Module - 5

Date 2 11 23

- **Conditional probability**

Let A and B be two ~~cases~~ events.

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

- **Theorem of Multiplication:**

Let A be an event such that $P(A) > 0$ then for any other event B.

$$P(A \cap B) = P(A) \cdot P(B/A)$$

- **Theorem of Total Probability:**

If $A_1, A_2, \ldots A_n$ are mutually exclusive events then ~~$P(A_1 \cup A_2$~~

$$P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots + P(A_n)$$

** Remark →

For any two events A and B

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Q If $P(A) = \frac{2}{3}$

$P(B) = \frac{1}{2}$

$P(A \cup B) = \frac{3}{4}$

Find i) $P(A \cap B)$

ii) $P(A^c \cup B^c)$

iii) $P(A^c \cap B^c)$

Ans ~~$P(A \cap B)$~~

i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{3}{4} = \frac{2}{3} + \frac{1}{2} - P(A \cap B)$

$P(A \cap B) = \frac{4+3}{6} - \frac{3}{4}$

$P(A \cap B) = \frac{7}{6} - \frac{3}{4}$

$= \frac{14-9}{12}$

$= \frac{5}{12}$

ii) $P(A^c \cup B^c) = P(A^c) \cup P(B^c)$

$= P(A \cap B)^c$ using de-morgan's law

$= 1 - \frac{5}{12}$

$= \frac{12-5}{12}$

$= \frac{7}{12}$

iii) $P(A^c \cap B^c) = P(A \cup B)^c$ using de-morgan's law

$= 1 - \frac{3}{4}$

$= \frac{4-3}{4}$

$= \frac{1}{4}$

Remark →

I $P(A - B) = P(A \cap B^c)$

$= P(A) - P(A \cap B)$

II $P(B - A) = P(B \cap A^c)$

$= P(B) - P(A \cap B)$

Q $P(A) = \frac{1}{2}$

$P(B) = \frac{1}{3}$

$P(A \cap B) = \frac{1}{4}$

Find i) $P(A \cup B)$

ii) $P(A \cap B^c)$

iii) $P(A^c \cup B^c)$

Ans i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = \frac{1}{2} + \frac{1}{3} - \frac{1}{4}$

