2.11 Worked Out Exercises

Exercise 2.4 Find the number of elements in the power set of $\{x, y, z, w, v\}$.

Solution

The power set of a set S is the set of all subsets of S, including the empty set and the set S itself.

If a set S has n elements, then the number of subsets of S, i.e., the number of elements in its power set, is given by:

$$\text{Number of subsets of } S = 2^n$$

In this case, the set $S = \{x, y, z, w, v\}$ has 5 elements. Therefore, the number of elements in the power set of S is:

$$2^5 = 32$$

Hence, the number of elements in the power set of $\{x, y, z, w, v\}$ is 32.

Exercise 2.5 If $A = \{2, 3\}$, then can we say $\{3\} \subset P(A)$?

Solution *First, recall that $P(A)$ denotes the power set of A, which is the set of all subsets of A. If $A = \{2, 3\}$, then the power set of A is:*

$$P(A) = \{\emptyset, \{2\}, \{3\}, \{2, 3\}\}$$

Now, we need to check if $\{3\} \subset P(A)$. A set X is a subset of Y, denoted $X \subset Y$, if every element of X is also an element of Y.

In this case, the set $\{3\}$ has only one element, which is 3. However, 3 is not an element of $P(A)$. Instead, $\{3\}$ (the set containing the element 3) is an element of $P(A)$, but 3 by itself is not. Therefore:

$$\{3\} \not\subset P(A)$$

So, we cannot say that $\{3\} \subset P(A)$.

Exercise 2.6 If $A = \{x : x \in \mathbb{Z},\ 1 \leq x \leq 5\}$, what is the size of the power set $P(A)$?

Solution

The set $A = \{1, 2, 3, 4, 5\}$, since A contains the integers from 1 to 5.

The power set of a set with n elements contains 2^n subsets. The size of the power set is the number of subsets of A, which is 2^n, where n is the number of elements in A.

Since A has 5 elements, the size of the power set $P(A)$ is:

$$2^5 = 32$$

Therefore, the size of the power set $P(A)$ is 32.

Exercise 2.7 If $A = \{x : x \text{ is a letter in the word STRESSED}\}$ and $B = \{y : y \text{ is a letter in the word DESSERTS}\}$, then what is $A \cap B$?

Solution *First, let's find the sets A and B.*

$$A = \{S, T, R, E, S, S, E, D\} = \{S, T, R, E, D\} \quad \textit{(after removing duplicates)}$$

$$B = \{D, E, S, S, E, R, T, S\} = \{D, E, S, R, T\} \quad \textit{(after removing duplicates)}$$

Therefore,

$$A \cap B = \{D, E, S, R, T\}$$

Exercise 2.8 Let $A = \{x \mid x \text{ is an integer and } x \geq 4\}$ and $B = \{x \mid x \text{ is an integer and } -2 \leq x \leq 8\}$. Find $A \cup B$ and $A \cap B$.

Solution $A = \{x \mid x \textit{ is an integer and } x \geq 4\} = \{4, 5, 6, \ldots\}$, $B = \{x \mid x \textit{ is an integer and } -2 \leq x \leq 8\} = \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8\}$

$A \cup B = \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \ldots\}$

$A \cap B = \{4, 5, 6, 7, 8\}$

Exercise 2.9 Verify that $(A \cup B) \cap (A \cup B') = A$, where $A = \{x \mid x \text{ is an integer and } x \geq 4\}$ and $B = \{x \mid x \text{ is an integer and } -2 \leq x \leq 8\}$.

Solution $A = \{x \mid x \textit{ is an integer and } x \geq 4\} = \{4, 5, 6, \ldots\}$, $B = \{x \mid x \textit{ is an integer and } -2 \leq x \leq 8\} = \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8\}$

$$A \cup B = \{4, 5, 6, \ldots\} \cup \{-2, -1, 0, \ldots, 8\} = \{-2, -1, 0, 1, 2, 3, 4, 5, \ldots\}.$$

$$B' = \{x \mid x < -2 \textit{ or } x > 8\},$$

$$A \cup B' = \{4, 5, 6, \ldots\} \cup \{x \mid x < -2 \textit{ or } x > 8\} = \{x \mid x \geq 4 \textit{ or } x < -2\}.$$

$$(A \cup B) \cap (A \cup B') = \{x \mid x \geq 4\} = A.$$

Exercise 2.10 If $U = \{x : x \in \mathbb{Z} \text{ and } 1 \leq x \leq 10\}$, $A = \{x : x \in U \text{ and } x \text{ is a prime number}\}$, $B = \{x : x \in U \text{ and } x \text{ is even}\}$, find $A \Delta B$.

Solution *Given that,* $U = \{x : x \in \mathbb{Z} \textit{ and } 1 \leq x \leq 10\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{x : x \in U \textit{ and } x \textit{ is a prime number}\} = \{2, 3, 5, 7\}$ *[Prime numbers between 1 and 10]*

$B = \{x : x \in U \textit{ and } x \textit{ is even}\} = \{2, 4, 6, 8, 10\}$ *[Even numbers between 1 and 10]*

$A \Delta B = (A \setminus B) \cup (B \setminus A) = \{3, 5, 7, 4, 6, 8, 10\}$