

Predictive 3D modelling of free oblique cutting with an ANN-based material constitutive model and experimental validation over a wide range of conditions

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Abstract

Modelling of the cutting process needs to move from 2D to 3D configurations to get closer to industrial applications. This study introduces a predictive 3D finite element model of free orthogonal and oblique cutting with an ANN-based material constitutive model and experimental validation in strictly the same conditions (cutting and geometrical). The developments are applied to the formation of continuous chips for the titanium alloy Ti6Al4V and an unseen broad range of 36 cutting conditions is considered: 2 cutting edge inclinations, 3 uncut chip thicknesses and 6 cutting speeds. The predictive performance of the model (i.e., the evaluation of the trends of fundamental variables with the absence of tuning of both numerical parameters and model features when cutting conditions are significantly modified) is high for the forces, mainly cutting and passive, and the chip thickness ratio on all 36 cutting conditions. The accuracy of the main cutting force is excellent: the mean difference with the experiments is 4%, within the experimental dispersion. No significant degradation of the results is brought by the apparition of the third, out-of-plane, force, which shows the ability of the model to handle orthogonal and oblique cutting configurations.

[FD:] Add a comment on ANN?

Keywords:

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1. Introduction

Selection of the tools and the cutting conditions in machining, but also comprehension of the influence of the process parameters on the quality of a component and its optimization, are still difficult to achieve because of the high level of complexity and linked nonlinear phenomena. In the frame of digital manufacturing and Industry 4.0, modelling the cutting process supports them, while remaining a challenging task. As highlighted by Arrazola et al. [1], most finite element (FE) models are developed in 2D (orthogonal cutting configuration usually) although industrial applications require 3D modelling.

Experimental validation of a model is a crucial step in the modelling of the cutting process. The experimental configuration must be as close as possible to the simulation. For orthogonal cutting validation, a rotating movement usually generates the cutting speed. This is often achieved in turning [2] or in milling [3] and the diameter of the rotating part must be large enough to reduce the influence of the curvature on the results. Experimental configurations in strictly orthogonal cutting conditions are less often adopted, for example, on broaching [4] or milling [5, 6] machines. While they remove assumptions linked to the rotating cutting movement, they usually allow for lower cutting speeds (except on a dedicated machine, such as in Afrasiabi et al. [7]). Free oblique cutting with a straight cutting edge has not been studied yet: all the efforts have been focussing on orthogonal cutting (mostly for 2D validation).

Lagrangian and Eulerian formulations are the most used for FE modelling of the cutting process. Combinations of formulations, such as Arbitrary Lagrangian-Eulerian (ALE) and Coupled Eulerian-Lagrangian (CEL) are increasingly used to avoid (or reduce) mesh distortions [8]. The Coupled Eulerian-Lagrangian (CEL) formulation has recently been successfully applied to the modelling of cutting (2D orthogonal configuration): it provides accurate results with a realistic chip shape and no mesh distortion [8]. First applications in 3D are found in recent works [3, 9–12]. They cover (free) orthogonal cutting or a simple 3D operation, while free oblique cutting still needs to be investigated.

The behaviour of the machined material is one of the key aspects of a FE model [1, 13]. Research is very intense in this field, which leads to a growing number of material constitutive models ranging from empirical to physical models, some

including microstructure effects [13]. The thermo-elasto-viscoplastic empirical model of Johnson-Cook (JC) [14] is still the most used so far:

$$\sigma^y = (A + B \varepsilon^{p^n}) \left(1 + C \ln \frac{\dot{\varepsilon}^p}{\dot{\varepsilon}_0^p} \right) \left(1 - \left[\frac{T - T_{\text{room}}}{T_{\text{melt}} - T_{\text{room}}} \right]^m \right) \quad (1)$$

31 In this model, the flow stress, σ^y , is a function of the plastic strain, ε^p , of the plas-
 32 tic strain rate, $\dot{\varepsilon}^p$, and of the temperature, T . It is composed of 3 terms describing
 33 independently plastic, viscous and thermal aspects. One of the points in favour
 34 of its adoption is the rather limited number of parameters to be identified, 5: A ,
 35 B , C , m and n . $\dot{\varepsilon}_0^p$ is the reference plastic strain rate, while T_{room} and T_{melt} are the
 36 room temperature and the melting temperature, respectively. More recent models
 37 developed based on it, such as the one of Calamaz et al. [15], increase this num-
 38 ber of parameters (for Calamaz's particular model to 9). The better (in theory)
 39 description of the behaviour is achieved at the cost of a greater complexity of the
 40 identification process and of a reduction of the link with the physical meaning of
 41 the model.

42 One issue of material behaviour modelling for cutting simulation is the iden-
 43 tification of the parameters, moreover, as the experimental equipment does not
 44 allow to reach the high levels of strains, strain rates and temperature of machining
 45 [13]. Inverse identification is an alternative, but the uniqueness of the solution
 46 is not always guaranteed [1, 13]. The early work of Özel and Altan [16] used
 47 the least squares method to inversely identify the input parameters of a FE model.
 48 Shrot and Bäker [17] then used the Levenberg–Marquardt algorithm for their iden-
 49 tification of the constitutive material parameters. They showed that similar results
 50 (cutting forces and chip morphology) could be obtained by different sets of param-
 51 eters and therefore highlighted the non-uniqueness of the solution of the inverse
 52 problem. In addition to the flow stress parameters, Klocke et al. [18] also iden-
 53 tified the damage parameters. In more recent works, such as Bosetti et al. [19]
 54 and Denkena et al. [20], the approach to tackle the inverse identification problem
 55 is moving from optimization to Artificial Intelligence (AI) based methods. The
 56 Downhill Simplex Algorithm (DSA) is adopted by Bergs et al. [21] and by Hardt
 57 et al. [22] for AISI 1045. Stampfer et al. [23] also selected the DSA when dealing
 58 with AISI 4140 tempered at 3 different temperatures. In [24], Hardt et al. showed
 59 that the Particle Swarm Optimization (PSO) was more efficient to solve the in-
 60 verse problem than the DSA, even if computation time is still significant. In an
 61 effort to reduce it, an Efficient Global Optimization (EGO) algorithm has recently
 62 been introduced by Kugalur Palanisamy et al. [25]. They simultaneously identi-
 63 fied the parameters of the material constitutive model and of the friction model for

64 Ti6Al4V. Most of these works highlight the non-uniqueness of the identification
65 and they all require the definition of the analytical expression of the constitutive
66 model.

67 This paper fills the literature gap on oblique cutting by investigating free or-
68 thogonal and oblique 3D cutting configurations from both the experimental and
69 the numerical points of view. An Artificial Neural Network (ANN), introduced
70 in Pantalé et al. [26], is implemented in a FE cutting model for the first time in-
71 stead of the analytical JC law. A broad range of cutting speeds (6), uncut chip
72 thicknesses (3) and cutting edge inclination angles (2) resulting in 36 different
73 conditions is considered to demonstrate the predictive ability of the FE model for
74 fundamental variables. The main goals of a predictive model are the accurate
75 modelling of the trends of the results when the conditions change and the good
76 agreement of the predicted values with the experimental ones (exact values are not
77 looked for due to experimental dispersions). This type of models aims at support-
78 ing future choices and developments without the need for experimental data. No
79 assumption is made about the geometry of the machined workpiece in the model
80 (i.e., its width is the same as in the experiments), while keeping computation time
81 relevant for industrial applications. The developments are applied to the formation
82 of continuous chips of titanium alloy Ti6Al4V.

83 2. Experimental setup

84 A 3-axis GF Mikron VCE 600 Pro milling machine is used to carry out dry
85 free orthogonal and oblique cutting tests on Ti6Al4V (grade 5 annealed at 750 °C
86 for 1 h followed by air cooling) with the same kinematics as a shaper. As shown
87 in Figure 1, the tungsten carbide tool (modified LCGN160602-0600-FG, CP500
88 from SECO) is fixed on a dedicated holder (modified CFHN-06 from SECO) and
89 the sample to be cut is clamped in the spindle (no rotation is allowed during the
90 test). The top of the sample includes 3 ribs of 1 mm in width (the width of the tool
91 is 6 mm) and 10 mm in length. The test consists of removing the upper layer (its
92 height is the uncut chip thickness, h) of one rib at the prescribed cutting speed, v_c .
93 The cutting speed is provided by the feed rate, v_f , of the machine (max. value of
94 40 m/min). The tool cutting edge inclination, λ_s , results from the relative angular
95 orientation of the tool and the sample. Table 1 shows the cutting conditions: 6
96 cutting speeds, 3 uncut chip thicknesses and 2 inclination angles, each is repeated
97 3 times.

