randomly selected student plays soccer or takes a foreign language class? If so, explain how. If not, what extra information would enable you to answer the question?

10. Construct viable arguments. When would you use the Addition Rule $P(A \cup B) = P(A) + P(B)$?

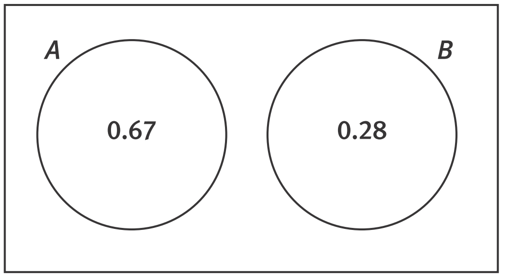
My Notes

LESSON 27-2 PRACTICE

11. **Construct viable arguments.** Determine if each of the following events are mutually exclusive or not mutually exclusive. Explain your reasoning.

- **a.** On a visit to Fire Island, see the lighthouse. See the historical mansion.
- **b.** Take a car to the shore. Take a train to the shore.
- **c.** On a fishing trip, catch bluefish, sea bass, and fluke.
- **d.** Attend a baseball game and attend a football game on the same Saturday afternoon.

12. Using the information given in the Venn diagram, calculate each probability.



- **a.** $P(A \cup B)$
- **b.** $P(A^C)$
- **c.** $P(B^C)$
- **d.** $P((A \cup B)^C)$
- **e.** $P(A \cap B)$

13. **Attend to precision.** At Annabel High School, 26.3% of the students are 9th graders and 22.8% of the students are 12th graders. If a student is selected at random from Annabel High School, what is the probability that the student is either a 9th grader or a 12th grader? Justify the method you use.

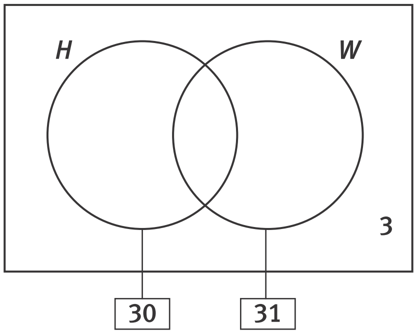
14. On average, 75% of the annual visitors to the lighthouse visit during the summer. Ten percent visit in the fall. The lighthouse is closed in the winter but open in the spring. Draw a Venn diagram that represents this information. Calculate the probability that a visitor selected at random from the visitor log last year visited the lighthouse in the spring or fall.

ACTIVITY 27 PRACTICE

Answer each item. Show your work.

Lesson 27-1

1. State the generalized Addition Rule.
2. There are 58 houses on Elm Avenue. Some of the houses have hardwood flooring and some do not. Some of the houses have wall-to-wall carpeting and some do not. The numbers of houses with various combinations of flooring are shown in the Venn diagram.



   a. Complete the Venn diagram.
   b. How many houses have hardwood flooring and carpeting?
   c. How many houses have hardwood flooring or carpeting?
3. When a house is selected at random from Hector Street, the probability that the house has gas-fired central heating is 0.25, the probability that the house has off-street parking is 0.76, and the probability that the house has both gas-fired central heating and off-street parking is 0.19. Find the probability that a randomly selected house has either gas-fired central heating or off-street parking.
4. When a non-commercial vehicle is selected at random from a city, the probability that it is an SUV is 0.28, the probability that it is a hybrid is 0.22, and the probability that it is neither of these is 0.54.
   a. Draw a Venn diagram to represent this information.
   b. Suppose that a noncommercial vehicle is selected at random from this city. Find the probability that it is:
      i. either an SUV or a hybrid.
      ii. a hybrid SUV.
      iii. a hybrid car that is not an SUV.
      iv. not a hybrid.

Lesson 27-2

5. Suppose that you have a set of 40 cards consisting of

   10 red cards labeled 1 through 10,

   10 blue cards labeled 1 through 10,

   10 green cards labeled 1 through 10, and

   10 yellow cards labeled 1 through 10.

   One card will be selected at random.

   a. Let R be the event that the card is red and let G be the event that the card is green. Are R and G mutually exclusive events? Explain.
   b. Let R be the event that the card is red and let T be the event that the card is a 10. Are R and T mutually exclusive events? Explain.
   c. Let L be the event that the number on the card is at least 6 and let E be the event that the card shows an even number. Are L and E mutually exclusive events? Explain.
6. State the Addition Rule for the probability of mutually exclusive events.

7. The class had to solve this problem for homework. "What is the probability of rolling at least a 4 or an odd number when rolling a number cube?" Mical wrote the following solution:

$$P(\text{at least } 4) = \frac{3}{6}$$

$$P(\text{odd number}) = \frac{3}{6}$$

$$P(\text{at least 4 or odd}) = \frac{3}{6} + \frac{3}{6} = \frac{6}{6}$$

Is Mical correct? Explain.

8. Colleen rolls a number cube and flips a coin. What is the probability that she will roll at least a 3 or flip heads?

A. $\frac{1}{6}$

B. $\frac{4}{6}$

C. $\frac{5}{6}$

D. $\frac{7}{6}$

9. At Annabel High School, 83% of the students take the bus and 6% drive themselves to school. If a student is selected at random from the school, what is the probability that the student either takes the bus or drives him/herself to school? Justify the method that you use.

MATHEMATICAL PRACTICES

Construct Viable Arguments and Critique the Reasoning of Others

10. The manager of an office building intends to replace the carpet in offices where the carpet is worn out and to replace the furniture in offices where the furniture is outdated. In order to find out how many offices need attention, the office manager plans to add the number of offices that need a new carpet to the number of offices that need new furniture. Is this correct? Explain.

Probability and the Addition Rule

DIANE'S BOOKS

1. Diane is organizing her collection of books. She has 91 novels in total, and has categorized them as shown in the table below.

	Hardcover	Paperback	Total
Detective Novels	12	20	
Romance Novels	8	28	
Other Novels	9	14	
Total			91

a. Complete the table by writing the totals in the empty cells.
b. Suppose that a book will be selected at random from these 91 novels. Find the probability that the book is:
  i. a detective novel.
  ii. a hardcover book.
  iii. a paperback detective novel.
  iv. a paperback or a detective novel.
  v. not a detective novel.
c. If Diane selects one of these novels at random, are the events "is a detective novel" and "is a hardcover" mutually exclusive? Explain.
d. If Diane selects one of these novels at random, are the events "is a detective novel" and "is a romance novel" mutually exclusive? Explain.

2. Diane also has a number of nonfiction books. Of those books, 28% are hardcover, 22% are reference books, and 13% are hardcover reference books. Diane will select a nonfiction book at random. Let the event that the selected book is a hardcover be *H* and the event that it is a reference book be *R*.
a. Draw a Venn diagram to represent this information. (Show the probabilities as decimals.)
b. Find the probability that the selected book is
  i. a hardcover *or* a reference book.
  ii. a hardcover that is not a reference book.
  iii. neither a hardcover nor a reference book.
c. Write each of the following using probability notation. Use the symbols $\cap$, $\cup$, and ^{C} as necessary. (Note: You do not have to find the values of the probabilities.)
  i. the probability that the selected book is a hardcover and a reference book
  ii. the probability that the selected book is a hardcover or a reference book
  iii. the probability that the selected book is not a reference book

