The OEIS mourns the passing of Jim Simons and is grateful to the Simons Foundation for its support of research in many branches of science, including the OEIS.

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A000931 Padovan sequence (or Padovan numbers): a(n) = a(n-2) + a(n-3) with a(0) = 1, a(1) = a(2) = 0. 243

(Formerly M0284 N0102)

1, 0, 0, 1, 0, 1, 1, 1, 2, 2, 3, 4, 5, 7, 9, 12, 16, 21, 28, 37, 49, 65, 86, 114, 151, 200, 265, 351, 465, 616, 816, 1081, 1432, 1897, 2513, 3329, 4410, 5842, 7739, 10252, 13581, 17991, 23833, 31572, 41824, 55405, 73396, 97229, 128801, 170625 (list; graph; refs; listen; history; text; internal format)

OFFSET 0,9

COMMENTS

Number of compositions of n into parts congruent to 2 mod 3 (offset -1). - Vladeta Jovovic, Feb 09 2005

a(n) is the number of compositions of n into parts that are odd and >= 3. Example: a(10)=3 counts 3+7, 5+5, 7+3. - David Callan, Jul 14 2006

Referred to as N0102 in R. K. Guy's "Anyone for Twopins?" - Rainer Rosenthal, Dec 05 2006

Zagier conjectures that a(n+3) is the maximum number of multiple zeta values of weight n > 1 which are linearly independent over the rationals. - Jonathan Sondow and Sergey Zlobin (sirg_zlobin(AT)mail.ru), Dec 20 2006

Starting with offset 6: (1, 1, 2, 2, 3, 4, 5, ...) = INVERT transform of A106510: (1, 1, -1, 0, 1, -1, 0, 1, -1, ...). - Gary W. Adamson, Oct 10 2008

Starting with offset 7, the sequence 1, 2, 2, 3, 4, 5, 7, 9, 12, 16, 21, 28, ... is called the Fibonacci quilt sequence by Catral et al., in Fib. Q. 2017. - N. J. A. Sloane, Dec 24 2021

Triangle A145462: right border = A000931 starting with offset 6. Row sums = Padovan sequence starting with offset 7. - Gary W. Adamson, Oct 10 2008

Starting with offset 3 = row sums of triangle A146973 and INVERT transform of [1, -1, 2, -2, 3, -3, ...]. - Gary W. Adamson, Nov 03 2008

a(n+5) corresponds to the diagonal sums of "triangle": 1; 1; 1,1; 1,1; 1,2,1; 1,2,1; 1,3,3,1; 1,3,3,1; 1,4,6,4,1; ..., rows of Pascal's triangle (A007318) repeated. - Philippe Deléham, Dec 12 2008

With offset 3: (1, 0, 1, 1, 1, 2, 2, ...) convolved with the tribonacci numbers prefaced with a "1": (1, 1, 1, 2, 4, 7, 13, ...) = the tribonacci numbers, A000073. (Cf. triangle A153462.) - Gary W. Adamson, Dec 27 2008

a(n) is also the number of strings of length (n-8) from an alphabet {A, B} with no more than one A or 2 B's consecutively. (E.g., n = 4: {ABAB,ABBA,BABA,BABB,BBAB} and a(4+8) = 5.) - Toby Gottfried, Mar 02 2010

p(n):=A000931(n+3), n >= 1, is the number of partitions of the numbers {1,2,3,...,n} into lists of length two or three containing neighboring numbers. The 'or' is inclusive. For n=0 one takes p(0)=1. For details see the W. Lang link. There the explicit formula for p(n) (analog of the Binet-de Moivre formula for Fibonacci numbers) is also given. Padovan sequences with different inputs are also considered there. - Wolfdieter Lang, Jun 15 2010

Equals the INVERTi transform of Fibonacci numbers prefaced with three 1's, i.e., (1 + x + x^2 + x^3 + x^4 + 2x^5 + 3x^6 + 5x^7 + 8x^8 + 13x^9 + ...). - Gary W. Adamson, Apr 01 2011

When run backwards gives (-1)^n*A050935(n).

a(n) is the top left entry of the n-th power of the 3 X 3 matrix [0, 0, 1; 1, 0, 1; 0, 1, 0] or of the 3 X 3 matrix [0, 1, 0; 0, 0, 1; 1, 1, 0]. - R. J. Mathar, Feb 03 2014

