Topic 1: Bisection and False Position Methods

Q1.1: Which method always halves the interval in which a continuous function changes sign?
a) Newton–Raphson method
b) Bisection method
c) Secant method
d) Fixed point iteration
Answer: b) Bisection method

Q1.2: What is the necessary condition for applying the bisection method?
a) f(x) must be differentiable on [a, b]
b) f(a) and f(b) must have opposite signs
c) f(x) must be monotonic
d) f(x) must be bounded
Answer: b) f(a) and f(b) must have opposite signs

Q1.3: Which formula is used in the false position (regula falsi) method to compute the next approximation?
$a)\quad x = (a+b)/2$

$b)\quad x = a - f(a) * (b-a)/(f(b) - f(a))$

$c)\quad x = b - f(b) * (b-a)/(f(b) - f(a))$

$d)\quad x = a + f(a) * (b-a)/(f(b) - f(a))$

C is the answer

Q1.4: What type of convergence does the bisection method exhibit?
a) Quadratic
b) Linear
c) Superlinear
d) Exponential
Answer: b) Linear

Q1.5: Which method guarantees convergence as long as the function is continuous and the endpoints have opposite signs?
a) Newton–Raphson method
b) Bisection method
c) Secant method
d) Fixed point iteration
Answer: b) Bisection method

Q1.6: In the false position method, which endpoint is updated at each iteration?
a) Both endpoints are updated
b) The endpoint whose function value has the larger magnitude

c) The endpoint with the same sign as f(x) at the new approximation
d) The endpoint that is closer to the midpoint
Answer: c) The endpoint with the same sign as f(x) at the new approximation

Q1.7: What is the main disadvantage of the bisection method?
a) It requires the derivative of the function
b) It may converge slowly
c) It is not guaranteed to converge
d) It cannot be used if f(x) is continuous
Answer: b) It may converge slowly

Q1.8: In the false position method, why might convergence slow down even though the method uses a weighted interpolation?
a) Because it always halves the interval
b) Because one endpoint might remain fixed over iterations
c) Because it requires computing second derivatives
d) Because it uses a non-iterative approach
Answer: b) Because one endpoint might remain fixed over iterations

Q1.9: Which of the following is a common feature of both the bisection and false position methods?
a) They require the derivative of the function
b) They use bracketing to locate the root
c) They are both iterative methods that use extrapolation
d) They both have quadratic convergence
Answer: b) They use bracketing to locate the root

Q1.10: The false position method relies on which type of approximation to choose the next estimate?
a) Quadratic interpolation
b) Linear interpolation
c) Exponential interpolation
d) Polynomial regression
Answer: b) Linear interpolation

Topic 2: Secant, Newton–Raphson, and Fixed Point Iteration

Q2.1: Which method uses two initial approximations and estimates the derivative numerically?
a) Bisection method
b) Secant method
c) Newton–Raphson method
d) Fixed point iteration
Answer: b) Secant method

Q2.2: Which method requires an explicit calculation of the derivative f′(x)f'(x)f′(x) at every iteration?
a) Secant method
b) Newton–Raphson method
c) Fixed point iteration
d) Bisection method
Answer: b) Newton–Raphson method

Q2.3: In fixed point iteration, the equation x=g(x)x = g(x)x=g(x) is used. What is the key condition for the method to converge?
a) ∣g(x)∣<1|g(x)| < 1∣g(x)∣<1
b) ∣g′(x)∣<1|g'(x)| < 1∣g′(x)∣<1
c) g(x)g(x)g(x) must be linear
d) g(x)=0g(x) = 0g(x)=0
Answer: b) ∣g′(x)∣<1|g'(x)| < 1∣g′(x)∣<1

Q2.4: Which method is typically more robust if the derivative is difficult to compute?
a) Newton–Raphson method
b) Fixed point iteration
c) Secant method
d) Bisection method
Answer: c) Secant method

Q2.5: Under ideal conditions and when starting near the root, which method converges quadratically?
a) Fixed point iteration
b) Secant method
c) Newton–Raphson method
d) Bisection method
Answer: c) Newton–Raphson method

Q2.6: Which method does not require the function to be bracketed by two initial guesses?
a) Newton–Raphson method
b) Secant method
c) Fixed point iteration
d) Both b and c
Answer: d) Both b and c

