The effect of immigration on the transmission of shocks in a currency union

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Abstract

The present paper is based on the work of Schmitt-Grohé and Uribe (2012) which proposes a downward-rigid wages model and studies the effects of exogenous shocks in an economy, when the latter is a member of a currency union. In the current paper I extend this model to allow for labour movement inside a two-country currency union and I study the effects of immigrants' degree of integration in the foreign society, denoted as δ, on the propagation of domestic exogenous shocks and their spillover effects in the foreign country, under flexible and downward-rigid wages. The value of δ is taken to be the share of foreign-earned income that immigrants spend in the foreign country, while the simulated exogenous shocks in the domestic economy consist of shocks in the tradable good's endowment and in labour productivity. I conclude that under downward-rigid wages the effect of δ is almost negligible. Under flexible wages, the effects of δ affect the propagation and spillover of domestic shocks but also depend on their type. Following a shock in productivity, I find a monotonic relation between δ and the effects it produces. However, this is not always true for a shock on the tradable endowment. Nonetheless, the larger δ is, the smaller is the variance in observed changes in the consumption levels of both goods in both economies.

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1 Introduction

Following the eruption of the global financial crisis in 2008 due to the bursting of the U.S. housing bubble, financial institutions were put under stress and were in need of taxpayers bailout funding, with some of these institutions going bankrupt, the primmest example being that of Lehman Brothers. Due to this, financial institutions became more considerate about their lending policies and at the same time began to revaluate their debtors' ratings.

The aforementioned stricter revaluation of ratings, coupled with the slowdown of the international market that followed the eruption of the crisis, greatly affected countries that were on a high debt to GDP ratio. Some of the countries mostly hit were, and still are, the European southern economies of Portugal, Italy, Greece and Spain. A further hurdle these countries had to overcome was the fact that they had adopted the Euro, becoming part of the European monetary union. This left them unable to devalue their currencies and hence decrease real wages and prices. Furthermore, their labour markets prevented nominal wages to adjust downwards, driving prices downwards as well, and therefore tackle the inability of a currency devaluation in these countries. The effects of these downward-wage rigidities is currently studied by Schmitt-Grohé and Uribe (2012) in their working paper *Pegs and Pain*, which is discussed in section 2.

This paper attempts to extend the aforementioned authors work, by modifying their proposed model and allow for labour movement between two countries, which are both part of the same monetary union. The motivation behind of this extension is the great increase in citizens of crisis-struck Euro-zone economies leaving their country of origin, and relocating inside the Euro-zone to economies that can provide them better expectations for the future. For example, the number of Greeks moving to Germany has increased by almost 50% between the years of 2008 and 2010 while that of Spanish citizens has also increased by 40% during this time[1]. This labour force movement on the one hand alleviates part of the stress imposed on the labour market of the crisis-struck economies, by decreasing the labour supply, but on the other hand decreases the revenues of labour-demanding entities, since aggregate consumption in the domestic country decreases. Similarly, this movement poses the same but reverse pressures on the economy the citizens move to, by increasing labour supply and aggregate consumption.

The question this paper tries to address is whether the consumption pattern of immigrant workers affects, and to what degree, the propagation of shocks taking place in their home country, as well as the spillover effects for the foreign country. To do so, my

[1]Source: OECD statistics

model assumes that citizens moving abroad will tend to be more frugal with their foreign-earned income, compared to an average citizen of their destination countries, due to having to support their families back home and/or their desire to accumulate savings and return back to their countries of origin in the long-run, once they feel more financially secure. This degree of 'frugality', which is represented by δ, could also be though as the degree of immigrants' integration in the foreign society, showing in effect the degree that immigrants' consumption pattern is similar to that of the citizens in the country of destination. I base my research on how does δ affect the dynamics of the proposed model, assuming either flexible or fully downward-rigid nominal wages.

Following simulations of two types of negative external shocks, in either the domestic tradable endowment or the domestic level of productivity, I conclude that under a downward-rigid wages' specification the effects of δ although existent are almost negligible, due to the fact that there is no labour movement following any of the aforementioned shocks. Nonetheless, under flexible wages, a greater value of δ results in a smaller variance of the post-shock changes in both households' consumption levels, irrespective of the type of shock. Furthermore, when a negative shock in the domestic tradable endowment is concerned, an increase in the value of δ increases (decreases) the number of immigrants and the real wages domestically while decreasing (increasing) real wages in the foreign country for small (large) values of δ. Finally, following a negative shock in domestic productivity, an increase in δ enlarges the negative effects of the shock in the domestic country, reduces them in the foreign country, increases the immigration rate and reduces the inter-country post-shock debt differential.

This paper moves on by presenting a literature review of recent papers studying the area of monetary unions and currency pegs in section 2. Following this, section 3 presents the proposed model which accounts for labour movement between countries, section 4 states the models parameters' calibration. Section 5 studies the dynamic optimisation's results under flexible wages, while section 6 studies these results under fully downward-rigid wages. Section 7 summarises the economic implications of the model by focusing on the value of δ, and finally section 8 concludes and summarises the paper.

2 Literature review

Schmitt-Grohé and Uribe (2012), in their paper *Pegs and Pain*, propose a small open economy model consisting of two goods, which faces nominal wage rigidities. The goods consist of a tradable one, supplied periodically through a stochastic endowment, and a non-tradable that is produced from each economy's firms after labour is provided. Furthermore, the exchange rate with the rest of the world is assumed to be pegged, and therefore the price of domestic and foreign tradable goods remains constant in relation to each other, by assuming that the law of one price holds. The novelty of the paper is that when modelling nominal wage rigidities, the authors chose to depart from the standard Calvo pricing mechanism and instead employ downward-wage rigidities which can provide, in contrast to more standard models, the ability to relate aggregate volatility with unemployment. They conclude that the median welfare cost of living under a currency peg, is in between 4 and 10 percent of lifetime consumption, compared to an optimal exchange rate policy.

Abbritti and Mueller (2012), analyse how the volatility of unemployment and inflation differentials affect the labour market institutions of a monetary union. They do so by proposing a dynamic model with two countries which are members of the same currency union. Both countries have a two sector economy, one of intermediate goods being produced under perfect competition with hiring frictions, and a retail goods sector operating under monopolistic competition with price rigidities. They focus on two types of rigidities, unemployment and real wage ones and ultimately result in 3 key findings. First of all, that the two aforementioned rigidities' types have opposite effects on the volatility of unemployment and inflation differentials. Secondly, that the volatility of both unemployment and inflation increases with the magnitude of the asymmetries of unemployment and real wage rigidities between the countries. Finally, the effects of these rigidities offset (reinforce) each other if they are intra-country positively (negatively) correlated. They conclude that labour market structures' asymmetries create obstacles in the post-shock recovery of a currency union.

Farhi, Gopinath and Itskhoki (2012), intend to bridge the gap between academic literature and economic policies undertaken in the Euro-zone to tackle the recent crisis, and attempt to overcome Mundell's impossible triangle. They do so by proposing two fiscal-devaluating policy alternatives under a fixed exchange rate regime, that accurately mimic the dynamics of a nominal exchange rate devaluation under a floating exchange rate regime. These involve on the one hand, a uniform increase in export and import subsidies and tariffs respectively, and on the other hand, an increase in value added tax coupled with a decrease in payroll taxation. According to the paper any of these policies is sufficient to mimic the dynamics of either a devaluation of the exchange rate in the cases where trade is balanced or, an unexpected policy change that takes place in an incomplete markets' environment. In all other cases, they need to be coupled by increased income and reduced consumption taxation. Nonetheless, these policies, which are government revenue neutral, can work independently of the degree of rigidities in the wage and price levels, after being tailored for a specific asset market structure.

Poilly and Sahuc (2011), examine the welfare implications of permanent reforms in the labour market institutions, when under a currency union with heterogeneous labour market. These are the scaling parameter in the matching function, the workers' bargaining power and the job destruction rate. They conclude that first of all, not all structural changes of the labour market result in similar welfare effects. Secondly, changes in the home country spillover in the foreign country. Thirdly, in all occasions the way policy is conducted by the authorities has a slight effect in the post-shock transition of the economy, and finally, permanent labour market changes have greater welfare effects the greater the degree of price rigidities is.

Schmitt-Grohé and Uribe (2012), in their paper *The Case of Temporary Inflation in the Eurozone*, apply the mathematical model they developed in *Pegs and Pain* (Schmitt-Grohé and Uribe, 2012) to propose an inflationary policy that according to their model of downward-rigid wages, could tackle the recession in peripheral Europe. More specifically, they believe that the current recession will be a protracted one unless sizeable policy action is undertaken. They argue that an ECB inflationary policy targeted at raising the inflation rate by 4 percent, for a duration of five years, will deflate real wages back to their pre-boom levels.

3 Model

3.1 Monetary union

Both countries' economies are assumed to belong to a currency union, therefore sharing the same currency. There are two goods available in each economy, the non-produced, tradable good that is periodically endowed and the non-tradable good that is produced by the firms that exist in each country. Tradable good follow the law of one price and hence,

$$P_t^T = E_t P_t^{T*}$$

where P_t^T is the price of the tradable good in the domestic country, P_t^{T*} is that in the foreign country[2] and E_t is the exchange rate between the two countries. Furthermore, due to the common currency the two countries share, the exchange rate is assumed constant

[2]It is assumed from here forth that all variables and equations refer to the domestic country, unless otherwise stated. Furthermore, unless stated otherwise, all variables or equations referring to the foreign country are denoted by a star (*) superscript and are symmetrical.

and is normalised to unity.

$$P_t^T = P_t^{T*}, \forall t$$

3.2 Households

Both the domestic and the foreign country's economies are populated by an infinite amount of identical households which maximise their expected utility of current and expected future consumption, subject to a common discount rate β,

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U(C_t).$$

Consumption C_t refers to a composite good made up by the consumption of the non-produced, tradable good c_t^T and that of the non-tradable good c_t^N. The utility received from consumption follows an increasing and concave aggregating function over the two available goods,

$$U(C_t) = F\left(c_t^T, c_t^N\right)$$

Consumption of the non-tradable good can take place either in the domestic or foreign country. Hence,

$$c_t^N = c_t^{N,H} + c_t^{N,F}$$

where $c_t^{N,H}$, and $c_t^{N,F}$ denote the consumption, by the domestic households, of non-tradable goods in the Home and Foreign country respectively.[3]

The price of the tradable good is common in both economies, due to the assumptions stated in section 3.1, and is normalised to 1. P_t denotes the relative price of the non-tradable good, for period t, in the home country.

In every period, households are endowed with an amount of the tradable good y_t which follows an AR(1) stochastic process of the form,

$$y_t = \bar{y} + \rho_1 \left(y_{t-1} - \bar{y}\right) + \epsilon_t \tag{1}$$

where $\bar{y}$ is the steady state endowment quantity of the tradable good, $\rho_1 \in (0,1)$ and ϵ_t is the stochastic shock that follows a normal distribution,

$$\epsilon_t \sim \mathcal{N}(0, \sigma_1^2)$$

[3]This indication of Home and Foreign is used in an identical fashion from here forth, wherever else necessary.

In period t, households can lend or borrow capital from the international market (d_t) at an interest rate $1+r_t$ which they settle in the next period (d_{t+1}). Furthermore, households own the firms producing the non-tradable good and hence receive the profits (or losses) Φ_t of the latter.

Households are free to supply their labour to firms in either their home or the foreign country. By doing so, they get in return the relevant wage rate W_t, or W_t^* when working abroad. Therefore, the household's total labour supply can be divided into labour provided to the domestic and the foreign firm,

$$h_t^s = h_t^H + h_t^F$$

The domestic household's budget constraint, in terms of tradable goods, is given by,

$$c_t^T + P_t c_t^{N,H} + P_t^* c_t^{N,F} + d_t \leq y_t + W_t h_t^H + W_t^* h_t^F + \frac{d_{t+1}}{1+r_t} + \Phi_t$$

3.3 Firms

Firms produce the non-tradable good using labour which can be supplied from both countries. They enumerate all labour supplied to them with the wage rate W_t, irrespective of the household supplying it.

The quantity of the non-tradable good produced by the firm, is subject to an increasing and concave production function $G(h_t^d)$, and the productivity coefficient A_t which follows an AR(1) stochastic process of the form,

$$A_t = \overline{A} + \rho_2 \left(A_{t-1} - \overline{A}\right) + \zeta_t \tag{2}$$

where $\overline{A}$ is the steady state productivity level, $\rho_2 \in (0,1)$ and ζ_t is the stochastic shock which follows a normal distribution,

$$\zeta_t \sim \mathcal{N}(0, \sigma_2^2)$$

Furthermore, due to the non-tradable nature of the good, all output must be consumed during the same time period and hence,

$$A_t G\left(h_t^d\right) = c_t^{N,H} + c_t^{N,F*} \tag{3}$$

Finally, the firms' profit function is given by

$$\Phi_t = A_t P_t G(h_t^d) - W_t h_t^d \tag{4}$$

3.4 Assumptions

The following assumptions are either necessary or are used to simplify the model's dynamics.

i. The consumption's utility aggregating function is of a Cobb-Douglas form and is identical for both the domestic and the foreign household.

$$F\left(c_t^T, c_t^N\right) = \left(c_t^T\right)^\alpha \left(c_t^N\right)^{1-\alpha}, \text{ where } \alpha \in (0,1)$$

ii. The production function of the domestic and foreign firms is identical and equal to

$$G\left(h_t^d\right) = \left(h_t^d\right)^\gamma, \text{ where } \gamma \in (0,1)$$

iii. The foreign country has a larger steady-state productivity level than the domestic country.

$$\overline{A}^* = \eta \overline{A}, \text{ where } \eta > 1$$

iv. Both countries have an equal steady state level of labour supply, which in addition is the maximum labour supply rate (full employment).

$$\bar{h}^s = \bar{h}^{s*} = \bar{h}$$

$$h_t^s,\, h_t^{s*} \leq \bar{h},\, \forall t$$

v. Households of the domestic country can provide labour to the foreign country, but households in the foreign country are obliged to only provide labour in their country.

$$h_t^s = h_t^H + h_t^F \leq \bar{h} \text{ and } h_t^{s*} = h_t^{H*} \leq \bar{h}$$

vi. The amount of non-tradables consumed by domestic households in the foreign country is a fraction of the income received in this country in terms of non-tradables. Specifically, it is equal to

$$c_t^{N,F} = \frac{\delta W_t^* h_t^F}{P_t^*}, \text{ where } \delta \in (0,1)$$

Thus, although the domestic household is in all cases able to work in the foreign country and earn part of its income there, the quantity of non-tradables consumed there depends on δ. More specifically, when $\delta = 0$ the domestic household is barred from consuming abroad. As the value of δ increases, the domestic household needs to spend part of its foreign earned income to consume non-tradables in the foreign country, ultimately leading to the case of $\delta = 1$ where all foreign earned income has to be consumed abroad.

In addition, it follows from assumption (v) that $c_t^{N,F*} = 0$.