Figure 4 of Brauchart et al., 2014, shows a way to "visualize the Padovan sequence as cuboid spirals, where the dimensions of each cuboid made up by the previous ones are given by three consecutive numbers in the sequence". - N. J. A. Sloane, Mar 26 2014

a(n) is the number of closed walks from a vertex of a unidirectional triangle containing an opposing directed edge (arc) between the second and third vertices. Equivalently the (1,1) entry of A^n where the adjacency matrix of digraph is A=(0,1,0;0,0,1;1,1,0). - David Neil McGrath, Dec 19 2014

Number of compositions of n-3 (n >= 4) into 2's and 3's. Example: a(12)=5 because we have 333, 3222, 2322, 2232, and 2223. - Emeric Deutsch, Dec 28 2014

The Hoffman (2015) paper "offers significant evidence that the number of quantities needed to generate the weight-n multiple harmonic sums mod p is" a(n). - N. J. A. Sloane, Jun 24 2016

a(n) gives the number of compositions of n-5 into odd parts where the order of the 1's does not matter. For example, a(11)=4 counts the following compositions of 6: (5,1)=(1,5), (3,3), (3,1,1,1)=(1,3,1,1)=(1,1,3,1)=(1,1,1,3), (1,1,1,1,1,1). - Gregory L. Simay, Aug 04 2016

For n > 6, a(n) is the number of maximal matchings in the (n-5)-path graph, maximal independent vertex sets and minimal vertex covers in the (n-6)-path graph, and minimal edge covers in the (n-5)-pan graph and (n-3)-path graphs. - Eric W. Weisstein, Mar 30, Aug 03, and Aug 07 2017

From James Mitchell and Wilf A. Wilson, Jul 21 2017: (Start)

a(2n + 5) + 2n - 4, n > 2, is the number of maximal subsemigroups of the monoid of order-preserving mappings on a set with n elements.

a(n + 6) + n - 3, n > 3, is the number of maximal subsemigroups of the monoid of order-preserving or reversing mappings on a set with n elements.

(End)

Has the property that the largest of any four consecutive terms equals the sum of the two smallest. - N. J. A. Sloane, Aug 29 2017 [David Nacin points out that there are many sequences with this property, such as 1,1,1,2,1,1,1,2,1,1,1,2,... or 2,3,4,5,2,3,4,5,2,3,4,5,... or 2,2,1,3,3,   4,1,4,    5,5,1,6,6,   7,1,7, 8,8,1,9,9,   10,1,10, ...  (spaces added for clarity), and a conjecture I made here in 2017 was simply wrong. I have deleted it. - N. J. A. Sloane, Oct 23 2018]

a(n) is also the number of maximal cliques in the (n+6)-path complement graph. - Eric W. Weisstein, Apr 12 2018

a(n+8) is the number of solus bitstrings of length n with no runs of 3 zeros. - Steven Finch, Mar 25 2020

Named after the architect Richard Padovan (b. 1935). - Amiram Eldar, Jun 08 2021

Shannon et al. (2006) credit a French architecture student Gérard Cordonnier with the discovery of these numbers.

For n >= 3, a(n) is the number of sequences of 0s and 1s of length (n-2) that begin with a 0, end with a 0, contain no two consecutive 0s, and contain no three consecutive 1s. - Yifan Xie, Oct 20 2022

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Index entries for linear recurrences with constant coefficients, signature (0,1,1).

