ES 637: Mathematical Foundations for Computer Vision and Graphics Programming Assignment-1

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Question:
Given a set of images containg registered faces, design a Facial Recognition System using eigenfaces.

Principal Component Analysis (PCA)

Principal Component Analysis is a method used for linear Dimensionality reduction. We are basically reducing the dimension by transforming the correlated variables into a smaller number of uncorrelated variables. We are removing the variables which have the variables which have least covariance.

The data of reduced dimension contains most of the information present in the initial data, but removes the redundancy. The eigenvalues with larger values in the covariance matrix correspond to the variable with the most variance.

PCA Algorithm

1. Create the data matrix from the image training set by extending each 2-dimensional array into a single column vector.

$$X = [\vec{x_1} \quad \vec{x_2} \quad ... \quad \vec{x_n}]$$

2. Compute the mean of every row of the data matrix.

$$\vec{q} = \frac{1}{N}\sum_{i=1}^{n} \vec{x_i}$$

3. Translate all the data points so that the origin is at the mean.

$$\vec{x_i'} = \vec{x_i} - \vec{q}$$

4. Construct the Covariance matrix C_x.

$$C_{\mathrm{x}} = \frac{1}{n} X X^{\mathrm{T}}$$

5. Compute the eigenvalues and eigenvectors of C_{x} using Eigen Value Decomposition.

$$C_{\mathrm{x}} = U D U^{\mathrm{T}}$$

6. Sort the eigenvalues in decreasing order $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_{\mathrm{n}}$.

7. Correspoding eigenvectors of $C_{\mathrm{x}}$are the principal components (soted in the order of significance of eigenvectors).

$$P = U^{\mathrm{T}}$$

8. Dimensionality Reduction (Y is the reduced data matrix).

$$Y = PX$$

Facial Recognition using PCA

1. For Facial recognition, we should have a dataset of geometrically aligned faces.

2. We perform PCA (Training Phase).

3. In test phase, we take every image of the test set,perform dimensionality reduction on it, and then subtract the meanface and the find which subject it is the closest to by finding its nearest neighbour in the training set. Over all the training set, find the minimum of $||\vec{(y)}_{\mathrm{test}} - \vec{(y)}_{\mathrm{i}}||_2$ for every image.

4. The subject corresponding to the minimum norm is the subject of the new test image.

Implementation Details

- The dataset used for Facial recognition is AT&T "The Database of Faces" (formerly "The ORL Database of Faces").
- There are 40 subjects and each subject has 10 images, which totals to 400 images.
- The dimension of each image is 112x92. So, the dimension of the data matrix becomes 10,304x280.
- The test training ratio used is 7:3. So, there are 280 images in the training set and 120 images in the test set.
- The number of principal components used for dimensionality reduction is 280.
- The dimension of the reduced data matrix becomes 280x280.

Results

Figure 1: This is the meanface of the training dataset.

- The accuracy on the test set of 120 images is 90%. This accuracy comes from the fact that we labeled 108 images of the testset correctly to the subject out of the 120 images.

References

[1] R. Gross, Face Databases, Handbook of Face Recognition, Stan Z. Li and Anil K. Jain, ed., Springer-Verlag, February 2005, 22 pages

[2] AT&T "The Database of Faces" (formerly "The ORL Database of Faces") from *"http://www.cl.cam.ac.uk/research/dtg/attarchive/facedatabase.html"*.

[3] Jonathan Shlens, "A Tutorial on Principal Component Analysis", arXiv:1404.1100 [cs.LG]

[4] Stéfan van der Walt, S. Chris Colbert and Gaël Varoquaux. The NumPy Array: A Structure for Efficient Numerical Computation, Computing in Science & Engineering, 13, 22-30 (2011), DOI:10.1109/MCSE.2011.37.

[5] John D. Hunter. Matplotlib: A 2D Graphics Environment, Computing in Science & Engineering, 9, 90-95 (2007), DOI:10.1109/MCSE.2007.55.