Furthermore, the non-tradable nature of the produced good depicted in equation (3), coupled with assumptions (ii) and (v), results in

$$c_t^{N,H} = A_t \left(h_t^d\right)^\gamma \text{ and } c_t^{N,F} + c_t^{N,H*} = A_t^* \left(h_t^{d*}\right)^\gamma \tag{5}$$

vii. The interest rate is assumed to be elastic to the amount of debt held during each period by the household, relative to the steady state amount of debt of each country. Therefore,

$$1 + r_t = \frac{1}{\beta}\left(\frac{d_{t+1}}{\bar{d}}\right)^\kappa$$

where $\bar{d}$ is the steady state level of debt and κ is a measurement of the interest rate's level of elasticity relative to the households' debt. Households are assumed to be ignorant about this rule.

viii. The amount of labour acquired by the firm $h_t^d = h_t^H + h_t^{F*}$, can be less than the amount demanded by it, and is the sum of the labour supplied by the domestic and foreign household. Nonetheless, this is not the case in this paper since, only negative shocks are examined and thus, the demand of labour should not exceed the supply.

4 Calibration

As it can also be seen in Table 4.1, I set the parameter for labour share at 0.62, equal to the EU-15 average as calculated by Arpaia, Pérez and Pichelmann (2009). The degree of relative productivity of the foreign country was set according to the following formula

$$\eta = \left(\frac{\text{Labour productivity per person employed (Germany)}}{\text{Labour productivity per person employed (Greece)}}\right)^\gamma = 1.06$$

where productivity values used were those provided by Eurostat for 2008.

I normalise the steady-state levels of output of tradables and productivity of the domestic country at unity, and hence due to the value of η the corresponding values for the foreign country, which are equal to those of the domestic country after being multiplied by η, are set to 1.06. In addition, labour hours are also normalised at unity.

Table 4.1: Parameters' calibration

Parameter	Value	Description
α	0.5	Cobb-Douglas aggregating function parameter
β	0.9926	Quarterly subjective discount factor
$\bar{h}$	1	Labour endowment
$\overline{A}$	1	Steady-state productivity (Home country)
$\overline{A}^*$	1.06	Steady-state productivity (Foreign country)
γ	0.62	Labour share
η	1.06	Degree of (greater) relative productivity of the foreign country
$\bar{y}$	1	Steady-state tradable output (Home country)
$\bar{y}^*$	1.06	Steady-state tradable output (Foreign country)
ρ_1	0.95	Persistance of tradable output shocks
ρ_2	0.95	Persistance of production shocks
$\bar{d}$	2.21	Steady-state debt level (Home country)
$\bar{d}^*$	0.84	Steady-state debt level (Foreign country)
κ	10^{-10}	Interest rate's elasticity relative to the households' debt

The persistence of both the tradable output and the production shocks are set to the standard value of $\rho_1 = \rho_2 = 0.95$. Similarly, the parameter of the Cobb-Douglas consumption aggregating function is set to a standard value of 0.5, implying an optimal of equal consumption shares of both the tradable and the non-tradable good.

Time is measured in quarters. The subjective discount factor β is set to 0.9926, implying a risk-less annual return of 3.3%, which was Euribor's annual interest rate in 2008 as provided by ECB.

The steady-state levels of debt for both countries are set in such a way so that they equal the debt to GDP[4] ratios provided by OECD for Greece (Home country) and Germany (Foreign country)[5]. It should be noted however that due to the endogenous nature in the model of the steady-state debt level and labour supply, for the home and foreign country, these values correctly reflect the aforementioned debt to GDP levels only for the case that $\bar{h}^H = \bar{h}^{H*} = \bar{h}$.

Finally, the interest rate's elasticity level to the household's debt (κ) is taken as (almost) zero[6] to proxy for the interest rate level that is provided by the ECB, which unless altered

[4]GDP is taken to be the sum of the tradable and non-tradable endowment and production respectively

[5]Debt to GDP ratio was equal to 110.6% (Greece) and 39.6% (Germany) in 2008

[6]In the case where κ is set to exactly zero the model fails to converge.

exogenously remains stable.

5 Flexible wages model

By assuming that wages are flexible, both the domestic and the foreign household are always willing to provide all their labour hours to the firms. Most importantly, firms are willing to hire all labour supplied at the equilibrium wage. Therefore, and due to assumption (v) of section 3.4 I get that

$$h_t^H + h_t^F = h_t^{H*} = \bar{h} \tag{6}$$

By taking the above into account and also following the assumptions of section 3.4 the inter-temporal Lagrangian of the domestic household is the following

$$\begin{aligned}\mathcal{L}\left(c_t^T, c_t^{N,H}, h_t^H, d_{t+1}, \lambda_t\right) = \mathbb{E}_t \sum_{s=t}^{\infty} \Bigg\{ \beta^{s-t} \left(c_s^T\right)^{\alpha} \left(c_s^{N,H} + \frac{\delta W_s^* \left(\bar{h} - h_s^H\right)}{P_s^*} \right)^{1-\alpha} \\ + \lambda_s \left[c_s^T + P_s c_s^{N,H} + d_s - W_s h_s^H - (1-\delta) W_s^* \left(\bar{h} - h_s^H\right) - y_s - \frac{d_{s+1}}{1+r_s} - \Phi_s \right] \Bigg\}\end{aligned} \tag{7}$$

while that of the foreign household is

$$\begin{aligned}\mathcal{L}^*\left(c_t^{T*}, c_t^{N,H*}, d_{t+1}^*, \lambda_t^*\right) = \mathbb{E}_t \sum_{s=t}^{\infty} \Bigg\{ \beta^{s-t} \left(c_s^{T*}\right)^{\alpha} \left(c_s^{N,H*}\right)^{1-\alpha} \\ + \lambda_s^* \left[c_s^{T*} + P_s^* c_s^{N,H*} + d_s^* - W_s^* \bar{h} - y_s^* - \frac{d_{s+1}^*}{1+r_s^*} - \Phi_s^* \right] \Bigg\}\end{aligned} \tag{8}$$

Furthermore, the profit functions of the domestic and foreign firm are

$$\Phi_t\left(h_t^d\right) = A_t P_t \left(h_t^d\right)^{\gamma} - W_t h_t^d \tag{9}$$

$$\Phi_t^*\left(h_t^{d*}\right) = A_t^* P_t^* \left(h_t^{d*}\right)^{\gamma} - W_t^* h_t^{d*} \tag{10}$$

for the domestic and foreign firm respectively.

5.1 Optimality conditions

5.1.1 Households' optimisation

By solving the household's problem the relative price equations can be retrieved. These solely depend on the consumption pattern of the respective household of each country.

Nonetheless, a change in the labour supply of the domestic household abroad, apart from changing the labour supply domestically and thus altering domestic production and hence consumption of non-tradables, will indirectly affect the relative price in the foreign country since there will be a change in the quantity of non-tradables produced abroad. This change in quantity will in turn create a change in the quantity of non-tradables consumed by the foreign household. This indirect effect of a change in the labour supplied by the domestic household in the foreign country (h_t^F) will decrease as δ increases. The reason is that the larger the value of δ, the larger will be the quantity of non-tradables consumed by the domestic household in the foreign country, hence resulting in a smaller change in the quantity of non-tradables consumed following a change in h_t^F. This behaviour of the model implies that as δ increases, the pressures inflicted by a change in h_t^F to the relative price of the foreign country will decrease in magnitude.

$$P_t = \frac{(1-\alpha)\, c_t^T}{\alpha\left(A_t\left(h_t^H\right)^{\gamma} + \frac{\delta W_t^*\left(\bar{h}-h_t^H\right)}{P_t^*}\right)} \tag{11}$$

$$P_t^* = \frac{(1-\alpha)\, c_t^{T*}}{\alpha\left(A_t^*\left(2\bar{h}-h_t^H\right)^{\gamma} - \frac{\delta W_t^*\left(\bar{h}-h_t^H\right)}{P_t^*}\right)} \tag{12}$$

Another feature that results from the household's optimisation is the following equation which shows the dependence of relative prices and wages in both countries

$$\frac{W_t}{P_t} = \delta\left(\frac{W_t^*}{P_t^*}\right) + (1-\delta)\left(\frac{W_t^*}{P_t}\right) \tag{13}$$

The above no-arbitrage rule shows that the domestic real wage level, which is taken to be equal to the domestic firm's wage level divided by the domestic relative price P_t, must always equal an weighted average of the foreign real wage level (computed in a symmetrical fashion with the domestic real wage level) and an 'inter-country real-wage' $\frac{W_t^*}{P_t}$.

The two terms of the aforementioned weighted average, exert opposing forces on the share of labour supplied abroad h_t^F, and depend on the degree of immigrants integration in the foreign society (δ). On the one hand, the first term shows the dependence on the real-wages of the foreign country, which as δ increases deters changes in h_t^F, since the flexibility of wages can reflect exogenous shocks in the economy, thus nullifying immigration opportunities. On the other hand, the second term increases the willingness of the domestic labour to increase its supply of labour to the foreign country following a shock, given that δ increases, since as described above an increased value of δ will result in lesser pressures for a change in h_t^F on the foreign relative price P_t^* and hence the wage level[7], thus creating more 'space' for immigration until the no-arbitrage condition is satisfied.

Furthermore, it should be noted that an increase (decrease) of the domestic household's foreign labour supply (h_t^F) results in the equivalent of a decrease (increase) in the foreign country's productivity level and an increase (decrease) in that of the domestic country. The reason behind this feature is that an increase in h_t^F will decrease the output per labour hour provided by the foreign household due to diminishing returns, coming from the labour share parameter (γ). A similar but opposing force will follow an increase in h_t^F and be exerted on domestic productivity. Nonetheless, the overall effect of this model's behaviour on prices and wages should be minimal compared to the effect of δ discussed previously.

Finally, the Euler equations and the budget constraints of the two households, can be also derived from the households optimisation

$$\left(\frac{c_t^T}{A_t\left(h_t^H\right)^{\gamma}+\frac{\delta W_t^*\left(\bar{h}-h_t^H\right)}{P_t^*}}\right)^{\alpha-1}=\beta\left(1+r_t\right)\mathbb{E}_t\left[\left(\frac{c_{t+1}^T}{A_{t+1}\left(h_{t+1}^H\right)^{\gamma}+\frac{\delta W_{t+1}^*\left(\bar{h}-h_{t+1}^H\right)}{P_{t+1}^*}}\right)^{\alpha-1}\right] \tag{14}$$

$$\left(\frac{c_t^{T*}}{A_t^*\left(2\bar{h}-h_t^H\right)^{\gamma}-\frac{\delta W_t^*\left(\bar{h}-h_t^H\right)}{P_t^*}}\right)^{\alpha-1}=\beta\left(1+r_t^*\right)\mathbb{E}_t\left[\left(\frac{c_{t+1}^{T*}}{A_{t+1}^*\left(2\bar{h}-h_{t+1}^H\right)^{\gamma}-\frac{\delta W_{t+1}^*\left(\bar{h}-h_{t+1}^H\right)}{P_{t+1}^*}}\right)^{\alpha-1}\right] \tag{15}$$

$$c_t^T+d_t=\left(1-\delta\right)W_t^*\left(\bar{h}-h_t^H\right)+y_t+\frac{d_{t+1}}{1+r_t} \tag{16}$$

$$c_t^{T*}+\left(1-\delta\right)W_t^*\left(\bar{h}-h_t^H\right)+d_t^*=y_t^*+\frac{d_{t+1}^*}{1+r_t^*} \tag{17}$$

5.1.2 Firms' optimisation

The two rules regarding the wage level can be retrieved through the domestic and foreign firms' optimisation problem.

$$W_t=\gamma A_tP_t\left(h_t^H\right)^{\gamma-1} \tag{18}$$

$$W_t^*=\gamma A_t^*P_t^*\left(2\bar{h}-h_t^H\right)^{\gamma-1} \tag{19}$$

[7]This can be seen from the wage level rule, equation (19) in section 5.1.2, that follows

It should be noted that since wages are taken to be flexible, they depend on the relevant country's productivity level, relative price and labour supply.

5.1.3 Equilibrium conditions

The equations (11)-(19) of sections 5.1.1 and 5.1.2, coupled with the four AR(1) stochastic processes (1), (2) and their symmetrical counterparts of the foreign country governing the propagation of shocks in the domestic and foreign economy, as well as the interest rate rules which result from assumption (vii) in section 3.4, fully describe this dynamic model.

5.2 Simulations

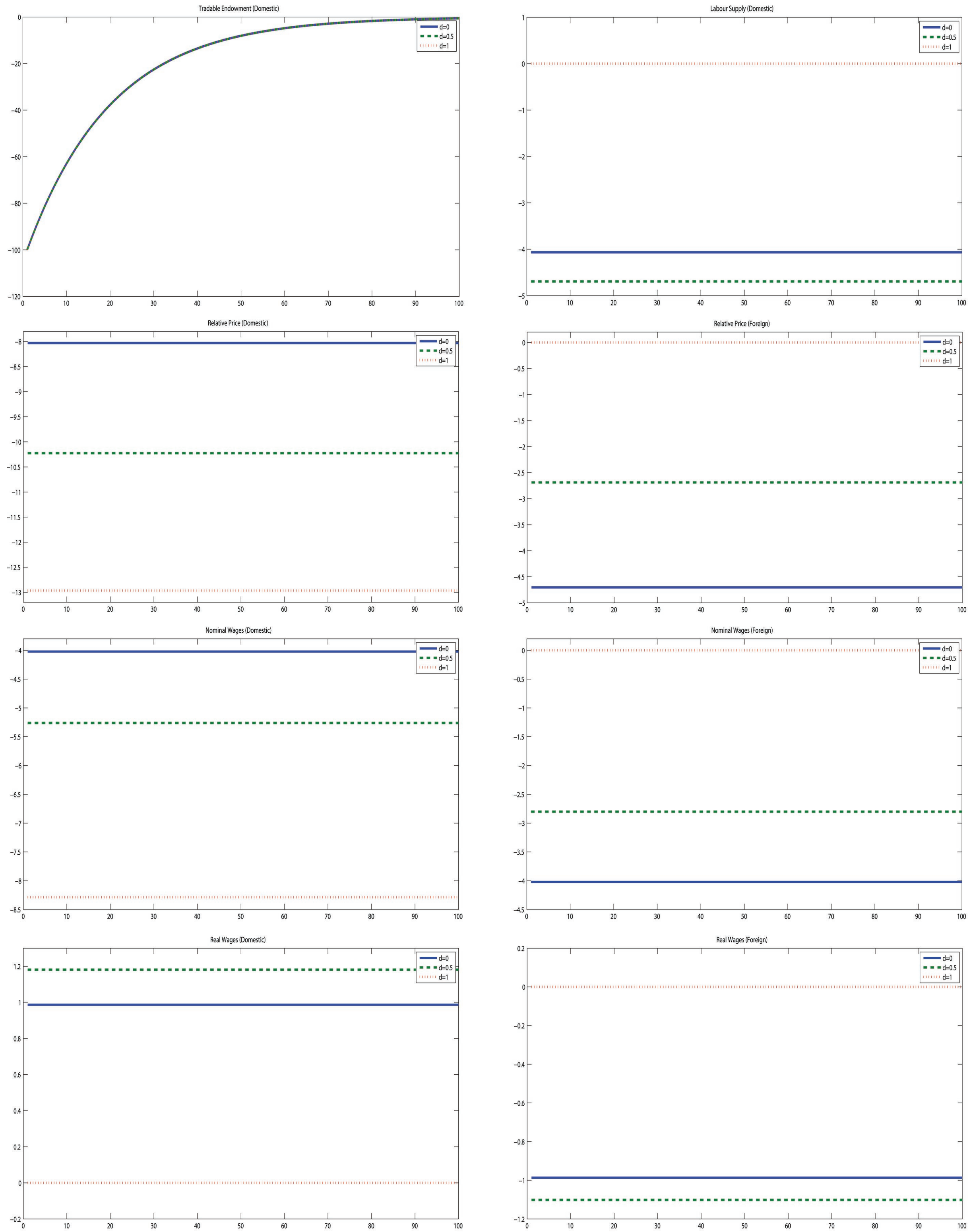
5.2.1 Negative shock in domestic tradable endowment

In this section the model is simulated 3 times for different values of δ, namely $\delta = 0$, $\delta = 0.5$ and $\delta = 1$, with all results depicted in figures 5.1 and 5.2. The model's dynamics for different values of δ are explained below.