98 Forces are measured with a 3-component Kistler 9257B dynamometer and are
99 amplified by a Kistler 5070A charge amplifier. The acquisition is performed at

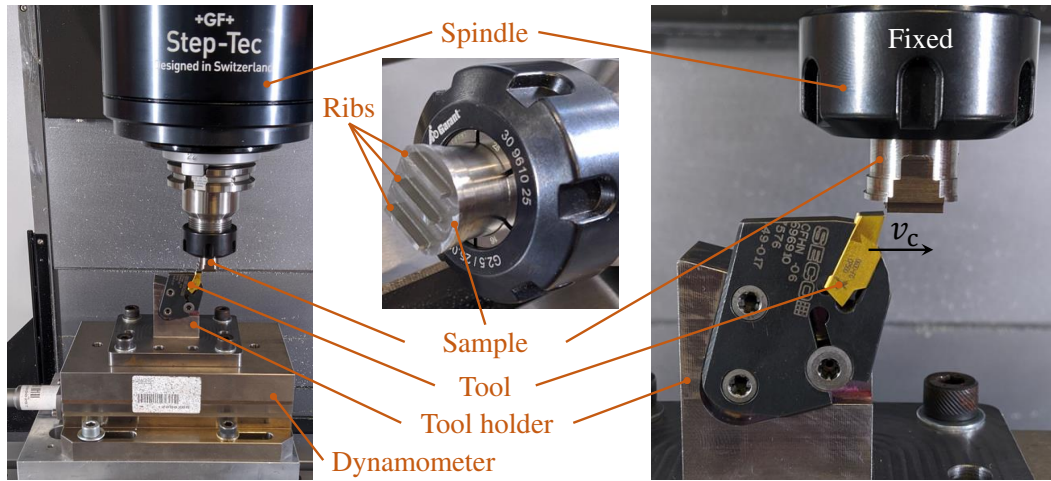


Figure 1: Experimental setup

Table 1: Cutting conditions of the study

Parameter	Values
Cutting speed, v_c (m/min)	5, 7.5, 10, 20, 30, 40
Uncut chip thickness, h (μm)	40, 60, 80
Cutting edge inclination, λ_s ($^\circ$)	0, 6
Width of the workpiece (mm)	1
Length of the workpiece (mm)	10
Width of the cutting edge (mm)	6 (1.1 in the model)
Cutting edge radius, r_β (μm)	20
Rake angle, γ_0 ($^\circ$)	15
Clearance angle, α_0 ($^\circ$)	2

100 3 kHz with a Kistler 5697A2 data acquisition system and the DynoWare software.
 101 Recorded forces are then filtered with a second-order low-pass Bessel filter at
 102 750 Hz before computing the mean value of the signal at the steady state.

103 All the chips are collected and observed with a Dino Lite digital microscope
 104 AM7013MZT (5 MP, magnification $20\times - 250\times$). Each chip is measured 3 times
 105 along its length to get a mean value representative of the whole chip.

3. Finite element model

3.1. Modelling choices

The CEL formulation is adopted to model the dry free orthogonal and oblique cutting tests with Abaqus/Explicit 2020. The 3D model is composed of a fixed Lagrangian tool and a Eulerian workpiece (Figure 2). Chip formation occurs by plastic flow across the Eulerian domain with no mesh distortion. The Eulerian formulation enables to form chips without damage properties, removing modelling assumptions. These two characteristics contribute to cutting models providing accurate results and realistic chips [8].

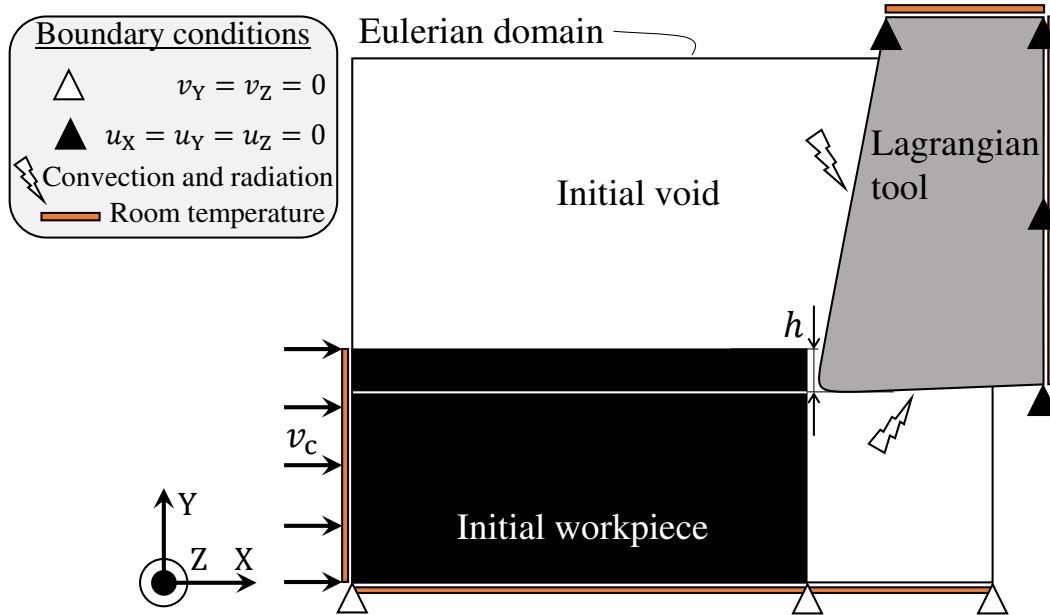


Figure 2: Boundary conditions and schematic initial geometry of the model

As shown in Figure 3, the full width of the workpiece (1 mm), i.e., a rib in the experiments, is modelled. To allow chip formation and side flow, the Eulerian domain is wider (it includes the volume in which material can move). The volume above the initial workpiece is also meshed with Eulerian elements for the same reasons. As in the experiments and to fulfil the hypothesis of free orthogonal and oblique cutting, the tool is wider than the workpiece (it is 1.1 mm in the model and 6 mm in the experiments). It is very important to stress that the models are the same for both inclination angles: they only differ by the rotation of

123 the tool of 6° about Y axis as in the experiments (Figure 3). This, coupled with
 124 the absence of assumptions when developing the models, contributes to make the
 125 models predictive: no input is changed when cutting conditions do.

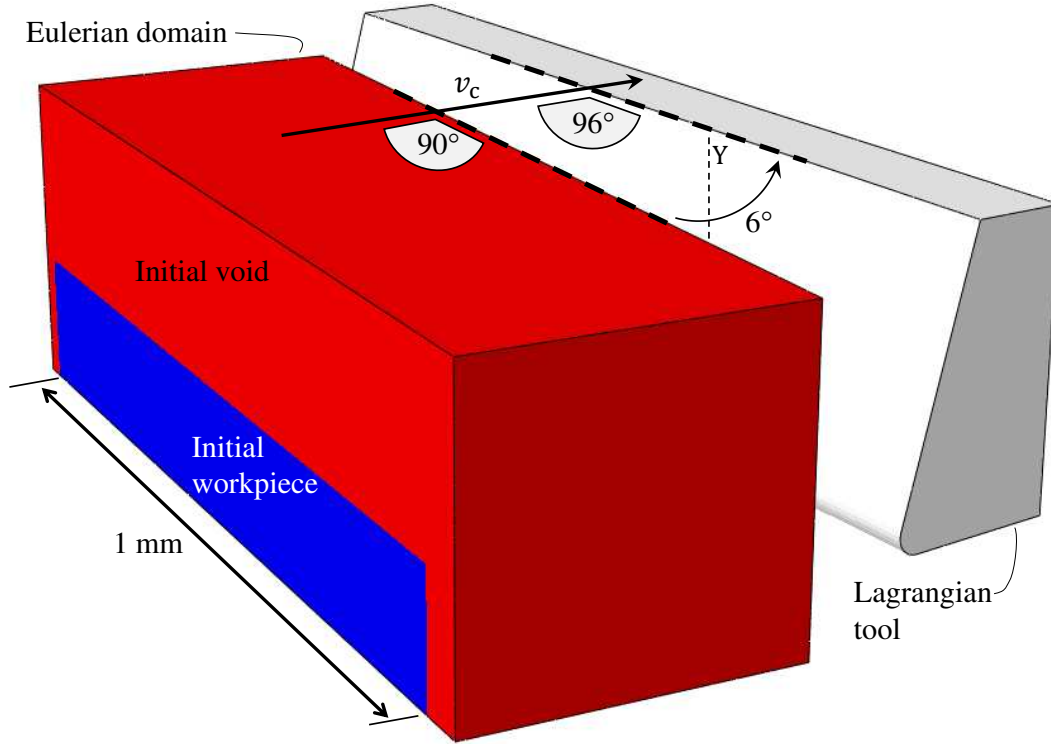


Figure 3: Configuration of the FE model for $\lambda_s = 6^\circ$

126 According to a previous mesh sensitivity study in orthogonal cutting with the
 127 CEL formulation [9], elements edge size is $5 \mu\text{m}$ in the plane parallel to the cutting
 128 speed. In the direction perpendicular to this plane, it is $5 \mu\text{m}$ in areas close to the
 129 lateral boundaries of the Eulerian domain and $50 \mu\text{m}$ in the middle of the work-
 130 piece. To reduce computation time, the size of the model depends on the value
 131 of the uncut chip thickness. This results in a Eulerian domain (EC3D8RT linear
 132 3D Eulerian elements with 8 nodes, coupled mechanical-thermal behaviour and
 133 reduced integration) composed of 216 550 to 273 350 nodes and a Lagrangian do-
 134 main (C3D8T linear 3D Lagrangian elements with 8 nodes, coupled mechanical-
 135 thermal behaviour) of 4650 nodes.

