UNIVERSIDADE ESTADUAL DE CAMPINAS
INSTITUTE OF COMPUTING

TOPICS IN COMBINATORIAL OPTIMIZATION
MO824

EXERCISE 1

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OBJETIVO

The objective of this activity is to model a linear programming problem and the solution of this model using the Gurobi software

DEVELOPMENT OF EXERCISE

1.- Mathematical model

1.1 Objetive Function

$$\min z = \sum_{p \in P} \sum_{f \in F} (\sum_{l \in L} x_{p,f,l} p_{p,l,f} + \sum_{j \in J} y_{p,f,j} t_{p,f,j})$$

The variable "x" represents the quantity in tons of the product "p" that a machine "l" produces in the factory "f" for the client "j".

The variable "y" represents the quantity in transport of the product "p" starting from the factory "f" to the client "j"

1.2 Restrictions

$$D_{j,p} \leq \sum_{f \in F} y_{p,f,j} \ \forall p \in P \ \forall j \in J.$$

$$\sum_{l \in L} x_{p,l,f} = \sum_{j \in J} y_{p,f,j} \ \forall p \in P \ \forall f \in F.$$

$$R_{m,f} \geq \sum_{p \in P} \sum_{l \in L} x_{p,l,f} r_{m,p,l} \ \forall f \in F \ \forall m \in M$$

$$C_{l,f} \geq \sum_{p \in P} x_{p,l,f} \forall l \in L \ \forall f \in F.$$

$$x_{p,l,f} \geq 0 \ \forall p \in P \ \forall l \in L \ \forall f \in F$$

$$y_{p,l,j} \geq 0 \ \forall p \in P \ \forall l \in L \ \forall j \in J.$$

1.3 Result

Instancia J	X variables	Y variables	Demmand	Prod_equals _transp	Material	Capacity	Objetive func	Time Ejecution (seconds)
100	4060	81200	700	812	580	580	1254910.0	1.22
200	9765	279000	1000	1395	1395	1953	1793768.0	3.66
300	39830	1707000	3000	5690	4552	3983	5409044.0	19.77
400	20610	916000	2000	2290	4122	4122	3593297.0	11.82
500	44750	2237500	2500	4475	6265	8950	4504440.0	29.56
600	26796	2296800	3600	3828	3828	4466	6438896.0	30.08
700	70488	5482400	5600	7832	5874	8811	10044034.0	139.15
800	104184	10418400	7200	13023	14470	11576	12993386.0	412.42
900	45720	8229600	5400	9144	10668	7620	9711961.0	490.7
1000	46160	9232000	8000	9232	5770	5770	14290822.0	475.86

1.2 Data analysis

When carrying out several tests we were able to verify that in a first attempt it did not manage to execute all the test data, it only stayed at 300 or maximum 500, so we used a computer with more RAM (16 RAM) and we managed to finish all the tests that the teacher requested.

Based on the table we can see that if there are J=100 clients the execution time is low (1.2 seconds) we can say that up to the instance J=600 the time is relatively low (30.08) but when we are in the instance J=700 there is a takeoff of almost five times more than the previous instance (139.15) that is where we can appreciate GUROBI's ability to solve problems that require many mathematical operations.

This is where the objective function is defined in the code:

```
# Variables declaration
```

```
x = [ [ [ model.addVar(lb=0.0, ub=GRB.INFINITY, obj=C_p[p][l][f] ,
vtype=GRB.INTEGER, name=f"x[{p}][{l}][{f}])") for f in range(F) ] for l
in range(L) ] for p in range(P) ]

y = [ [ [ model.addVar(lb=0.0, ub=GRB.INFINITY, obj=C_t[p][f][j] ,
vtype=GRB.INTEGER, name=f"y[{p}][{f}][{j}])") for j in range(J) ] for f
in range(F) ] for p in range(P) ]
```

Here we define the constraints:

```
# RESTRICTIONS

model.addConstrs(( sum([y[p][f][j] for f in range(F)]) == D[j][p] for p
in range(P) for j in range(J)),name="demmand")



model.addConstrs((sum(x[p][l][f] for l in range(L)) - sum(y[p][f][j]
for j in range(J)) == 0 for f in range(F) for p in
range(P)),name="prod_equals_transp")



model.addConstrs((sum([x[p][l][f] for l in range(L) for p in range(P)])
<= R[m][f] for f in range(F) for m in range(M)),name="material")



model.addConstrs((sum([x[p][l][f] for p in range(P)]) <= C[l][f] for f
in range(F) for l in range(L)),name="capacity")
```