Case of $\delta = 0$ In this case, the domestic household does not consume at all in the foreign country, irrespective of its income share earned there.

As can be seen from the no arbitrage condition in equation (13), nominal wages of the two countries equal each other before the shock in the domestic endowment. Following this shock however, the tradable endowment naturally becomes more scarce in the domestic country, which pushes the relative price of non-tradables downwards since the latter now become relatively more abundant. This drop in price coupled with wage flexibility makes the domestic firm decrease wages, as can be seen from the wage level rule in equation (18) so that it continues maximising profits. This drop in domestic wages will lead to an imbalance of the no-arbitrage condition mentioned above since now the nominal wage level of the foreign country will be higher than the domestic one. This will trigger an increase in the share of labour that the domestic household is providing to the foreign firm and therefore decrease its share supplied to the domestic firm. The result of this is that the shock in domestic tradables causes a decrease in nominal wages domestically and thus an increase in immigration will be triggered, which will increase production of non-tradables in the foreign country. As was discussed in section 5.1.1, this will also push down prices (equation (12)) and therefore wages (equation (19)) in the foreign country. Ultimately,

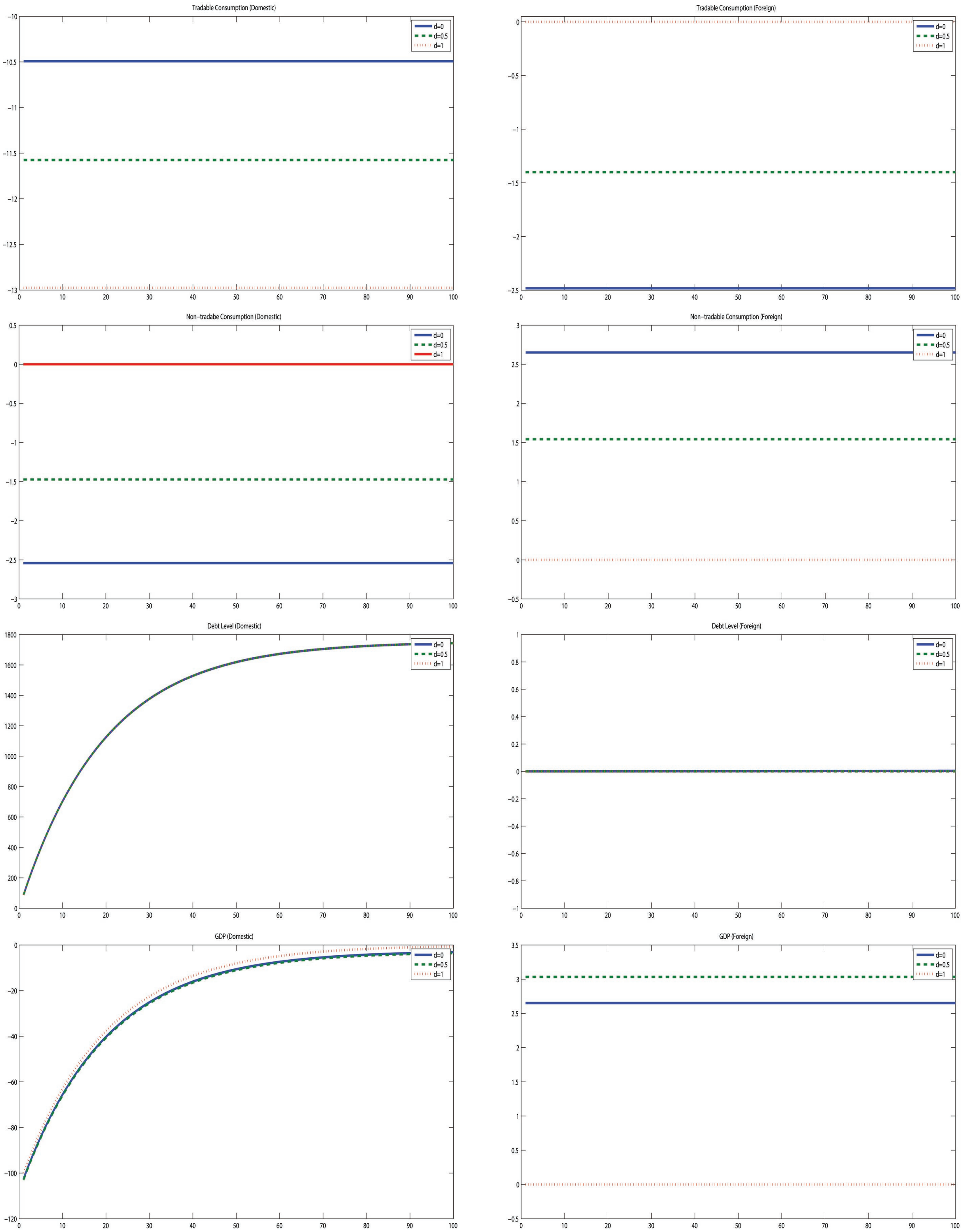
Figure 5.1: Shock in domestic tradable endowment (Flexible wages)



wages in both countries will be equalised once again, satisfying the no-arbitrage condition and thus ending this increase in immigration.

Following the shock in the domestic endowment domestic prices and wages will drop, as

Figure 5.2: Shock in domestic tradable endowment (Flexible wages)



described above, which will trigger an increase in the supply of labour of the domestic household to the foreign country. Moreover, since the shock is in the tradable good, which the households can lend or borrow if needed, and its propagation can also be foreseen, the domestic household will instantly jump to a new steady state after the shock. Therefore, the aforementioned decrease in labour provided domestically by the domestic household will be permanent and therefore the production and hence consumption of non-tradables by the domestic household will permanently decrease, therefore permanently decreasing the domestic GDP level as well. Furthermore, this drop in consumption of non-tradables by the domestic household, will lead to a reduction in the household's consumption of the tradable good as well, due to imperfect substitutability of the two goods. The reduction of tradable's consumption will also be necessary due to the increased price of the tradable good. Finally, the new stable consumption pattern that the domestic household will jump to instantaneously following the shock, will be partly financed through an increase in the debt level until the shock in domestic tradables diminishes. Therefore, the domestic household will move to a new steady state level of debt in the long-term which will be higher than the previous one.

Regarding the foreign country, as discussed previously the shock will trigger a permanent increase in the amount of labour supplied abroad by the domestic household, resulting in a permanent increase in the foreign GDP level. The increase in immigration will permanently increase the foreign household's consumption of non-tradables, thus reducing the relative price of non-tradables in this country due to the dynamics of equation (12) previously discussed. The drop of the foreign price level will lead the foreign firm to decrease the foreign wage level according to the profit maximising rule of equation (19). Finally, the increased consumption of non-tradables by the foreign household will be financed solely by a decrease in its consumption of the tradable good, which following the shock has become relatively more expensive, and there will be no need for an increase in the foreign debt level.

Case of $\delta = 0.5$ Similar with the previous case, following the shock there will be an increase in immigration from the domestic to the foreign country which will now be even larger than when $\delta = 0$. The reason for this, which is described in detail in section 5.1.1, arises from the no-arbitrage condition, equation (13), and is due to the fact that under the current setup, when $\delta = 0.5$[8] there is less pressure applied to the foreign country's wage level, thus creating more 'room' for immigration and although there also exists an opposing force that limits immigration due to the dependence of the no-arbitrage condition in the foreign real-wages which are flexible.

Due to the previously described permanent decrease of domestic labour supply, the domestic GDP is permanently reduced. Moreover, since the domestic household will also

[8]For this value of δ half of the domestic household's earnings in the foreign country will have to be spent in non-tradable consumption in that country

consume non-tradables in the foreign country ($\delta > 0$), the drop in the domestic household's total consumption of non-tradables will not be as great as in the case where $\delta = 0$. Therefore, the domestic relative price of non-tradables will have to further adjust downwards, according to equation (11), and hence so will the domestic wage level (equation (18)). Finally, due to this smaller in magnitude decrease of non-tradable consumption, compared to the case where $\delta = 0$, the domestic household will choose to further decrease it's consumption of tradable goods since otherwise the extra debt burden imposed would not be optimal.

Although the increase in labour supplied by the domestic household abroad will be greater now than the case where $\delta = 0$, the drop in foreign relative prices will be smaller in magnitude. The reason is that because $\delta = 0.5$ the domestic household now spends half of its earnings in the foreign country to consume non-tradables there. Hence, this partly alleviates the downward pressures on the foreign relative price level since now the increase in immigration results in a smaller increase in the foreign household's non-tradable consumption (equation (12)). Due to this, the foreign wage level also decreases by less, as can be seen by equation (19). In addition, due to the aforementioned lesser increase of non-tradables' consumption by the foreign household, a smaller decrease in its consumption of tradables is now required to finance this consumption pattern change. Finally, an increase in the foreign GDP level is observed again, being also greater in magnitude than the previous case since the labour supplied abroad by the domestic household is larger.

Case of $\delta = 1$ Now the domestic household is obliged to spend the whole amount of wages earned abroad in foreign non-tradables' consumption. Due to this, the no-arbitrage condition, shown in equation (13), is now satisfied when real wages in both countries are exactly equal.

Following the shock, domestic wages and prices will adjust, resulting in the real wages of the domestic country to remain unchanged, since the drop in wages is perfectly matched by a drop in prices and thus no further opportunities for immigration arise for the domestic household, resulting in the labour supplied domestically and abroad by it to remain unchanged. A straightforward implication of this is that the GDP level of the domestic country will revert back to its initial value in the long term after the shock in tradables has diminished. Furthermore, since the production of non-tradables by the domestic household will remain unchanged, so will its consumption, and therefore the downward pressures to the domestic relative price level (equation (11)) will be maximal compared to the two previous cases. Thus, the domestic wage level will also move further downwards than before, according to equation (18). Finally, in contrast with the previous cases the consumption of non-tradables will remain unchanged and due to this, the household will choose to decrease its consumption of non-tradables by even more in this case.

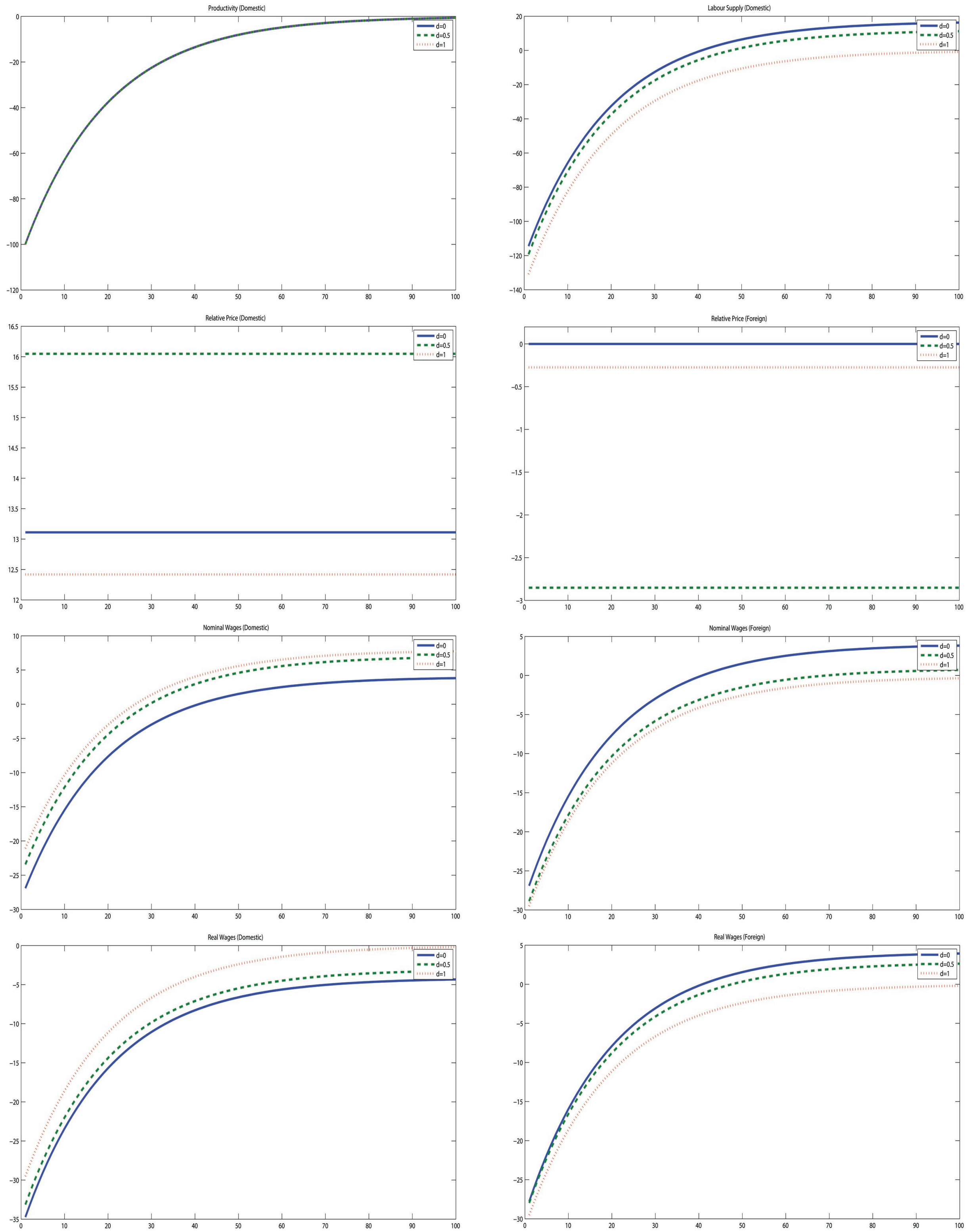
5.2.2 Negative shock in domestic productivity

As it can be seen in figures 5.3 and 5.4, the shock in domestic productivity is in all cases making the domestic household increase its supply of labour to the foreign country and take advantage of the inter-country productivity differential. However, since this increase is due to the post-shock productivity levels' differential, it alters as the shock propagates.