136 The Ti6Al4V workpiece is assumed to be thermo-elasto-viscoplastic (isotropic)
 137 and the inelastic heat fraction is 0.9. JC set of parameters from Seo et al. [27] is

138 adopted as the value of A corresponds to the typical yield stress value of Ti6Al4V
 139 and this set proved to provide the best results among 20 sets available in the lit-
 140 erature [28]. The tungsten carbide tool with TiN coating is assumed to be linear
 141 elastic. Material properties are provided in Table 2.

Table 2: Materials properties [27, 29, 30]

Young's modulus, E (GPa)	Ti6Al4V	113.8*
	WC	650
Poisson's ratio, ν	Ti6Al4V	0.34
	WC	0.2
Density, ρ (kg/m ³)	Ti6Al4V	4430
	WC	14 850
Conductivity, k (W/m K)	Ti6Al4V	6.3*
	WC	100
Expansion, α (1/K)	Ti6Al4V	8.6E−6*
	WC	5E−6
Specific heat, c_p (J/kg K)	Ti6Al4V	531*
	WC	202
JC constitutive model	A (MPa)	997.9
	B (MPa)	653.1
	C	0.0198
	m	0.7
	n	0.45
	$\dot{\epsilon}_0$ (1/s)	1
	T_{room} (K)	293
	T_{melt} (K)	1873

*: Dependence on the temperature, value provided at 293 K

Following the experimental results of Rech et al. [31], Coulomb's friction is assumed to occur at the tool-workpiece interface and both friction, μ , and heat partition, β , coefficients depend on the cutting speed. Limiting shear stress, τ_{max} , is included and is given by

$$\tau_{\text{max}} = \frac{\text{yield stress}}{\sqrt{3}} = \frac{A}{\sqrt{3}} \quad (2)$$

142 All the friction energy is converted into heat. Table 3 shows the friction coeffi-
 143 cients adopted in this study.

Table 3: Friction and heat transfer coefficients [29, 31]

Cutting speed, v_c (m/min)	μ	β
5	0.24	1
7.5	0.22	0.89
10	0.21	0.80
20	0.19	0.63
30	0.18	0.55
40	0.17	0.50
Limiting shear stress, τ_{\max} (MPa)	576	
Convection, U (W/m ² K)	50	
Radiation, ϵ	0.3	

144 A room temperature of 293 K is imposed on the upper and right surfaces of the
145 tool and on the left and bottom surfaces of the workpiece (Figure 2). Radiation
146 and convection are assumed to occur on the rake and clearance faces of the tool.
147 The initial temperature of the tool and the workpiece is set to room temperature
148 (293 K). Heat transfer coefficients are provided in Table 3.

149 3.2. Material constitutive model of Ti6Al4V

150 The constitutive model of the Ti6Al4V material used in all the numerical sim-
151 ulations proposed in section 4 is a thermo-elasto-viscoplastic law using a flow cri-
152 terion based on an ANN identified for the selected material and implemented in
153 the Abaqus/Explicit code via a Fortran routine VUHARD as proposed by Pantalé
154 et al. in [26]. The principle of this approach consists in replacing the analyti-
155 cal formulation of the flow law, based on a Johnson-Cook or Zerilli-Armstrong
156 type model, and allowing the calculation of the flow stress σ^y as a function of
157 the plastic strain ε^p , of the plastic strain rate, $\dot{\varepsilon}^p$, and of the temperature T , by
158 a multi-layer ANN serving as a universal approximator. Thus, the parameters of
159 the neural network can directly be identified from the experimental data without
160 having to postulate a behavioural model, which simplifies the procedure and al-
161 lows more flexibility in the definition of the model. The proposed approach also
162 allows, as shown in Pantalé et al. [26], to compute the derivatives of the flow
163 stress σ^y with respect to the three input variables of the model, a necessary step to
164 implement this model as a flow law in the form of a VUHARD subroutine in the
165 Abaqus/Explicit FEM code.

166 In order to verify the influence of the complexity of the neural network on the
 167 numerical results of the simulation and on the computing time, several architec-
 168 tures of ANN are tested thereafter (in 3.4). The chosen global architecture has
 169 2 hidden layers with a variable number of neurons for the first hidden layer ($\zeta =$
 170 9 to 17) and 7 neurons for the second hidden layer, 3 inputs (plastic strain, ε^p ,
 171 plastic strain rate, $\dot{\varepsilon}^p$, and temperature, T) and one output (the yield stress, σ^y).
 172 The global architecture of this kind of ANN is given in Figure 4 for 9 neurons in
 173 the first hidden layer. Conforming to Pantalé et al. [26], this ANN is referred to
 174 after the terminology ANN 3-9-7-1-sig, because it has 3 inputs, 9 neurons in the
 175 first hidden layer, 7 neurons in the second hidden layer, 1 output and a sigmoid
 176 activation function.

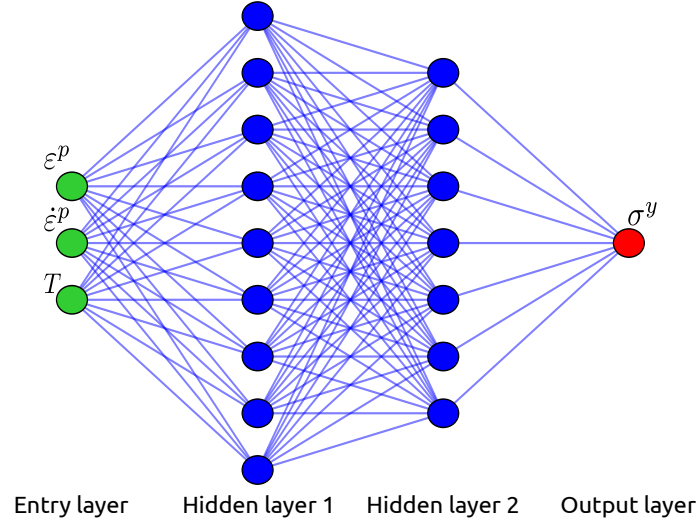


Figure 4: Architecture of the ANN 3-9-7-1-sig used for the flow law

177 The main advantage of this approach (the use of an ANN), after the training
 178 phase, is that the output σ^y of the network is related to the inputs ε^p , $\dot{\varepsilon}^p$, and T
 179 through equations (3) to (7). The first step is to scale the input data to the interval
 180 $[0, 1]$ using the following equation:

$$\vec{x} = \begin{cases} x_1 = \frac{\varepsilon^p - [\varepsilon^p]_{min}}{[\varepsilon^p]_{max} - [\varepsilon^p]_{min}} \\ x_2 = \frac{\ln(\dot{\varepsilon}^p) - [\ln(\dot{\varepsilon}^p)]_{min}}{[\ln(\dot{\varepsilon}^p)]_{max} - [\ln(\dot{\varepsilon}^p)]_{min}} \\ x_3 = \frac{T - [T]_{min}}{[T]_{max} - [T]_{min}} \end{cases} \quad (3)$$

181 The outputs of the neurons in the first hidden layer are given by the following

182 equation:

$$\vec{y}_1 = \text{sig}\left(\mathbf{w}_1 \cdot \vec{x} + \vec{b}_1\right) \quad (4)$$

where, $\text{sig}()$ is the sigmoid activation function defined by equation (5):

$$\text{sig}(x) = \frac{1}{1 + e^{-x}} \quad (5)$$

Then, the output of the neurons in the second hidden layer are given by equation (6):

$$\vec{y}_2 = \text{sig}\left(\mathbf{w}_2 \cdot \vec{y}_1 + \vec{b}_2\right) \quad (6)$$

So, the output of the ANN is therefore given by equation (7).

$$\sigma^y = ([\sigma^y]_{max} - [\sigma^y]_{min})\left(\vec{w}^T \cdot \vec{y}_2 + b\right) + [\sigma^y]_{min} \quad (7)$$

183 In equations (3) to (7), quantities \mathbf{w}_1 , \mathbf{w}_2 , \vec{w} , \vec{b}_1 , \vec{b}_2 and b are given by the
 184 training procedure of the ANN. Corresponding values for an ANN containing 9
 185 neurons in the first hidden layer are reported in Appendix A. Quantities $[\]_{min}$ and
 186 $[\]_{max}$ are the boundaries of the range of the corresponding field during the training
 187 phase, values are also given in Appendix A.

188 Because of the large number of identified parameters for all the ANN models
 189 (from 114 to 202 for 9 and 17 neurons for the first hidden layer, respectively),
 190 the other 4 sets of ANN parameters used in this publication can be found in [32].
 191 [FD:] Inclure dans corps du texte puisque article normal et rq reviewer?