Probability and the Addition Rule
DIANE'S BOOKS

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates the following characteristics:			
Mathematics Knowledge and Thinking (Items 1, 2)	• Clear and accurate understanding of determining probabilities using a table of values or a Venn diagram • Clear and accurate understanding of the term *mutually exclusive*	• A functional understanding of determining probabilities using a table of values or a Venn diagram • A functional understanding of the term *mutually exclusive*	• Partial understanding of determining probabilities using a table of values or a Venn diagram • Partial understanding of the term *mutually exclusive*	• Little or no understanding of determining probabilities using a table of values or a Venn diagram • Little or no understanding of the term *mutually exclusive*
Problem Solving (Items 1, 2b)	• An appropriate and efficient strategy that results in correct answers	• A strategy that results in mostly correct answers	• A strategy that results in some correct answers	• No clear strategy when solving problems
Mathematical Modeling / Representations (Items 2a, 2c)	• Clear and accurate understanding of using probabilities to create a Venn diagram • Clear and accurate understanding of writing probability statements using appropriate mathematical symbols	• Mostly accurate understanding of using probabilities to create a Venn diagram • Mostly accurate understanding of writing probability statements using appropriate mathematical symbols	• Partial understanding of using probabilities to create a Venn diagram • Partial understanding of writing probability statements using appropriate mathematical symbols	• Little or no understanding of using probabilities to create a Venn diagram • Little or no understanding of writing probability statements using appropriate mathematical symbols
Reasoning and Communication (Items 1c, 1d)	• Precise use of appropriate mathematics and language to explain why the statements are or are not mutually exclusive	• Mostly correct use of appropriate mathematics and language to explain why the statements are or are not mutually exclusive	• Misleading or confusing use of appropriate mathematics and language to explain why the statements are or are not mutually exclusive	• Incomplete or inaccurate use of appropriate mathematics and language to explain why the statements are or are not mutually exclusive

Dependent Events

Coco Wildlife Conservation Trust

Lesson 28-1 Understanding Conditional Probability

My Notes

Learning Targets:

- Understand the conditional probability of *A* given *B*.
- Determine conditional probabilities using two-way frequency tables and Venn diagrams.
- Interpret the answer in terms of the model.

SUGGESTED LEARNING STRATEGIES: Summarizing, Paraphrasing, Think Aloud, Create Representations, Visualization, Think-Pair-Share, Debriefing

1. There are 90 cranes that belong to the Coco Wildlife Conservation Trust. The cranes have been categorized by gender and type as shown in the two-way frequency table.

	Male	Female	Total
Gray-crowned	16	13	
White-naped	17	14	
Stanley	11	19	
Total			90

a. Complete the table by entering the other totals.

b. If a female crane is selected at random, what is the probability that it is a Stanley crane?

c. Calculate the probability that a crane is a Stanley crane given that it is female.

You found the probability that a crane is a Stanley crane *given that it is female*. This is a ***conditional probability***. You know that the crane is female. Then you can find the probability that it is a Stanley crane.

In probability notation you can show conditional probabilities using a vertical line. The probability that a crane is a Stanley crane given that it is female is written *P*(Stanley | female). If you denote the events by their initial letters, this conditional probability is written $P(S \mid F)$. $P(S \mid F)$ is read "the probability of *S* given *F*."

MATH TERMS

A **conditional probability** is the probability that an event occurs given that another event occurs.

WRITING MATH

Probability Notation

The probability of *A* given *B* is written $P(A \mid B)$.

My Notes

2. Write the following probabilities using probability notation.

a. the probability that a crane is male given that it is a gray-crowned crane

b. the probability that a crane is a gray-crowned crane given that it is male

c. the probability that a randomly selected white-naped crane is female

Check Your Understanding

3. The Coco Trust also has flamingos. The flamingos are categorized as shown in the table.

	Male	Female	Total
Chilean	16	13	
Greater	17	14	
Total			

a. Complete the table by entering the totals.

b. Suppose that a flamingo will be selected at random. The various possible events are denoted by their initial letters. Calculate the following probabilities to the nearest thousandth.

i. $P(M)$

ii. $P(M \mid C)$

iii. $P(C \mid M)$

c. If a female flamingo is selected at random, to the nearest thousandth, what is the probability that it is a greater flamingo? Use conditional probability notation to write your answer.

My Notes

4. As part of their math course, Kate and Rob were required to do a statistical study. They selected a random sample of 80 students from their school. They asked all the selected students their grade level (9, 10, 11, or 12) and whether or not they had $5 or more with them. The results are shown in the table.

	9th Grade	10th Grade	11th Grade	12th Grade
Less than $5	13	10	9	3
$5 or more	8	9	17	11
Total				

a. Complete the table by entering the totals.

b. **Attend to precision.** A student will be selected at random from those in the sample. Find the following probabilities to the nearest hundredth.

i. P($5 or more | 9th grade)

ii. P($5 or more | 10th grade)

iii. P($5 or more | 11th grade)

iv. P($5 or more | 12th grade)

c. Since the sample was selected randomly, the students in the sample are likely to be representative of the school as a whole. What do the answers to part b suggest to Kate and Rob about the amounts of money carried by students at the school?

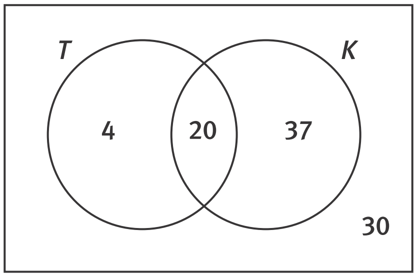
My Notes

Check Your Understanding

5. Suppose that a student is selected at random from your school. What would be the probabilities of each of the following events?
  a. the probability that the student plays the trumpet
  b. the probability that the student plays the trumpet given that the student is in the school band

6. Make sense of problems. Let T be the event that the student plays trumpet and let B be the event that the student is in the school band. Explain in words the meaning of each of the following probabilities.
  a. $P(T \cap B)$
  b. $P(T \mid B)$
  c. Are the probabilities in parts a and b the same? Explain. (*Hint:* Use numbers.)

7. Some of the Trust's animals are tortoises. Some are not. Some of the animals are of known gender. Some are not. Let the set of tortoises be T and the set of animals of known gender be K. The numbers of animals falling into these categories are shown.



  a. How many animals are there at this location?

  b. How many of these animals are tortoises?

  c. How many of these animals are of known gender?

  d. How many of the animals of known gender are tortoises?

My Notes

e. Find each of the following probabilities to the nearest thousandth. Use probability notation to write your answer.

i. Suppose an animal of known gender is selected at random. What is the probability that this animal is a tortoise?

ii. Calculate $P(K \mid T)$.

iii. Do $P(T \mid K)$ and $P(K \mid T)$ represent the same region on the Venn diagram? Are the probabilities equal? Explain.

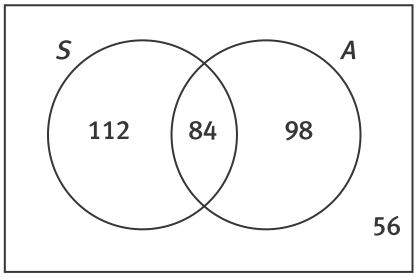
iv. Suppose a tortoise is selected at random. What is the probability that its gender is not known?

MATH TIP

The phrase *its gender is not known* means that you should look for the complement of K.

Check Your Understanding

8. Reason quantitatively. At Annabel High School, some students take Spanish and some do not. Some students take an art class, and some do not. Let S be the set of students who take Spanish and A be the set of students who take an art class. The numbers of students in various combinations of these classes are given in the Venn diagram.



a. If a student is selected at random from the school, what is the probability that the student takes Spanish?

b. Find $P(S \mid A)$.

c. Find $P(A \mid S)$.

d. Which of the answers to parts a and b is larger? Explain in your own words what this means.

e. Find $P(S \cap A \mid S^C)$.

My Notes

LESSON 28-1 PRACTICE

9. Recall the phone store. Some of the phones have caller ID. Some do not. Some have a speakerphone. Some do not. The numbers of phones with and without these features are shown in the table.

a. Write the totals in the table.

	Speakerphone	No Speakerphone	Total
Caller ID	18	2	
No Caller ID	16	12	
Total			

Let C be the event that a phone has caller ID and let S be the event that a phone has a speakerphone. Calculate each probability to the nearest thousandth.

b. If a phone is selected at random from those available at the store, what is the probability that it has a speakerphone?

c. If you are now told that a phone has caller ID, does this make it more or less likely that it has a speakerphone?

d. Find $P(C \mid S^C)$.

10. a. Draw a Venn diagram to illustrate the information in the table in Item 9.

b. If a phone is selected at random from those available at the store, what is the probability that it has caller ID?

c. If you are now told that a phone has a speakerphone, does this make it more or less likely that it has caller ID?