Case of $\delta = 0$ Following the shock, the production of non-tradable goods in the domestic country is greatly reduced, therefore so does the consumption of these goods by the domestic household, and hence from equation (11) the relative price of non-tradables will shoot upwards as they become relatively scarcer in the domestic country. However, since now the shock is in productivity the wage level will decrease according to equation (18). This will create an incentive for the domestic household to provide more labour in the foreign country and take advantage of the larger nominal wage there. Nonetheless, due to the shock and the following drop in domestic productivity, consumption of non-tradables by the domestic household will necessarily decrease. This will also decrease the consumption of tradables by the domestic household due to imperfect substitutability, which will in turn result in a greater degree of thrift domestically, thus decreasing in the long-term the domestic household's debt level. In addition, due to this long-term decrease in debt the domestic household will increase its long-term consumption of both goods. To achieve this, since wages earned in the foreign country cannot be spent on consumption of non-tradables there, because $\delta = 0$, the labour supply of the domestic household to the foreign firm will decrease in the long-run, so that more is produced and hence consumed at home. This will imply that the GDP level will increase domestically in the long-run, while that in the foreign country will decrease.

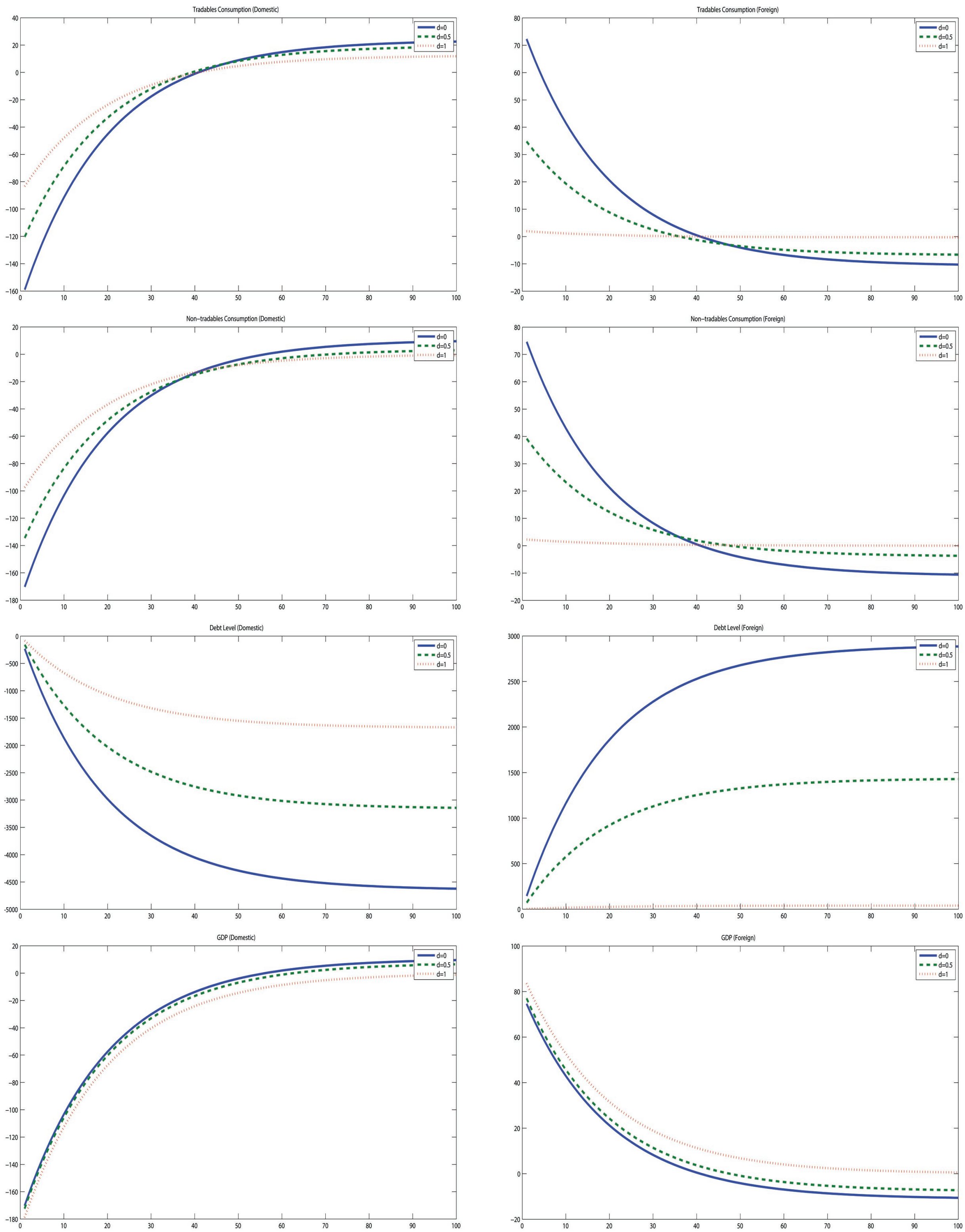
Regarding the foreign household, the increase in labour supplied by the domestic household abroad will greatly increase the production of non-tradables. Therefore, since $\delta = 0$ there will be an equal increase in the consumption of non-tradables by the domestic household, which will be matched by a similar in magnitude increase in the consumption of non-tradables. However, as can be seen by the no-arbitrage condition in equation (13), relative prices abroad will remain unchanged as they do not need to move for this condition to be satisfied. Furthermore, foreign wages will jump downwards and then increase

Figure 5.3: Shock in domestic productivity (Flexible wages)



as the shock propagates, because firstly there is an indirect decrease in the foreign productivity level related with the increase in the amount of labour supplied to the foreign firm, as was discussed in section 5.1.1, and secondly, due to the temporal nature of the change in labour supplied abroad by the domestic household. Finally, the increase in consumed

Figure 5.4: Shock in domestic productivity (Flexible wages)



quantities of both goods by the foreign household will lead to an increased amount of borrowing so that this consumption pattern change is financed. This will increase the foreign household's debt level.

Case of $\delta = 0.5$ Following the unexpected drop in non-tradables' production in the domestic country, the domestic relative price has to increase while the wage level has to decrease. Similar to the case of a shock in the domestic tradable endowment, when $\delta = 0.5$ there is an increase in the amount of labour supplied to the foreign firm by the domestic household, compared with the case of $\delta = 0$. Moreover, because $\delta = 0.5$ the consumption of non-tradables by the domestic household will not decrease by as much as in the previous case since now the domestic household will also consume non-tradables in the foreign country. Due to imperfect substitutability, the consumption of tradables' by the domestic household will also not decrease by the same magnitude in this case, resulting in a smaller amount of saving domestically and therefore to a smaller decrease in the steady state amount of debt in the long-run.

Regarding the foreign household, the increase in immigration will again result in an increase in the amount of non-tradables consumed. However, since now $\delta = 0.5$ part of the increased production, due to immigration, will be consumed by the domestic household itself, and therefore there will be a smaller increase in the consumption of non-tradables by the foreign household compared with the case of $\delta = 0$. It follows from imperfect substitutability of the two goods that also tradable consumption by the foreign household will also increase by less, hence the necessary increase in the foreign debt level to finance this change in the consumption pattern will be smaller. In addition, due to the increased amount of immigrants providing labour in the foreign country the wage level will have to decrease by even more than the previous case. Finally, since in the long-run there will be a reduced supply of domestic household's labour to the foreign firm which household, because $\delta > 0$, partly consumes non-tradables in the foreign country, the foreign relative prices will also have to move downwards.

Case of $\delta = 1$ In contrast to the case of a shock in the tradable endowment of the domestic country, a shock in the productivity level creates a dynamic change in the real wage level in the domestic country, and due to this the wage flexibility does not bar the domestic household to increase its labour supply abroad. In other words, the force that increasingly bars the movement of labour following a shock, as explained in section 5.1.1, could be though as being lifted due to the dynamic adjustment of real wages. Therefore, the increase that is observed for $\delta = 1$ in the amount of labour supplied abroad by the domestic household is greater than the last two cases. Nonetheless, since the domestic household is obliged to spend all its foreign-earned wage in consuming non-tradables in the foreign country, the decrease that is observed in its consumption of non-tradables is smaller compared to the previous values of δ and is associated solely with the shock in productivity. This also results in a smaller decrease in the consumption of tradables by the domestic household, compared to the other cases, and hence a smaller decrease in the household's debt level is observed.

Concerning the foreign household, the change in the consumption of non-tradables that it experiences is negligible. This is due to the domestic household spending all its foreign-earnings in consuming non-tradables in the foreign country, and in effect not increasing the amount of non-tradables that have to be consumed by the foreign household. Hence, foreign consumption of tradables and debt level remain almost unchanged.

6 Downward-rigid wages model

In this section I am assuming that wages experience downward rigidities. More specifically, wages are assumed to be fully rigid downwards, therefore only able to increase in nominal terms. However, in the dynamic analysis that follows I am only looking into exogenous shocks that pressure wages downwards and thus for reasons of simplicity, wages are modelled as constant during this section.

$$W_t = \overline{W} \text{ and } W_t^* = \overline{W}^*, \forall t$$

Due to the aforementioned (negative) exogenous shocks, and since firms are not able to optimally set the wage level due to the downward rigidities prevailing in the model, following the shocks, the labour demanded (h_t^d) by the domestic firms necessarily decreases. Due to this, the domestic household is eager to supply more labour in the foreign country which in turn results in an excess supply of labour in this country. Therefore, the demand for labour in the foreign country needs to be conveniently split between the domestic and the foreign household's supply of labour. To achieve this, the following rationing rule is followed

$$h_t^F = \theta h_t^{d*} \leq \bar{h} - h_t^H \text{ and } h_t^{H*} = (1-\theta)\, h_t^{d*} \leq \bar{h}, \text{ where } \theta \in (0,1) \tag{20}$$

where θ is the domestic household's share, of the total labour supplied to the foreign firm in the steady state,

$$\theta = \frac{\bar{h}^F}{\bar{h}^{d*}}$$

Finally, since the exogenous shocks are negative, which implies that labour is in excess supply, the labour demanded by firms is always met. On the other hand, households wish to supply more labour than the firms demand, and hence the supply of labour is suboptimal. Therefore, since following a shock households always provide less than the optimal amount of labour, they can be assumed to always meet the firms' demand for labour, using the above rationing rule whenever needed.

By taking the above rationing rule into account and also following the assumptions of section 3.4 the inter-temporal Lagrangian of the domestic household is the following

$$\mathcal{L}\left(c_t^T, c_t^{N,H}, d_{t+1}, \lambda_t\right) = \mathbb{E}_t \sum_{s=t}^{\infty} \left\{ \beta^{s-t} \left(c_s^T\right)^{\alpha} \left(c_s^{N,H} + \frac{\delta \overline{W}^* \theta h_s^{d*}}{P_s^*}\right)^{1-\alpha} \right.$$
$$\left. + \lambda_s \left[c_s^T + P_s c_s^{N,H} + d_s - \overline{W} h_s^d - (1-\delta) \overline{W}^* \theta h_t^{d*} - y_s - \frac{d_{s+1}}{1+r_s} - \Phi_s \right] \right\} \tag{21}$$

while that of the foreign household is

$$\mathcal{L}^*\left(c_t^{T*}, c_t^{N,H*}, d_{t+1}^*, \lambda_t^*\right) = \mathbb{E}_t \sum_{s=t}^{\infty} \left\{ \beta^{s-t} \left(c_s^{T*}\right)^{\alpha} \left(c_s^{N,H*}\right)^{1-\alpha} \right.$$
$$\left. + \lambda_s^* \left[c_s^{T*} + P_s^* c_s^{N,H*} + d_s^* - \overline{W}^* (1-\theta) h_t^{d*} - y_s^* - \frac{d_{s+1}^*}{1+r_s^*} - \Phi_s^* \right] \right\} \tag{22}$$

Furthermore, the profit functions of the domestic and foreign firm are,

$$\Phi_t\left(h_t^d\right) = A_t P_t \left(h_t^d\right)^{\gamma} - \overline{W} h_t^d \tag{23}$$
$$\Phi_t^*\left(h_t^{d*}\right) = A_t^* P_t^* \left(h_t^{d*}\right)^{\gamma} - \overline{W}^* h_t^{d*} \tag{24}$$

for the domestic and foreign firm respectively.

6.1 Optimality conditions

6.1.1 Households' optimisation

By solving the household's problem the relative price equations can be retrieved. Similar with the flexible wages model, these depend only on the consumption pattern of the respective household in each country. However, in this case a change in the supply of labour by the domestic household to the foreign country (h_t^F), and hence in the consumption pattern of the foreign household, depends on the demand of labour by the foreign firm.

$$P_t = \frac{(1-\alpha) c_t^T}{\alpha \left(A_t \left(h_t^d\right)^{\gamma} + \frac{\delta \overline{W}^* \theta h_t^{d*}}{P_t^*}\right)} \tag{25}$$

$$P_t^* = \frac{(1-\alpha)\, c_t^{T*}}{\alpha \left(A_t^* \left(h_t^{d*} \right)^{\gamma} - \frac{\delta \overline{W}^* \theta h_t^{d*}}{P_t^*} \right)} \tag{26}$$

Furthermore, the Euler equations and the budget constraints of the two households, can also be derived from the households optimisation

$$\left(\frac{c_t^T}{A_t \left(h_t^d \right)^{\gamma} + \frac{\delta \overline{W}^* \theta h_t^{d*}}{P_t^*}} \right)^{\alpha-1} = \beta \left(1 + r_t\right) \mathbb{E}_t \left[\left(\frac{c_{t+1}^T}{A_{t+1} \left(h_{t+1}^d \right)^{\gamma} + \frac{\delta \overline{W}^* \theta h_{t+1}^{d*}}{P_{t+1}^*}} \right)^{\alpha-1} \right] \tag{27}$$

$$\left(\frac{c_t^{T*}}{A_t^* \left(h_t^{d*} \right)^{\gamma} - \frac{\delta \overline{W}^* \theta h_t^{d*}}{P_t^*}} \right)^{\alpha-1} = \beta \left(1 + r_t^*\right) \mathbb{E}_t \left[\left(\frac{c_{t+1}^{T*}}{A_{t+1}^* \left(h_{t+1}^{d*} \right)^{\gamma} - \frac{\delta \overline{W}^* \theta h_{t+1}^{d*}}{P_{t+1}^*}} \right)^{\alpha-1} \right] \tag{28}$$

$$c_t^T + d_t = y_t + \frac{d_{t+1}}{1 + r_t} + (1-\delta)\, \overline{W}^* \theta h_t^{d*} \tag{29}$$

$$c_t^{T*} + d_t^* = y_t^* + \frac{d_{t+1}^*}{1 + r_t^*} - (1-\delta)\, \overline{W}^* \theta h_t^{d*} \tag{30}$$

6.1.2 Firms' optimisation

The two rules regarding the labour demanded by the domestic and foreign firm can be retrieved through the domestic and foreign firms' optimisation problem.

$$h_t^d = \left(\frac{\overline{W}}{\gamma A_t P_t} \right)^{\frac{1}{\gamma-1}} \text{ and } h_t^{d*} = \left(\frac{\overline{W}^*}{\gamma A_t^* P_t^*} \right)^{\frac{1}{\gamma-1}} \tag{31}$$

It can be seen by these equations, that the labour demanded only depends on the respective country's productivity and price level. Due to this, and since as described above relative prices only depend on the same economy's demand for labour, a shock in the domestic country will not spillover to the foreign country.