$P(A \cup B) = \frac{3+2}{6} - \frac{1}{4}$

$P(A \cup B) = \frac{5}{6} - \frac{1}{4}$

$P(A \cup B) = \frac{10-3}{12}$

$P(A \cup B) = \frac{7}{12}$

ii) $P(A \cap B^c) = P(A) - P(A \cap B)$

$= \frac{1}{2} - \frac{1}{4}$

$= \frac{2-1}{4} = \frac{1}{4}$

iii) $P(A^c \cup B^c) = P(A \cap B)^c$ using de-morgan's law

$= 1 - \frac{1}{4}$

$= \frac{4-1}{4}$

$= \frac{3}{4}$

Q $P(A) = 0.6$

$P(A \cap B) = 0.3$

$P(A/B) = 0.75$

find i) $P(A^c)$ v) $P(A^c/B)$

ii) $P(B^c)$ vi) $P(B/A^c)$

iii) $P(A \cap B^c)$ vii) $P(B^c/A^c)$

iv) $P(A^c \cap B^c)$ viii) $P(A^c \cap B)$

Ans i) $P(A^c) = 1 - 0.6$

$= 0.4$

ii) $P(A/B) = \frac{P(A \cap B)}{P(B)}$

$P(B) = \frac{0.3}{0.75}$

$P(B) = 0.4$

$$P(B^c) = 1 - 0.4$$

$$= 0.6$$

iii) $$P(A \cap B^c) = P(A) - P(A \cap B)$$

$$= 0.6 - 0.3$$

$$= 0.3$$

iv) $$P(A^c \cap B^c) = P(A \cup B)^c$$

So, $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = 0.6 + 0.4 - 0.3$$

$$= 1 - 0.3$$

$$= 0.7$$

$$P(A \cup B)^c = 1 - 0.7$$

$$= 0.3$$

v) $$P(A^c / B) = \frac{P(A^c \cap B)}{P(B)}$$

$$= \frac{P(B) - P(A \cap B)}{P(B)}$$

$$= \frac{0.4 - 0.3}{0.4}$$

$$= \frac{0.1}{0.4}$$

$$= 0.25$$

vi) $$P(B/A^c) = \frac{P(B \cap A^c)}{P(A^c)}$$

$$= \frac{P(\cancel{B}) - P(A \cap B)}{P(A^c)}$$

$$= \frac{0.4 - 0.3}{0.4}$$

$$= \frac{0.1}{0.4}$$

$$= 0.25$$

vii) $$P(B^c/A^c) = \frac{P(A^c \cap B^c)}{P(A^c)}$$

$$= \frac{P(A \cup B)^c}{P(A^c)}$$

$$= \frac{1 - 0.7}{0.4} = \frac{0.3}{0.4} = 0.75$$

viii) $P(A^c \cap B) = P(B) - P(A \cap B)$

$= 0.4 - 0.3$

$= 0.1$

- **Statistical Independence of events:**

A no. of events are said to be independent if the probability of occurance of any one of them is not affected by the supplymentary information regarding the occurance of any no. of the remaining events.

Two events A & B are said to be independent if:

$$P(A \cap B) = P(A) \cdot P(B)$$

Q An integer is choosen at random from 50 integers. 1, 2, ... , 50. What is the probability that the selected integer is divisible by 7 or 10.

7) A be the selected integer is divisible by 7

& B be the selected integer is divisible by 10.

$$P(A) = \frac{7}{50}$$

$$P(B) = \frac{5}{50}$$

Since none of the integers from 1 to 50 is a multiple of 7 as well as that of 10.

So, A & B are mutually exclusive events

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{7}{50} + \frac{5}{50} - 0$$

$$= \frac{\cancel{12}^{6}}{\cancel{50}_{25}} = \frac{6}{25}$$

8) Two newspapers X & Y are published in a city. It is estimated from a survey that 16% people read X, 14% read Y & 5% read both X & Y. Find the probability that a randomly selected person

i) doesnot read any newspaper

ii) reads only Y.

Ans) i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$P(A \cup B) = \frac{16}{100} + \frac{14}{100} - \frac{5}{100}$$

$$= \frac{30}{100} - \frac{5}{100}$$

$$= \frac{25}{100}$$

$$P(A \cup B)^c = 1 - \frac{25}{100}$$

$$= \frac{100-25}{100}$$

$$= \frac{75}{100}$$

$= 75\% \longrightarrow$ reads none.

ii) $P(B) = \frac{14}{100} - \frac{5}{100}$

$$= \frac{9}{100}$$

$= 9\% \longrightarrow$ reads only Y

4/11/23

- ## Baye's Theorem :

Suppose the events $E_1, E_2 \ldots E_n$ are mutually exclusive and exhaustive and none of them has 0 probability. Further let B be an event which too has non zero probability then

$$P(A_i/B) = \frac{P(B/A_i) \cdot P(A_i)}{\sum_{i=1}^{n} P(A_i) P(B/A_i)}$$

Q1 A picnic is arranged to be held on a particular day. The weather forecast says that there is 80% chance of rain on that day. If it rains, the probability of a good picnic is 0.3 and if it doesnot, the probability is 0.4. What is the probability that the picnic will be good.

Ans Let, A_1 denote the event that there will be rain

& $A_2 \rightarrow$ there will be no rain.

& $B \rightarrow$ picnic will good.

So,
$$P(B) = P(B \cap A_1) + P(B \cap A_2)$$

$$P(B) = P(A_1)\,P(B/A_1) + P(A_2)\,P(B/A_2)$$

$$= (0.8 \times 0.3) + (0.2 \times 0.9)$$

$$= 0.24 + 0.18$$

$$= 0.42$$

Q2 The probabilities of solving a problem by two students A & B are $\frac{3}{7}$ & $\frac{3}{8}$ respectively. If all of them try independently, find the probability that the problem could be solved by one student only. Also find the probability that the problem is not solved.