11. Model with mathematics. Recall the SpringBoard Superstar contest. This table shows four categories into which the contestants can be placed.

	Male	Female
Under 20	1	2
20 or Over	5	4

a. Suppose that a contestant will be selected at random. Find the following probabilities to the nearest thousandth.

i. $P(\text{female})$

ii. $P(\text{female} \mid \text{under } 20)$

iii. $P(\text{female} \mid 20 \text{ or over})$

b. Suppose you are a reporter for a newspaper. Write an article about the three probabilities you found in part a. Compare these probabilities.

My Notes

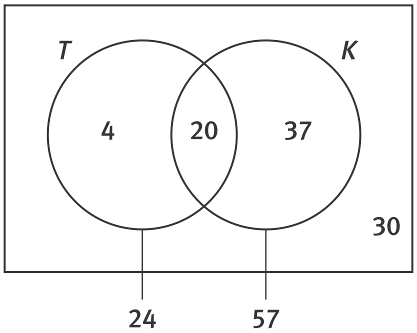
Learning Targets:

- Develop the conditional probability formula.
- Use conditional probability for everyday situations.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Predict and Confirm, Summarizing, Paraphrasing, Discussion Groups, Think-Pair-Share, Note Taking

You have calculated conditional probabilities using *counts*. You can develop a formula to find conditional probability given probabilities. Begin with a Venn diagram showing counts.

Recall the tortoises and animals of known gender in the Coco Wildlife Preserve. The relevant counts are given in this Venn diagram where T is the set of tortoises and K is the set of animals of known gender.



1. Write each probability as a fraction.

a. $P(T \mid K)$ **b.** $P(T \cap K)$ **c.** $P(K)$

d. Calculate $\dfrac{P(T \cap K)}{P(K)}$. What do you notice?

MATH TIP

The favorable outcomes of $P(T \mid K)$ and $P(T \cap K)$ are the same but the total possible outcomes are different. So the probabilities will be different.

To generalize, conditional probability can be found using this formula.

For any two events A and B,

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

READING MATH

The formula for conditional probability is read:

"The probability of event A given event B is equal to the probability of event A and event B divided by the probability of event B."

My Notes

2. Do you think the likelihood of the animal being a tortoise *depends* on whether the gender of the animal is known? Explain.

3. Make use of structure. Suppose A and B are ***dependent events***. Write a formula for $P(A \text{ and } B)$ by rearranging the conditional probability formula.

MATH TERMS

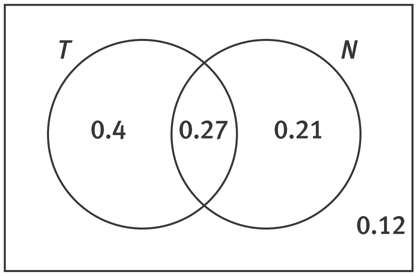
Two events are said to be **dependent** if the result of the second event is affected by the result of the first event.

The reasoning above leads to the **Multiplication Rule** for dependent events:

> If A and B are dependent events, then
>
> $$P(A \text{ and } B) = P(B) \cdot P(A \mid B)$$

Check Your Understanding

4. The Coco Foundation takes care of a number of ***primates***. When a primate is selected at random, let T be the event that it is a tamarin and let N be the event that it is at least nine years old. Some of the relevant probabilities are shown in the Venn diagram.



Write your answer to each question using probability notation.

a. What is the probability that a randomly selected primate is a tamarin *and* is at least nine years old?

b. What is the probability that a randomly selected primate is at least nine years old?

c. What is the probability that a randomly selected primate is a tamarin given that it is at least nine years old? Are the events dependent? Use the probability formula.

CONNECT TO BIOLOGY

The ***primates*** are an order of placental mammals containing humans, apes, and monkeys.

My Notes

LESSON 28-2 PRACTICE

5. Of the Coco Foundation's lemurs, 39% are female, 54% are at least eight years old, and 21% are females who are at least eight years old. When a lemur is selected at random, let the event that it is female be F and let the event that it is at least eight years old be E.

a. Write the values of $P(F)$, $P(E)$, and $P(F \cap E)$.

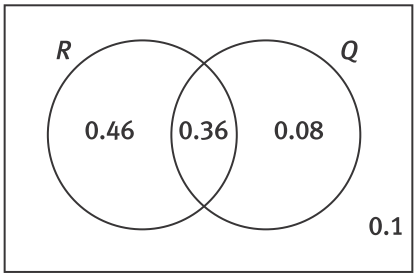
b. **Model with mathematics.** Illustrate the information you have been given using a Venn diagram. Write decimals in your diagram, not percentages.

c. When a lemur is selected at random, what is the probability that it is female given that it is at least eight years old?

d. When a lemur is selected at random, what is the probability that it is at least eight years old given that it is female?

e. When a lemur is selected at random, what is the probability that it is either a female or at least eight years old? (*Hint:* You may use the Addition Rule if you wish.)

6. Recall the information you were given about a local baseball team. R is the event that the player is a right-handed thrower and Q is the event that the player is a pitcher. The probabilities of various combinations of these events are shown in the Venn diagram.



A player is selected at random.

a. Calculate $P(R \mid Q)$.

b. **Construct viable arguments.** What is the probability that the player is a pitcher given that he is a right-handed thrower? Explain the difference between this probability and the probability that a randomly selected player is a pitcher and a right-handed thrower.

c. If a right-handed thrower is selected, what is the probability that he is not a pitcher? Are these events dependent?

d. If a player who is not a pitcher is selected, what is the probability that he is a right-handed thrower? How would this probability be written using conditional probability notation?

My Notes

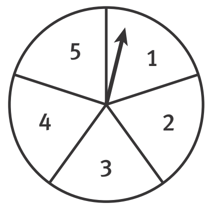
Learning Targets:

- Use tree diagrams to determine conditional probabilities.
- Apply the general Multiplication Rule.

SUGGESTED LEARNING STRATEGIES: Role Play, Shared Reading, Visualization, Discussion Groups, Debriefing, Close Reading, Marking the Text, Sharing and Responding

When problems involve conditional probabilities, it is often beneficial to use tree diagrams to illustrate the problem. A tree diagram can help you to find the conditional probabilities.

Anson is concerned that video games are interfering with his connecting with his friends. He has decided that, every day, when he's finished his homework, he will spin a spinner to decide if he should play video games or connect with his friends. The five outcomes on the spinner are equally likely.

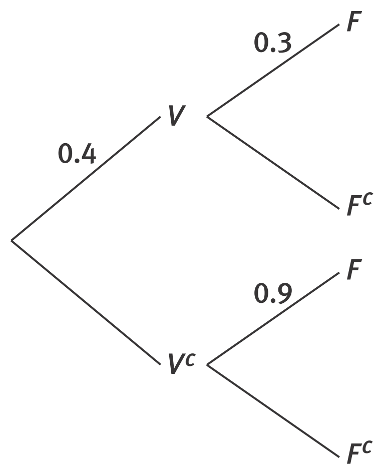


He decided that if the result of the spin is a 4 or a 5, he will play video games; otherwise, he will not.

1. **Attend to precision.** What is the probability that Anson will play video games? Write your answer as a decimal. Use probability notation.

Anson found that if he plays video games, then the probability that he will connect with his friends is 0.3. If he doesn't play video games, the probability he'll connect with his friends is 0.9. You can use a ***tree diagram*** to illustrate this information.

Let V be the event that Anson plays video games, F be the event that he connects with his friends, V^C be the event that he doesn't play video games, and F^C be the event that he doesn't connect with his friends.



My Notes

MATH TERMS

A **tree diagram** is a graphic organizer for listing the possible outcomes of an experiment.

2. Find each probability. Write the answer using probability notation. Then write the probability in the appropriate place on the tree diagram.