6.1.3 Equilibrium conditions

The equations (25)-(31) of sections 6.1.1 and 6.1.2, coupled with the four AR(1) stochastic processes (1), (2) and their symmetrical counterparts for the foreign country governing the propagation of shocks in the domestic and foreign economy, as well as the interest rate rules which result from assumption (vii) in section 3.4, fully describe this dynamic model.

6.2 Simulations

As can be seen in figures 6.1, 6.2, 6.3 and 6.4, when wages are downward rigid there is no spillover of the domestic shocks to the foreign country. The reason of this is explained in detail in section 6.1.2. In a nutshell, when prices are modelled to be rigid, the domestic economy behaves as a closed economy with a constant part of its household's labour supply provided abroad, preventing any shock to be spilled over to the foreign country since there is no change in the labour demanded by the foreign firm.

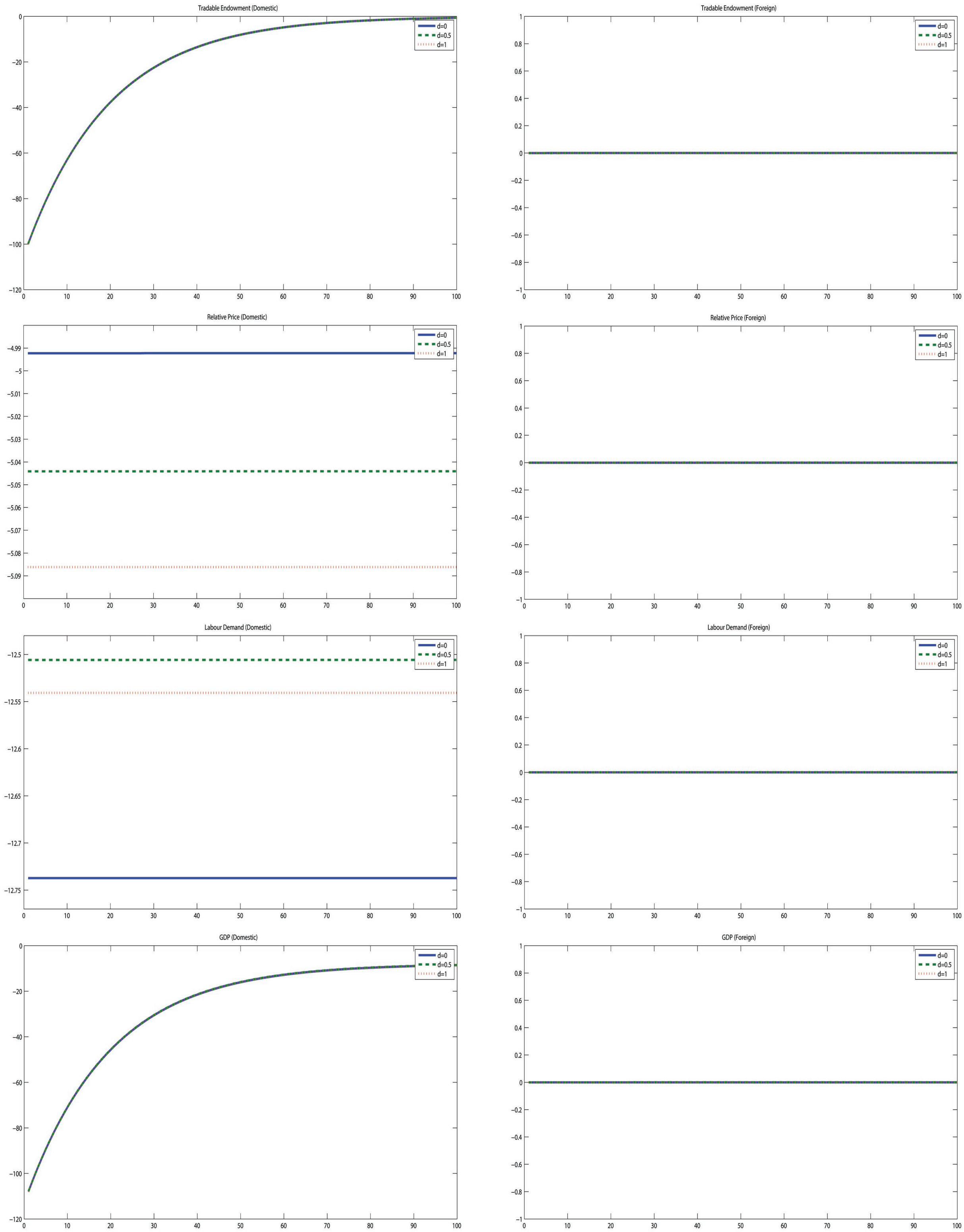
During this section only the baseline scenario of $\delta = 0.5$ will be studied since the change in the model's dynamics arising from a change in δ are identical with the flexible model, but of a smaller magnitude. To remind the reader, in the case where $\delta = 0.5$ the domestic household has to consume half of its foreign-earned income in the foreign country by consuming non-tradables.

6.2.1 Negative shock in domestic tradable endowment

Baseline case of $\delta = 0.5$ Following the shock in the domestic tradable endowment, the relative price of non-tradables decreases since the tradable good becomes more scarce. Due to this, and since the wage level cannot jump downwards, the labour demanded by the domestic firms also decreases therefore creating involuntary unemployment (equation (31)), which in turn decreases the domestic production of the tradable good. Furthermore, as explained previously the shock does not affect the foreign economy, hence there is no change in the demand for labour by the foreign firm and thus the domestic household although willing, is unable to increase its provision of labour abroad, as can be seen by the rationing rule in equation (20). Due to this, the reduction in domestic output of non-tradables also decreases the consumption of non-tradables by the domestic household, and thus due to imperfect substitutability of the two goods, tradable consumption is also reduced. Nonetheless, this new consumption pattern of the domestic household requires an increased amount of finance until the shock is diminished, resulting in an increased level of domestic debt in the long-run.

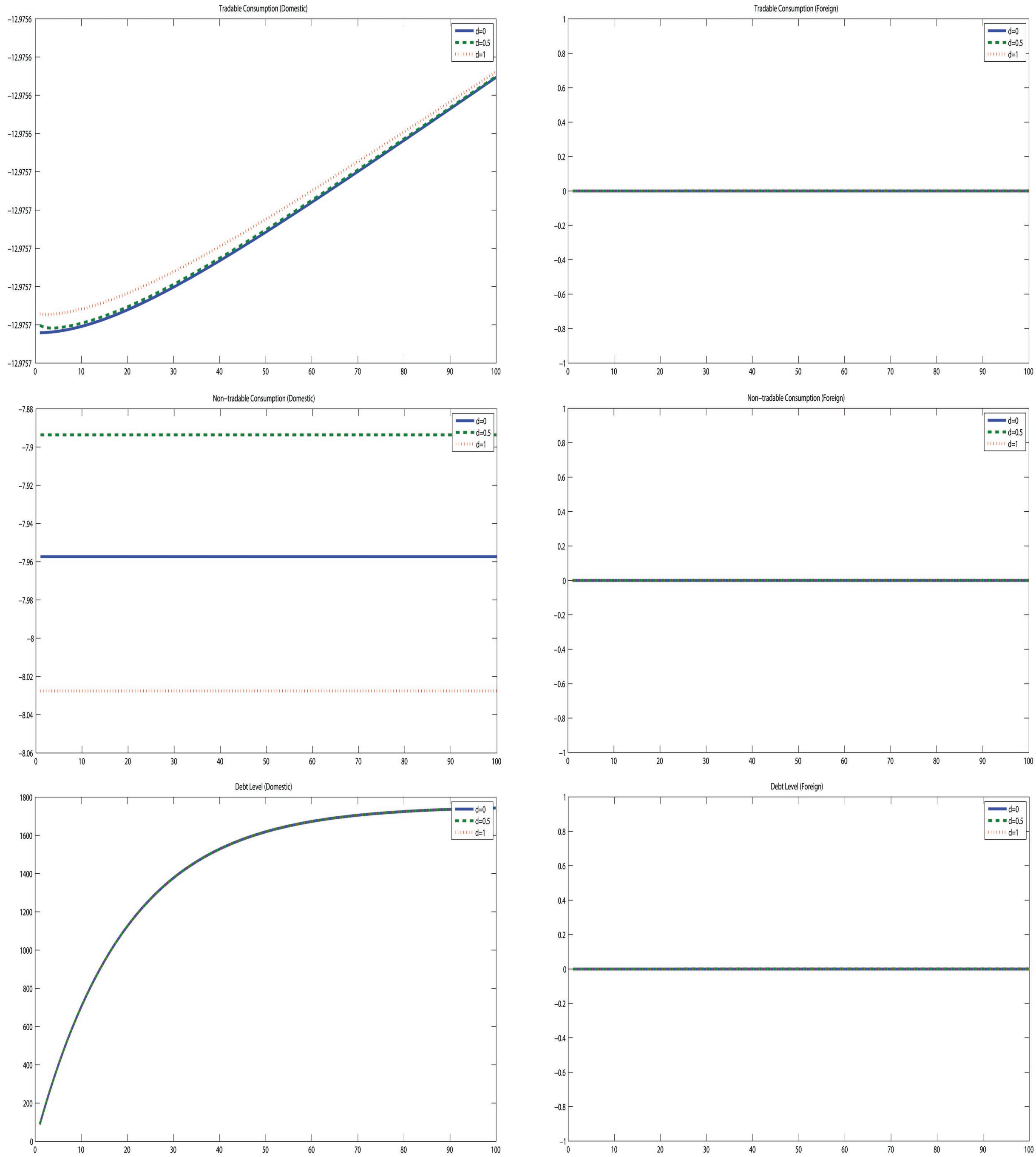
For other values of δ there are small changes in the domestic labour demand, price level and consumption of non-tradables. The reason is that first of all, the steady state amount of labour provided to the domestic firm by the domestic household depends on the value of δ, thus altering the amount of labour demand adjustments needed following the shock. Secondly, different values of δ also alter the share of income that does not have to be spent in non-tradables' consumption in the foreign country, resulting in changes in the

Figure 6.1: Shock in domestic tradable endowment (Downward-rigid wages)



steady state amount of foreign-earned income that can be saved and used in favour of the domestic household following the shock.

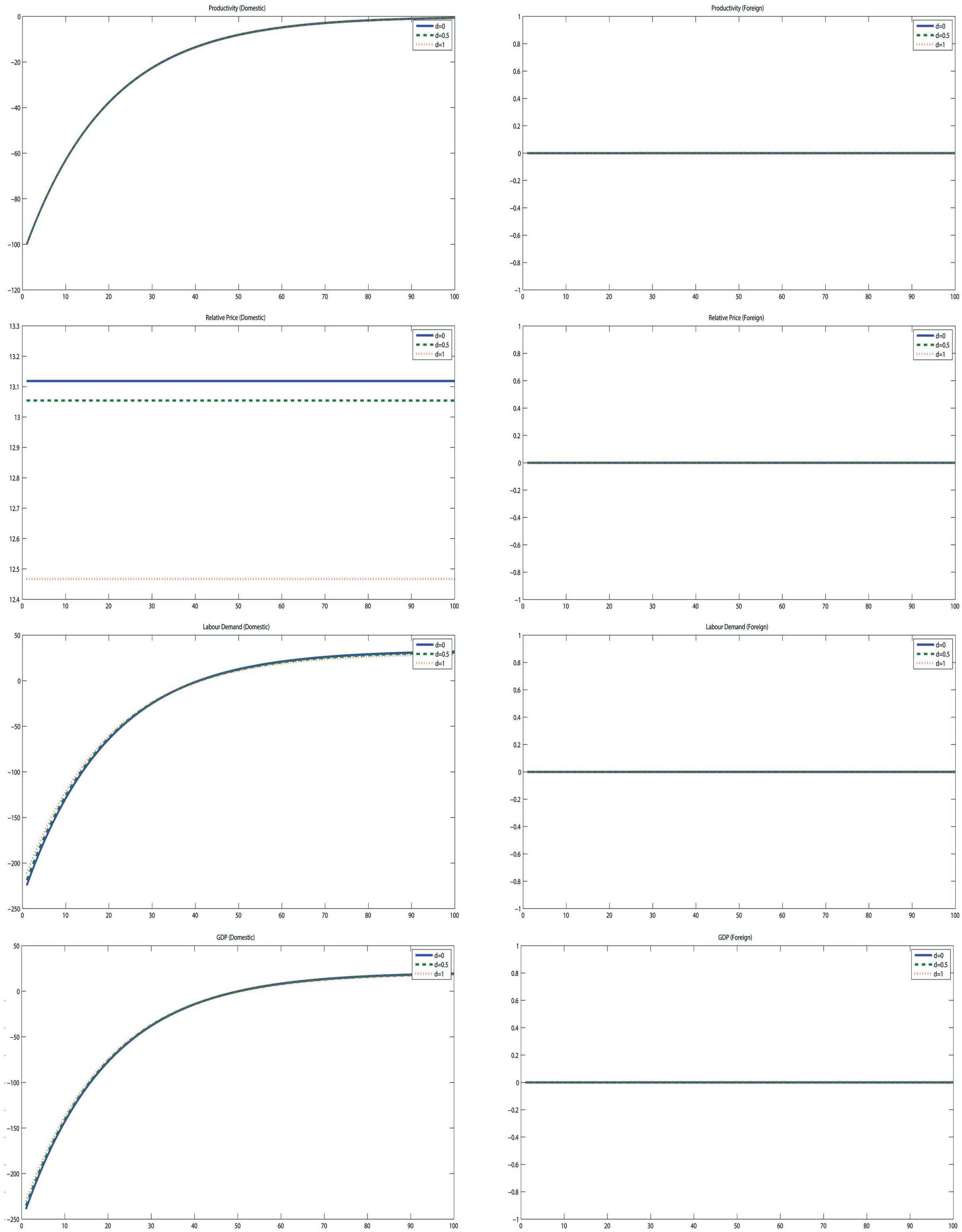
Figure 6.2: Shock in domestic tradable endowment (Downward-rigid wages)