FORMULA

G.f.: (1-x^2)/(1-x^2-x^3).
a(n) is asymptotic to r^n / (2*r+3) where r = 1.3247179572447... = A060006, the real root of x^3 = x + 1. - Philippe Deléham, Jan 13 2004
a(n)^2 + a(n+2)^2 + a(n+6)^2 = a(n+1)^2 + a(n+3)^2 + a(n+4)^2 + a(n+5)^2 (Barniville, Question 16884, Ed. Times 1911).
a(n+5) = a(0) + a(1) + ... + a(n).
a(n) = central and lower right terms in the (n-3)-th power of the 3 X 3 matrix M = [0 1 0 / 0 0 1 / 1 1 0]. E.g., a(13) = 7. M^10 = [3 5 4 / 4 7 5 / 5 9 7]. - Gary W. Adamson, Feb 01 2004
G.f.: 1/(1 - x^3 - x^5 - x^7 - x^9 - ...). - Jon Perry, Jul 04 2004
a(n+4) = Sum_{k=0..floor((n-1)/2)} binomial(floor((n+k-2)/3), k). - Paul Barry, Jul 06 2004
a(n+3) = Sum_{k=0..floor(n/2)} binomial(k, n-2k). - Paul Barry, Sep 17 2004, corrected by Greg Dresden and Zi Ye, Jul 06 2021
a(n+3) is diagonal sum of A026729 (as a number triangle), with formula a(n+3) = Sum_{k=0..floor(n/2)} Sum_{i=0..n-k} (-1)^(n-k+i)*binomial(n-k, i)*binomial(i+k, i-k). - Paul Barry, Sep 23 2004
a(n) = a(n-1) + a(n-5) = A003520(n-4) + A003520(n-13) = A003520(n-3) - A003520(n-9). - Henry Bottomley, Jan 30 2005
a(n+3) = Sum_{k=0..floor(n/2)} binomial((n-k)/2, k)(1+(-1)^(n-k))/2. - Paul Barry, Sep 09 2005
The sequence 1/(1-x^2-x^3) (a(n+3)) is given by the diagonal sums of the Riordan array (1/(1-x^3), x/(1-x^3)). The row sums are A000930. - Paul Barry, Feb 25 2005
a(n) = A023434(n-7) + 1 for n >= 7. - David Callan, Jul 14 2006
a(n+5) corresponds to the diagonal sums of A030528. The binomial transform of a(n+5) is A052921. a(n+5) = Sum_{k=0..floor(n/2)} Sum_{k=0..n} (-1)^(n-k+i)*binomial(n-k, i)binomial(i+k+1, 2k+1). - Paul Barry, Jun 21 2004
r^(n-1) = (1/r)*a(n) + r*(n+1) + a(n+2), where r = 1.32471... is the real root of x^3 - x - 1 = 0. Example: r^8 = (1/r)*a(9) + r*a(10) + a(11) = ((1/r)*2 + r*3 + 4 = 9.483909... - Gary W. Adamson, Oct 22 2006
a(n) = (r^n)/(2r+3) + (s^n)/(2s+3) + (t^n)/(2t+3) where r, s, t are the three roots of x^3-x-1. - Keith Schneider (schneidk(AT)email.unc.edu), Sep 07 2007
a(n) = -k*a(n-1) + a(n-2) + (k+1)a(n-2) + k*a(n-4), n > 3, for any value of k. - Gary Detlefs, Sep 13 2010
From Francesco Daddi, Aug 04 2011: (Start)
a(0) + a(2) + a(4) + a(6) + ... + a(2*n) = a(2*n+3).
a(0) + a(3) + a(6) + a(9) + ... + a(3*n) = a(3*n+2)+1.
a(0) + a(5) + a(10) + a(15) + ... + a(5*n) = a(5*n+1)+1.
a(0) + a(7) + a(14) + a(21) + ... + a(7*n) = (a(7*n) + a(7*n+1) + 1)/2. (End)
a(n+3) = Sum_{k=0..floor((n+1)/2)} binomial((n+k)/3,k), where binomial((n+k)/3,k)=0 for noninteger (n+k)/3. - Nikita Gogin, Dec 07 2012
a(n) = A182097(n-3) for n > 2. - Jonathan Sondow, Mar 14 2014
a(n) = the k-th difference of a(n+5k) - a(n+5k-1), k>=1. For example, a(10)=3 => a(15)-a(14) => 2nd difference of a(20)-a(19) => 3rd difference of a(25)-a(24)... - Bob Selcoe, Mar 18 2014
Construct the power matrix T(n,j) = [A^*j]*[S^*(j-1)] where A=(0,0,1,0,1,0,1,...) and S=(0,1,0,0,...) or A063524. [* is convolution operation] Define S^*0=I with I=(1,0,0,...). Then a(n) = Sum_{j=1...n} T(n,j). - David Neil McGrath, Dec 19 2014
If x=a(n), y=a(n+1), z=a(n+2), then x^3 + 2*y*x^2 - z^2*x - 3*y*z*x + y^2*x + y^3 - y^2*z + z^3 = 1. - Alexander Samokrutov, Jul 20 2015
For the sequence shifted by 6 terms, a(n) = Sum_{k=ceiling(n/3)..ceiling(n/2)} binomial(k+1,3*k-n) [Doslic-Zubac]. - N. J. A. Sloane, Apr 23 2017
From Joseph M. Shunia, Jan 21 2020: (Start)
a(2n) = 2*a(n-1)*a(n) + a(n)^2 + a(n+1)^2, for n > 8.
a(2n-1) = 2*a(n)*a(n+1) + a(n-1)^2, for n > 8.
a(2n+1) = 2*a(n+1)*a(n+2) + a(n)^2, for n > 7. (End)
0*a(0) + 1*a(1) + 2*a(2) + ... + n*a(n) = n*a(n+5) - a(n+9) + 2. - Greg Dresden and Zi Ye, Jul 02 2021
From Greg Dresden and Zi Ye, Jul 06 2021: (Start)
2*a(n) = a(n+2) + a(n-5) for n >= 5.
3*a(n) = a(n+4) - a(n-9) for n >= 9.
4*a(n) = a(n+5) - a(n-9) for n >= 9. (End)

