SESA3029 Aerothermodynamics coursework 3

ERIK VALLS MUÑOZ ID: 30405351

1. Introduction

This work covers the comparison of Mach angle and Prandtl-Meyer functions calculated using different methods, as well as a minimum length nozzle designed for argon gas operation at Mach 2.1. In order to achieve this, the method of characteristics has been used.

The method of Characteristics (MoC) starts by assuming homentropic flow, and using momentum, mass conservation, Crocco's and Prandtl-Meyer equations it derives a system of equations where the variables are the Prandtl Meyer angle ν and the flow angle θ. From here, two eigenvalues represent the slope of the Mach lines in the flow, referred as 'characteristics', one positive (C^+) and one negative(C^-). They represent the limits of sound propagation in the flow.

Decoupling the system of equations, we find two other equations which have only two variables: ν-θ and ν+θ, which are preserved along the same characteristic line. This leads to two conditions:

- ν-θ is preserved along C^+
- ν+θ is preserved along C^-

These two quantities are known as Reimann invariants, the main pillar of the method of characteristics. Knowing the Reimann invariants of points A and B, we can easily calculate flow conditions of a point P that lies in the intersection of the two characteristic lines that come from A and B, one of them being the positive and the other one the negative, as seen in figure 1.

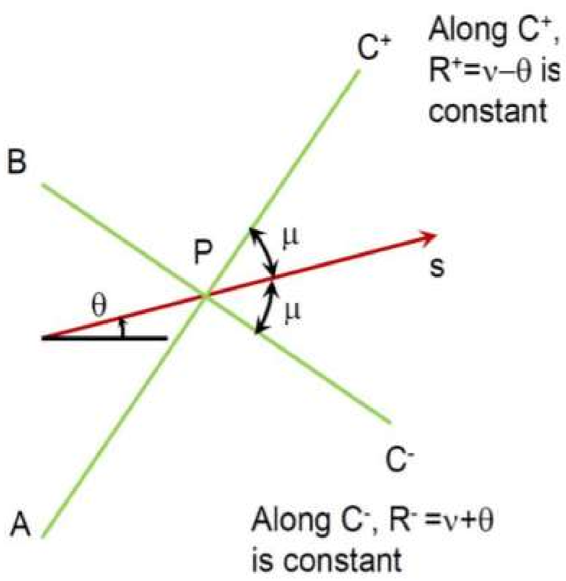

Figure 1. Characteristic lines crossing the point of interest P

Therefore:

$$R_A^+ = \nu_A - \theta_A = \nu_P - \theta_P$$

$$R_B^- = \nu_A + \theta_A = \nu_P + \theta_P$$

Two equations with two unknowns. Solving them the Prandtl-Meyer value at P can be solved, and therefore the Mach number and Mach angle μ.

To find out the P coordinates, average angles are defined as by:

$$\alpha_{AP} = \frac{1}{2}[(\theta+\mu)_A + (\theta+\mu)_P]$$

$$\alpha_{BP} = \frac{1}{2}[(\theta-\mu)_B + (\theta-\mu)_P]$$

And the coordinates are calculated using:

$$x_P = \frac{x_B \tan\alpha_{BP} - x_A\, tan\alpha_{AP} + y_A - y_B}{\tan\alpha_{BP} - tan\alpha_{AP}}$$

$$y_P = y_A + \left(x_p - x_A\right)\tan\alpha_{AP}$$

This is called unit process. In the method of characteristics, we apply the unit process for a certain number of points conforming a grid. As shown in figure 2.

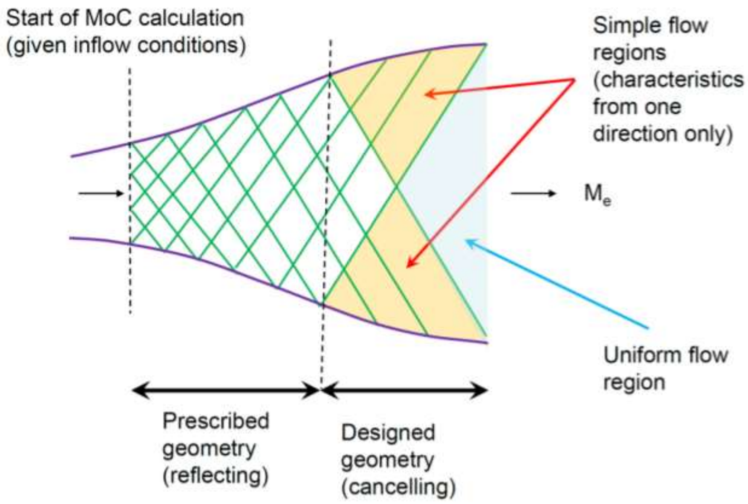

Figure 2. Method of Characteristics applied for a nozzle

For the nozzle design, we assume symmetry along the X axis (nozzle centreline) and use it as a reflection point. This means that whenever a characteristic arrives to the centreline it is transformed from positive to negative and vice versa.

