Module: 4 Introductory Probability

Page No.
Date 14 9 23

1) **Random Experiment** :- By random experiment we mean an experiment of whose all result or outcomes are known, but which one is going to be happened at a particular performance is unknown.

eg: 1) tossing a coin
2) throwing a die

2) **Sample Space** :- The set of all possible outcomes of ~~an~~ random experiment is called sample space. It is denoted by S.

eg: suppose a coin is tossing twice the sample space.

$$S = \{HH, HT, TH, TT\}$$

There are 3 types of sample space:-

i) **Finite sample space:**

If a sample space contains a finite no. of elements then it is called a finite sample space.

ii) **Countably infinite sample space:**

By countably infinite sample space we mean, that it is possible to put the all possible outcomes ~~of~~ a random experiment in a sequence & count them in a step-by-step manner.

eg: No. of ~~tumor~~ typing errors in a book we can have 0, 1, 2, ... so on errors. Theoritically speaking there can be any large no. of errors up to ∞.

iii) <u>Uncountably infinite sample space</u>:

By this we mean that the no. of all possible outcomes is so large that it is impossible to count them in a sequencial manner.

eg: The set of real nos.

3) <u>Event</u> :- The outcome of a ~~rons~~ random experiment is called an event.

There are different types of event :-

1) <u>Mutually exclusive events</u> :- several events $E_1, E_2, E_3, \ldots E_n$ are said to be mutually exclusive or mutually disjoint when no two of them can occur simultaneously.

2) Exhaustive event :- Several events $E_1, E_2, \ldots, E_n$ are said to be "exhaustive" events if atleast one of them necessarily occurs whenever the random experiment is performed.

3) Equally likely event :- Several events $E_1, E_2, \ldots, E_n$ are said to be equally likely if non-of them can be expected to occur in preference to the others, when all relevent evidents is taken into account.

OR

if there is no reason to believe that any one of them is more likely to happen than the other.

4) Complementary event :- In any random experiment E be any event then the event, then the event "Not E" is known as complementary event which is denoted by $\bar{E}$ or E' or E^c

$P(E') = 1 - P(E)$

5) Impossible event :- In any random exp. an event is said to be an impossible event if it can never happen.

eg: possibility of getting "7" if after throwing a die.

~~ii)~~ $P(\text{Impossible event}) = 0$

★ $\boxed{0 \leq P(\underbrace{E}_{\text{any event}}) \leq 1}$

6) Sure event :- In any random exp. an event is said to be an sure event if it ~~all~~ always happens.

$P(\text{sure event}) = 1$

- ## Algebra of Events :-

The algebra of events in probability comprises the following

1) The event "A or B" is the union of two sets A & B denoted by $A \cup B$. containing all those elements in A or B or Both.

eg: Consider the throwing of two die at the same time. The sample space is given by.

$$S = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)(1,6)$$
$$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)(2,6)$$
$$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)(3,6)$$
$$(4,1)\ (4,2)\ (4,3)\ (4,4),(4,5)\ (4,6)$$
$$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$$
$$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$$

A = be the event of getting a sum of two face value will be a multiple of (3)

$$A = \{(1,2)\ (2,1)\ (6,3)\ (6,6)$$
$$(2,4)\ (4,2)\ (5,4)\ (4,5)\}$$
$$(1,5)\ (5,1)$$
$$(3,3)\ (3,6)$$

B = same score on both die

B = { (1,1) (2,2) (3,3), (4,4) (5,5)
(6,6) }

A∪B = { (1,2) (2,1) (2,4) (4,2)
(1,5) (5,1) (3,3) (3,6)
(6,3) (6,6) (5,4) (4,5)
(1,1) (2,2) ~~(3,3)~~ (4,4)
(5,5) ~~(6,6)~~ }

2) The event "A and B" is the intersection of two sets ~~A & B~~
'A ∩ B' is the set of those common elements on set A & B.

eg: A = be the event of getting a score on the second die is 5 ~~B~~

B = be the event of getting sum of scores on both die is 10 or more than 10.

A = { (1,5) (2,5) (3,5) (4,5) (5,5) (6,5) }

B = { (5,5), (6,4) (4,6), (5,6) (6,5) (6,6) }

A ∩ B = { (5,5), (6,5) }.