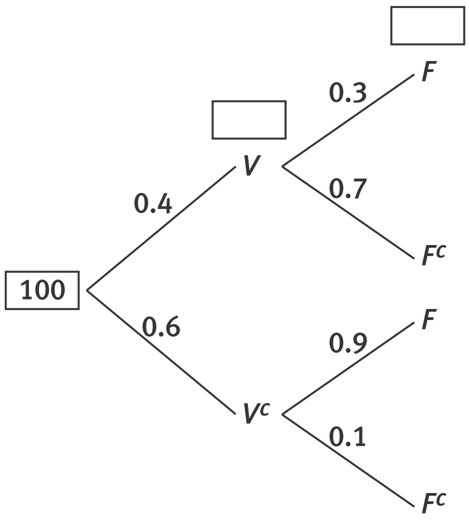
a. What is the probability that Anson does not play video games?

b. If Anson plays video games, what is the probability that he will not connect with his friends?

c. If Anson doesn't play video games, what is the probability that he will not connect with his friends?

My Notes

Now imagine 100 days, as shown in the tree diagram.



3. On how many of these 100 days would you expect Anson to play video games? Write this number in the box above "*V*."

4. On how many days would you expect Anson to play video games *and* connect with his friends? Write this number in the box above "*F*."

5. What is the probability of the event in Item 4?

6. What two probabilities can you apply the Multiplication Rule to in order to find the probability of the event in Item 4?

7. Find each of the following probabilities using the Multiplication Rule.
a. Anson plays video games *and* does not connect with his friends.

b. Anson does not play video games *and* connects with friends.

c. Anson does not play video games *and* does not connect with his friends.

You can work out the probability that Anson connects with his friends. In order to connect with his friends, Anson has to either play video games and connect with his friends *or* not play video games and connect with his friends.

8. What is the probability that Anson connects with his friends? Use the Addition Rule for mutually exclusive events.

9. Notice the probability 0.3 in the tree diagram. This is a conditional probability. It is the probability of F given V. What is the probability of F given V^C? Use probability notation to write your answer.

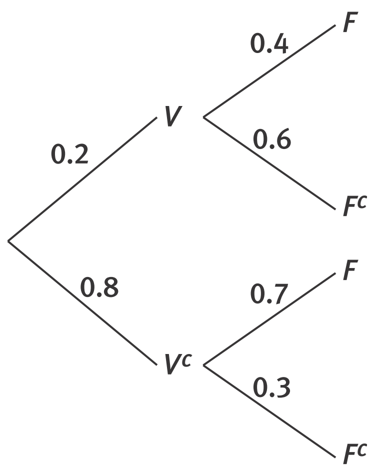
My Notes

MATH TIP

These two events are mutually exclusive. You cannot *play* video games and *not play* video games at the same time.

Check Your Understanding

Bernadette also plays video games or does not, and connects with her friends or does not. The probabilities are shown in the diagram.



10. Find the probability that Bernadette plays video games and connects with her friends.

11. To find the probability that Bernadette connects with her friends, she has to either play video games and connect with her friends *or* not play video games and connect with her friends. What is the probability that she connects with her friends?

12. Why is it possible to add to find the probability in Item 11?

13. Write the probability that Bernadette connects with her friends given she does not play video games. Use probability notation to write your answer.

My Notes

Consider another probability experiment involving dependent events.

In this experiment, the two events occur without replacement.

14. Reason quantitatively. Ms. Troy's math class consists of 18 girls and 12 boys. Ms. Troy is planning to choose two students at random to record the results of a survey on a poster. To do this, Ms. Troy will write the names of the students on identical slips of paper and will place the slips in a hat. She will randomly pick one slip from the hat, and then, without replacing the first slip, will randomly pick a second slip.

a. What is the probability that the first student selected is a girl?

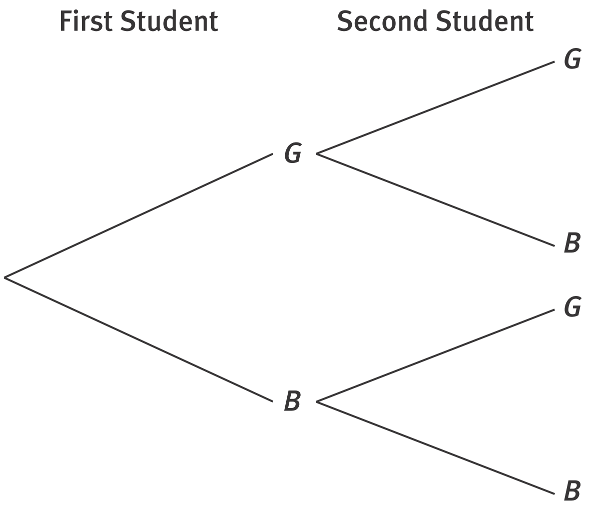
MATH TIP

Since the first slip of paper is not replaced, the sample space of the second event will be changed.

b. If the first student selected is a girl, how many slips of paper for girls are now left in the hat? How many slips of paper are left in the hat? So, if the first student is a girl, what is the probability that the second student will also be a girl?

c. If the first student selected is a boy, how many slips of paper for girls are now left in the hat? How many slips of paper are left in the hat? So, if the first student is a boy, what is the probability that the second student will be a girl?

d. Write the relevant probabilities on the tree diagram below.



e. Find the probability that both students chosen are girls. Use probability notation to write your answer.

f. Find the probability that a girl and a boy are chosen to represent the class. Use probability notation to write your answer.

My Notes

MATH TIP

Follow the branches of the tree diagram from *G* to *G* and multiply.

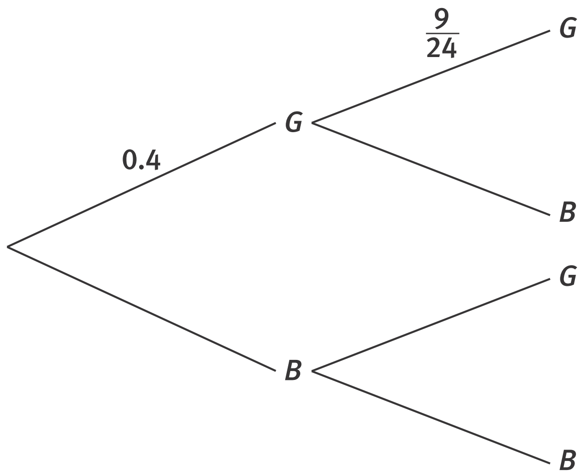
MATH TIP

The probability that a girl and a boy are chosen can occur if the first student chosen is a girl *or* if the first student chosen is a boy.

Check Your Understanding

15. Mr. Santaella's class consists of 10 girls and 15 boys. He will choose two students at random to be hall monitors.

a. Complete the tree diagram.



b. What is the probability that the students chosen will be:

i. two girls?
ii. a girl and a boy?
iii. two boys?

c. Why is this a probability experiment without replacement?

16. Owen has a bag of marbles with 3 red, 5 white, and 7 green marbles. He picks a marble from the bag without looking. He picks a second marble without putting the first marble back in the bag.

a. Draw a tree diagram to represent this experiment.

b. What is the probability that Owen will pick:

i. two white marbles?
ii. a red and a white marble?
iii. two green marbles?

My Notes

LESSON 28-3 PRACTICE

17. Make sense of problems. Gerry is developing a small ski resort. He has worked out that if there's a lot of snow this coming winter, the probability that the resort will make a profit is 0.9. However, if there's not a lot of snow this winter, then the probability that the resort will make a profit is 0.6. According to information Gerry found on the Internet, the probability that there will be a lot of snow this winter is 0.7.

a. Draw a tree diagram to represent these possible outcomes. Include all the relevant probabilities.

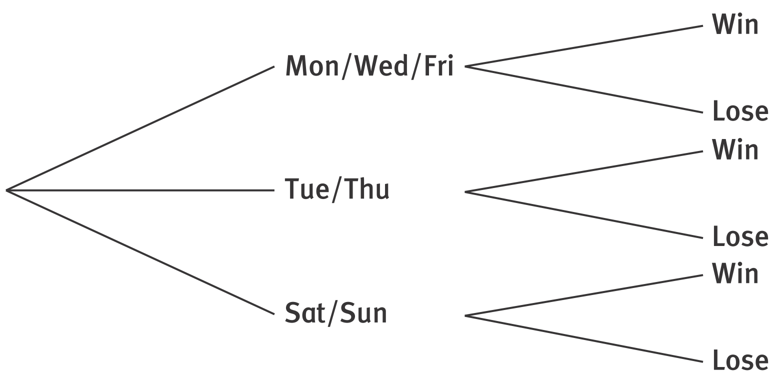
b. What is the probability that there will be a lot of snow and the resort will make a profit?

c. What is the probability that there will not be a lot of snow and the resort will not make a profit?

d. What is the probability that the resort will make a profit?