I use this code in python to be able to verify that my variables have the correct dimensions:

```
for i in range(1,11):

        J, F, L, M, P, D, r, R, C, C_p, C_t = g.gen(i*100)

        print("====================  INICIO  ======================")

        print("Instância = ",J, F, L, M, P)

        print("D => ", numpy.shape(D))

        print("r => ", numpy.shape(r))
```

```
        print("R => ", numpy.shape(R))

        print("C => ", numpy.shape(C))

        print("C_p => ", numpy.shape(C_p))

        print("C_t => ", numpy.shape(C_t))

        print("-------------------------------------------------------")

        s.solve(J, F, L, M, P, D, r, R, C, C_p, C_t)

        print("=======================  FIN  ========================")
```

ANEXO

Result of the execution of the program in python gurobi

```
====================  INICIO  ====================
Instância =  100 116 5 5 7
D =>  (100, 7)
r =>  (5, 7, 5)
R =>  (5, 116)
C =>  (5, 116)
C_p =>  (7, 5, 116)
C_t =>  (7, 116, 100)
-------------------------------------------------
Gurobi Optimizer version 9.5.1 build v9.5.1rc2 (win64)
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 2672 rows, 85260 columns and 190820 nonzeros
Model fingerprint: 0xaab893b6
Variable types: 0 continuous, 85260 integer (0 binary)
Coefficient statistics:
  Matrix range  [1e+00, 1e+00]
  Objective range  [1e+01, 1e+02]
  Bounds range          [0e+00, 0e+00]
  RHS range             [1e+01, 1e+03]
Found heuristic solution: objective 747546.00000
Presolve removed 580 rows and 0 columns
Presolve time: 0.26s
Presolved: 2092 rows, 85260 columns, 170520 nonzeros
Variable types: 0 continuous, 85260 integer (0 binary)
Found heuristic solution: objective 772716.00000
```

Root relaxation: objective 1.254910e+06, 7174 iterations, 0.58 seconds (0.30 work units)

Nodes		Current Node		Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
* 0	0		0		1254910.0000	1254910.00	0.00%	-	1s

Explored 1 nodes (7174 simplex iterations) in 1.22 seconds (0.86 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 1.25491e+06 772716 747546

Optimal solution found (tolerance 1.00e-04)
Best objective 1.254910000000e+06, best bound 1.254910000000e+06, gap 0.0000%
========== FIN ==========
========== INICIO ==========
Instância = 200 279 7 5 5
D => (200, 5)
r => (5, 5, 7)
R => (5, 279)
C => (7, 279)
C_p => (5, 7, 279)
C_t => (5, 279, 200)

Gurobi Optimizer version 9.5.1 build v9.5.1rc2 (win64)
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 5743 rows, 288765 columns and 626355 nonzeros
Model fingerprint: 0x7396817c
Variable types: 0 continuous, 288765 integer (0 binary)
Coefficient statistics:
Matrix range [1e+00, 1e+00]
Objective range [1e+01, 1e+02]
Bounds range [0e+00, 0e+00]
RHS range [1e+01, 1e+03]
Found heuristic solution: objective 1041047.0000
Presolve removed 1395 rows and 0 columns
Presolve time: 1.21s
Presolved: 4348 rows, 288765 columns, 577530 nonzeros
Variable types: 0 continuous, 288765 integer (0 binary)
Found heuristic solution: objective 1094267.0000

Root simplex log...

Iteration	Objective	Primal Inf.	Dual Inf.	Time
11687	1.9670600e+06	1.629288e+04	0.000000e+00	5s
15733	1.7937680e+06	0.000000e+00	0.000000e+00	6s

Root relaxation: objective 1.793768e+06, 15733 iterations, 3.66 seconds (2.07 work units)

Nodes		Current Node		Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
* 0	0		0		1793768.0000	1793768.00	0.00%	-	6s

Explored 1 nodes (15733 simplex iterations) in 6.43 seconds (4.06 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 1.79377e+06 1.09427e+06 1.04105e+06

