Current Injection

November 2023

Formula Derivation

$$\Delta I_{RK} = I_{RK} - \sum_{i=1}^{n}(G_{ki}V_{Ri} - B_{ki}V_{Mi}) = 0 \tag{1}$$

$$\Delta I_{MK} = I_{MK} - \sum_{i=1}^{n}(G_{ki}V_{Mi} + B_{ki}V_{Ri}) = 0 \tag{2}$$

where,

$$I_{RK} = \frac{P_k^{sh}V_{RK} + Q_k^{sh}V_{Mk}}{V_{Rk}^2 + V_{Mk}^2} \tag{3}$$

$$I_{MK} = \frac{P_k^{sh}V_{MK} - Q_k^{sh}V_{Rk}}{V_{Rk}^2 + V_{Mk}^2} \tag{4}$$

Since, the equations (3) and (4) are non linear so we linearized them using Taylor Series.

For PQ Bus

Now, Let us take the partial derivatives of the equations (3) and (4) with respect to V_{Rk} and V_{Mk}. As, the for the PQ Bus voltage magnitude and angle is unknown so we take partial derivative with real and imaginary component of Voltage. The linearized form of Equation (3) and (4) are given by:

$$I_{Rk} = I_{Rk}^0 + \frac{\partial I_{Rk}}{\partial V_{Rk}}(V_{Rk} - V_{Rk}^0) + \frac{\partial I_{Rk}}{\partial V_{Mk}}(V_{Mk} - V_{Mk}^0) \tag{5}$$

$$I_{Mk} = I_{Mk}^0 + \frac{\partial I_{Mk}}{\partial V_{Rk}}(V_{Rk} - V_{Rk}^0) + \frac{\partial I_{Mk}}{\partial V_{Mk}}(V_{Mk} - V_{Mk}^0) \tag{6}$$

Where,

$$\frac{\partial I_{Rk}}{\partial V_{Rk}} = \frac{P_k^{sh}(V_{Mk}^2 - V_{Rk}^2) - 2Q_k^{sh}V_{Rk}V_{Mk}}{(V_{Rk}^2 + V_{Mk}^2)^2}$$

$$\frac{\partial I_{Rk}}{\partial V_{Mk}} = \frac{Q_k^{sh}(V_{Rk}^2 - V_{Mk}^2) - 2P_k^{sh}V_{Rk}V_{Mk}}{(V_{Rk}^2 + V_{Mk}^2)^2}$$

$$\frac{\partial I_{Mk}}{\partial V_{Rk}} = \frac{Q_k^{sh}(V_{Rk}^2 - V_{Mk}^2) - 2P_k^{sh} V_{Rk} V_{Mk}}{(V_{Rk}^2 + V_{Mk}^2)^2}$$

$$\frac{\partial I_{Mk}}{\partial V_{Mk}} = \frac{P_k^{sh}(V_{Rk}^2 - V_{Mk}^2) + 2Q_k^{sh} V_{Rk} V_{Mk}}{(V_{Rk}^2 + V_{Mk}^2)^2}$$

Now, using equation (5) and (6), equation (1) and (2) can be written as,

$$I_{Rk}^0 + \frac{\partial I_{Rk}}{\partial V_{Rk}}(V_{Rk} - V_{Rk}^0) + \frac{\partial I_{Rk}}{\partial V_{Mk}}(V_{Mk} - V_{Mk}^0) - I_{Rk}^{calc} = 0 \tag{7}$$

$$I_{Mk}^0 + \frac{\partial I_{Mk}}{\partial V_{Rk}}(V_{Rk} - V_{Rk}^0) + \frac{\partial I_{mk}}{\partial V_{Mk}}(V_{Mk} - V_{Mk}^0) - I_{Mk}^{calc} = 0 \tag{8}$$

where,

$$I_{Rk}^{calc} = \sum_{i=1}^{n} (G_{ki} V_{Ri} - B_{ki} V_{Mi})$$

$$I_{Mk}^{calc} = \sum_{i=1}^{n} (G_{ki} V_{Mi} + B_{ki} V_{Ri})$$

$$I_{Rk}^{calc} + \frac{\partial I_{Rk}}{\partial V_{Rk}}(V_{Rk}^0) + \frac{\partial I_{Rk}}{\partial V_{Mk}}(V_{Mk}^0) - I_{Rk}^0 = \frac{\partial I_{Rk}}{\partial V_{Rk}}(V_{Rk}) + \frac{\partial I_{Rk}}{\partial V_{Mk}}(V_{Mk})$$

$$Or, \Delta I_{RK} = \frac{\partial I_{Rk}}{\partial V_{Rk}}(V_{Rk}) + \frac{\partial I_{Rk}}{\partial V_{Mk}}(V_{Mk})$$

$$I_{Mk}^{calc} + \frac{\partial I_{Mk}}{\partial V_{Rk}}(V_{Rk}^0) + \frac{\partial I_{Mk}}{\partial V_{Mk}}(V_{Mk}^0) - I_{Mk}^0 = \frac{\partial I_{Mk}}{\partial V_{Rk}}(V_{Rk}) + \frac{\partial I_{Mk}}{\partial V_{Mk}}(V_{Mk})$$

$$Or, \Delta I_{MK} = \frac{\partial I_{Mk}}{\partial V_{Rk}}(V_{Rk}) + \frac{\partial I_{Mk}}{\partial V_{Mk}}(V_{Mk})$$