18. Coach Caroline's softball team has made the playoffs. The first game has not been scheduled yet, but Caroline knows that the first game is equally likely to be on any one of the seven days of the week. Caroline has different players available on different days of the week, and she has estimated that if the game is on a Monday, a Wednesday, or a Friday, the probability that her team will win is 0.6. If the game is on a Tuesday or a Thursday, the probability that her team will win is 0.5. If the game is on a Saturday or a Sunday, the probability that her team will win is 0.45.

a. Complete the diagram below by writing the appropriate probabilities above the lines of the tree diagram.



b. What is the probability that Coach Caroline's team will win?

ACTIVITY 28 PRACTICE

Answer each item. Show your work.

Lesson 28-1

1. Recall the high school current affairs club that has members in all four grades (9, 10, 11, and 12) and of both genders (male and female). The numbers of students in the various grade/gender categories are given in the table.

	Male	Female	Total
9	8	6	
10	4	2	
11	3	7	
12	9	9	
Total			

   a. Complete the table.
   b. If a student is selected at random from the club's female members, what is the probability that she is a 9th grader?
   c. When selecting a club member at random, what is the probability that the student is female given that the student is a 9th grader?
   d. When selecting a club member at random, what is $P(\text{11th grade} \mid \text{male})$?

2. A random sample of 100 people was selected from the adults at a mall. Each person selected was asked whether he or she had used a mobile phone that day; people's ages were also noted. The results of the survey are summarized below.

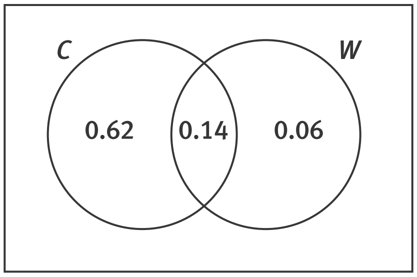
	Age		
	18–35	36–55	56 or over
Had Used a Mobile Phone	34	27	10
Had Not Used a Mobile Phone	3	11	15
Total			

   a. Complete the table.
   b. Suppose that a person will be selected at random from those included in the sample. Find the following probabilities to two decimal places.
      i. $P(\text{used a mobile phone} \mid 18\text{–}35)$
      ii. $P(\text{used a mobile phone} \mid 36\text{–}55)$
      iii. $P(\text{used a mobile phone} \mid 56 \text{ or over})$
   c. Since the sample was selected randomly from the adults at the mall, the results of the survey are likely to represent the population of all the adults at the mall. What do your answers in part b suggest about this population?
   d. Do you think it would be reasonable to use the sample result to make conclusions about all the adults in the town where the mall is located? Explain.

3. Suppose that you will select an American adult at random. Let S be the event that the person visited a supermarket last Wednesday, and let F be the event that the person is female.
   a. Roughly, what do you think would be the value of $P(S)$? (Just make a rough guess! Any answer that is in any way reasonable will be accepted.)
   b. Now think about the probability $P(S \cap F)$. Would this probability be larger or smaller than your answer to part a? Explain.
   c. Now think about the probability $P(S \mid F)$. Do you think this probability would be larger or smaller than your answer to part b? Explain.

Lesson 28-2

4. In an earlier exercise, a vehicle was to be selected at random from a city. Let the event that the vehicle is a car be C and the event that the vehicle is white be W. Some of the relevant probabilities are given in the Venn diagram.



   a. Calculate the value of the probability that has been omitted from the Venn diagram, and write that probability in the appropriate place.
   b. Calculate $P(C \mid W)$. Are these events dependent?
   c. What proportion of cars in this city is white?
   d. If a non-white vehicle is selected at random, what is the probability that it is a car?

Lesson 28-3

5. Janet is keeping her eye on a stock market index. If the index rises today, then the probability that she will make an investment tomorrow is 0.84. However, if the index does not rise today, then the probability that Janet will make an investment tomorrow is 0.58. This particular index rises on 70% of days.
   a. Draw a tree diagram to represent this information. Complete the tree diagram by writing all the relevant probabilities next to the branches. Use decimals, not percentages, for the probabilities.
   b. What is the probability that the index rises but Janet does not make an investment tomorrow? Show your work. Use probability notation.
   c. What is the probability that Janet makes an investment tomorrow? Show your work. Use probability notation.

MATHEMATICAL PRACTICES
Reason Abstractly and Quantitatively

6. You spend your life making decisions. Though you may not realize it, many of your decisions are based on conditional probabilities. For example, a taxi driver thinks to himself, "If I take the interstate, it's likely that I'll get stuck in traffic, but if I take the back streets, it's much less likely that I'll get stuck in traffic. Therefore, I'll take the back streets." Describe a recent decision you've made that was based on conditional probability.

Independent Events

ACTIVITY 29

The Caribou, the Bear, and the Tyrannosaurus

Lesson 29-1 The Multiplication Rule

My Notes

Learning Targets:

- Understand when two events are independent.
- Use the Multiplication Rule to determine if two events are independent.
- Understand independent and dependent events in real-world situations.

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Interactive Word Wall, Close Reading, Think Aloud, Shared Reading, Create Representations, Visualization, Think-Pair-Share, Debriefing

In the previous lesson, you learned the Multiplication Rule for dependent events. How might this rule change if two events are not dependent?

1. Suppose we have a set of 40 cards consisting of blue, green, red, and yellow cards. Each color has ten cards labeled 1 through 10, as indicated in the chart below.

Blue:	1	2	3	4	5	6	7	8	9	10
Green:	1	2	3	4	5	6	7	8	9	10
Red:	1	2	3	4	5	6	7	8	9	10
Yellow:	1	2	3	4	5	6	7	8	9	10

If a card selected at random is an 8, a 9, or a 10, you will win a prize.

a. What is the probability that you will win a prize?

b. Construct viable arguments. Suppose someone selects a card for you and tells you that the card is blue, but doesn't tell you what number is on the card. Does this change the probability that you will win a prize? Show the calculation that leads you to your answer.

c. Using probability notation, the probability you found in part a is written as P(prize). How would you use probability notation to write the probability in part b?

The fact that $P(\text{prize} \mid \text{blue})$ and P(prize) are equal tells us that the events "winning a prize" and "picking a blue card" are ***independent***. Knowing that the card is blue does not change the probability that you win a prize. If two events are independent, the following is true.

> Two events A and B are **independent** if $P(A \mid B) = P(A)$ and $P(B \mid A) = P(B)$.

MATH TERMS

Independent Events
Two events are **independent** if the result of the second event is not affected by the result of the first event.

My Notes

We use this relationship to determine if two events are independent.

2. Suppose that you roll a cube with faces numbered 1 through 6.
Let E be the event that the cube shows an even number.
Let F be the event that the cube shows a number greater than 2.
Let G be the event that the cube shows a number greater than 3.

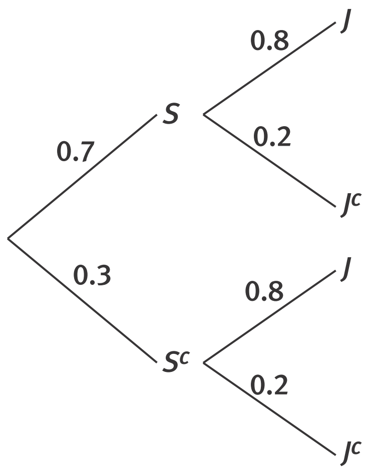
a. Find $P(E)$.

b. Find $P(E \mid F)$. Are E and F independent events?

c. Find $P(E \mid G)$. Are E and G independent events?

Sometimes you are only given the probabilities and you would like to determine if the events are independent or not. Based on the situation, you might think that the events are dependent. You need to do an analysis of the data to be sure.