192 3.3. Sensitivity study of the results to mass scaling

FE modelling of the cutting process is very expensive in CPU computation time because of the coupling of many nonlinear phenomena and the large amount of tiny finite elements. Mass scaling (MS) is introduced in the model to reduce the CPU computation time while checking that it does not influence the results (forces and energies) via a mass scaling sensitivity study. MS factors, MS_f , ranging from 1E6 (theoretical scaling of CPU time of $\sqrt{MS_f} = 1000$) to 1 (no scaling) have been used for one cutting condition ($\lambda_s = 0^\circ$, $v_c = 30$ m/min and $h = 60$ μm). The same signal processing procedure is applied to the numerical forces as to the experimental forces (cf. 2): they are filtered with a second-order low-pass Bessel

filter at 750 Hz before computing the mean value at steady state. Table 4 gives the results of the model with MS normalized (\hat{F}_i) by these of the model without MS:

$$\hat{F}_i = \frac{F_i \text{ with MS}}{F_i \text{ without MS}} \quad (8)$$

with $i = c$ for the cutting force and $i = f$ for the feed force. As expected, actual speed-up does not increase linearly with the MS_f , but it is still significant. MS_f of 1E6 leads to unstable computation and MS_f of 1E5 results in erratic forces evolutions. These results are confirmed by high values of the kinetic (KE) on internal (IE) energies ratio (it should not exceed a few % [33, 34]). An MS_f value of 1E3 is selected as it provides a good balance between computation time reduction and impact on forces, while keeping $\frac{KE}{IE}$ below 1%. To provide an order of magnitude of CPU computation time, between 10 h and 50 h (depending on the value of h) are needed on 4 cores of an Intel i7-5700HQ CPU at 2.7–3.5 GHz.

Table 4: MS sensitivity study (selected MS factor, MS_f , in bold, \hat{F}_c : normalized cutting force, \hat{F}_f : normalized feed force, KE : kinetic energy, IE : internal energy)

MS_f	CPU scaling	Speed-up	\hat{F}_c	\hat{F}_f	$\frac{KE}{IE}$ (%)
1	1	1	1	1	2.3E−4
1E2	10	9	1.006	0.982	2.2E−2
1E3	32	21	1.008	0.940	2.2E−1
1E4	100	61	1.012	0.921	2.4
1E5	316	173	Erratic	Erratic	22
1E6	1000	207	Unstable	Unstable	58

3.4. Sensitivity study of the results to the number of neurons

The number of neurons on the hidden layers may influence the results. A sensitivity study on the number of neurons for the first hidden layer, ζ , is carried out to select the ANN providing the best balance between CPU computation time and quality of the results. The results of the study are provided in Table 5. \check{F}_i corresponds to the results of the model with ANN normalized by these of the model with the built-in JC model:

$$\check{F}_i = \frac{F_i \text{ with ANN}}{F_i \text{ with JC}} \quad (9)$$

203 They show no influence on the forces when compared to the built-in Johnson-
 204 Cook model, only computation time is influenced. A first hidden layer with 9
 205 neurons is therefore selected as it leads to the lowest CPU computation time in-
 206 crease.

Table 5: Sensitivity of the forces to the number of neurons of the first layer, ζ (selection in bold, \check{F}_c : normalized cutting force, \check{F}_f : normalized feed force)

ζ	Time increase (%)	\check{F}_c	\check{F}_f
Built-in	0	1.000	1.000
9	6	1.000	0.999
11	6	1.001	1.000
13	7	1.000	0.998
15	8	1.001	1.001
17	10	1.000	1.000

207 4. Experimental and numerical results

An example of temporal evolutions of numerical and experimental forces is plotted for the 3 directions in Figure 5 at $\lambda_s = 6^\circ$, $v_c = 10$ m/min and $h = 40$ m/min. Computation of the FE models is carried out until a few microseconds after the steady state is reached. Then, linear extrapolation (dashed line between the two last markers in Figure 5) is used to provide numerical values during the same time range as the experimental values. Mean and standard deviation (2σ) are computed from the 3 experimental values. The resulting dispersion is plotted in Figure 5 around the mean values of each force. Steady state takes more time to be reached for the experiments than in the numerical model, particularly for the cutting force. Dispersion around the mean force evolution is larger for the feed force than for the cutting force, while the mean feed force value is 46% of the mean cutting force value. The numerical cutting force is very close to the experimental mean cutting force; it is only 4% larger. This difference, Δj , is computed by

$$\Delta j = \frac{|j^{(\text{sim})} - j^{(\text{exp})}|}{j^{(\text{exp})}} \times 100 \quad (10)$$

208 with j the cutting force, the feed force, the passive force or the chip thickness.
 209 $j^{(\text{sim})}$ is the mean value from the simulation, while $j^{(\text{exp})}$ is the mean experimental

210 value.

211 The numerical feed force is underestimated by the model, but it is at the bound-
 212 ary of the 95% experimental confidence interval. The numerical passive force
 213 difference is also underestimated and it is not in the narrower experimental dis-
 214 persion. The difference between the mean values of experimental and numerical
 215 feed and passive forces is 25%. A feed force less well modelled than the cutting
 216 force is typical of FE models of the cutting process and the difference with the
 217 experimental value is similar to other studies for a less extensive range of cutting
 218 conditions [7, 35–38]. Hardt and Bergs [12] also obtained higher differences for
 219 the feed and the passive force than for the cutting force. The difference for the
 220 passive force was higher than for the feed force, which is the opposite observation
 221 from this work.

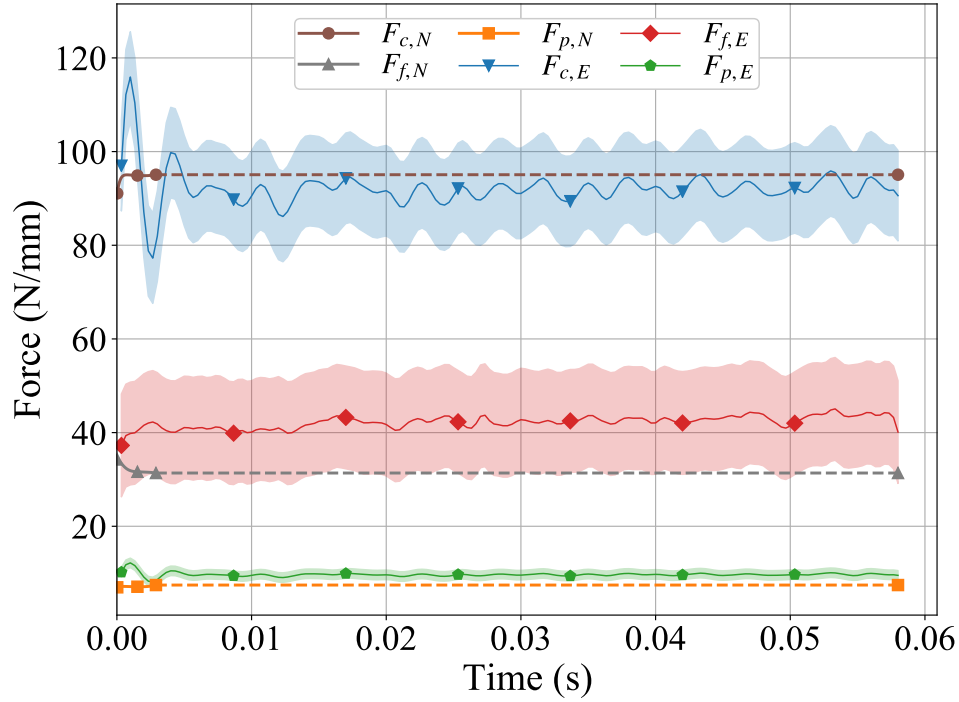


Figure 5: Temporal evolutions of experimental (E) and numerical (N) forces at $\lambda_s = 6^\circ$, $v_c = 10$ m/min and $h = 40$ μ m with dispersion around mean experimental values (linear extrapolation of numerical values in dashed)

222 Numerical chips at $v_c = 10$ m/min and $h = 40$ μ m for $\lambda_s = 0^\circ$ and $\lambda_s = 6^\circ$ are
 223 provided in Figures 6 and 7. When the cutting edge inclination is 0° , both sides
 224 of the chip are identical and a symmetry plane can be drawn in the middle of the
 225 workpiece (Figure 7 (a)). On the contrary, for the cutting edge inclination of 6° ,
 226 the chip is not aligned with the workpiece any more. The chip bends on one side
 227 due to the orientation of the tool and the symmetry is lost for both the geometry,
 228 and the thermal and mechanical fields as highlighted in Figure 7 (b). This pro-
 229 duces helical chips for the inclination angle of 6° as in the experiments. Figure 8
 230 shows the variation of the chip thickness across its width: it is thicker in the mid-
 231 dle (i.e., the body of the chip) than on its sides. This stresses the importance of 3D
 232 modelling, even for the orthogonal cutting configuration as previously highlighted
 233 [9]. 3D modelling also allows to reproduce the side flow occurring in the experi-
 234 ments for both cutting edge inclination values (Figure 6), contrary to a 2D model
 235 [3, 9, 10]. Although this leads to higher computation times, future cutting models
 236 should be in 3D, even when orthogonal cutting is considered. In this case, it is
 237 recommended to take advantage of the symmetry of the configuration to reduce
 238 computation time. This simplification has not been included in this study to avoid
 239 any difference in the FE models between the 2 cutting edge inclinations.