6.2.2 Negative shock in domestic productivity

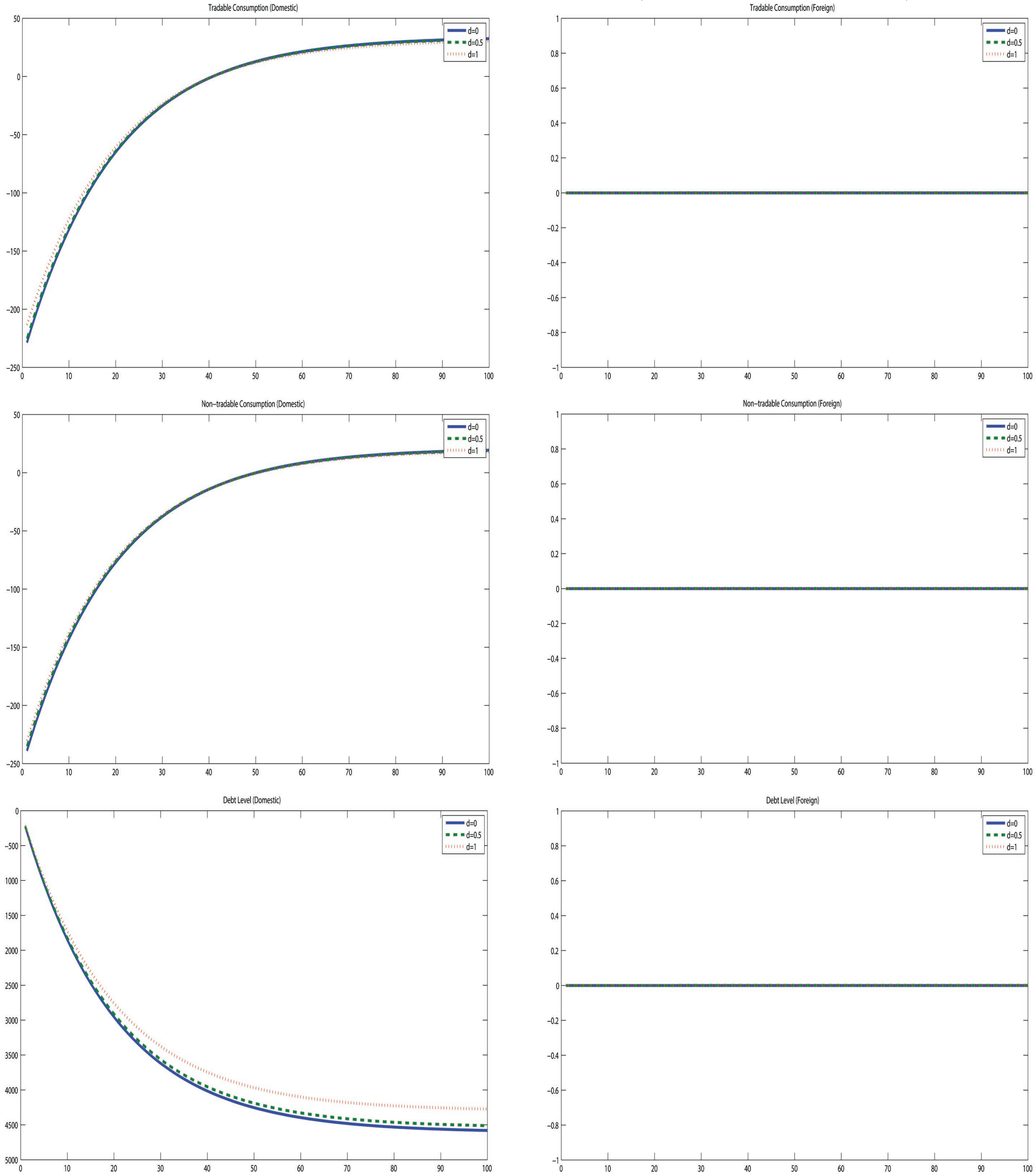
Baseline case of $\delta = 0.5$ A shock in productivity on the one hand affects positively the domestic price of non-tradables as these become relatively more scarce. Also, since the domestic productivity per labour hour supplied decreases, the firm will decrease its labour demand according to equation (31). It follows that this will decrease domestic production and hence domestic consumption of non-tradables, as explained previously in the case of a shock in the tradable endowment under downward-rigid wages. Once again,

Figure 6.3: Shock in domestic productivity (Downward-rigid wages)



the observed decrease in consumption of non-tradables will be followed by a decrease in consumption of tradables, ultimately resulting in a decrease in the debt level of the domestic household.

Figure 6.4: Shock in domestic productivity (Downward-rigid wages)



However, it should be noted that the simulation of a productivity shock should not be considered to be taking place under downward-rigid wages, but under constant wages instead. The reason is that after the economy gets into a new steady state path with decreased debt financing, it not only recovers (following the productivity shock's propagation), but also reaches a point where the wage level is unable to increase (since it is modelled as constant). This leads to an increased demand of labour which surpasses the steady-state (and upper limit) of labour supply ($\bar{h}$).

7 Discussion

7.1 Flexible wages model

As discussed in the previous chapter, a shock in the tradable endowment has a different effect on the currency union's economies than a shock in productivity. Nonetheless, the effect of δ in both countries' economies is found to alter the propagation of a shock and its spillover effects in the foreign country in two ways.

First of all, the smaller the value of δ is, the larger is the share of the 'immigrant-produced' non-tradables that the foreign household is obliged to consume. Therefore, following an equal in magnitude change in the immigration rate, a smaller value of δ will cause a greater change in the consumption pattern of the foreign household, and hence the spillover effects of the domestic shock will be larger. Also, an increased value of δ will decrease the resilience of the domestic household in domestically-produced non-tradables and therefore decrease the magnitude of occurring consumption pattern changes, following a domestic shock.

Secondly, the amount of labour movement that takes place following a shock, and which alters in magnitude the previous effects described, is also affected by δ. However, the effect δ has on labour movement depends on the type of shock occurring in the domestic economy.

In the case of a shock in the tradable endowment, the value of δ, which can be considered as the degree of immigrant 'integration' into the foreign country, is found to affect the change in the immigration rate through the following two channels.

i. As δ increases so does the share of foreign-earned income that the domestic household has to spend in non-tradables' consumption in the foreign country. This results in an alleviation of the pressures imposed on the foreign household to consume the 'immigrant-produced' non-tradables, leading in turn to a decrease in the pressures on the foreign relative price and wage level. The decreased pressures on the wage level increase the labour provided abroad by the domestic household following a shock in the domestic endowment, since now there is more 'space' for immigrants to take advantage of a wage differential.

ii. An increase in δ makes immigrants in the foreign country rely more in the foreign real wages, which due to wage flexibility are always equal in the two countries. Therefore, it decreases the willingness of domestic households to provide labour in the foreign country.

In the case of a shock in productivity real and nominal wages in the two countries cannot adjust instantly and instead have to do so over time. This in effect nullifies the second channel described above through which δ affects the change in the immigration rate. Therefore following a shock in productivity, a greater δ, will increase the amount of labour supplied abroad by the domestic household.

The above described behaviour of δ results in the following changes in the model's dynamics under flexible wages. Regarding a shock in the endowment of tradable goods, an increase of δ, or in other words a greater degree of integration of immigrants in the foreign country, seems to trigger an increase (decrease) in the amount of labour provided by the domestic household abroad, as well as in real domestic wages for small (large) values of δ. The relation is reversed when real foreign wages are concerned as an increase in δ triggers a decrease (increase) in their level for small (large) values of δ. In addition, larger values of δ reduce the magnitude of the resulting changes in consumption levels following a shock in the tradable endowment. Concerning a shock in productivity, an increased value of δ increases the immigration rate and reduces the inter-country debt differential caused by the shock, by reducing in magnitude both the decrease of the domestic country's debt and the increase in the foreign country's debt. Furthermore, it reduces the magnitude of negative (positive) changes in the domestic (foreign) country's short-term consumption levels of both goods, while at the same time reducing as well their long-term permanent displacement relative to the pre-shock levels. Finally, a larger value of δ decreases real wages by more (less) in the domestic (foreign) country.

7.2 Downward-rigid wages model

First of all, it should be mentioned that in contrast to Schmitt-Grohé and Uribe (2012), the wages are modelled to be fully rigid instead of just rigid downwards. The reason is the much greater difficulty that downward rigid wages would pose in the model's programming. Unfortunately, although the case of a shock in the tradable endowment is representative of a downward-rigid model, the case of a shock in productivity is not, since, as discussed in section 6.2.2, it leads to an increased demand for labour, above the steady state (and maximal) value of $\bar{h}$, due to the model's inability to accommodate for an increase in the wage level.

Moreover, as can be seen in figures 6.1, 6.2, 6.3 and 6.4, when downward-rigid wages are assumed, the foreign country is not at all affected by any of the domestic shocks. Therefore, the dynamics observed in the model resemble those of a closed country economy. Furthermore, the effects that δ has on the model's dynamics are only minimal and emanate from two sources.

On the one hand, as δ increases, the steady-state amount of labour provided abroad increases, providing a larger share of the total income to the domestic household, hence minimising the effects of the shock, while at the same time decreasing the amount of population that is affected by this shock. On the other hand however, an increased value of δ results in less income being transferred back to the domestic country, therefore reducing the aforementioned positive effects.

8 Conclusion

The proposed two country model, which assumes the existence of a currency union, shows a relation between the degree of immigrants' integration in the foreign society, represented as δ in the model, and the effects that a negative shock occurring in the domestic country will have in both countries, when under a flexible wages specification. Under a downward-rigid wages specification there is no change observed in the number of immigrants working abroad, and the effects of δ although existent are almost negligible.

Concerning the flexible wages specification, the effects of δ take place through two different channels. First of all, following the occurrence of a negative shock in the domestic country, the value of δ alters the resulting changes in the consumption patterns of the two households. Secondly, the value of δ affects the share of domestic labour supply that will be provided both domestically and abroad, therefore increasing or decreasing in magnitude the effects of the previous point.

Under flexible wages, a greater value of δ results in a smaller variance of the post-shock changes in both households' consumption levels, irrespective of the type of shock. Furthermore, when a negative shock in the domestic tradable endowment is concerned, an increase in the value of δ increases (decreases) the number of immigrants and the real wages domestically while decreasing (increasing) real wages in the foreign country for small (large) values of δ. Finally, following a negative shock in domestic productivity, an increase in δ enlarges the negative effects of the shock in the domestic country, reduces these in the foreign country, increases the immigration rate and reduces the inter-country post-shock debt differential.

References

A. Arpaia, E. Pérez and K. Pichelmann, *Understanding Labour Income Share Dynamics in Europe*
Economic Papers 379, European Commision, (May 2009)

Céline Poilly and Jean-Guillaume Sahuc, *Welfare Implications of Heterogeneous Labor Markets in a Currency Area*
Macroeconomic Dynamics, (2011)

Emmanuel Farhi, Gita Gopinath and Oleg Itskhoki, *Fiscal Devaluations*
Working paper, (May 2012)

Mirko Abbritti and Andreas I. Mueller, *Asymmetric Labor Market Institutions in the EMU and the Volatility of Inflation and Unemployment Differentials*
IZA Discussion Paper No. 6488, (April 2012)

Stephanie Scmitt-Grohé and Martín Uribe, *Pegs and Pain*
Working paper (August 2012)

Stephanie Scmitt-Grohé and Martín Uribe, *The Case For Temporary Inflation in the Eurozone*
Unpublished paper, (August 2012)

Appendices

A Proofs

A.A Optimisation of the flexible model

By applying equation (6) the optimality conditions derived from the Lagrangians and profit equationss (7)-(10) are the following

$$\frac{\partial \mathcal{L}}{\partial c_t^T} = 0 \Rightarrow \alpha \left(\frac{c_t^T}{c_t^{N,H} + \frac{\delta W_t^* \left(\bar{h} - h_t^H \right)}{P_t^*}} \right)^{\alpha - 1} + \lambda_t = 0$$

$$\frac{\partial \mathcal{L}}{\partial c_t^{N,H}} = 0 \Rightarrow (1-\alpha) \left(\frac{c_t^T}{c_t^{N,H} + \frac{\delta W_t^* \left(\bar{h} - h_t^H \right)}{P_t^*}} \right)^{\alpha} + \lambda_t P_t = 0$$

$$\frac{\partial \mathcal{L}}{\partial h_t^H} = 0 \Rightarrow \frac{\delta (1-\alpha)}{P_t^*} \left(\frac{c_t^T}{c_t^{N,H} + \frac{\delta W_t^* \left(\bar{h} - h_t^H \right)}{P_t^*}} \right)^{\alpha} + \lambda_t \left(\frac{W_t}{W_t^*} - (1-\delta) \right) = 0$$

$$\frac{\partial \mathcal{L}}{\partial d_{t+1}} = 0 \Rightarrow \frac{\lambda_t}{1+r_t} = \mathbb{E}_t \left[\lambda_{t+1} \right]$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = 0 \Rightarrow c_t^T + P_t c_t^{N,H} + d_t = W_t h_t^H + (1-\delta) W_t^* \left(\bar{h} - h_t^H \right) + y_t + \frac{d_{t+1}}{1+r_t} + \Phi_t$$

$$\frac{\partial \mathcal{L}^*}{\partial c_t^{T*}} = 0 \Rightarrow \alpha \left(\frac{c_t^{T*}}{c_t^{N,H*}} \right)^{\alpha-1} + \lambda_t^* = 0$$

$$\frac{\partial \mathcal{L}^*}{\partial c_t^{N,H*}} = 0 \Rightarrow (1-\alpha) \left(\frac{c_t^{T*}}{c_t^{N,H*}} \right)^{\alpha} + \lambda_t^* P_t^* = 0$$

$$\frac{\partial \mathcal{L}^*}{\partial d_{t+1}^*} = 0 \Rightarrow \frac{\lambda_t^*}{1+r_t^*} = \mathbb{E}_t \left[\lambda_{t+1}^* \right]$$

$$\frac{\partial \mathcal{L}^*}{\partial \lambda_t^*} = 0 \Rightarrow c_t^{T*} + P_t^* c_t^{N,H*} + d_t^* = W_t^* \bar{h} + y_t^* + \frac{d_{t+1}^*}{1+r_t^*} + \Phi_t^*$$

$$\frac{\partial \Phi_t}{\partial h_t^d} = 0 \Rightarrow W_t = \gamma A_t P_t \left(h_t^d \right)^{\gamma-1}$$

$$\frac{\partial \Phi_t^*}{\partial h_t^{d*}} = 0 \Rightarrow W_t^* = \gamma A_t^* P_t^* \left(h_t^{d*}\right)^{\gamma-1}$$

Furthermore, equation (6), coupled with assumption (v) of section 3.4 and since wages are flexible leads to

$$h_t^d = h_t^H \text{ and } h_t^{d*} = \left(\bar{h} - h_t^H\right) + h_t^{H*} = 2\bar{h} - h_t^H$$

which when combined with assumption (vi) and equation (5), section 3.4, leads to

$$c_t^N = c_t^{N,H} + c_t^{N,F} = A_t \left(h_t^H\right)^\gamma + \frac{\delta W_t^* \left(\bar{h} - h_t^H\right)}{P_t^*}$$

$$c_t^{N*} = c_t^{N,H*} = A_t^* \left(2\bar{h} - h_t^H\right)^\gamma - \frac{\delta W_t^* \left(\bar{h} - h_t^H\right)}{P_t^*}$$

Also, by taking into account the above result for labour demand, one can express the profit functions (9) and (10) as

$$\Phi_t = A_t P_t \left(h_t^H\right)^\gamma - W_t h_t^H \text{ and } \Phi_t^* = A_t^* P_t^* \left(2\bar{h} - h_t^H\right)^\gamma - W_t^* \left(2\bar{h} - h_t^H\right)$$

for the domestic and foreign firms respectively.

