Dynamic Portfolio Selection in the U.S.

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Abstract

Dynamic portfolio choice strategies take into account time-varying economic conditions to optimally select investment weights. I apply the Brandt and Santa Clara (2006) method of dynamic portfolio selection by augmenting the asset space for stock and bond returns in the U.S. markets from 1990 to 2015, and compare these results to the static Markowitz method.

1 Introduction

Developing an optimal investment strategy for selecting a portfolio is a fundamental area of study in financial economics. The static Markowitz model of portfolio selection involves a mean-variance optimization over a set of asset expected returns variances, and covariances. However, the static method also creates portfolio weights that are time-constant, not affected by economic conditions or time-varying risk premia. Fama and French (1989,1991) and many others have identified the existence of time-varying economic risk factors that play a part in explaining the cross-section of stock returns. By taking into account these predictors for returns, it is possible to construct dynamic trading strategies which offer superior risk and expected returns relative to standard static portfolios.

Hansen and Richard (1987) were the first to develop the idea of incorporating dynamic strategies into portfolio choice. They looked at conditional managed portfolios, which account for conditioning variables that affect the conditional distribution of returns, and invests in each basis asset an amount proportional to the level of the variable. More recent research includes Bansal, Hsieh, and Viswanathan (1993), Bansal and Harvey (1996), Ferson and Siegel (2001), Cochrane (2001), Bansal, Dahlquist, and Harvey (2004), and Brandt and Santa Clara (2003, 2006).

Brandt and Santa Clara (2003, 2006) apply a method of expanding the asset space past the static model to include simple mechanically managed portfolios, and compute the optimal static portfolio within this extended asset space. The idea is that a static choice of managed portfolios is equivalent to a dynamic strategy. Given an expanded asset space of conditional variables with managed portfolios, Santa Clara and Brandt find the optimal strategy as a combination of managed portfolios.

I use the single period method of Brandt and Santa Clara (2006) with historical data in the U.S. from 1990-2015 to choose optimal portfolio weights for stocks, bonds, and cash. I use Federal Reserve senior officer opinion survey results, dividend-price ratio, short-term interest rate, term spread, and default spread as time-varying predictors.

2 Methodology

I give a brief overview of the methodology covered in Santa Clara and Brandt (2006) They first consider an investor who maximizes the conditional expected value of a quadratic utility function over next period's wealth, W_{t+1}, with a utility function as follows:

$$max : E_t\left[W_{t+1} - \frac{b_t}{2}W_{t+1}^2\right] \tag{1}$$

where b_t is positive and sufficiently small to ensure that marginal utility of wealth is positive. When returns are independent and identically distributed and the portfolio weights are constant over time, weights can be solved for with the Markowitz equation

$$x = \frac{1}{\gamma} E\big[r_{t+1} r_{t+1}^{\top}\big]^{-1} E[r_{t+1}] \tag{2}$$

where γ is a positive constant, $x_t = x$ is a vector of portfolio weights on N risky assets at time t, and r_{t+1} is the vector of excess returns on the risky assets. We assume the risk free rate is holding cash and therefore 0. This can be implemented in practice by replacing population moments with sample averages

$$x = \frac{1}{\gamma} \left[\sum_{t=1}^{T-1} r_{t+1} r_{t+1}^{\top}\right]^{-1} \left[\sum_{t=1}^{T-1} r_{t+1}\right] \tag{3}$$

Now, assuming that the optimal portfolio policies are linear in a vector of K state variables ($x_t \neq x_t$), then

$$x_t = \theta z_t \tag{4}$$

where θ is an $N \times K$ matrix of coefficients for a vector of state variables z_t, the first of which is a constant. After some linear algebra, the solution for the optimal unconditional portfolio weights $\tilde{x}$ is

$$\tilde{x} = \frac{1}{\gamma} E\big[(z_t z_t^{\top}) \otimes (r_{t+1} r_{t+1}^{\top})\big]^{-1} E[z_t \otimes r_{t+1}] \tag{5}$$

where $\otimes$ is the Kronecker product. Again, this can be applied in practice by using sample averages

$$\tilde{x} = \frac{1}{\gamma} \left[\sum_{t=0}^{T} (z_t z_t^{\top}) \otimes (r_{t+1} r_{t+1}^{\top})\right]^{-1} \left[\sum_{t=0}^{T} z_t \otimes r_{t+1}\right] \tag{6}$$

From this solution the optimal weight invested in each of the basis assets can be solved for by adding the corresponding products of elements of $\tilde{x}$ and z_t.