3) **The event A but not B** :-

This the set $(A - B)$ ie difference of sets A & B. It is the set of all those elements which are in set A but not in B.

eg: consider a experiment of
$S = \{1, 2, 3, 4, 5, 6\}$

A → Prime no.
B → even no.

$A = \{2, 3, 5\}$
$B = \{2, 4, 6\}$

$(A - B) = \{3, 5\}$
& $(B - A) = \{4, 6\}$

HW

Q Consider the experiment of throwing a die & the events are.

A = no. less than 4
B = no. ~~less of~~ greater than 2 but less than 5

find i) A or B
ii) A & B.
iii) A Not B ⟺ (A − B)
iv) A' and B'

Ans $S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 2, 3\}$

$B = \{3, 4\}$

i A or $B = A \cup B = \{1, 2, 3, 4\}$

ii A and $B = A \cap B = \{3\}$

iii A not $B = (A - B) = \{1, 2\}$

iv A' and $B' = (A' \cap B')$

$A' = \{4, 5, 6\}$

$B' = \{1, 2, 5, 6\}$

$(A' \cap B') = \{5, 6\}$

16/9/23

Ⓐ **Commutative laws :-**

i) $A \cup B = B \cup A$

ii) $A \cap B = B \cap A$

Ⓑ **Associative laws :-**

i) $(A \cup B) \cup C = A \cup (B \cup C)$

ii) $(A \cap B) \cap C = A \cap (B \cap C)$

(c) Distributive laws :-

i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(d) De-Morgan's law :-

i) $(A \cup B)' = A' \cap B'$

ii) $(A \cap B)' = A' \cup B'$

- Classical Definition of Probability :-

Let in any random experiment A, has n possible outcomes, which are mutually exclusive, exhaustive and equally likely, and out of n outcomes 'm' are favourable outcomes to an event E then the probability of E, denoted by $P(E) = \frac{m}{n}$

$= \frac{\text{Favourable no. of outcomes}}{\text{Total no. of outcomes}}$

Q1. Two unbiased coins are tossed. what is the probability of obtaining

i) Both heads

ii) One head & One tail

iii) Both Tail

iv) atleast one head.

Ans Two coin are ~~tost~~ tossed.

Total no. of outcomes $= 2^2 = 4$.

Event	Favourable case	No. of favourable cases
1) Both heads	HH	1
2) One H & One tail	HT, TH	2
3) Both Tails	TT	1
4) at least 1 head	TH, HT, HH	3

$P(1) = \frac{1}{4}$

$P(2) = \frac{2}{4}$

$P(3) = \frac{1}{4}$

$P(4) = \frac{3}{4}$

Q2 A fair coin is tossed thrice ~~twice~~ find the probability that there are

i) atmost 1 tail

ii) at least one head

Ans Total no. of outcomes $= 2^3 = 8$

Event	Favourabe case	No. of favourable cases
1 atmost 1 tail	HHH, HHT, HTH, THH.	4
2 at least one hed	no head = 1 = (A)'	

So, P(atleast one head) $= 1 - P(A')$

$= 1 - \frac{1}{8}$

$= \frac{8-1}{8}$

$= \frac{7}{8}$

Q3 A bag contains 6 white & 4 black balls. 1 ball is drawn what is the probability that it is white.

Ans Total no. of balls $= 6+4 = 10$

P(white ball) $= \frac{\cancel{6}\,3}{\cancel{10}\,5} = \frac{3}{5}$

Q4 If two balls are drawn one after another from a bag containing 3 white & 5 black balls. i) What is the probability that the first ball is white & the second is black.

ii) One is white & another is black.

Ans Total no. of events $= {}^8C_1 \times {}^7C_1 = 56$

1) No. of favorable event $= {}^3C_1 \times {}^5C_1$

$= 15$

2) F.C $= ({}^3C_1 \times {}^5C_1) + ({}^5C_1 \times {}^3C_1)$

$= 15 + 15 = 30$

Q5 Two unbiased dice are thrown. Find the probability of obtaining total of 8 ~~coins~~ points.

Ans Total no. of outcomes $= 6^2 = 36$

F.C $= \{(2,6), (6,2), (4,4), (5,3), (3,5)\}$

$P(F.C) = \frac{5}{36}$

Q6 A club consisting of 15 married couple chooses a president & then a secretary by random selection. i) What is the probability that both are men.

ii) One is man & one is woman.

iii) president is man & secretary is woman.

