

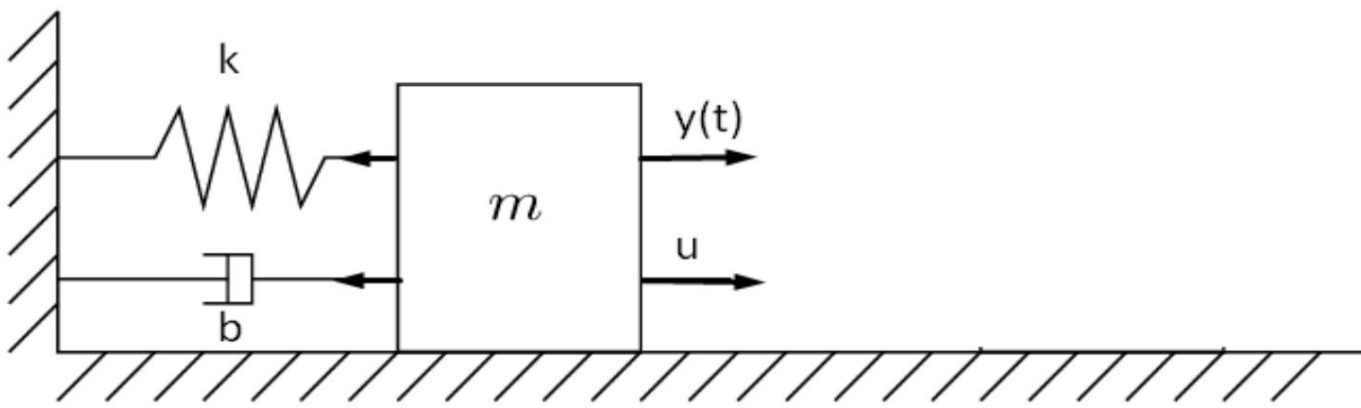
# Simulation and Modeling of Dynamic Systems Task 1

## Linear Parameterization, Estimation of Unknown Parameters, Method of Least Squares

The aim of this laboratory exercise is to estimate unknown parameters using the least squares method.

### Topic 1

Given the mass-spring-damper system shown in the figure below:



where  $b$  is the damping constant,  $k$  is the spring constant,  $u$  an external force, and  $y(t)$  is the displacement of the mass  $m$  due to the force applied to it.

- a) Find the mathematical model that describes the dynamic behavior of the system and parameterize it linearly, in the form:

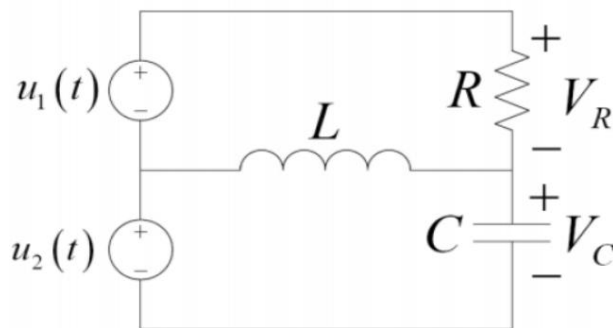
$$y = \theta^T \zeta$$

where the signal  $\zeta$  is produced by measurements of the external force  $u$  and the displacement  $y$ .

- b) Design the least squares algorithm to estimate the unknown parameters  $m$ ,  $b$  and  $k$  when we measure only the displacement and the external force applied to the mass.
- c) Simulate the algorithm you designed in the previous step assuming that  $m = 10\text{kg}$ ,  $b = 0.3\text{kg/s}$ ,  $k = 1.5\text{kg/s}^2$  and  $u = 10\sin(3t) + 5\text{N}$ . Use samples every  $0.1\text{s}$  for  $10\text{s}$  assuming zero initial conditions for the system states. Comment on the results.

## Topic 2

Given the circuit of the figure below:



where  $u_1(t) = 3\sin(2t)$  V and  $u_2(t) = 2$  V. Furthermore, we can only measure the voltages  $V_R$ ,  $V_C$  across resistor and capacitor respectively.

- a) Estimate with the method of least squares the transfer matrix of the above circuit. The voltages  $V_R$ ,  $V_C$  are produced from the vp file by calling the function as follows:

$$[V_R, V_C] = v(t);$$

where the parameter  $t$  can be either a scalar (eg  $t = 2.5$ ;) or a matrix (eg  $t = [1 \dots 10]$ ).

Note: The simulations should be done with Matlab functions that you will choose based on the quality of the results they give.

- b) Assume that the  $V_R$ ,  $V_C$  measurements are taken incorrectly (eg generate the  $V_R$ ,  $V_C$  signals as before and add at 3 random times some random numbers of a much larger order of magnitude than the normal values). Notice the impact this error has on the parameter estimates via the least squares method.

Submit the codes of the programs you wrote, the theoretical analyses (where needed) and a report containing your comments and graphs.