Since the portfolio problem is in an estimation context, it is possible to compute standard errors for the portfolio weights in order to test their significance in the portfolio policy. Similar to Britten and Jones (1999), who interpret the solution as being proportional to the coefficients of a standard ordinary least squares regression of a vector of ones on the excess returns $\tilde{r}_{t+1}$, allowing us to compute standard errors for $\tilde{x}$ from the standard errors of the regression coefficients. The constant of proportionality is $1/\gamma$.

3 Data

I use the dynamic portfolio selection strategy above to choose an optimal portfolio weight allocation between stocks, long-term bonds, short-term bonds, and cash. The predictive variables I use are the Federal Reserve Senior Loan Officer Opinion Survey on Bank Lending Practices results (FROS) and default spread (DEF).

I use returns for the S&P 500 index to represent stocks, US government 10 year government bond to represent bonds, and the three-month Treasury bill as cash. The FROS variable is based on responses to the Senior Loan Officer Opinion Survey on Bank Lending that has been given quarterly since the second quarter of 1990. The numbers I use are the "Net Percentage of Domestic Respondents Tightening Standards for C&I Loans" for large and medium firms. DEF is the difference between the yield on BAA and AAA rated corporate bonds.

Data is collected from Bloomberg and the Federal Reserve website, and the predictors are standardized similar to Santa Clara and Brandt (2006) to ease the interpretation of the coefficients of the portfolio policy. The sample period is June 1990 - March 2015, covering a total of 101 quarters. Results are calculated in R.

4 Portfolio Choice Application

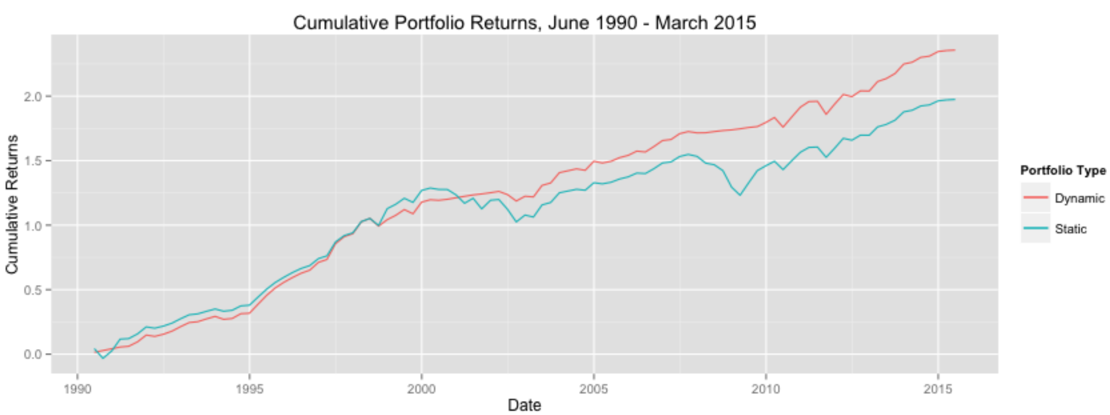
Results for the static (unconditional portfolio) weights versus the dynamic (conditional) portfolio weights are displayed in Table 1. I have scaled the weights with $\gamma = \sum \tilde{x}$ to reflect the fact that the investor invests all of their assets in either stocks, bonds, or cash. Standard errors for the coefficients of the portfolio policies are in parentheses next to their respective coefficients.

Table 1: Quarterly Portfolio Weights: June 1990 - March 2015

	State Variable	Unconditional	Conditional
Stock	Constant	.5912 (.17582*)	1.5730 (.4569*)
	FROS		-1.1685 (.4735$^+$)
	DEF		-.3800 (-.8837)
Bond	Constant	.2437 (0.0044*)	.6485 (.0104*)
	FROS		.0163 (.0040*)
	DEF		-.0429 (.0077*)
Cash	Constant	.1651 (0.0103*)	.4391 (.0226*)
	FROS		.0333 (.0074*)
	DEF		-.1188 (.0175*)
Mean excess return (%)		2.33	1.95
Cumulative excess return (%)		235.7	197.4
SD return		3.49	4.69
Sharpe Ratio		0.669	0.561