EXAMPLE

G.f. = 1 + x^3 + x^5 + x^6 + x^7 + 2*x^8 + 2*x^9 + 3*x^10 + 4*x^11 + ...

MAPLE

```
A000931 := proc(n) option remember; if n = 0 then 1 elif n <= 2 then 0 else
  procname(n-2)+procname(n-3); fi; end;
A000931:=-(1+z)/(-1+z^2+z^3); # Simon Plouffe in his 1992 dissertation; gives
  sequence without five leading terms
a[0]:=1; a[1]:=0; a[2]:=0; for n from 3 to 50 do a[n]:=a[n-2]+a[n-3]; end do; #
  Francesco Daddi, Aug 04 2011
```

MATHEMATICA

```
CoefficientList[Series[(1-x^2)/(1-x^2-x^3), {x, 0, 50}], x]
a[0]=1; a[1]=a[2]=0; a[n_]:= a[n]= a[n-2] + a[n-3]; Table[a[n], {n, 0, 50}] (*
  Robert G. Wilson v, May 04 2006 *)
LinearRecurrence[{0, 1, 1}, {1, 0, 0}, 50] (* Harvey P. Dale, Jan 10 2012 *)
Table[RootSum[-1 -# +#^3 &, 5#^n -6#^(n+1) +4#^(n+2) &]/23, {n, 0, 50}] (* Eric W.
  Weisstein, Nov 09 2017 *)
```

PROG

```
(Haskell)
a000931 n = a000931_list !! n
a000931_list = 1 : 0 : 0 : zipWith (+) a000931_list (tail a000931_list)
-- Reinhard Zumkeller, Feb 10 2011
(PARI) Vec((1-x^2)/(1-x^2-x^3) + O(x^50)) \\ Charles R Greathouse IV, Feb 11 2011
(PARI) {a(n) = if( n<0, polcoeff(1/(1+x-x^3) + x * O(x^-n), -n), polcoeff( (1 -
  x^2)/(1-x^2-x^3) + x * O(x^n), n))}; /* Michael Somos, Sep 18 2012 */
(Magma) I:=[1, 0, 0]; [n le 3 select I[n] else Self(n-2) + Self(n-3): n in
  [1..60]]; // Vincenzo Librandi, Jul 21 2015
(Sage)
def A000931_list(prec):
    P.<x> = PowerSeriesRing(ZZ, prec)
    return P( (1-x^2)/(1-x^2-x^3) ).list()
A000931_list(50) # G. C. Greubel, Dec 30 2019
(GAP) a:=[1, 0, 0];; for n in [4..50] do a[n]:=a[n-2]+a[n-3]; od; a; # G. C.
  Greubel, Dec 30 2019
(Python)
def aupton(nn):
    alst = [1, 0, 0]
    for n in range(3, nn+1): alst.append(alst[n-2]+alst[n-3])
    return alst
print(aupton(49)) # Michael S. Branicky, Mar 28 2022
```

CROSSREFS

The following are basically all variants of the same sequence: A000931, A078027, A096231, A124745, A133034, A134816, A164001, A182097, A228361 and probably A020720. However, each one has its own special features and deserves its own entry.

Closely related to A001608.

Cf. A000073, A005682-A005691, A103372-A103380, A106510, A145462, A146973, A153462.

Doubling every term gives A291289.

Sequence in context: A018124 A124745 A133034 * A078027 A134816 A228361

Adjacent sequences: A000928 A000929 A000930 * A000932 A000933 A000934

KEYWORD

nonn,easy,nice

AUTHOR

N. J. A. Sloane

EXTENSIONS

Edited by Charles R Greathouse IV, Mar 17 2010

Deleted certain dangerous or potentially dangerous links. - N. J. A. Sloane, Jan 30 2021

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