Q2.7: What is a common disadvantage of fixed point iteration?
a) It requires evaluating derivatives
b) Its convergence is not guaranteed if ∣g′(x)∣≥1|g'(x)| \geq 1∣g′(x)∣≥1
c) It uses two initial approximations
d) It always converges slowly
Answer: b) Its convergence is not guaranteed if ∣g′(x)∣≥1|g'(x)| \geq 1∣g′(x)∣≥1

Q2.8: Which method can be seen as an approximation to Newton–Raphson when the derivative is estimated using finite differences?
a) Secant method
b) Bisection method
c) Fixed point iteration
d) Simpson's method
Answer: a) Secant method

Q2.9: In Newton–Raphson, what is the update formula for the next approximation xn+1x_{n+1}xn+1?
$a) \quad xn+1 = x_n - f(x_n)/f'(x_n)$

$b) \quad xn+1 = (x_n + x_(n-1))/2$

$c) \quad xn+1 = x_n - f(x_n) * (x_n - x_(n-1))$

$d) \quad xn+1 = g(x_n)$

Ans is a

Q2.10: Which method may exhibit slow convergence if the initial guess is far from the true root?
a) Newton–Raphson method
b) Secant method
c) Fixed point iteration
d) All of the above
Answer: c) Fixed point iteration

Topic 3: Interpolation

Q3.1: Which interpolation method constructs a polynomial using the formula that sums Lagrange basis polynomials?
a) Newton's divided differences
b) Lagrange interpolation
c) Spline interpolation
d) Linear interpolation
Answer: b) Lagrange interpolation

Q3.2: Which interpolation technique uses divided differences to construct the polynomial incrementally?
a) Lagrange interpolation
b) Newton's divided difference interpolation
c) Spline interpolation

d) Linear interpolation
Answer: b) Newton's divided difference interpolation

Q3.3: What is a common problem associated with high-degree polynomial interpolation?
a) Underfitting
b) Overfitting
c) Runge's phenomenon
d) Lack of continuity
Answer: c) Runge's phenomenon

Q3.4: Interpolation is best defined as:
a) Approximating values within the range of given data points
b) Extrapolating values outside the given data range
c) Minimizing the error in differential equations
d) Finding the derivative at a given point
Answer: a) Approximating values within the range of given data points

Q3.5: Which method is more sensitive to the addition of new data points, requiring a complete recalculation of the polynomial?
a) Newton's divided difference interpolation
b) Lagrange interpolation
c) Spline interpolation
d) Linear interpolation
Answer: b) Lagrange interpolation

Q3.6: In Lagrange interpolation, the functions used to construct the interpolating polynomial are called:
a) Basis functions
b) Kernel functions
c) Weight functions
d) Spline functions
Answer: a) Basis functions

Q3.7: Which interpolation method allows for an incremental update if an additional data point is introduced?
a) Lagrange interpolation
b) Newton's divided difference interpolation
c) Cubic spline interpolation
d) Nearest neighbor interpolation
Answer: b) Newton's divided difference interpolation

Q3.8: Which interpolation technique uses piecewise low-degree polynomials for a smoother approximation?
a) Lagrange interpolation
b) Newton's interpolation
c) Spline interpolation

d) Polynomial regression
Answer: c) Spline interpolation

Q3.9: What is the primary purpose of interpolation in numerical analysis?
a) To solve differential equations
b) To approximate unknown function values between known data points
c) To calculate definite integrals
d) To optimize functions
Answer: b) To approximate unknown function values between known data points

Q3.10: Which of the following is a disadvantage of polynomial interpolation when the number of data points is high?
a) The interpolating polynomial may oscillate significantly between points
b) It always produces a smooth curve
c) It is computationally inexpensive
d) It eliminates the need for data smoothing
Answer: a) The interpolating polynomial may oscillate significantly between points

Topic 4: Newton's Forward/Backward Difference Table and Curve Fitting

Q4.1: Which formula is used for interpolation when the data points are equally spaced and the desired value is near the beginning of the table?
a) Newton's backward formula
b) Newton's forward formula
c) Lagrange formula
d) Simpson's rule
Answer: b) Newton's forward formula

Q4.2: Which formula is more suitable when the desired value is near the end of the data set?
a) Newton's forward formula
b) Newton's backward formula
c) Secant method
d) Trapezoidal rule
Answer: b) Newton's backward formula

Q4.3: In Newton's forward interpolation formula, the variable ppp is defined as:

$a)\quad p = h/(x - x_0)$

$b)\quad p = (x - x_0)/h$

$c)\quad p = (x_0 - x)/h$

$d)\quad p = h * (x - x_0)$

D is the answer

Q4.4: What is the primary goal of curve fitting?
a) To interpolate between data points exactly
b) To determine a function that best approximates a set of data points
c) To compute derivatives numerically
d) To find the root of a function
Answer: b) To determine a function that best approximates a set of data points

Q4.5: Which method is commonly used for curve fitting by minimizing the sum of the squares of the errors?
a) Newton's interpolation
b) Least squares method
c) Lagrange interpolation
d) Cubic spline interpolation
Answer: b) Least squares method

Q4.6: In the context of a forward difference table, what do the first differences represent?
a) The slopes between successive data points
b) The second derivative
c) The cumulative sum of data values
d) The weighted averages of the data
Answer: a) The slopes between successive data points

Q4.7: Which interpolation method is preferable when the interpolation point lies far from the beginning of the data set?
a) Newton's forward formula
b) Newton's backward formula
c) Lagrange interpolation
d) Linear interpolation
Answer: b) Newton's backward formula

Q4.8: In curve fitting, what does the "residual" refer to?
a) The difference between the actual data and the fitted curve
b) The slope of the curve at a given point
c) The integration error

d) The derivative of the curve
Answer: a) The difference between the actual data and the fitted curve

Q4.9: Which characteristic distinguishes interpolation from curve fitting?
a) Interpolation approximates noisy data, while curve fitting passes exactly through all points
b) Interpolation passes through all given data points, while curve fitting seeks a best-fit approximation
c) Curve fitting is only used for polynomial functions
d) Interpolation is only used for linear functions
Answer: b) Interpolation passes through all given data points, while curve fitting seeks a best-fit approximation

Q4.10: What is one advantage of using Newton's forward/backward formulas in numerical interpolation?
a) They can handle non-uniform data spacing without adjustments
b) They allow for easy incremental addition of new data points (especially in the divided difference form)
c) They do not require computation of differences
d) They converge exponentially
Answer: b) They allow for easy incremental addition of new data points (especially in the divided difference form)

Topic 5: Euler's, RK2, and RK4 Methods

Q5.1: Which method is known as a first-order method for solving ordinary differential equations (ODEs)?
a) RK2 method
b) Euler's method
c) RK4 method
d) Trapezoidal rule
Answer: b) Euler's method

Q5.2: Which method improves on Euler's method by taking an average of slopes to achieve higher accuracy?
a) RK2 method
b) RK4 method
c) Secant method
d) Fixed point iteration
Answer: a) RK2 method

Q5.3: The RK4 method is also known as the:
a) Midpoint method
b) Heun's method
c) Classical Runge–Kutta method
d) Backward Euler method
Answer: c) Classical Runge–Kutta method

Q5.4: What is the main drawback of Euler's method?
a) It requires multiple derivative evaluations per step
b) It is computationally intensive
c) It has low accuracy and can suffer from stability issues
d) It cannot be applied to linear ODEs
Answer: c) It has low accuracy and can suffer from stability issues

Q5.5: How many derivative (slope) evaluations are performed per step in the RK4 method?
a) Two
b) Three
c) Four
d) One
Answer: c) Four

Q5.6: What is the order of accuracy for the RK2 method?
a) First order
b) Second order
c) Third order
d) Fourth order
Answer: b) Second order

Q5.7: Which method would generally be preferred when high accuracy is required for solving ODEs?
a) Euler's method
b) RK2 method
c) RK4 method
d) Fixed point iteration
Answer: c) RK4 method

Q5.8: The error in Euler's method decreases proportionally to which power of the step size hhh?
a) hhh
b) h2h^2h2
c) h3h^3h3
d) h4h^4h4
Answer: a) hhh

Q5.9: In RK2, the new value is computed by taking the initial slope and a midpoint slope. This technique is used to:
a) Decrease the computational cost
b) Improve the order of accuracy to second order
c) Avoid computing the derivative
d) Ensure convergence in stiff equations
Answer: b) Improve the order of accuracy to second order

Q5.10: Compared to Euler's method, both RK2 and RK4 methods:
a) Require fewer function evaluations

b) Are explicit methods with improved accuracy
c) Always use adaptive step sizing
d) Are implicit methods
Answer: b) Are explicit methods with improved accuracy

Topic 6: Trapezoidal, Simpson’s 1/3, and Simpson’s 3/8 Rules

Q6.1: Which rule approximates the area under a curve by summing the areas of trapezoids?
a) Simpson’s 1/3 rule
b) Trapezoidal rule
c) Simpson’s 3/8 rule
d) Midpoint rule
Answer: b) Trapezoidal rule