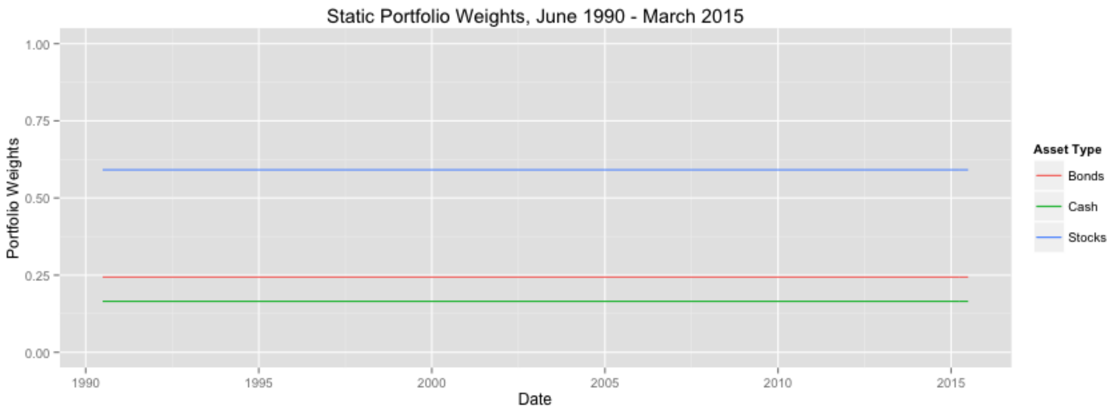
Note: $^+$ and * denote significance at the .01 and .001 level respectively

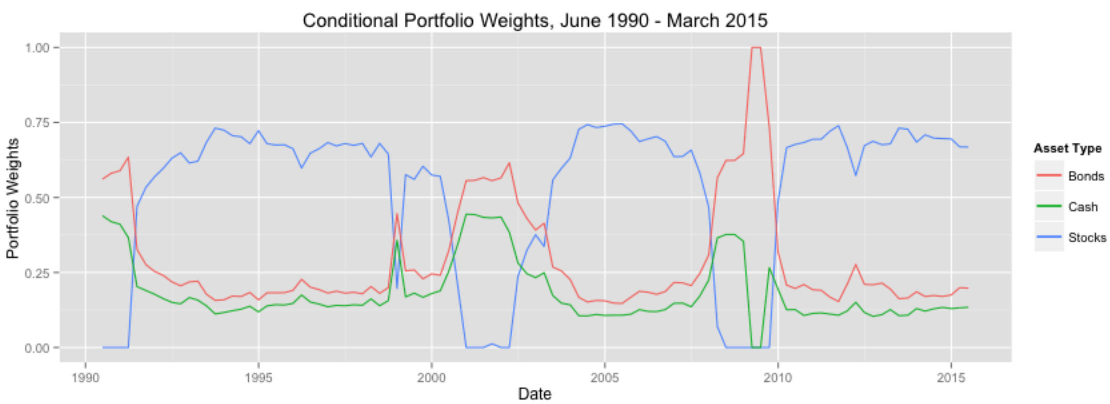
There is very significant evidence that the predictors affect the portfolio weights, with the exception of the effect of the default rate on stock return. The dynamic portfolio conditioning on only two predictive variables outperforms the static portfolio. The conditional portfolio provides a 235.7% increase in assets compared to the unconditional portfolio's 197.4% increase, and provides less volatile returns with a greater Sharpe ratio.



The graph above compares cumulative ex post returns using the conditional portfolio weights. We see that the dynamic portfolio outperforms the static portfolio, mainly in the post-2001 period. This may be because the predictive variables I chose have greater predictive power in this later period. For greater performance, investors can condition on other significant predictive variables for more accurate economic forecasting.

Below, the graphs of portfolio weights allow a comparison of how the conditional portfolio weights change over time, compared to the static portfolio weights. According to the weights, investing in stocks during in the periods of approximately 1992-1999, 2003-2007, and 2010-2015 provide greater returns. Historically, this is consistent with avoiding the stock market during the aftermath of the dot-com bubble and the U.S. housing bubble.





Using the conditional portfolio weights calculated above, we can now calculate the current optimal portfolio using predictor values from July 2015. The current quarter has a FROS of -7 and DEF of .94. After standardizing and plugging in the conditional portfolio weights, we obtain an asset selection strategy of allocating 67.69% of the investment in stocks, 19.43% in bonds, and 12.88% in cash.

5 Conclusion

I find that dynamically optimizing a portfolio of stocks, bonds, and cash using two predictors outperforms the static Markowitz model of portfolio selection. There is significant evidence that the two conditioning variables I chose–the Federal Reserve Senior Officer Opinion Survey on Bank Lending Practices for large and medium firms, and the default spread–play a part in explaining past returns. With quarterly rebalancing, the conditional portfolio historically provided lower volatility and a higher Sharpe ratio. After applying the portfolio weights to the current June 2015 quarter and solving for an optimal allocation strategy, I find that the current optimal strategy using these two conditioning variables is an allocation of 67.69% in stocks, 19.43% in bonds, and 12.88% in cash. For greater expected returns in excess of the static portfolio, we can include other significant predictive variables in the analysis.

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