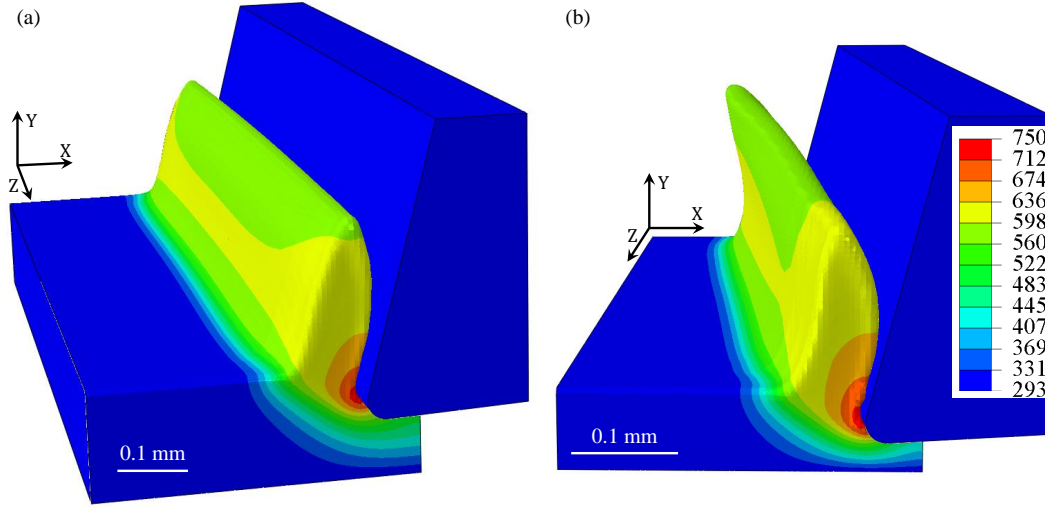


Figure 6: Temperature contours (in K) of the numerical chip after 1.5 ms at $v_c = 10$ m/min, $h = 40$ μ m and (a) $\lambda_s = 0^\circ$, (b) $\lambda_s = 6^\circ$

240 Mean values of the experimental forces and their dispersion are shown in Fig-
 241 ures 9 to 13 together with the mean numerical values. Passive force values are of

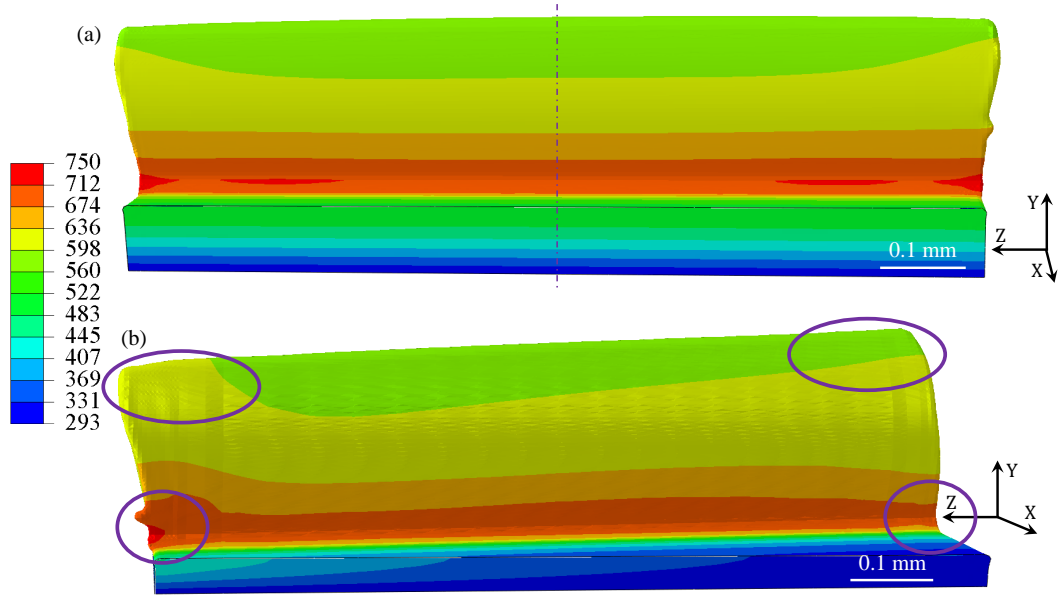


Figure 7: Temperature contours (in K) of the back of the numerical chip (tool is removed) after 1.5 ms at $v_c = 10$ m/min, $h = 40$ μ m and (a) $\lambda_s = 0^\circ$, (b) $\lambda_s = 6^\circ$

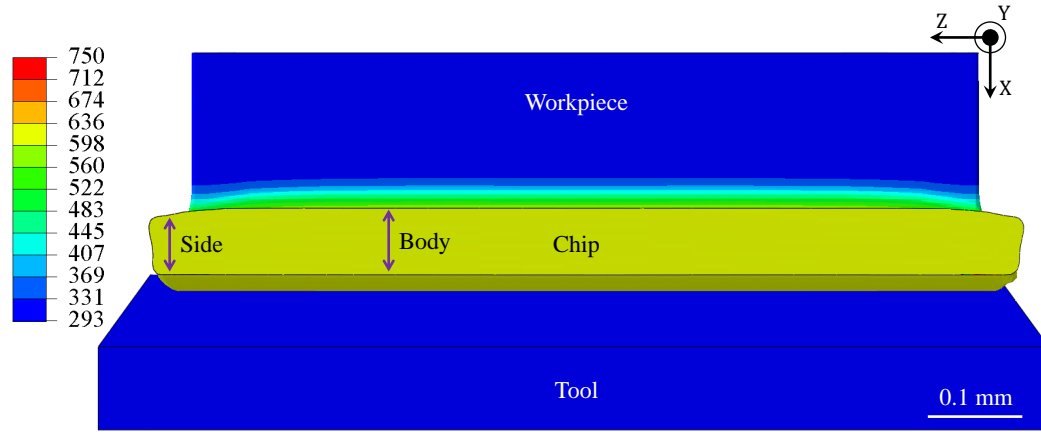


Figure 8: Temperature contours (in K) of the top of the numerical chip after 1.5 ms at $v_c = 10$ m/min, $h = 40$ μ m and $\lambda_s = 0^\circ$

course only plotted for $\lambda_s = 6^\circ$ as they are equal to zero when $\lambda_s = 0^\circ$.

The increase of the cutting force with the uncut chip thickness is clearly observed in Figures 9 and 10 for both experimental and numerical results at the 2

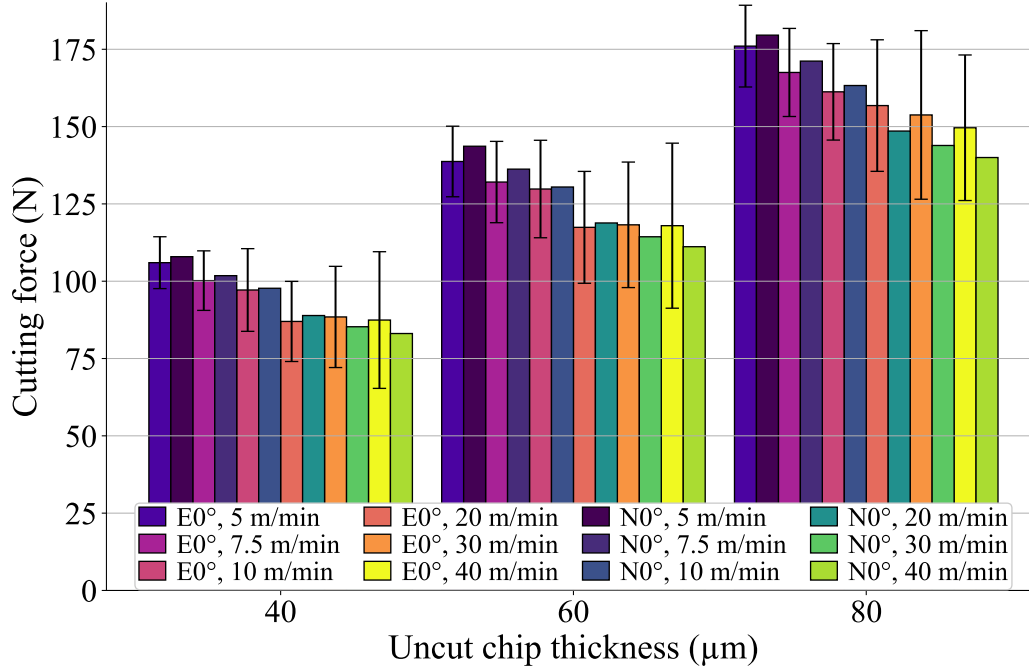


Figure 9: Comparison of experimental (E) and numerical (N) cutting forces at the cutting edge inclination of 0° for the 3 uncut chip thicknesses (40, 60 and 80 μm) and the 6 cutting speeds (5, 7.5, 10, 20, 30 and 40 m/min)

inclination angles, as well as the decrease of the force with the increase in the cutting speed. This shows temperature softening domination on strain rate hardening for Ti6Al4V and that it is accurately modelled. The increase of the inclination angle from 0° to 6° slightly reduces the cutting force; this is well captured by the model. For cutting speeds of 20–40 m/min and inclination angle of 0°, F_c is almost constant with the cutting speed for uncut chip thicknesses of 40 μm and 60 μm, while it slightly decreases for 80 μm; this small stabilization is less marked for the modelling.