By substituting these results where applicable in the above optimality conditions, one results in the following set of equations.

$$\alpha \left(\frac{c_t^T}{A_t \left(h_t^H\right)^\gamma + \frac{\delta W_t^* \left(\bar{h} - h_t^H\right)}{P_t^*}}\right)^{\alpha-1} + \lambda_t = 0 \tag{32}$$

$$(1-\alpha) \left(\frac{c_t^T}{A_t \left(h_t^H\right)^\gamma + \frac{\delta W_t^* \left(\bar{h} - h_t^H\right)}{P_t^*}}\right)^{\alpha} + \lambda_t P_t = 0 \tag{33}$$

$$\frac{\delta (1-\alpha)}{P_t^*} \left(\frac{c_t^T}{A_t \left(h_t^H\right)^\gamma + \frac{\delta W_t^* \left(\bar{h} - h_t^H\right)}{P_t^*}}\right)^{\alpha} + \lambda_t \left(\frac{W_t}{W_t^*} - (1-\delta)\right) = 0 \tag{34}$$

$$\frac{\lambda_t}{1+r_t} = \mathbb{E}_t \left[\lambda_{t+1}\right] \tag{35}$$

$$c_t^T + d_t = (1-\delta) W_t^* \left(\bar{h} - h_t^H\right) + y_t + \frac{d_{t+1}}{1+r_t} \tag{36}$$

$$\alpha \left(\frac{c_t^{T*}}{A_t^* \left(2\bar{h} - h_t^H\right)^\gamma - \frac{\delta W_t^* \left(\bar{h} - h_t^H\right)}{P_t^*}}\right)^{\alpha-1} + \lambda_t^* = 0 \tag{37}$$

$$(1-\alpha)\left(\frac{c_t^{T*}}{A_t^*\left(2\bar{h}-h_t^H\right)^{\gamma}-\frac{\delta W_t^*\left(\bar{h}-h_t^H\right)}{P_t^*}}\right)^{\alpha}+\lambda_t^* P_t^*=0 \tag{38}$$

$$\frac{\lambda_t^*}{1+r_t^*}=\mathbb{E}_t\left[\lambda_{t+1}^*\right] \tag{39}$$

$$c_t^{T*}+(1-\delta)\,W_t^*\left(\bar{h}-h_t^H\right)+d_t^*=y_t^*+\frac{d_{t+1}^*}{1+r_t^*} \tag{40}$$

$$W_t=\gamma A_t P_t\left(h_t^H\right)^{\gamma-1} \tag{41}$$

$$W_t^*=\gamma A_t^* P_t^*\left(2\bar{h}-h_t^H\right)^{\gamma-1} \tag{42}$$

By combining equations (32), (33) and (37), (38) one can retrieve the optimality conditions for the relative price of non-tradables in the domestic and foreign economy. These are respectively

$$P_t=\frac{(1-\alpha)\,c_t^T}{\alpha\left(A_t\left(h_t^H\right)^{\gamma}+\frac{\delta W_t^*\left(\bar{h}-h_t^H\right)}{P_t^*}\right)}$$

$$P_t^*=\frac{(1-\alpha)\,c_t^{T*}}{\alpha\left(A_t^*\left(2\bar{h}-h_t^H\right)^{\gamma}-\frac{\delta W_t^*\left(\bar{h}-h_t^H\right)}{P_t^*}\right)}$$

Furthermore, by combining equations (32) for the domestic household and (37) for the foreign household, along with the respective optimality conditions of tradable goods one time period ahead $(t+1)$ and equations (35) and (39), I arrive at the Euler equations for both economies. These are the following.

$$\left(\frac{c_t^T}{A_t\left(h_t^H\right)^{\gamma}+\frac{\delta W_t^*\left(\bar{h}-h_t^H\right)}{P_t^*}}\right)^{\alpha-1}=\beta\left(1+r_t\right)\mathbb{E}_t\left[\left(\frac{c_{t+1}^T}{A_{t+1}\left(h_{t+1}^H\right)^{\gamma}+\frac{\delta W_{t+1}^*\left(\bar{h}-h_{t+1}^H\right)}{P_{t+1}^*}}\right)^{\alpha-1}\right]$$

$$\left(\frac{c_t^{T*}}{A_t^*\left(2\bar{h}-h_t^H\right)^{\gamma}-\frac{\delta W_t^*\left(\bar{h}-h_t^H\right)}{P_t^*}}\right)^{\alpha-1}=\beta\left(1+r_t^*\right)\mathbb{E}_t\left[\left(\frac{c_{t+1}^{T*}}{A_{t+1}^*\left(2\bar{h}-h_{t+1}^H\right)^{\gamma}-\frac{\delta W_{t+1}^*\left(\bar{h}-h_{t+1}^H\right)}{P_{t+1}^*}}\right)^{\alpha-1}\right]$$

Finally, by combining equations (33) and (34) the no arbitrage rule can be found

$$\frac{W_t}{P_t} = \delta \left(\frac{W_t^*}{P_t^*} \right) + (1-\delta) \left(\frac{W_t^*}{P_t} \right)$$

A.B Optimisation of the downward-rigid model

By taking rule (20) into account the optimality conditions derived from the Lagrangian and profit equations (21)-(24) are the following

$$\frac{\partial \mathcal{L}}{\partial c_t^T} = 0 \Rightarrow \alpha \left(\frac{c_t^T}{c_t^{N,H} + \frac{\delta \overline{W}^* \theta h_t^{d*}}{P_t^*}} \right)^{\alpha-1} + \lambda_t = 0$$

$$\frac{\partial \mathcal{L}}{\partial c_t^{N,H}} = 0 \Rightarrow (1-\alpha) \left(\frac{c_t^T}{c_t^{N,H} + \frac{\delta \overline{W}^* \theta h_t^{d*}}{P_t^*}} \right)^{\alpha} + \lambda_t P_t = 0$$

$$\frac{\partial \mathcal{L}}{\partial d_{t+1}} = 0 \Rightarrow \frac{\lambda_t}{1+r_t} = \mathbb{E}_t \left[\lambda_{t+1} \right]$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = 0 \Rightarrow c_t^T + P_t c_t^{N,H} + d_t = \overline{W} h_t^d + (1-\delta) \overline{W}^* \theta h_t^{d*} + y_t + \frac{d_{t+1}}{1+r_t} + \Phi_t$$

$$\frac{\partial \mathcal{L}^*}{\partial c_t^{T*}} = 0 \Rightarrow \alpha \left(\frac{c_t^{T*}}{c_t^{N,H*}} \right)^{\alpha-1} + \lambda_t^* = 0$$

$$\frac{\partial \mathcal{L}^*}{\partial c_t^{N,H*}} = 0 \Rightarrow (1-\alpha) \left(\frac{c_t^{T*}}{c_t^{N,H*}} \right)^{\alpha} + \lambda_t^* P_t^* = 0$$

$$\frac{\partial \mathcal{L}^*}{\partial d_{t+1}^*} = 0 \Rightarrow \frac{\lambda_t^*}{1+r_t^*} = \mathbb{E}_t \left[\lambda_{t+1}^* \right]$$

$$\frac{\partial \mathcal{L}^*}{\partial \lambda_t^*} = 0 \Rightarrow c_t^{T*} + P_t^* c_t^{N,H*} + d_t^* = \overline{W}^* (1-\theta) h_t^{d*} + y_t^* + \frac{d_{t+1}^*}{1+r_t^*} + \Phi_t^*$$

$$\frac{\partial \Phi_t}{\partial h_t^d} = 0 \Rightarrow h_t^d = \left(\frac{\overline{W}}{\gamma A_t P_t} \right)^{\frac{1}{\gamma-1}}$$

$$\frac{\partial \Phi_t^*}{\partial h_t^{d*}} = 0 \Rightarrow h_t^{d*} = \left(\frac{\overline{W}^*}{\gamma A_t^* P_t^*} \right)^{\frac{1}{\gamma-1}}$$

Furthermore, assumption (vi) and equation (5) of section 3.4, lead to

$$c_t^N = c_t^{N,H} + c_t^{N,F} = A_t \left(h_t^d\right)^\gamma + \frac{\delta \overline{W}^* \theta h_t^{d*}}{P_t^*}$$

$$c_t^{N*} = c_t^{N,H*} = A_t^* \left(h_t^{d*}\right)^\gamma - \frac{\delta \overline{W}^* \theta h_t^{d*}}{P_t^*}$$

and by substituting where applicable the above result, one can derive the following set of equations.

$$\alpha \left(\frac{c_t^T}{A_t \left(h_t^d\right)^\gamma + \frac{\delta \overline{W}^* \theta h_t^{d*}}{P_t^*}} \right)^{\alpha-1} + \lambda_t = 0 \tag{43}$$

$$(1-\alpha) \left(\frac{c_t^T}{A_t \left(h_t^d\right)^\gamma + \frac{\delta \overline{W}^* \theta h_t^{d*}}{P_t^*}} \right)^{\alpha} + \lambda_t P_t = 0 \tag{44}$$

$$\frac{\lambda_t}{1+r_t} = \mathbb{E}_t \left[\lambda_{t+1}\right] \tag{45}$$

$$c_t^T + d_t = y_t + \frac{d_{t+1}}{1+r_t} + (1-\delta)\, \overline{W}^* \theta h_t^{d*} \tag{46}$$

$$\alpha \left(\frac{c_t^{T*}}{A_t^* \left(h_t^{d*}\right)^\gamma - \frac{\delta \overline{W}^* \theta h_t^{d*}}{P_t^*}} \right)^{\alpha-1} + \lambda_t^* = 0 \tag{47}$$

$$(1-\alpha) \left(\frac{c_t^{T*}}{A_t^* \left(h_t^{d*}\right)^\gamma - \frac{\delta \overline{W}^* \theta h_t^{d*}}{P_t^*}} \right)^{\alpha} + \lambda_t^* P_t^* = 0 \tag{48}$$

$$\frac{\lambda_t^*}{1+r_t^*} = \mathbb{E}_t \left[\lambda_{t+1}^*\right] \tag{49}$$

$$c_t^{T*} + d_t^* = y_t^* + \frac{d_{t+1}^*}{1+r_t^*} - (1-\delta)\, \overline{W}^* \theta h_t^{d*} \tag{50}$$

$$h_t^d = \left(\frac{\overline{W}}{\gamma A_t P_t} \right)^{\frac{1}{\gamma-1}} \tag{51}$$

$$h_t^{d*} = \left(\frac{\overline{W}^*}{\gamma A_t^* P_t^*} \right)^{\frac{1}{\gamma-1}} \tag{52}$$

By combining equations (43), (44) and (47), (48) one can retrieve the optimality conditions for the relative price of non-tradables in the domestic and foreign economy. These are respectively

$$P_t = \frac{(1-\alpha)\, c_t^T}{\alpha \left(A_t \left(h_t^d\right)^\gamma + \frac{\delta \overline{W}^* \theta h_t^{d*}}{P_t^*} \right)}$$

$$P_t^* = \frac{(1-\alpha)\, c_t^{T*}}{\alpha \left(A_t^* \left(h_t^{d*}\right)^\gamma - \frac{\delta \overline{W}^* \theta h_t^{d*}}{P_t^*} \right)}$$

Furthermore, by combining equations (43) for the domestic household and (47) for the foreign household, along with the respective optimality conditions of tradable goods one time period ahead $(t+1)$ and equations (45) and (49), I arrive at the Euler equations for both economies. These are the following.

$$\left(\frac{c_t^T}{A_t \left(h_t^d\right)^\gamma + \frac{\delta \overline{W}^* \theta h_t^{d*}}{P_t^*}} \right)^{\alpha-1} = \beta \left(1+r_t\right) \mathbb{E}_t \left[\left(\frac{c_{t+1}^T}{A_{t+1} \left(h_{t+1}^d\right)^\gamma + \frac{\delta \overline{W}^* \theta h_{t+1}^{d*}}{P_{t+1}^*}} \right)^{\alpha-1} \right]$$

$$\left(\frac{c_t^{T*}}{A_t^* \left(h_t^{d*}\right)^\gamma - \frac{\delta \overline{W}^* \theta h_t^{d*}}{P_t^*}} \right)^{\alpha-1} = \beta \left(1+r_t^*\right) \mathbb{E}_t \left[\left(\frac{c_{t+1}^{T*}}{A_{t+1}^* \left(h_{t+1}^{d*}\right)^\gamma - \frac{\delta \overline{W}^* \theta h_{t+1}^{d*}}{P_{t+1}^*}} \right)^{\alpha-1} \right]$$

A.C Steady State calculation of the flexible model for $\delta = 0$

To calculate the steady state, to be fed to Dynare's computation, I assume that domestic households, while allowed to work abroad, are barred from consuming in the foreign country ($\delta = 0$). The reason behind this is first of all to greatly reduce the difficulty of calculations and secondly, due to the fact that Dynare only requires to be given values close to the actual steady state values of the variables, but not necessarily the actual ones.

In the steady state, $y_t = \bar{y}$, $y_t^* = \bar{y}^*$, $A_t = \overline{A}$ and $A_t^* = \overline{A}^*$ from equations (1) and (2). In addition, from assumption (iii) of section 3.4, $\overline{A} = \overline{A}^*$ since $\eta = 1$. Furthermore, since in the steady state $d_t = \bar{d}$ and $d_t^* = \bar{d}^*$,

$$1 + r_t = 1 + r_t^* = 1 + \bar{r} = \frac{1}{\beta}$$

due to assumption (vii) of section 3.4.