Optimal solution found (tolerance 1.00e-04)
Best objective 1.793768000000e+06, best bound 1.793768000000e+06, gap 0.0000%
========== FIN ==========
========== INICIO ==========
Instância = 300 569 7 8 10
D => (300, 10)
r => (8, 10, 7)
R => (8, 569)
C => (7, 569)
C_p => (10, 7, 569)
C_t => (10, 569, 300)

Gurobi Optimizer version 9.5.1 build v9.5.1rc2 (win64)
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 17225 rows, 1746830 columns and 3812300 nonzeros
Model fingerprint: 0x54c95b7b
Variable types: 0 continuous, 1746830 integer (0 binary)
Coefficient statistics:
  Matrix range [1e+00, 1e+00]
  Objective range [1e+01, 1e+02]
  Bounds range [0e+00, 0e+00]
  RHS range [1e+01, 1e+03]
Found heuristic solution: objective 3186812.0000
Presolve removed 4552 rows and 0 columns (presolve time = 5s) ...
Presolve removed 4552 rows and 0 columns
Presolve time: 5.44s
Presolved: 12673 rows, 1746830 columns, 3493660 nonzeros
Variable types: 0 continuous, 1746830 integer (0 binary)
Found heuristic solution: objective 3364526.0000

Deterministic concurrent LP optimizer: primal and dual simplex
Showing first log only...

Root simplex log...

```
Iteration         Objective       Primal Inf.       Dual Inf.          Time
       0        3.3351060e+06   1.709125e+03   1.260644e+12     15s
  141469        5.4090440e+06   0.000000e+00   0.000000e+00     19s
Concurrent spin time: 0.02s

Solved with primal simplex

Root relaxation: objective 5.409044e+06, 141469 iterations, 9.01 seconds (2.97 work units)

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

*    0        0               0    5409044.0000 5409044.00  0.00%     -   19s

Explored 1 nodes (141469 simplex iterations) in 19.77 seconds (15.61 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 5.40904e+06 3.36453e+06 3.18681e+06

Optimal solution found (tolerance 1.00e-04)
Best objective 5.409044000000e+06, best bound 5.409044000000e+06, gap 0.0000%
======================  FIN  ======================
====================  INICIO  ====================
Instância =  400 458 9 9 5
D =>  (400, 5)
r =>  (9, 5, 9)
R =>  (9, 458)
C =>  (9, 458)
C_p =>  (5, 9, 458)
C_t =>  (5, 458, 400)
-----------------------------------------------------
Gurobi Optimizer version 9.5.1 build v9.5.1rc2 (win64)
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 12534 rows, 936610 columns and 2058710 nonzeros
Model fingerprint: 0x8720ba59
Variable types: 0 continuous, 936610 integer (0 binary)
Coefficient statistics:
  Matrix range  [1e+00, 1e+00]
  Objective range  [1e+01, 1e+02]
  Bounds range          [0e+00, 0e+00]
  RHS range             [1e+01, 1e+03]
Found heuristic solution: objective 2072624.0000
Presolve removed 3955 rows and 0 columns
Presolve time: 2.88s
Presolved: 8579 rows, 936610 columns, 1880735 nonzeros
Variable types: 0 continuous, 936610 integer (0 binary)
Found heuristic solution: objective 2211262.0000
```

Root simplex log...

Iteration	Objective	Primal Inf.	Dual Inf.	Time
0	2.3933504e+08	4.715451e+06	0.000000e+00	6s
15842	3.6321860e+06	5.158475e+04	0.000000e+00	10s
21578	3.5932970e+06	0.000000e+00	0.000000e+00	12s

Root relaxation: objective 3.593297e+06, 21578 iterations, 6.09 seconds (2.81 work units)

Nodes		Current Node		Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node Time
* 0	0		0		3593297.0000	3593297.00	0.00%	- 11s

Explored 1 nodes (21578 simplex iterations) in 11.82 seconds (9.53 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 3.5933e+06 2.21126e+06 2.07262e+06

Optimal solution found (tolerance 1.00e-04)
Best objective 3.593297000000e+06, best bound 3.593297000000e+06, gap 0.0000%
========== FIN ==========
========== INICIO ==========
Instância = 500 895 10 7 5
D => (500, 5)
r => (7, 5, 10)
R => (7, 895)
C => (10, 895)
C_p => (5, 10, 895)
C_t => (5, 895, 500)