An increase in the deviation around the mean value with the cutting speed is noted for values above 10 m/min. All numerical values are within a confidence interval of 95% of the experiments (35 out of 36 conditions are within a confidence interval of 68%). The mean difference with the experiments is 4%, which is remarkable, moreover given the wide range of cutting conditions considered and the absence of tuning of the model. This highlights the predictive ability and the accuracy of the FE model for both inclination angles.

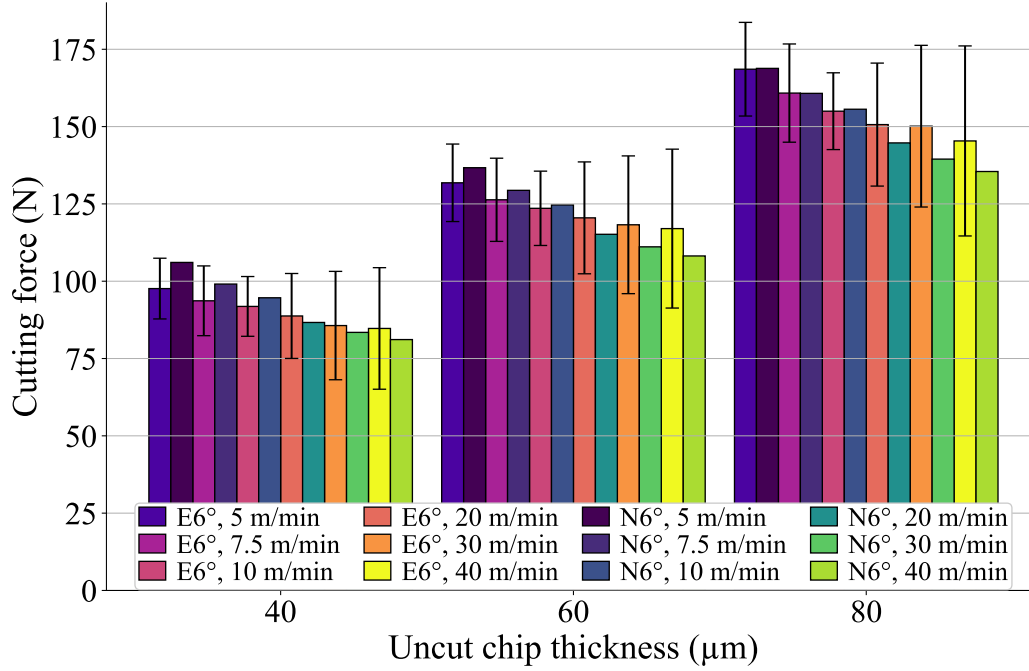


Figure 10: Comparison of experimental (E) and numerical (N) cutting forces at the cutting edge inclination of 6° for the 3 uncut chip thicknesses (40, 60 and $80\ \mu\text{m}$) and the 6 cutting speeds (5, 7.5, 10, 20, 30 and $40\ \text{m/min}$)

260 Figures 11 and 12 show the results for the feed force, where the two clearest
 261 trends for the experiments are its decrease with inclination angle and its increase
 262 with the uncut chip thickness (even if it is smaller than the expectations). For
 263 $80\ \mu\text{m}$, F_f globally decreases with v_c in the experiments. For $40\ \mu\text{m}$ and $60\ \mu\text{m}$,
 264 the force decreases at lower v_c and then increases for 0° , while a decrease is ob-
 265 served at all v_c for 6° (experimental dispersion is high for both inclination angles,
 266 but the mean trend with the cutting speed is clear at 6° , not at 0°). For the numeri-
 267 cal values, the global trend is the same for the 3 uncut chip thicknesses and both
 268 inclination angles: a decrease for the lowest v_c values and then an increase. It must
 269 be noted that the numerical model does not handle correctly the feed force trends:
 270 as clearly shown by Figure 12, the numerical forces have globally an increas-
 271 ing trend with the cutting speed, while their mean value mostly decreases when
 272 the uncut chip thickness increases. Differences between the mean numerical and
 273 experimental values increase with the uncut chip thickness: forces are closer at
 274 $40\ \mu\text{m}$ than at $80\ \mu\text{m}$. The numerical values are mostly not within the 95% confi-

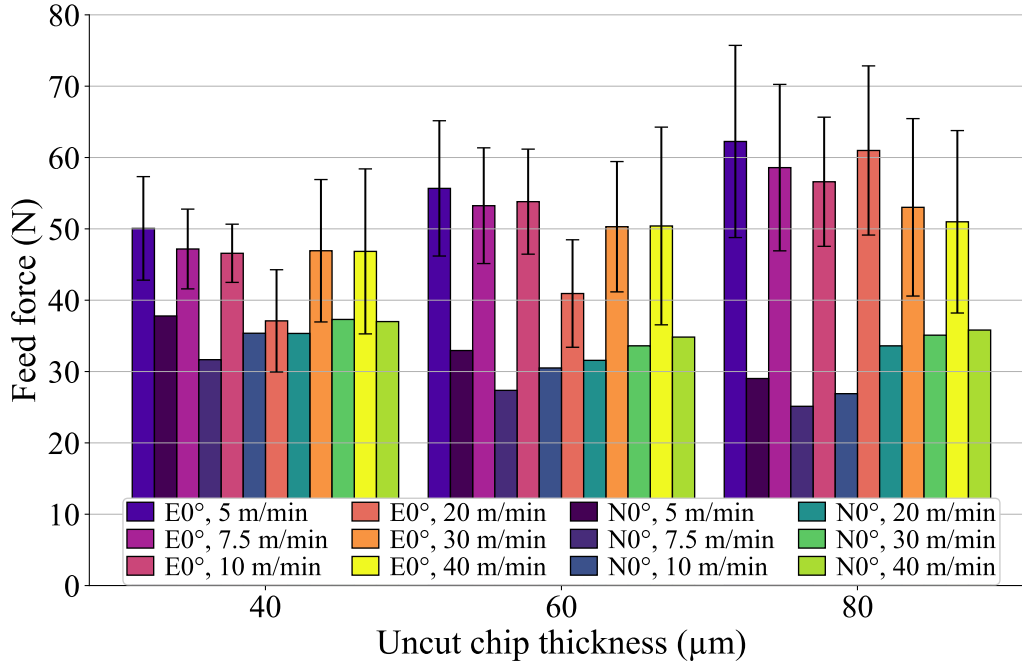


Figure 11: Comparison of experimental (E) and numerical (N) feed forces at the cutting edge inclination of 0° for the 3 uncut chip thicknesses (40, 60 and 80 μm) and the 6 cutting speeds (5, 7.5, 10, 20, 30 and 40 m/min)

275 dence interval (it has no clear evolution with the cutting conditions). Coupled with
 276 the differences in trends, it shows that F_f is less well modelled (mean difference
 277 is 39%) than F_c as usual in FE modelling of the cutting process and even more
 278 so in 3D [12]. The influence of the uncut chip thickness on the feed force should
 279 therefore be improved. Material constitutive model parameters are known to have
 280 an impact on the forces (and on the chip morphology) [25, 28]. The friction model
 281 should be improved as well to enhance the results [12].

282 Passive force is non-zero for the inclination angle of 6° (Figure 13). As the
 283 cutting force, it increases with the uncut chip thickness and it decreases with the
 284 cutting speed. Comparison with the experiments is globally the same as for F_c ,
 285 except for a larger difference in the magnitude of F_p (mean difference is 26%, but
 286 it is small in absolute – less than 5 N). The numerical values are mostly not in the
 287 95% experimental confidence interval. A lower magnitude of the passive force
 288 from the simulation than from the experiments with the right trends when the cut-
 289 ting conditions changed was also observed by Hardt and Bergs [12]. Differences

