Assignment-IGRF

Instructions

- Write your information (name, id, section department etc.) on the front page.
- Submit a PDF version of this file using the link : https://forms.gle/4devZDwgeYVFtTym7 by the mentioned date.
- Name your pdf file as "YourName_ID". e.g., Raj_1022

Deadline: within 27/11/2024

Solve all the problems:

1. Find the value of x^3 in the group of integers under the operation $+$ (arithmetic addition).
2. In the group of integers with respect to $+$, what does $2x$ represent?
3. How many binary operations are needed for a set to satisfy the ring axioms?
4. How many binary operations are required for a set to form a group?
5. Show that if a group has two identity elements e and f, then $e = f$.
6. Let e be the identity element of a group G. Verify that $g^{-1} \cdot g = e$ for all $g \in G$.
7. Determine the number of binary operations required for a set to form a field.
8. What is the minimum number of binary operations needed for a set to form a semigroup?
9. In any group, how many trivial subgroups can be identified?
10. Verify that $\{e\}$ and G are the only trivial subgroups of a group G.
11. Verify that the set of roots of $x^4 - 1 = 0$ satisfies the group axioms under the operation of multiplication.
12. Show that the set of roots of $x^3 - 1 = 0$ forms a group under multiplication.
13. Prove that $(G, *)$ is abelian if and only if $(a * b)^2 = a^2 * b^2$ holds for all $a, b \in G$.
14. Prove that a group $(G, *)$ is abelian if $(a * b)^3 = a^3 * b^3$ for all $a, b \in G$.
15. Check whether $(\mathbb{Z}, +, \cdot)$ forms a commutative ring with unity under addition and multiplication.
16. Show that $(\mathbb{R}, +, \cdot)$ is a commutative ring with unity.
17. Prove that the set $\{1, \omega, \omega^2\}$ is an abelian group with respect to multiplication.
18. Determine if $\{1, -1, i, -i\}$ is an abelian group under multiplication.