3. Let the event that it is sunny tomorrow be S and let the event that Emily will go jogging be J. The probabilities are shown.



a. Find $P(J)$. Show your work.

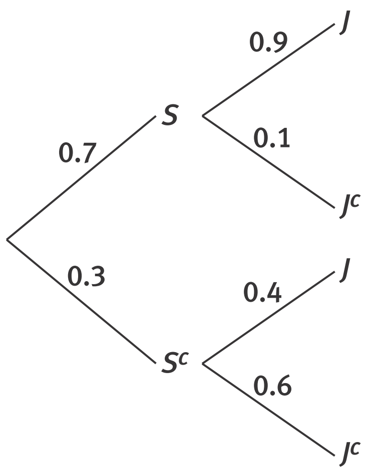
b. Find $P(J \mid S)$ and $P(J \mid S^C)$. Are J and S independent events? Are J and S^C independent events?

c. **Reason abstractly.** What conclusion can you draw regarding the probability of Emily jogging? Explain.

My Notes

Now let's look at the probabilities of another jogger.

4. Let the event that it is sunny tomorrow be S and let the event that Chaya will go jogging be J. Suppose that the probabilities associated with these events are as shown in this tree diagram.



a. Find $P(J)$.

b. Find $P(J \mid S)$. Are J and S independent events?

c. Is Chaya more or less likely than Emily to jog tomorrow? Explain.

CONNECT TO BIOLOGY

Biologists use this technique to interpret the results of clinical trials. They can determine if certain events affect the trial.

Check Your Understanding

5. A question in the previous activity told us about the Coco Trust's cranes. The cranes are categorized as shown in the table below.

	Male	Female	Total
Gray-crowned	16	13	29
White-naped	17	14	31
Stanley	11	19	30
Total	44	56	90

Suppose that a crane is selected at random. Find $P(\text{male})$ and $P(\text{male} \mid \text{Stanley})$. Are the events "is male" and "is a Stanley crane" in this population independent? Explain briefly.

My Notes

6. In an earlier exercise, you were told that when a house is selected at random from Hector Street, the probability that the house has gas-fired central heating is 0.25, the probability that the house has off-street parking is 0.76, and the probability that the house has both gas-fired central heating and off-street parking is 0.19. Let G be the event that a house has gas-fired central heating and F be the event that it has off-street parking.
   a. **Model with mathematics.** Draw a Venn diagram to represent this information.
   b. Are the events G and F independent? Show the calculations that lead you to your conclusion.
7. In the country of Millipotamia, 68% of the population do strenuous exercise, 50% walk for pleasure, and 21% do neither of these two things.
   a. When a person is selected at random from Millipotamia, let E be the event that the person engages in strenuous exercise and W be the event that the person walks for pleasure. Draw a Venn diagram to illustrate the information you've been given.
   b. Find the probability that a randomly selected person from Millipotamia engages in strenuous exercise and walks for pleasure. (*Hint:* The Addition Rule could be used to answer this question.)
   c. Are the events E and W independent? Explain.

In the first part of this lesson, we showed that for two independent events,

$$P(A \mid B) = P(A).$$

In a previous activity we showed that for any two events A and B,

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}.$$

Substitute $P(A)$ for $P(A \mid B)$ and rewrite the equation:

$$\frac{P(A \cap B)}{P(B)} = P(A)$$

Next, multiply both sides of the equation by $P(B)$. We see that this is equivalent to saying that

$$P(A \cap B) = P(A) \cdot P(B).$$

Compare this equation with the Multiplication Rule for dependent events from Lesson 28-3: $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$. Notice that the equations are equivalent if $P(B)$ is the same as $P(B \mid A)$, which would mean A and B are independent events.

Lesson 29-1
The Multiplication Rule

My Notes

Now you know the following:

The events A and B are independent if and only if $P(A \cap B) = P(A) \cdot P(B)$.

So for independent events, in order to find the probability that both events will happen, you multiply the probabilities of the events. That is,

if A and B are independent events, then $P(A \text{ and } B) = P(A) \cdot P(B)$.

Note that this form of the Multiplication Rule tells us two things:
First, if A and B are independent events, then $P(A \cap B) = P(A) \cdot P(B)$.
Second, if $P(A \cap B) = P(A) \cdot P(B)$, then A and B are independent events.

8. Recall the information about the houses on Hector Street, $P(G) = 0.25$, $P(F) = 0.76$, and $P(G \cap F) = 0.19$, and use the formula above to show that G and F are independent events.

MATH TIP

Do not confuse two events being *independent* with their being *mutually exclusive*.

If two events are independent, then

$$P(A \cap B) = P(A) \cdot P(B).$$

If two events are mutually exclusive, then

$$P(A \cap B) = 0.$$

9. Suppose that you will flip a coin and that you will roll a cube with faces numbered 1 through 6.

a. **Make sense of problems.** Explain briefly in words how you know that the events “the coin lands heads up” and “the cube shows a 6” are independent.

b. What is the probability that the coin will land heads up *and* the coin will show a six? Show your work.

10. When a student is selected at random from Annabel High School, let G be the event that the student’s average grade is over 90 and let S be the event that the student lives in the southern part of town. You are given that $P(G) = 0.29$, $P(S) = 0.42$, and that G and S are independent events.

a. When a student is selected at random from Annabel High School, what is the probability that the student has an average grade over 90 and lives in the southern part of town?

MATH TIP

Some ways of finding out whether A and B are independent events are to check whether

- $P(A \mid B) = P(A)$,
- $P(B \mid A) = P(B)$,
- $P(A \mid B) = P(A \mid B^C)$, and
- $P(A \cap B) = P(A) \cdot P(B)$.

b. When a student is selected at random from Annabel High School, what is the probability that the student has an average grade over 90 *or* lives in the southern part of town? (Note: The Addition Rule can be used to answer this question.)

My Notes

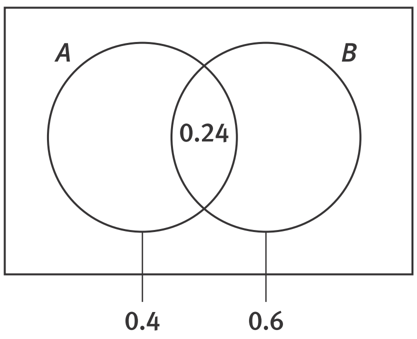
Check Your Understanding

11. Suppose that you roll a cube with faces numbered 1 through 6. Let A be the event that the result for the cube is a 5, and let B be the event that the result is a 3.

a. Explain in words how you know that A and B are independent events.

b. What is the probability that event A is a 5 and event B is a 3?

12. Construct viable arguments. Suppose that the events A and B have probabilities as shown in the Venn diagram below. Are A and B independent events? Explain.



LESSON 29-1 PRACTICE

13. Kathy is still eating Crisp breakfast cereal, where each box contains a plastic monkey, elephant, caribou, or bear, and where these four animals appear with equal probability. Brian is eating Wheats breakfast cereal. Each box of Wheats contains one of five different plastic dinosaurs: Tyrannosaurus, Stegosaurus, Argentinosaurus, Giganotosaurus, and Iguanodon, and these five dinosaurs appear with equal probability. Each of the kids has one new box of cereal.

a. Let C be the event that Kathy gets a caribou and let T be the event that Brian gets a Tyrannosaurus. Explain in words how you know that C and T are independent events.

b. What is the probability that Kathy gets a caribou and Brian gets a Tyrannosaurus?

c. What is the probability that Kathy gets a bear and Brian gets a Giganotosaurus?

d. Remember that when two events are mutually exclusive you can find the probability that one event happens or the other event happens by adding the probabilities of the two events. What is the probability that Kathy gets a caribou and Brian gets a Tyrannosaurus *or* Kathy gets a bear and Brian gets a Giganotosaurus?

My Notes

14. Reason quantitatively. Kathy has two new boxes of Crisp.
- **a.** What is the probability that both boxes contain caribous?
- **b.** What is the probability that the first box she opens contains a caribou and the second box she opens contains a bear?
- **c.** To get a caribou and a bear, Kathy can either get a caribou in the first box and a bear in the second box *or* she can get a bear in the first box and a caribou in the second box. What is the probability that Kathy gets a caribou and a bear?