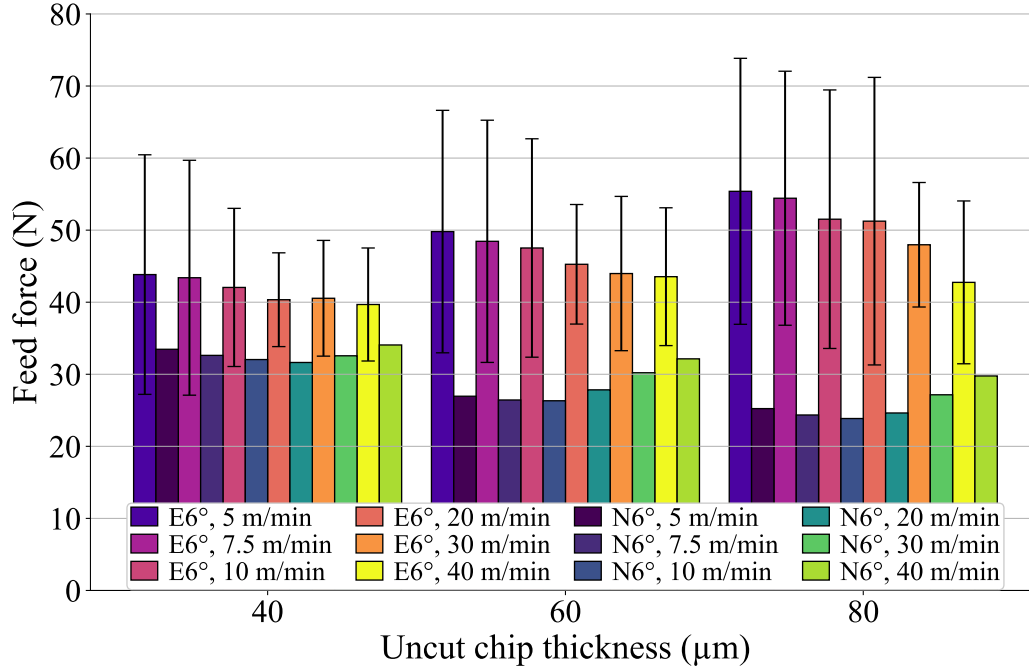


Figure 12: Comparison of experimental (E) and numerical (N) feed forces at the cutting edge inclination of 6° for the 3 uncut chip thicknesses (40, 60 and 80 μm) and the 6 cutting speeds (5, 7.5, 10, 20, 30 and 40 m/min)

290 were mainly attributed to differences in the cutting edge radius, the modelling of
 291 friction and the material constitutive model. In this work, impact of the cutting
 292 edge radius can be neglected as it is the same in the model as in the experiments.

Regarding the chips morphology, all the chips are continuous. For both the simulation and the experiments, the chip thickness ratio, λ_h :

$$\lambda_h = \frac{h'}{h} \quad (11)$$

293 with h the uncut chip thickness and h' the chip thickness, is almost independent of
 294 the uncut chip thickness (Figures 14 and 15). It is slightly reduced from $\lambda_s = 0^\circ$ to
 295 $\lambda_s = 6^\circ$, meaning that the chip thickness decreases with the inclination angle. This
 296 influence is underestimated by the model: the reduction of λ_h is lower than in the
 297 experiments. The mean difference between experimental and numerical λ_h is 17%
 298 across the whole range of cutting conditions. The chip thickness ratio decreases
 299 with the cutting speed because of the reduction of friction, which is correctly

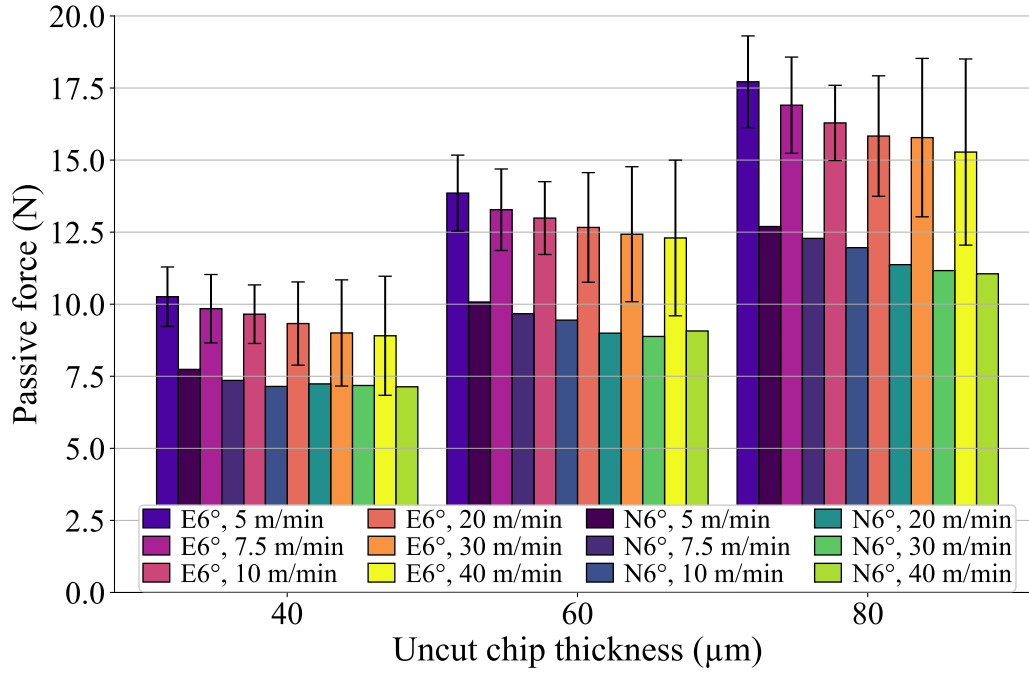


Figure 13: Comparison of experimental (E) and numerical (N) passive forces at the cutting edge inclination of 6° for the 3 uncut chip thicknesses (40, 60 and 80 μm) and the 6 cutting speeds (5, 7.5, 10, 20, 30 and 40 m/min)

300 captured by the model. As for the feed force, the results should be improved by
 301 more complex friction models and a set of material constitutive parameters for
 302 which the identification includes the forces and the chip thickness [25].

303 Differences computed according to equation 10 are shown in Table 6 to pro-
 304 vide a quantitative overview of the results. The cutting force is the best modelled
 305 quantity as observed in the literature. This results could be expected as the set
 306 of parameters of the constitutive model was selected mainly thanks to its good
 307 approximation of the cutting force [28]. As that selection was carried out with a
 308 2D model, the results show the capacity of the model to correctly handle the third
 309 (passive) force. Based on the mean differences, the performance of the model is
 310 very close for the cutting and the feed forces for both cutting edge inclinations,
 311 even if a small degradation (1% and 2%, respectively) is noted for 6°. This degra-
 312 dation is larger (7%) for the chip thickness ratio and should be linked with the
 313 difference in the passive force. Indeed, the modellings of the chip thickness and
 314 of the out-of-plane force are deeply linked. Improving the friction at the tool –

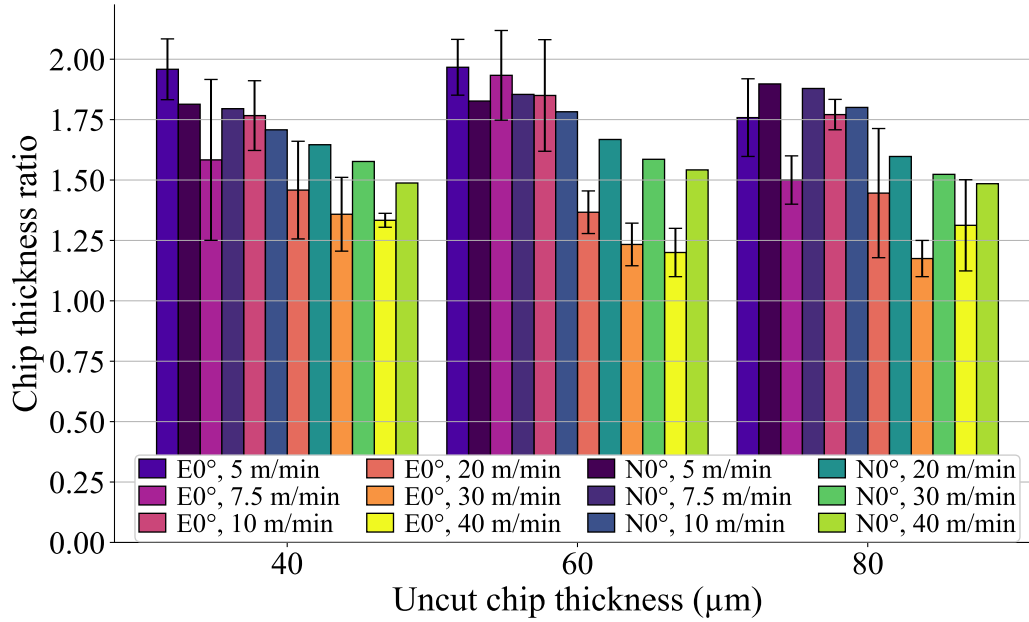


Figure 14: Comparison of experimental (E) and numerical (N) chip thickness ratios at the cutting edge inclination of 0° for the 3 uncut chip thicknesses (40, 60 and $80\ \mu\text{m}$) and the 6 cutting speeds (5, 7.5, 10, 20, 30 and $40\ \text{m/min}$)

workpiece interface should be a key point. It must be noted that the chip thickness is very well modelled in some cutting conditions with a minimal difference of 2 %. The difference is larger for the feed force than for the passive force, an opposite trend to Hardt and Bergs' [12]. Both mean and range (min – max) of differences are larger for the feed force. The smaller range of the passive force confirms an offset for all the cutting conditions, similarly to the results of Hardt and Bergs [12]. Again, the modelling of friction should be the first aspect of the model to improve in future developments.