Ans $P(A) = \frac{3}{7}$ | $A \rightarrow$ student A solves

$P(B) = \frac{3}{8}$ | $B \rightarrow$ student B solves.

$$P(A^c) = 1 - \frac{3}{7}$$

$$= \frac{7-3}{7} = \frac{4}{7}$$

$$P(B^c) = 1 - \frac{3}{8}$$

$$= \frac{8-3}{8} = \frac{5}{8}$$

X = the problem is solved only by one stud[illegible]

$$P(X) = P(A \cap B^c) + P(A^c \cap B)$$

$$= P(A)P(B') + P(A')P(B)$$

$$= \frac{3}{7} \times \frac{5}{8} + \frac{4}{7} \times \frac{3}{8}$$

$$= \frac{15}{56} + \frac{12}{56}$$

$$= \frac{27}{56}$$

$$P(X^c) = 1 - \frac{27}{56}$$

$$= \frac{56-27}{56}$$

$$= \frac{29}{56}$$

Q3 3 boxes of the same appearance have the following proportion of black & white balls.

Box - 1	Box - 2	Box - 3
B → 5	B → 6	B → 3
W → 3	W → 2	W → 5

One of the boxes is selected at random and one ball is drawn randomly from it. what find the probability that the ~~drawn~~ drawn ball came from box-3 given that the ball is black.

Ans Let, $A_1 \rightarrow$ event from Box 1

$A_2 \rightarrow$ event from Box 2

$A_3 \rightarrow$ event from Box 3

& $B \rightarrow$ Ball is drawn which is black.

$$P(A_1) = P(A_2) = P(A_3) = \frac{1}{3}$$

$$P(A_3/B) = \frac{P(B/A_3) \cdot P(A_3)}{\sum_{i=1}^{3} P(A_i)\, P(B/A_i)}$$

$$= \frac{P(B/A_3) \cdot P(A_3)}{P(A_1)\,P(B/A_1) + P(A_2)\,P(B/A_2) + P(A_3)\,P(B/A_3)}$$

$$= \frac{\frac{3}{8} \times \frac{1}{3}}{\frac{1}{3}\left[\frac{5}{8} + \frac{6}{8} + \frac{3}{8}\right]}$$

$$= \frac{3/8}{\frac{1}{3}\cancel{0} \times \frac{14}{8}} = \frac{3}{14}$$

Q3 The probabilities of X, Y and Z becoming the principal of the certain collage is 0.3, 0.5 & 0.2. The probabilities that student aid fund will be introduced in the college if X, Y, Z become principle are 0.4, 0.6 & 0.1 respectively. Given that student aid has been introduced, find the probability that Y has been appointed as the principal

Ans A_1 = X becomes principal

A_2 = Y becomes principal

A_3 = Z becomes principle

B = student aid fund is started.

$P(A_1) = 0.3$; $P(A_2) = 0.5$; $P(A_3) = 0.2$

$P(B/A_1) = 0.4$; $P(B/A_2) = 0.6$; $P(B/A_3) = 0.1$

$$P(A_2/B) = \frac{P(B/A_2) \cdot P(A_2)}{\sum_{i=1}^{3} P(A_i)\ P(B/A_i)}$$

$$= \frac{0.6 \times 0.5}{P(A_1)\ P(B/A_1) + P(A_2)\ P(B/A_2) + P(A_3)\ P(B/A_3)}$$

$$= \frac{0.3}{0.3\times0.4 + 0.5\times0.6 + 0.2\times0.1}$$

$$= \frac{0.3}{0.12 + 0.3 + 0.02}$$

$$= \frac{0.3}{0.44}$$

$$= \underline{\underline{0.68}}$$