15. Suppose that Kathy still doesn't have a caribou, and she really wants one. So her mother buys three boxes of Crisp, and Kathy opens the boxes one by one.
- **a.** What is the probability that Kathy gets three caribous?
- **b.** What is the probability that Kathy gets three bears?
- **c.** What is the probability that Kathy gets three caribous or three bears?
- **d.** What is the probability that Kathy gets three of the same type of animal?
- **e.** What will be the probability that Kathy doesn't get any caribous in the three boxes?

16. If Brian opens three boxes of Wheats, what is the probability that he gets:
- **a.** three Tyrannosauruses?
- **b.** no Tyrannosaurus?

17. If Brian opens two boxes of Wheats, what is the probability that he gets exactly one Tyrannosaurus? (*Hint:* The answer to this item is *not* 0.16.)

18. A survey found that 65 percent of Americans get enough exercise to meet a certain physical activity guideline. Suppose that two Americans will be selected at random. According to this guideline, find the probability that:
- **a.** they both get enough exercise.
- **b.** neither of them gets enough exercise.
- **c.** exactly one of them gets enough exercise.

19. Suppose that you have a red cube with faces numbered 1 through 6 and a blue cube with faces numbered 1 through 6. You roll both of the cubes. Find the probability that your total score is:
- **a.** 12.
- **b.** 11.
- **c.** 10.

My Notes

Learning Targets:

- Determine the probability of an event involving area.
- Understand independent and dependent events in geometric situations.

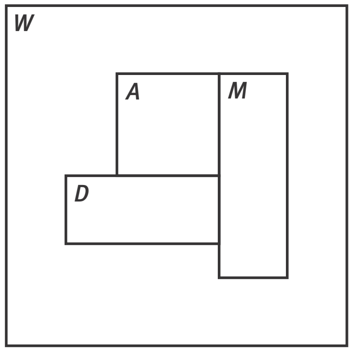
SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Visualization, Discussion Groups, Debriefing

Kathy and Brian built a game to play with the toys they collected. The board for their game is shown below. The board is divided into four sections that represent the approximate areas of the landform covering Earth's surface. A is arable land, suitable for growing crops; M is mountainous and/or snow-covered land; D is desert; and W is water. The goal is to be the first player to land in each section.

1. The outer square measures 10 in. by 10 in. Square A measures 3 in. by 3 in. Rectangle M measures 2 in. by 6 in. Rectangle D measures 2 in. by 4.5 in. Find the area of each section.

a. A **b.** M

c. D **d.** W



2. Kathy and Brian play their game by tossing their plastic toys onto the game board. Each toss must result in a toy landing in one of the areas. If it is partially in one area, the toss is repeated.

a. **Make sense of problems.** Kathy decides that when a player tosses a toy, it will be removed from the board before the same player tosses the second toy. Explain why these two events are independent events.

b. Kathy played first. Her first toy landed on D and the second on M. What is $P(D \cap M)$?

c. To land on D and M, Kathy could land on D and then M, or M and then D. What is the probability that Kathy lands on D and M?

MATH TIP

In part c, you are finding the sum of two equal probabilities, because $D \cap M = D \cap M$. The order of the independent events does not matter.

My Notes

3. Brian thought the game would be more interesting if the second toss must result in landing in a different section than the first toss. That is, if a toy landed in D on the first toss, then on the second toss the toy cannot land in D to be counted.

a. **Make sense of problems.** Using Brian's rules, is the second toss independent of the first toss?

b. On Brian's first toss, he lands in W. What is the total area of the sections that he can land in on his second toss for his second toss to count?

c. Given that Brian lands in W on his first toss, what is the probability he lands in D on his second toss?

d. What is the probability that Brian's toys land in W on the first toss, and in D on the second toss, using Brian's rules?

e. What is the probability that Brian's toys land in W on the first toss, and in D on the second toss, using Kathy's rules?

f. What is the probability that Brian's two tosses land in W and D based on his rules? Round to the nearest thousandth.

My Notes

Check Your Understanding

Refer to the game board from Item 1.

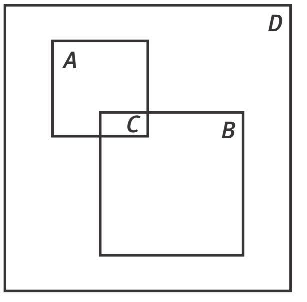
4. Find $P(M \cap A)$ using Kathy's rules.

5. Find $P(M \cap A)$ using Brian's rules. Round to the nearest thousandth.

6. **Construct viable arguments.** Explain why the results are different in Items 4 and 5.

LESSON 29-2 PRACTICE

The dartboard at Square Corners Teen Club is shown at the right. Square D is 12 in. by 12 in. Square A is 4 in. by 4 in. Square B is 6 in. by 6 in. Rectangle C is 1 in. by 2 in. Assume each dart thrown hits the dartboard.



7. Find the area of each of the following.
a. A **b.** B
c. C **d.** D

8. Find each probability for a dart landing in the given sections. Round to the nearest thousandth, if necessary.
a. $P(A)$ **b.** $P(B)$
c. $P(C)$ **d.** $P(D)$

9. A dart is thrown at the board, then is removed, and a second dart is thrown. Find each probability to the nearest thousandth.
a. $P(A \cap B)$ **b.** $P(B \cap B)$
c. $P(C \cap D)$ **d.** $P(D \cap A)$

10. To make the dart game more challenging, two players decide that if a dart lands in one section, then the dart is left in place and the second dart thrown must land in another section to be counted. For this game, find each probability.
a. $P(A \cap B \mid A)$ **b.** $P(C \cap D \mid C)$ **c.** $P(D \cap A \mid D)$

11. **Make sense of problems.** Consider the games described in Items 9 and 10. If two darts are thrown and land in D and then A, which game would result in a higher probability? Explain.

ACTIVITY 29 PRACTICE

Answer each item. Show your work.

Lesson 29-1

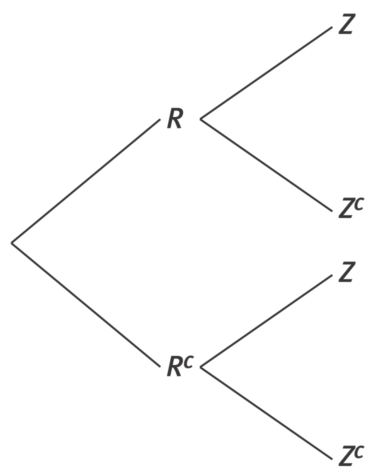
1. When a student is selected at random from Annabel High School, the probability that the student plays soccer is 0.18, the probability that the student plays basketball is 0.32, and the probability that the student plays at least one of these sports is 0.42.
   a. Find the probability that a student selected at random from Annabel High School plays soccer, plays basketball, or plays soccer and basketball.
   b. Determine whether *S* and *B* are independent events. Explain.

2. The zoo's penguins are categorized as shown in this table.

	Known to Be Male	Known to Be Female	Unknown Gender	Total
Northern Rockhopper Penguin	15	12	5	
Magellanic Penguin	18	15	7	
Total				

   a. Copy the table above, and complete it by entering the totals.
   b. Suppose that a penguin will be selected at random.
      i. Find the probability that the selected penguin is known to be female.
      ii. Find the probability that the selected penguin is known to be female given that it is a northern rockhopper penguin.
      iii. Are the events "is known to be female" and "is a northern rockhopper penguin" independent? Explain using your answers to parts b(i) and b(ii).