5. Conclusions

An experimental and numerical study of the free orthogonal and oblique cutting of Ti6Al4V has been carried out for a wide range of cutting conditions using an ANN-based material constitutive model. The following main conclusions are drawn:

- The experimental study has been carried out with the same setup in free or-

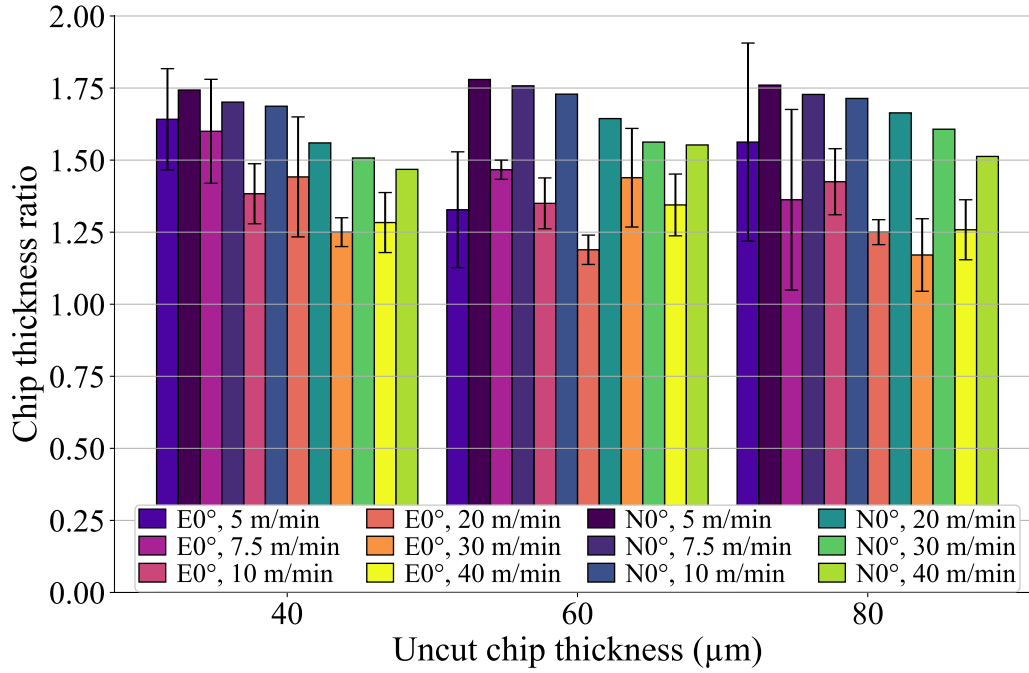


Figure 15: Comparison of experimental (E) and numerical (N) chip thickness ratios at the cutting edge inclination of 6° for the 3 uncut chip thicknesses (40, 60 and $80 \mu\text{m}$) and the 6 cutting speeds (5, 7.5, 10, 20, 30 and 40 m/min)

Table 6: Synthetic quantitative overview of the results: differences between the experimental and the numerical results (mean difference for each cutting edge inclination, and maximal, minimal and mean differences for all the conditions) for the cutting force, ΔF_c , the feed force, ΔF_f , the passive force, ΔF_p , and the chip thickness ratio, $\Delta \lambda_h$

Difference	ΔF_c (%)	ΔF_f (%)	ΔF_p (%)	$\Delta \lambda_h$ (%)
Mean $\lambda_s = 0^\circ$	3	38	–	14
Mean $\lambda_s = 6^\circ$	4	40	26	21
Max global	10	60	29	38
Min global	1	10	19	2
Mean global	4	39	26	17

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thogonal and free oblique cutting for the titanium alloy Ti6Al4V (the only change is the cutting edge inclination). It is a reference to assess the perfor-

330

mances of the 3D FE model introducing an ANN-based constitutive model and developed in the same conditions. An unpreviously seen wide range of cutting conditions, 36, is considered, including 2 cutting edge inclinations.

- Accurate evaluation of fundamental variables and their trends in 3D with the absence of tuning of both numerical parameters and model features when cutting conditions and inclination angle are significantly modified is a strong novelty of this work. Only changing the inclination angle to move from free orthogonal to oblique cutting while maintaining the quality of the results has no equivalent in the current literature, moreover as no study (experimental or numerical) on free oblique cutting is available.
- [FD:] Add one bullet on ANN and the associated future works?
- The cutting force is the best modelled quantity with a mean difference of 4 % with the experiments. The chip thickness ratio and the passive force show a higher deviation with the experiments (17% and 26%, respectively), while their trends when the cutting conditions change are accurate. This corresponds to the expected results provided by a predictive model. The difference for the feed force is higher (39%), and opposite trends by comparison with the experimental reference are observed. The absence of influence of the uncut chip thickness on friction in the model seems to be one of the aspects to include first in future works. The model turns out to handle the apparition of the third, out-of-plane, force well with no significant degradation of the results.
- Predictive abilities of the model make it adequate for the development of tools with a straight cutting edge, for example. This work moreover demonstrates the ability to model materials' behaviour with ANN and opens possibilities in this promising direction.

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481 **Appendix A. Coefficients of the ANN 3-9-7-1-sig**

482 In this Appendix, we present the values obtained after the training phase of an
483 ANN containing 9 neurons in the first hidden layer and 7 neurons in the second
484 hidden layer.

485 The training of the neural network was performed using a data set containing
486 3 430 data points defined by:

- 487 • 70 values for $\varepsilon^p \in [0.0, 3.0]$, so that $[\varepsilon^p]_{min} = 0$ and $[\varepsilon^p]_{max} = 3$.
- 488 • 7 plastic strain rates $\dot{\varepsilon}^p \in [1, 10, 50, 500, 5\,000, 50\,000, 500\,000]$, so that
489 $[\ln(\dot{\varepsilon}^p)]_{min} = 0$ and $[\ln(\dot{\varepsilon}^p)]_{max} = 13.12236$. [FD:] Add units to values?

490 • 7 temperatures $T \in [293, 400, 500, 700, 900, 1\,200, 1\,500]$, so that $[T]_{min} =$
 491 293 and $[T]_{max} = 1\,500$. [FD:] Add units to values?

492 Stresses in the training dataset ranges from $[\sigma^y]_{min} = 171.44$ to $[\sigma^y]_{max} =$
 493 2606.12. The results of the training process are given below for quantities \mathbf{W}_1 ,
 494 \mathbf{W}_2 , \vec{w} , \vec{b}_1 , \vec{b}_2 and b .

$$\mathbf{W}_1 = \begin{bmatrix} -0.87229 & -0.47675 & -1.50771 \\ -0.95762 & -0.25619 & 1.65222 \\ -10.61660 & 0.22003 & -0.11539 \\ 3.67883 & 0.37146 & -1.51069 \\ -63.39468 & 0.15466 & -0.95431 \\ 0.54807 & 0.25959 & -5.44355 \\ -1.33883 & 0.36089 & -1.66735 \\ -0.68125 & 1.02121 & 0.34242 \\ 0.08740 & 0.18764 & -41.32542 \end{bmatrix}$$

$$\mathbf{W}_2^T = \begin{bmatrix} 1.66285 & -0.59645 & -3.17333 & 0.20706 & 1.18760 & 2.01250 & -0.82147 \\ -0.26237 & -2.50330 & -1.45941 & -1.59833 & 4.05169 & -1.21146 & 1.05610 \\ -0.12958 & 0.67119 & -5.85989 & -2.55061 & 4.85245 & 4.31876 & 3.24070 \\ -2.12890 & 0.68296 & 0.71183 & 0.81706 & -0.09405 & 0.34919 & -1.41223 \\ 2.33631 & -0.08089 & 14.65789 & 0.12531 & 23.66363 & 2.55872 & 2.15338 \\ 0.11567 & 1.77629 & -1.80448 & 0.77825 & -1.58254 & 1.90442 & 1.23152 \\ 1.49265 & 0.41821 & -3.53803 & -0.48705 & -0.23671 & 0.75887 & -0.37441 \\ 0.95990 & 0.69041 & 0.43870 & 0.28393 & -1.40101 & -0.64569 & -0.38964 \\ 5.89937 & -0.13015 & 2.99264 & 1.78534 & -3.90189 & 1.17494 & -3.78854 \end{bmatrix}$$

$$\vec{w} = \begin{bmatrix} 0.34701 \\ 1.42079 \\ -0.96564 \\ 0.62467 \\ -0.56322 \\ 0.40960 \\ -0.42810 \end{bmatrix}$$

$$\vec{b}_1 = \begin{bmatrix} 2.57141 \\ 0.22673 \\ -1.16985 \\ -0.11246 \\ -0.82210 \\ -2.13264 \\ 0.78794 \\ 1.20434 \\ -3.48681 \end{bmatrix}$$

$$\vec{b}_2 = \begin{bmatrix} -0.36566 \\ -1.14445 \\ -0.79065 \\ -0.50670 \\ 1.30136 \\ 0.04521 \\ -0.29995 \end{bmatrix}$$

$$b = 0.04213$$