Gurobi Optimizer version 9.5.1 build v9.5.1rc2 (win64)
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 22190 rows, 2282250 columns and 4877750 nonzeros
Model fingerprint: 0xeafdd8f1
Variable types: 0 continuous, 2282250 integer (0 binary)
Coefficient statistics:
  Matrix range [1e+00, 1e+00]
  Objective range [1e+01, 1e+02]
  Bounds range [0e+00, 0e+00]
  RHS range [1e+01, 1e+03]
Found heuristic solution: objective 2619057.0000
Presolve removed 5384 rows and 0 columns (presolve time = 5s) ...
Presolve removed 5384 rows and 0 columns
Presolve time: 7.31s
Presolved: 16806 rows, 2282250 columns, 4608550 nonzeros
Variable types: 0 continuous, 2282250 integer (0 binary)
Found heuristic solution: objective 2829883.0000

Deterministic concurrent LP optimizer: primal and dual simplex
Showing first log only...

Root simplex log...

Iteration	Objective	Primal Inf.	Dual Inf.	Time
0	7.6702900e+05	1.748063e+04	1.481978e+11	22s
158434	4.0640166e+06	1.049820e+03	2.601233e+11	26s
351682	4.3819012e+06	3.834180e+02	8.278577e+10	32s
429523	4.4587591e+06	1.759514e+02	1.460430e+11	36s
532392	4.5023649e+06	1.920674e+01	1.029599e+07	41s
552248	4.5044400e+06	0.000000e+00	0.000000e+00	45s

Concurrent spin time: 0.02s

Solved with primal simplex

Root relaxation: objective 4.504440e+06, 552248 iterations, 29.56 seconds (7.31 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
* 0	0		0		4504440.0000	4504440.00	0.00%	-	45s

Explored 1 nodes (552248 simplex iterations) in 45.77 seconds (23.81 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 4.50444e+06 2.82988e+06 2.61906e+06

Optimal solution found (tolerance 1.00e-04)
Best objective 4.504440000000e+06, best bound 4.504440000000e+06, gap 0.0000%
========== FIN ==========
========== INICIO ==========
Instância = 600 638 7 6 6
D => (600, 6)
r => (6, 6, 7)
R => (6, 638)
C => (7, 638)
C_p => (6, 7, 638)
C_t => (6, 638, 600)

Gurobi Optimizer version 9.5.1 build v9.5.1rc2 (win64)
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 15722 rows, 2323596 columns and 4807968 nonzeros
Model fingerprint: 0x837ea99f
Variable types: 0 continuous, 2323596 integer (0 binary)
Coefficient statistics:

```
  Matrix range  [1e+00, 1e+00]
  Objective range  [1e+01, 1e+02]
  Bounds range     [0e+00, 0e+00]
  RHS range        [1e+01, 1e+03]
Found heuristic solution: objective 3824102.0000
Presolve removed 3828 rows and 0 columns (presolve time = 5s) ...
Presolve removed 3828 rows and 0 columns
Presolve time: 7.21s
Presolved: 11894 rows, 2323596 columns, 4647192 nonzeros
Variable types: 0 continuous, 2323596 integer (0 binary)
Found heuristic solution: objective 3999479.0000

Deterministic concurrent LP optimizer: primal and dual simplex
Showing first log only...


Root simplex log...

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0   4.1033910e+06   2.944375e+03   1.920905e+12     22s
  138067   6.3114180e+06   0.000000e+00   1.027192e+06     25s
  202374   6.4388960e+06   0.000000e+00   0.000000e+00     29s
Concurrent spin time: 0.02s

Solved with primal simplex

Root relaxation: objective 6.438896e+06, 202374 iterations, 14.93 seconds (4.32 work units)

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

*    0        0               0    6438896.0000 6438896.00  0.00%     -   29s

Explored 1 nodes (202374 simplex iterations) in 30.08 seconds (20.79 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 6.4389e+06 3.99948e+06 3.8241e+06

Optimal solution found (tolerance 1.00e-04)
Best objective 6.438896000000e+06, best bound 6.438896000000e+06, gap 0.0000%
====================  FIN  ====================
====================  INICIO  ====================
Instância =  700 979 9 6 8
D =>  (700, 8)
r =>  (6, 8, 9)
R =>  (6, 979)
C =>  (9, 979)
C_p =>  (8, 9, 979)
```

C_t => (8, 979, 700)