Now, we can write in matrix form,

$$\begin{bmatrix} \Delta I_{MK} \\ \Delta I_{RK} \end{bmatrix} = \begin{bmatrix} \frac{\partial I_{Mk}}{\partial V_{Rk}} & \frac{\partial I_{Mk}}{\partial V_{Mk}} \\ \frac{\partial I_{Rk}}{\partial V_{Rk}} & \frac{\partial I_{Rk}}{\partial V_{Mk}} \end{bmatrix} \begin{bmatrix} V_{Rk} \\ V_{Mk} \end{bmatrix} \tag{9}$$

For PV Bus

Now, Let us take the partial derivatives of the equations (3) and (4) with respect to V_{Mk} and Q_k^{sh}. As, the for the PV Bus, voltage angle and reactive power is unknown so we take partial derivative with imaginary component of Voltage and reactive power. The linearized form of Equation (3) and (4) are given by:

$$I_{Rk} = I_{Rk}^0 + \frac{\partial I_{Rk}}{\partial V_{Mk}}(V_{Mk} - V_{Mk}^0) + \frac{\partial I_{Rk}}{\partial Q_k^{sh}}(Q_k^{sh} - Q_k^{sh0}) \tag{10}$$

$$I_{Mk} = I_{Mk}^0 + \frac{\partial I_{Mk}}{\partial V_{Mk}}(V_{Mk} - V_{Mk}^0) + \frac{\partial I_{Mk}}{\partial Q_k^{sh}}(Q_k^{sh} - Q_k^{sh0}) \tag{11}$$

Where,

$$\frac{\partial I_{Rk}}{\partial Q_k^{sh}} = \frac{V_{Mk}}{(V_{Rk}^2 + V_{Mk}^2)}$$

$$\frac{\partial I_{Mk}}{\partial Q_k^{sh}} = \frac{-V_{Rk}}{(V_{Rk}^2 + V_{Mk}^2)}$$

Now, using equation (5) and (6), equation (1) and (2) can be written as,

$$I_{Rk}^0 + \frac{\partial I_{Rk}}{\partial V_{Mk}}(V_{Mk} - V_{Mk}^0) + \frac{\partial I_{Rk}}{\partial Q_k^{sh}}(Q_k^{sh} - Q_k^{sh0}) - I_{Rk}^{calc} = 0 \tag{12}$$

$$I_{Mk}^0 + \frac{\partial I_{Mk}}{\partial V_{Mk}}(V_{Mk} - V_{Mk}^0) + \frac{\partial I_{Mk}}{\partial Q_k^{sh}}(Q_k^{sh} - Q_k^{sh0}) - I_{Mk}^{calc} = 0 \tag{13}$$

$$Or, \Delta I_{RK} = \frac{\partial I_{Rk}}{\partial V_{Mk}}(V_{Mk}) + \frac{\partial I_{Rk}}{\partial Q_k^{sh}}(Q_k^{sh})$$

$$Or, \Delta I_{MK} = \frac{\partial I_{Mk}}{\partial V_{Mk}}(V_{Mk}) + \frac{\partial I_{Mk}}{\partial Q_k^{sh}}(Q_k^{sh})$$

Now, we can write in matrix form,

$$\begin{bmatrix} \Delta I_{MK} \\ \Delta I_{RK} \end{bmatrix} = \begin{bmatrix} \frac{\partial I_{Mk}}{\partial V_{Mk}} & \frac{\partial I_{Mk}}{\partial Q_k^{sh}} \\ \frac{\partial I_{Rk}}{\partial V_{Mk}} & \frac{\partial I_{Rk}}{\partial Q_k^{sh}} \end{bmatrix} \begin{bmatrix} V_{Mk} \\ Q_k^{sh} \end{bmatrix} \tag{14}$$

$$\begin{bmatrix} \begin{bmatrix} \Delta I_{M1} \\ \Delta I_{R1} \end{bmatrix} \\ \begin{bmatrix} \Delta I_{M2} \\ \Delta I_{R2} \end{bmatrix} \\ \begin{bmatrix} \Delta I_{M3} \\ \Delta I_{R3} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \frac{\partial I_{Mk}}{\partial V_{Rk}} & \frac{\partial I_{Mk}}{\partial V_{Mk}} \\ \frac{\partial I_{Rk}}{\partial V_{Rk}} & \frac{\partial I_{Rk}}{\partial V_{Mk}} \end{bmatrix} & 0 & 0 \\ 0 & \begin{bmatrix} \frac{\partial I_{Mk}}{\partial V_{Mk}} & \frac{\partial I_{Mk}}{\partial Q_k^{sh}} \\ \frac{\partial I_{Rk}}{\partial V_{Mk}} & \frac{\partial I_{Rk}}{\partial Q_k^{sh}} \end{bmatrix} & 0 \\ 0 & 0 & \begin{bmatrix} \frac{\partial I_{Mk}}{\partial V_{Rk}} & \frac{\partial I_{Mk}}{\partial V_{Mk}} \\ \frac{\partial I_{Rk}}{\partial V_{Rk}} & \frac{\partial I_{Rk}}{\partial V_{Mk}} \end{bmatrix} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} V_{Rk} \\ V_{Mk} \end{bmatrix} \\ \begin{bmatrix} V_{Mk} \\ Q_k^{sh} \end{bmatrix} \\ \begin{bmatrix} V_{Rk} \\ V_{Mk} \end{bmatrix} \end{bmatrix}$$

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