Ans) Total no. of event $= {}^{30}P_2$

= no. of ways in which two positions can be occupied by $= 870$

i) The no. of events favourable to the event that both are men $=$

$${}^{15}C_2$$

$$= 105$$

$$P(i) = \frac{105}{870}$$

ii) $${}^{15}C_1 \times {}^{15}C_1 \times 2!$$

$$= 450$$

$P(II) = \frac{45\cancel{0}}{87\cancel{0}} = \frac{45}{87}$

III) ${}^{15}C_1 \times {}^{15}C_1$

$= 225$

$P(III) = \frac{225}{870}$

Q7 ~~A box~~ Two cards are drawn from a full pack of 52 cards find the probability that

I) Both are red cards

II) One is heart & other is diamond.

Ans Total no. of cases $= {}^{52}C_2$

$= 1326$

I) $FC = {}^{26}C_2 = 325$

$P(I) = \frac{325}{1326}$

II) $FC = {}^{13}C_1 \times {}^{13}C_1$

$P(II) = \frac{169}{1326}$

- **Frequency Definition of probability :-**

Suppose a random experiment is repeated n times under same essential conditions and an event E (connected to that random experiment) i.e. is found to occur $n(E)$ times then the ratio

$$\frac{n(E)}{n} = \text{relative frequency of } E.$$

The limiting value of $\frac{n(E)}{n}$ as $n \to \infty$ is called the probability of event E.

Denoted by $P(E) = \lim_{n \to \infty} \frac{n(E)}{n}$

Page No.
Date 21 9 23

imp Q Write the difference between primary data and secondary data.

—x—

module 4

• **Axiomatic definition of probability:-**

let S be a sample space of a random experiment and let E be any event connected with that random experiment, ie. $E \subseteq S$. The probability of E is a no. associated with E to be denoted by P(E) such that the following axioms are satisfied.

i) $P(E) \geq 0$

ii) If E be a sure event $P(E) = 1$

iii) ~~for~~ For ~~uncountably~~ any ~~infinite~~ finite number ~~of~~ or countably infinite no. of mutually exclusive events $E_1, E_2, \ldots$ of 'S' then

$$P(E_1 \cup E_2 \cup E_3 \ldots) = P(E_1) + P(E_2) + P(E_3) + \ldots$$

Permutations & Combination

23 | 9 | 23

- **Permutation** : Each of the arrangments which can be made by taking some or all of a no. of objects is called a permutation.

Note that in a permutation, the order of the arrangements of objects is taken into account. when the order is changed, a different permutation is obtained.

Q 7 candidates are to be examined, 2 in mathematic and the remaining in diff. subjects. In how many ways can they be sitted in a row so that the two examinees in maths may not sit together.

Ans when there is no restriction the total no. of ways in which 7 candidates can sit $= {}^7P_7$

$$\boxed{{}^nP_r = \frac{n!}{(n-r)!}}$$

$$= \frac{7!}{(7-7)!} = 7! = 5040$$

When two candidates of maths sit together, we consider them as 1 candidate, Now the total candidate become 6.
They can sit in way $= {}^6P_6$

$$= \frac{6!}{(6-6)!}$$

$$= 720.$$

The total no. of ways in which mathematics stu. can sit together.

$$= {}^6P_6 \times 2! = 1440$$

∴ No. of ways in which 2 maths student do not sit together

$$= 5040 - 1440$$

$$= 3600 \text{ ways.}$$

Q2 In how many ways 5 boys & 3 ~~boys~~ girls can be sitted in a row so that ~~no~~ no two girls are together.

Ans

× B × B × B × B × B ×

So No. of ways $= {}^5P_5 \times {}^6P_3$

$= 120 \times 120$

$= 14400$

- ~~Combination~~ :- Each of the groups or selections which can be made by taking sum or all of a no. of objects without reference to the order of objects in each group is called a combination.

Q1 A ~~comittee~~ commitee of 7 has to be formed from 9 boys & 4 girls. ~~How~~ In how many ways can this be done when the comittee

i) consists of exactly 3 girls

ii) atleast 3 girls.

Ans. i) ${}^4C_3 \times {}^9C_4$

$= 4 \times 126 = 504$

ii) ${}^4C_3 \times {}^9C_4 + {}^4C_4 \times {}^9C_3$

$= 504 + 1 \times 84$

$= 588.$