Furthermore, our main objective is to achieve uniform flow at Mach 2.1 in the exit, this means we need wave cancelation, which is achieved when the flow angle is the same at both points A and P.

A maximum angle θ_{max} is needed to know the aperture of the nozzle from the throat, as seen in Figure 3. This is given by the design mach number (in this case 2.1) by $\theta_{max} = \frac{\nu_e}{2}$. Using these tools, the minimum length nozzle can be designed by using one reflection starting at the expansion fan.

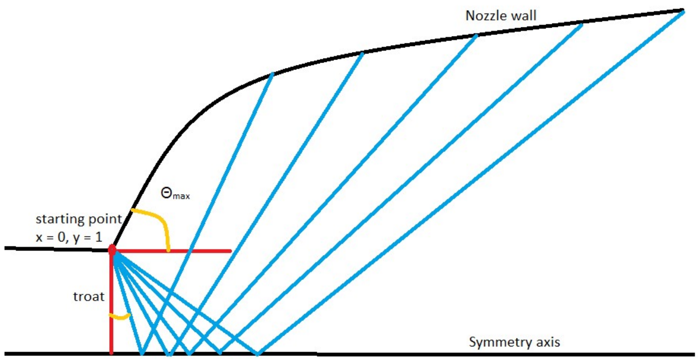

Figure 3. Nozzle design exercise using 1 reflection and 5 characteristics

2. Validation tables

M = 2.1	MACH ANGLE	PRANDTL-MEYER FUNCTION	
HAND CACULATOR	28.4364683	23.8699947	
SPREADSHEET	28.4368901	23.8700	
ONLINE CALCULATOR	28.43689	24.984	https://www.grc.nasa.gov/www/k-12/airplane/pranmyer.html

Table 1. Validation table for Mach angle and Prandtl-Meyer angle from different sources

If we look at the values for the Mach angle, there is practically no errors until the 4th decimal place, while the Prandtl-Meyer values differ even a whole unit from one another. This is caused by the Mach angle being a straightforward formula, while the Prandtl-Meyer function is calculated using different iteration methods, which may cause the values to differ depending on the iteration method used.

3. MoC template

Point	R^+	R^-	θ	ν	M	μ	x	y
a	0	0.8	0.4	0.4	1.0471	72.74974	0	1
b	0	6.5648	3.2824	3.2824	1.206619	55.97178	0	1
c	0	12.3346	6.1673	6.1673	1.332747	48.61897	0	1
d	0	18.10212	9.05106	9.05106	1.452027	43.52669	0	1
e	0	23.86951	11.93476	11.93476	1.57037	39.55308	0	1
1	0.8	0.8	0	0.8	1.0758	68.36335	0.352969	0
2	0.8	6.5648	2.88368	3.6836	1.225	54.71874	0.520166	0.32789991
3	0.8	12.3346	5.7673	6.56737	1.3495	47.81797	0.604437	0.45094572
4	0.8	18.10212	8.651062	9.451062	1.4684	42.92288	0.67254	0.53995517
5	0.8	23.86951	11.53475	12.3347	1.5869	39.06189	0.734361	0.61653037
6	0.8	-	11.53475	12.3347	1.5869	39.06189	1.260758	1.25730058
7	6.5648	6.5648	0	6.5661	1.3495	47.81797	0.782736	0
8	6.5648	12.3346	2.88497	9.44977	1.4684	42.92288	0.935478	0.16272693
9	6.5648	18.10212	5.76866	12.33346	1.5869	39.06189	1.055298	0.28389001
10	6.5648	23.86951	8.652349	15.21715	1.7075	35.84903	1.165897	0.39320701
11	6.5648	-	8.652349	15.21715	1.7075	35.84903	2.188874	1.39853244
12	12.3346	12.3346	0	12.334	1.5868	39.06482	1.122506	0
13	12.3346	18.10212	2.88406	15.21806	1.7075	35.84903	1.286967	0.13269891
14	12.3346	23.86951	5.76775	18.10175	1.8323	33.07676	1.437215	0.25345233
15	12.3346	-	5.76775	18.10175	1.8323	33.07676	2.955271	1.4759444
16	18.10212	18.10212	0	18.10175	1.8323	33.07676	1.480134	0
17	18.10212	23.86951	2.883694	20.98582	1.9626	30.63241	1.6763	0.12883946
18	18.10212	-	2.883694	20.98582	1.9626	30.63241	3.772465	1.51710848
19	23.86951	23.86951	0	23.86951	2.1	28.43689	1.903511	0
20	23.86951	-	0	23.86951	2.1	28.43689	4.705032	1.51710848

Table 2. MoC template completed for designing an argon gas nozzle with 5 characteristic lines at Mach 2.1