Q6.2: Simpson’s 1/3 rule approximates the integrand using which type of polynomial?
a) Linear
b) Quadratic
c) Cubic
d) Quartic
Answer: b) Quadratic

Q6.3: Simpson’s 3/8 rule approximates the integrand using:
a) A linear function
b) A quadratic polynomial
c) A cubic polynomial
d) A quartic polynomial
Answer: c) A cubic polynomial

Q6.4: What is the requirement on the number of subintervals for Simpson’s 1/3 rule?
a) Any number of subintervals
b) An odd number of subintervals
c) An even number of subintervals
d) A multiple of 3 subintervals
Answer: c) An even number of subintervals

Q6.5: Simpson’s 3/8 rule requires that the number of subintervals be a multiple of:
a) 2
b) 3
c) 4
d) 5
Answer: b) 3

Q6.6: Compared to the trapezoidal rule, Simpson’s rules generally offer:
a) Lower accuracy for smooth functions
b) The same level of accuracy
c) Higher accuracy for smooth functions

d) Instability in the computed integral
Answer: c) Higher accuracy for smooth functions

Q6.7: Simpson's 1/3 rule is derived by integrating a Lagrange polynomial of which degree?
a) 1
b) 2
c) 3
d) 4
Answer: b) 2

Q6.8: The error term in the trapezoidal rule is proportional to:
a) hhh
b) h2h^2h2
c) h3h^3h3
d) h4h^4h4
Answer: b) h2h^2h2

Q6.9: The error in Simpson's 3/8 rule is typically proportional to which power of the subinterval width hhh?
a) h2h^2h2
b) h3h^3h3
c) h4h^4h4
d) h5h^5h5
Answer: c) h4h^4h4

Q6.10: For Simpson's rules to be effective, the function being integrated should be:
a) Highly oscillatory
b) Discontinuous
c) Sufficiently smooth
d) Linear
Answer: c) Sufficiently smooth

Topic 7: Gauss Elimination, Gauss–Jordan Elimination, and the Power Method

Q7.1: Which method is a direct approach for solving systems of linear equations by eliminating variables?
a) Gauss elimination
b) Gauss–Jordan elimination
c) Power method
d) Jacobi method
Answer: a) Gauss elimination

Q7.2: Which method transforms the coefficient matrix into reduced row echelon form?
a) Gauss elimination
b) Gauss–Jordan elimination
c) LU decomposition
d) Power method
Answer: b) Gauss–Jordan elimination

Q7.3: In Gauss elimination, the matrix is transformed into which form?
a) Diagonal form
b) Lower triangular form
c) Upper triangular form
d) Identity matrix
Answer: c) Upper triangular form

Q7.4: The power method is primarily used to approximate:
a) The inverse of a matrix
b) The determinant of a matrix
c) The dominant eigenvalue and its eigenvector
d) The solution to linear systems
Answer: c) The dominant eigenvalue and its eigenvector

Q7.5: Which technique is often used with Gauss elimination to improve numerical stability?
a) Scaling
b) Partial pivoting
c) Extrapolation
d) Back substitution
Answer: b) Partial pivoting

Q7.6: The key difference between Gauss elimination and Gauss–Jordan elimination is that Gauss–Jordan:
a) Only eliminates elements below the pivots
b) Reduces the matrix to an upper triangular form

c) Eliminates elements both above and below the pivot elements
d) Does not require pivoting
Answer: c) Eliminates elements both above and below the pivot elements

Q7.7: For the power method to converge, the matrix must have:
a) All positive entries
b) A unique dominant eigenvalue in magnitude
c) Symmetry
d) A zero determinant
Answer: b) A unique dominant eigenvalue in magnitude

Q7.8: Which method is an iterative algorithm rather than a direct solution method?
a) Gauss elimination
b) Gauss–Jordan elimination
c) Power method
d) LU decomposition
Answer: c) Power method

Q7.9: Which elimination method can be directly used to compute the inverse of a matrix?
a) Gauss elimination
b) Gauss–Jordan elimination
c) LU decomposition
d) Cholesky decomposition
Answer: b) Gauss–Jordan elimination

Q7.10: When solving a linear system, the power method is not used to:
a) Approximate the dominant eigenvalue
b) Find the eigenvector corresponding to the dominant eigenvalue
c) Directly solve for the system's unique solution
d) Analyze the convergence properties of iterative methods
Answer: c) Directly solve for the system's unique solution