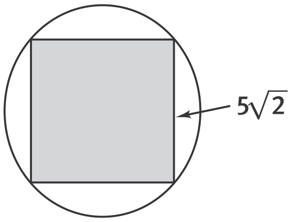
3. Dinah, one of the ducks at the zoo, is sick. The probability that Dinah will recover by next Thursday is 0.8. If Dinah recovers by next Thursday, the probability that Mr. Lee will go to the National Zoological Conference is 0.9. If Dinah doesn't recover by next Thursday, the probability that Mr. Lee will go to the conference is 0.4. In the tree diagram below, *R* denotes the event that Dinah recovers and *Z* denotes the event that Mr. Lee will go to the zoological conference.
   a. Write the appropriate probabilities next to the six branches.



   b. Find the probability that Dinah recovers but Mr. Lee doesn't go to the conference.
   c. Find the probability that Mr. Lee goes to the conference.
   d. Are *Z* and *R* independent events? Explain.

Lesson 29-2

4. The shaded region is a square with side length of $5\sqrt{2}$ units on a circular dartboard. Melanie and Jayne play a game using the dartboard. They take turns throwing their own darts at the board. The gray section is worth 5 points, and the white sections are worth 10 points. Assume the darts are randomly thrown at the board, and land on the board.



a. Find the probability that a single dart scores 5 points, and the probability that it scores 10 points.
b. For two thrown darts, find P(gray and white).
c. For two thrown darts, find P(gray and gray).
d. Do you think the probability of scoring more points than the opponent is the same for each player? Explain.

5. Suppose now that only the first dart to land in a white section scores points. If the next dart lands in the same white section, it scores no points.
a. For two thrown darts, find P(white and white).
b. For two thrown darts, find P(gray and white).
c. Do you think the probability of scoring more points than the opponent is the same for each player? Explain.

6. A skateboard company decides to build a skateboard park for a randomly selected state among the 48 states in the contiguous United States.
a. Devise a method that does not use geometric probability to select the state. Find P(Texas) and P(Rhode Island).
b. The president of the company decides to throw a dart at random at a map of the United States. Will the resulting geometric probabilities agree with your answers from part a? Explain.
c. Suppose the company decides to select two different states in which to build a skateboard park. Find P(California and Maine) using each method.

MATHEMATICAL PRACTICES
Look For and Make Use of Structure

7. Choose an area you're interested in. It could be a sport, listening to or playing music, some other area of the arts, or any other interest.
a. In your chosen context, describe two events that are mutually exclusive, and explain why they are mutually exclusive.
b. In your chosen context, describe two events that you believe to be independent, and explain why you think this is the case.

Conditional Probability and Independent Events

DIANE'S e-BOOKS

Embedded Assessment 2

Use after Activity 29

1. Diane has a collection of 91 novels. They are categorized as shown in the table below.

	Hardcover	Paperback	Total
Detective Novels	12	20	32
Romance Novels	8	28	36
Other Novels	9	14	23
Total	29	62	91

a. If Diane selects one of her novels at random, what is the probability that it is:

i. a romance novel?

ii. a romance novel given that it is a paperback?

b. Use your answers to parts a(i) and a(ii) to decide whether the two events are independent. Explain your reasoning.

2. Diane is looking at her collection of e-books. She has worked out that 60% of her e-books were downloaded from Company A, 68% are works of fiction, and 36% are works of fiction that were downloaded from Company A. When Diane selects one of her e-books at random, let the event that it was downloaded from Company A be A and the event that it is a work of fiction be F.

a. Draw a Venn diagram to represent the information you have been given. (As always, be sure to write the probabilities in your diagram and in your work as decimals, not percentages.)

b. Complete your Venn diagram by writing on it any additional relevant probabilities.

c. When Diane selects one of her e-books at random, what is the probability that it is neither a work of fiction nor was downloaded from Company A?

d. If Diane randomly selects a work of fiction, what is the probability that it was downloaded from Company A?

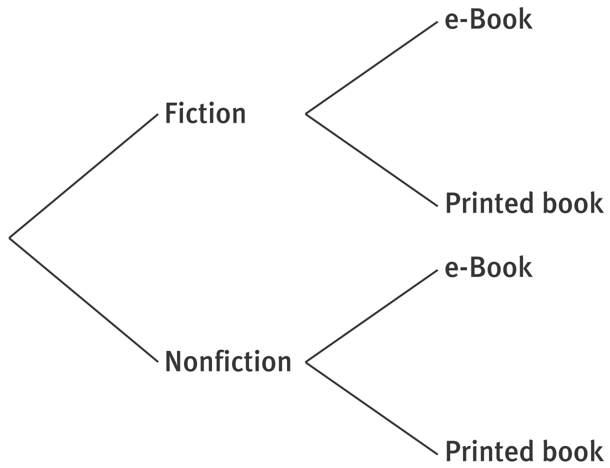
e. If Diane randomly selects a book that was downloaded from Company A, what is the probability that it is not a work of fiction?

f. What are the values of $P(A)$, $P(F)$, and $P(A \cap F)$?

g. Use your answers to part e to decide whether the events A and F are independent. Be sure to show any calculation that leads to your answer.

3. Books are classified as fiction or nonfiction, and Company A offers its books as either e-books or printed books. When Diane goes to Company A's website, the probability that she will choose a work of fiction is 0.6. If she chooses a work of fiction, the probability that she will buy it as an e-book is 0.8. If she chooses a book that is nonfiction, then the probability that she will buy it as an e-book is 0.3.

a. Write the relevant probabilities next to the six branches on the tree diagram.



b. When Diane goes to Company A's website, what is the probability that she will buy a work of fiction as a printed book?

c. When Diane goes to Company A's website, what is the probability that she will buy an e-book?

4. Company B has a very large number of books on its website, and 42% of the books are available as e-books. Diane will select two books at random from Company B's website.

a. Consider the event that the first book Diane selects is available as an e-book and the event that the second book Diane selects is available as an e-book. Explain in words how you know that these two events are independent.

b. Find each probability. Write your answer as a percent to the nearest tenth of a percent.

i. Both books are available as e-books.

ii. Neither book is available as an e-book.

iii. Exactly one of the two books is available as an e-book.

5. Diane belongs to a book club. The club has just read its 10th novel and is planning a party to celebrate. Diane decides to make a copy of the front cover of one of the detective novels to use as a dartboard. The trim size of the cover is $5\frac{1}{2}$ by $8\frac{1}{2}$. On the cover is a moon with a $2\frac{1}{2}$ in. diameter. Estimate the probability that a club member will hit the moon on the dartboard.

Conditional Probability and Independent Events

DIANE'S e-BOOKS

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates the following characteristics:			
Mathematics Knowledge and Thinking (Items 1, 2, 3, 4, 5)	• Clear and accurate understanding of determining probabilities using a table of values, a Venn diagram, a tree diagram, or area of plane figures • Clear and accurate understanding of independent events	• A functional understanding of determining probabilities using a table of values, a Venn diagram, a tree diagram, or area of plane figures • A functional understanding of independent events	• Partial understanding of determining probabilities using a table of values, a Venn diagram, a tree diagram, or area of plane figures • Partial understanding of independent events	• Little or no understanding of determining probabilities using a table of values, a Venn diagram, a tree diagram, or area of plane figures • Little or no understanding of independent events
Problem Solving (Items 1a, 2b–f, 3b–c, 4b, 5)	• An appropriate and efficient strategy that results in correct answers	• A strategy that results in mostly correct answers	• A strategy that results in some correct answers	• No clear strategy when solving problems
Mathematical Modeling / Representations (Items 2a, 3a)	• Clear and accurate understanding of using probabilities to create a Venn diagram • Clear and accurate understanding of using probabilities to complete a tree diagram	• Mostly accurate understanding of using probabilities to create a Venn diagram • Mostly accurate understanding of using probabilities to complete a tree diagram	• Partial understanding of using probabilities to create a Venn diagram • Partial understanding of using probabilities to complete a tree diagram	• Little or no understanding of using probabilities to create a Venn diagram • Little or no understanding of using probabilities to complete a tree diagram
Reasoning and Communication (Items 1b, 2g, 4a)	• Precise use of appropriate mathematics and language to explain why or why not the events are independent	• Mostly correct use of appropriate mathematics and language to explain why or why not the events are independent	• Misleading or confusing use of appropriate mathematics and language to explain why or why not the events are independent	• Incomplete or inaccurate use of appropriate mathematics and language to explain why or why not the events are independent