4. Nozzle Design

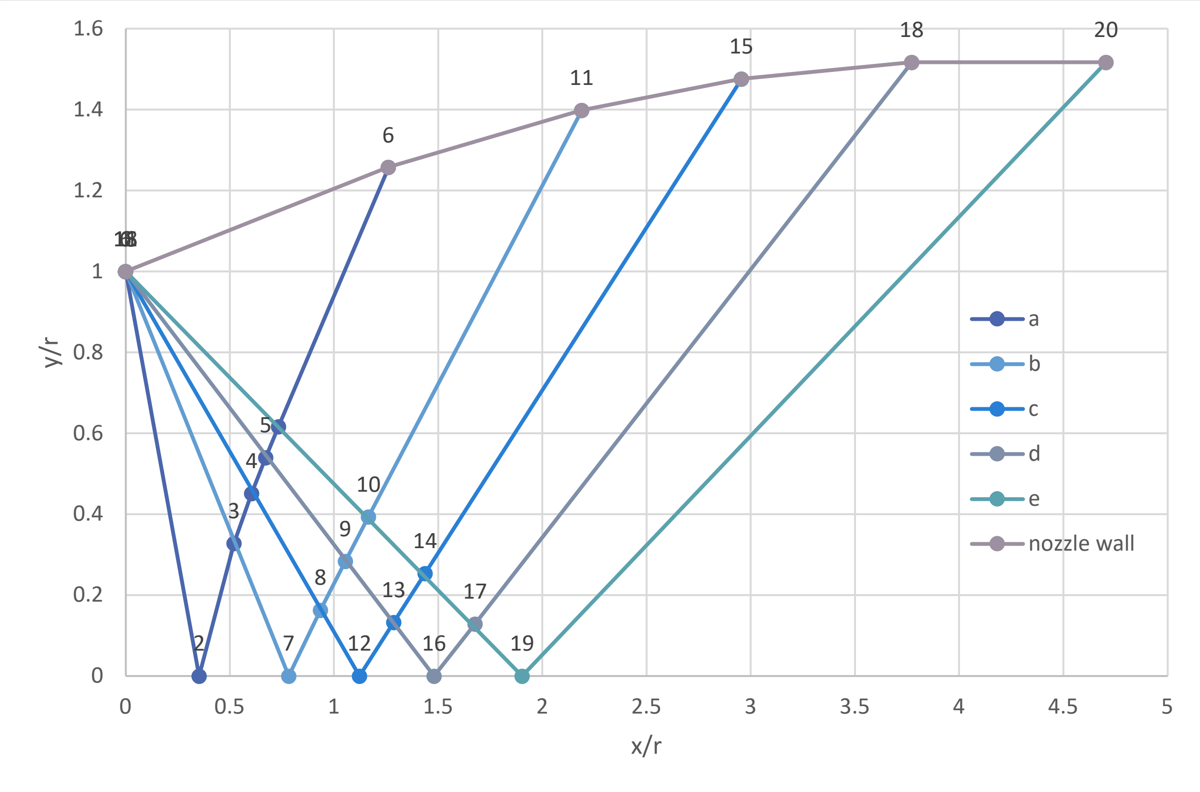

Figure 4. Final nozzle design and characteristic lines

MoC concluded that for an argon gas nozzle working at Mach 2.1, the minimum relative length ($\frac{x}{r}$) would be 4.7.

In terms of limitations, this is the optimal length was designed for Mach 2.1. Operational Mach number might vary, so its efficiency might be conditioned on the use speed. Furthermore, in this case only 5 characteristics were used, while the number of characteristics is a critical factor in the accuracy of the method. This means that the results might not be very optimal and might be considered as an approximation more than an exact design. Arbitrary decisions were taken, for example, when selecting the first characteristic flow angle and therefore there is a lot of room for improvement in those aspects. This angle cannot be 0 since it will create and endless reflection of characteristics at the throat, but it is clear that a good selection of this angle can be determinant in reducing the nozzle length.

Looking at the area ratios. For our designed nozzle, A/A* gives 1.5171, while the design downstream condition for argon gas at Mach 2.1 from shock calculators is 1.6341. this difference means that optimum performance is not achieved, as the area ratio is a design condition. This might be caused by the slight deviation from Mach 1 at the throat, as observed in Figure 5.

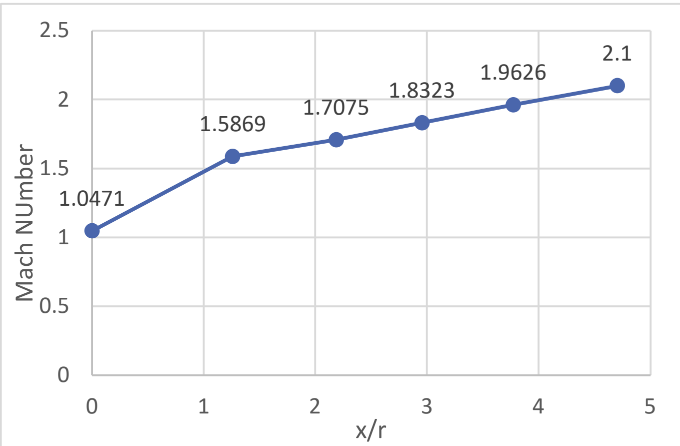

Figure 5. Mach number at each of the nozzle wall locations.