Before proceeding, I supply below for the readers convenience the seven remaining equations, after applying the steady state values mentioned above, that will be used to calculate the steady state values of the remaining seven control variables.

$$\bar{c}^{T} + (1-\beta)\,\bar{d} = \bar{y} + \overline{W}^{*}\left(\bar{h} - \bar{h}^{H}\right) \tag{53}$$

$$\bar{c}^{T*} + (1-\beta)\,\bar{d}^{*} = \bar{y}^{*} - \overline{W}^{*}\left(\bar{h} - \bar{h}^{H}\right) \tag{54}$$

$$\overline{P} = \frac{(1-\alpha)\,\bar{c}^{T}}{\alpha\overline{A}\left(\bar{h}^{H}\right)^{\gamma}} \tag{55}$$

$$\overline{P}^{*} = \frac{(1-\alpha)\,\bar{c}^{T*}}{\alpha\eta\overline{A}\left(2\bar{h} - \bar{h}^{H}\right)^{\gamma}} \tag{56}$$

$$\frac{\overline{W}}{\overline{W}^{*}} = 1 \tag{57}$$

$$\overline{W} = \gamma\overline{A}\overline{P}\left(\bar{h}^{H}\right)^{\gamma-1} \tag{58}$$

$$\overline{W}^{*} = \gamma\eta\overline{A}\overline{P}^{*}\left(2\bar{h} - \bar{h}^{H}\right)^{\gamma-1} \tag{59}$$

First of all, it is straightforward to see from equation (57) that

$$\overline{W} = \overline{W}^{*}$$

Then by equating equations (58) and (59), one can get

$$\frac{\overline{P}}{\overline{P}^{*}} = \eta\left(\frac{2\bar{h} - \bar{h}^{H}}{\bar{h}^{H}}\right)^{\gamma-1}$$

This relation combined with equations (55) and (56) results in

$$\frac{\bar{c}^{T}}{\bar{c}^{T*}} = \left(\frac{\bar{h}^{H}}{2\bar{h} - \bar{h}^{H}}\right)$$

which when used with (53) and (54) leads to

$$\bar{c}^{T*} = \left(\frac{2\bar{h} - \bar{h}^H}{2\bar{h}}\right)\left(\bar{y} + \bar{y}^* - (1-\beta)\left(\bar{d} + \bar{d}^*\right)\right)$$

Applying this result in equations (54) and (55) respectively produces

$$\overline{W} = \overline{W}^* = \frac{\left(\bar{y}^* - (1-\beta)\,\bar{d}^*\right)\bar{h}^H - \left(\bar{y} - (1-\beta)\,\bar{d}\right)\left(2\bar{h} - \bar{h}^H\right)}{2\bar{h}\left(\bar{h} - \bar{h}^H\right)}$$

and

$$\overline{P}^* = \left(\frac{1-\alpha}{2\alpha\eta}\right)\left(\frac{\left(2\bar{h} - \bar{h}^H\right)^{1-\gamma}}{\overline{A}\bar{h}}\right)\left(\bar{y} + \bar{y}^* - (1-\beta)\left(\bar{d} + \bar{d}^*\right)\right)$$

Finally by using these values in equation (59) the fully parametrised function for $\bar{h}^H$ can be retrieved

$$\bar{h}^H = \left(\frac{\left(\bar{y} - (1-\beta)\,\bar{d}\right)(2\alpha + \gamma(1-\alpha)) + \gamma(1-\alpha)\left(\bar{y}^* - (1-\beta)\,\bar{d}^*\right)}{\left(\bar{y} + \bar{y}^* - (1-\beta)\left(\bar{d} + \bar{d}^*\right)\right)(\alpha + \gamma(1-\alpha))}\right)\bar{h}$$

The parametrised steady state values of the rest of the variables follow.

$$\overline{W} = \overline{W}^* = \frac{\gamma(1-\alpha)\left(\bar{y} + \bar{y}^* - (1-\beta)\left(\bar{d} + \bar{d}^*\right)\right)}{2\alpha\bar{h}}$$

$$\overline{P} = \left(\frac{(1-\alpha)}{2\alpha\overline{A}\bar{h}^\gamma}\right)\left(\frac{(2\alpha + \gamma(1-\alpha))\left(\bar{y} - (1-\beta)\,\bar{d}\right) + \gamma(1-\alpha)\left(\bar{y}^* - (1-\beta)\,\bar{d}^*\right)}{(\alpha + \gamma(1-\alpha))\left(\bar{y} + \bar{y}^* - (1-\beta)\left(\bar{d} + \bar{d}^*\right)\right)^{\frac{\gamma}{\gamma-1}}}\right)^{1-\gamma}$$

$$\overline{P}^* = \left(\frac{(1-\alpha)}{2\alpha\eta\overline{A}\bar{h}^\gamma}\right)\left(\frac{(2\alpha + \gamma(1-\alpha))\left(\bar{y}^* - (1-\beta)\,\bar{d}^*\right) + \gamma(1-\alpha)\left(\bar{y} - (1-\beta)\,\bar{d}\right)}{(\alpha + \gamma(1-\alpha))\left(\bar{y} + \bar{y}^* - (1-\beta)\left(\bar{d} + \bar{d}^*\right)\right)^{\frac{\gamma}{\gamma-1}}}\right)^{1-\gamma}$$

$$\bar{c}^T = \frac{(2\alpha + \gamma(1-\alpha))\left(\bar{y} - (1-\beta)\,\bar{d}\right) + \gamma(1-\alpha)\left(\bar{y}^* - (1-\beta)\,\bar{d}^*\right)}{2(\alpha + \gamma(1-\alpha))}$$

$$\bar{c}^{T*} = \frac{(2\alpha + \gamma(1-\alpha))\left(\bar{y}^* - (1-\beta)\,\bar{d}^*\right) + \gamma(1-\alpha)\left(\bar{y} - (1-\beta)\,\bar{d}\right)}{2(\alpha + \gamma(1-\alpha))}$$

B Dynare's pre-ample text files

The simulation of the model presented in this paper was the outcome of 12 different text files that were used in Dynare. Half of them were used for the flexible model and the rest for the downward-rigid wages model. Out of the 6 different text files concerning each model, 3 introduced a shock in the tradable endowment for the 3 different values of δ (0, 0.5 and 1), while the other 3 introduced a productivity shock for the same 3 values of δ. To save space, here I present only two of the text files presented, one for the flexible model when $\delta = 0$, introducing a tradable endowment shock, and one for the downward-rigid model for $\delta = 0.5$, introducing a shock in productivity. The other 5 text files that are not present regarding the flexible model are almost identical with the one provided, the only difference being the change in the value of δ itself and/or the shock's source and magnitude, so as to ultimately result in an identical shock in the domestic GDP (as stated in section 4). Concerning the 5 non-present text files of the downward-rigid model, apart from having the same slight differences with the flexible model, they also differ in the steady-state variables' values. These values can be retrieved from Dynare's output after running the respective flexible model's text file.

B.A Flexible model, shock in endowment ($\delta = 0$)

```
// 1. Declare variables and parameters
// ----------------------------------

var  y ystar a astar r rstar d dstar p pstar h c cstar w wstar hstar cn
 cnstar gdp gdpstar real realstar;
varexo e estar z zstar;
parameters ybar ybarstar rho1 rho2 abar abarstar beta dbar dbarstar
 kappa alpha delta hbar eta gamma ;

// 2. Calibrate parameters values
// ----------------------------------

eta         = 1.06;
ybar        = 1.00;
ybarstar    = eta*ybar;
rho1        = 0.95;
rho2        = 0.95;
abar     = 1.00;
```

```
beta    = 0.9926;
dbar       = 2.21;
dbarstar   = 0.84;
kappa      = 0.0000000001;
alpha      = 0.50;
delta    = 0.000000000;
hbar       = 1.00;
gamma      = 0.62;
abarstar   = eta*abar;

// 3. Declare the model's dynamics
// -----------------------------------

model;

y=ybar+rho1*(y(-1)-ybar)-e;
ystar=ybarstar+rho1*(ystar(-1)-ybarstar)-estar;
a=abar+rho2*(a(-1)-abar)-z;
astar=abarstar+rho2*(astar(-1)-abarstar)-zstar;

r=(1/beta)*(d/dbar)^kappa;
rstar=(1/beta)*(dstar/dbarstar)^kappa;

p=((1-alpha)*c)/(alpha*(a*((h)^gamma)+((delta*wstar*(hbar-h))/(pstar))));
pstar=((1-alpha)*cstar)/(alpha*(astar*(2*hbar-h)^gamma-(delta*wstar*
(hbar-h))/(pstar)));

(c/(a*h^gamma+(delta*wstar*(hbar-h))/pstar))^(alpha-1)=beta*r*((c(1))/
((a(1))*(h(1))^gamma+(delta*(wstar(1))*(hbar-(h(1))))/(pstar(1))))^
(alpha-1);
(cstar/(astar*(2*hbar-h)^gamma-(delta*wstar*(hbar-h))/pstar))^(alpha-1)
=beta*rstar*((cstar(1))/((astar(1))*(2*hbar-h(1))^gamma-(delta*
(wstar(1))*(hbar-(h(1))))/(pstar(1))))^(alpha-1);

w/wstar=(delta*p)/pstar+(1-delta);

h=(w/(gamma*a*p))^(1/(gamma-1));
2*hbar-h=(wstar/(gamma*astar*pstar))^(1/(gamma-1));
```

```
c+d(-1)=(1-delta)*wstar*(hbar-h)+y+d/r;
cstar+dstar(-1)=(delta-1)*wstar*(hbar-h)+ystar+dstar/r;

hstar=2*hbar-h;
cn=a*h^gamma+(delta*wstar*(hbar-h))/pstar;
cnstar=astar*(2*hbar-h)^gamma-(delta*wstar*(hbar-h))/pstar;

gdp=y+a*h^gamma;
gdpstar=ystar+astar*hstar^gamma;
real=p/w;
realstar=pstar/wstar;

end;

// 4. Define initial (steady-state) values
// ---------------------------------------

initval;

a=abar;
astar=abarstar;
y=ybar;
ystar=ybarstar;
d=dbar;
dstar=dbarstar;

r=(1/beta);
rstar=r;

h=hbar*((((ybar-(1-beta)*dbar)*(2*alpha+gamma*(1-alpha)*(ybarstar-
(1-beta)*dbarstar))))/((ybar+ybarstar-(1-beta)*(dbar+dbarstar))*
(alpha+gamma*(1-alpha))));

c=((2*alpha+gamma*(1-alpha))*(ybar-(1-beta)*dbar)+gamma*(1-alpha)*
(ybarstar-(1-beta)*dbarstar))/(2*(alpha+gamma*(1-alpha)));
cstar=((2*alpha+gamma*(1-alpha))*(ybarstar-(1-beta)*dbarstar)+gamma*
(1-alpha)*(ybar-(1-beta)*dbar))/(2*(alpha+gamma*(1-alpha)));

p=((1-alpha)/(2*alpha*abar*hbar^gamma))*((((2*alpha+gamma*(1-alpha))
*(ybar-(1-beta)*dbar)+gamma*(1-alpha)*(ybarstar-(1-beta)*dbarstar))
```

```
/((alpha+gamma*(1-alpha))*(ybar+ybarstar-(1-beta)*(dbar+dbarstar))^
(gamma/(gamma-1))))^(1-gamma);
pstar=((1-alpha)/(2*alpha*abar*hbar^gamma))*((((2*alpha+gamma
*(1-alpha))*(ybarstar-(1-beta)*dbarstar)+gamma*(1-alpha)*(ybar
-(1-beta)*dbar)))/((alpha+gamma*(1-alpha))*(ybar+ybarstar-
(1-beta)*(dbar+dbarstar))^(gamma/(gamma-1))))^(1-gamma);

w=(gamma*(1-alpha)*(ybar+ybarstar-(1-beta)*(dbar+dbarstar)))/
(2*alpha*hbar);
wstar=w;

hstar=2*hbar-h;
cn=abar*h^gamma+(delta*wstar*(hbar-h))/pstar;
cnstar=abarstar*(2*hbar-h)^gamma-(delta*wstar*(hbar-h))/pstar;
gdp=ybar+abar*h^gamma;
gdpstar=ybarstar+abarstar*hstar^gamma;
real=p/w;
realstar=pstar/wstar;

end;

steady;

// 5. Simulate the stochastic model
// ---------------------------------------

shocks;
var e=0.061;

end;

check;
stoch_simul(order=1,irf=100, relative_irf);
```

B.B Downward-rigid wages model, shock in productivity ($\delta = 0.5$)

```
// 1. Declare variables and parameters
// ----------------------------------
```

```
var  y ystar a astar r rstar d dstar p pstar h c cstar hstar cn
 cnstar gdp gdpstar;
varexo e estar z zstar;
parameters ybar ybarstar rho1 rho2 abar abarstar beta dbar
dbarstar kappa alpha delta hbar eta gamma w wstar theta;

// 2. Calibrate parameters values
// ----------------------------------

eta        = 1.06;
ybar       = 1.00;
ybarstar   = eta*ybar;
rho1       = 0.95;
rho2       = 0.95;
abar    = 1.00;
beta    = 0.9926;
dbar       = 2.21;
dbarstar   = 0.84;
kappa      = 0.0000000001;
alpha      = 0.50;
delta    = 0.5;
hbar       = 1.00;
gamma      = 0.62;
abarstar   = eta*abar;
w          = 0.639142;
wstar      = 0.624326;
theta      = 1-0.953518;

// 3. Declare the model's dynamics
// -----------------------------------

model;

y=ybar+rho1*(y(-1)-ybar)-e;
ystar=ybarstar+rho1*(ystar(-1)-ybarstar)-estar;
a=abar+rho2*(a(-1)-abar)-z;
astar=abarstar+rho2*(astar(-1)-abarstar)-zstar;

r=(1/beta)*(d/dbar)^kappa;
```

```
rstar=(1/beta)*(dstar/dbarstar)^kappa;

p=((1-alpha)*c)/(alpha*(a*((h)^gamma)+((delta*wstar*theta
*hstar)/(pstar))));
pstar*(alpha*(astar*(hstar)^gamma-(delta*wstar*theta*hstar)/
(pstar)))=((1-alpha)*cstar);

(c/(a*h^gamma+(delta*wstar*theta*hstar)/pstar))^(alpha-1)=beta*
r*((c(1))/((a(1))*(h(1))^gamma+(delta*(wstar(1))*theta*hstar(1)
)/(pstar(1))))^(alpha-1);

(cstar/(astar*hstar^gamma-(delta*wstar*theta*hstar)/pstar))^(al
pha-1)=beta*rstar*((cstar(1))/(((astar(1))*hstar(1)^gamma-(del
ta*(wstar(1))*theta*hstar(1)))/(pstar(1))))^(alpha-1);

c+d(-1)=(1-delta)*wstar*theta*hstar+y+d/r;
cstar+dstar(-1)=(delta-1)*wstar*theta*hstar+ystar+dstar/r;

h=(w/(gamma*a*p))^(1/(gamma-1));
hstar=(wstar/(gamma*astar*pstar))^(1/(gamma-1));

cn=a*h^gamma+(delta*wstar*theta*hstar)/pstar;
cnstar=astar*(1-theta)*hstar-(delta*wstar*theta*hstar)/pstar;

gdp=y+a*h^gamma;
gdpstar=ystar+astar*hstar^gamma;

end;

// 4. Define initial (steady-state) values
// ---------------------------------------

initval;

a=abar;
astar=abarstar;
y=ybar;
ystar=ybarstar;

d=dbar;
```

```
dstar=dbarstar;

r=(1/beta);
rstar=r;

h=0.953518;
c=0.998156;
cstar=1.03927;
p=1.0124;
pstar=0.966523;
hstar=2-0.953518;

cn=abar*h^gamma+(delta*wstar*theta*hstar)/pstar;
cnstar=abarstar*(1-theta)*hstar-(delta*wstar*theta*hstar)
/pstar;
gdp=ybar+abar*h^gamma;
gdpstar=ybarstar+abarstar*hstar^gamma;

end;

steady;

// 5. Simulate the stochastic model
// ----------------------------------------

shocks;
var z=0.062;

end;

check;
stoch_simul(order=1,irf=100, relative_irf);
```