Gurobi Optimizer version 9.5.1 build v9.5.1rc2 (win64)
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 28117 rows, 5552888 columns and 11528704 nonzeros
Model fingerprint: 0xb9e4f6a0
Variable types: 0 continuous, 5552888 integer (0 binary)
Coefficient statistics:
  Matrix range [1e+00, 1e+00]
  Objective range [1e+01, 1e+02]
  Bounds range [0e+00, 0e+00]
  RHS range [1e+01, 1e+03]
Found heuristic solution: objective 5848963.0000
Presolve removed 4895 rows and 0 columns (presolve time = 5s) ...
Presolve removed 5619 rows and 0 columns (presolve time = 10s) ...
Presolve removed 5619 rows and 0 columns (presolve time = 15s) ...
Presolve removed 5619 rows and 0 columns
Presolve time: 19.48s
Presolved: 22498 rows, 5552888 columns, 11124136 nonzeros
Variable types: 0 continuous, 5552888 integer (0 binary)
Found heuristic solution: objective 6244403.0000

Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...

Root barrier log...

Elapsed ordering time = 5s
Elapsed ordering time = 10s
Elapsed ordering time = 15s
Elapsed ordering time = 20s
Elapsed ordering time = 25s
Elapsed ordering time = 30s
Elapsed ordering time = 35s
Elapsed ordering time = 40s
Elapsed ordering time = 45s
Ordering time: 47.67s

Barrier performed 0 iterations in 135.37 seconds (57.43 work units)
Barrier solve interrupted - model solved by another algorithm

Concurrent spin time: 0.55s

Solved with primal simplex

Root relaxation: objective 1.004403e+07, 700610 iterations, 96.67 seconds (13.18 work units)

```
     Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

*    0        0               0        1.004403e+07 1.0044e+07  0.00%   -  137s

Explored 1 nodes (700610 simplex iterations) in 139.15 seconds (53.38 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 1.0044e+07 6.2444e+06 5.84896e+06

Optimal solution found (tolerance 1.00e-04)
Best objective 1.004403400000e+07, best bound 1.004403400000e+07, gap 0.0000%
======================== FIN ========================
====================== INICIO ======================
Instância =  800 1447 8 10 9
D =>  (800, 9)
r =>  (10, 9, 8)
R =>  (10, 1447)
C =>  (8, 1447)
C_p =>  (9, 8, 1447)
C_t =>  (9, 1447, 800)
-----------------------------------------------------
Gurobi Optimizer version 9.5.1 build v9.5.1rc2 (win64)
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 46269 rows, 10522584 columns and 22087008 nonzeros
Model fingerprint: 0xb1741133
Variable types: 0 continuous, 10522584 integer (0 binary)
Coefficient statistics:
  Matrix range  [1e+00, 1e+00]
  Objective range  [1e+01, 1e+02]
  Bounds range          [0e+00, 0e+00]
  RHS range             [1e+01, 1e+03]
Found heuristic solution: objective 7623229.0000
Presolve removed 0 rows and 0 columns (presolve time = 5s) ...
Presolve removed 0 rows and 0 columns (presolve time = 10s) ...
Presolve removed 13023 rows and 0 columns (presolve time = 15s) ...
Presolve removed 14470 rows and 0 columns (presolve time = 20s) ...
Presolve removed 14470 rows and 0 columns (presolve time = 25s) ...
Presolve removed 14470 rows and 0 columns (presolve time = 30s) ...
Presolve removed 14470 rows and 0 columns (presolve time = 37s) ...
Presolve removed 14470 rows and 0 columns
Presolve time: 39.64s
Presolved: 31799 rows, 10522584 columns, 21045168 nonzeros
Variable types: 0 continuous, 10522584 integer (0 binary)
Found heuristic solution: objective 8104147.0000

Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
```

Root barrier log...

Elapsed ordering time = 5s
Elapsed ordering time = 10s
Elapsed ordering time = 15s
Elapsed ordering time = 24s
Elapsed ordering time = 25s
Elapsed ordering time = 30s
Elapsed ordering time = 35s
Elapsed ordering time = 40s
Elapsed ordering time = 45s
Elapsed ordering time = 50s
Elapsed ordering time = 55s
Elapsed ordering time = 60s
Elapsed ordering time = 65s
Elapsed ordering time = 70s
Elapsed ordering time = 75s
Elapsed ordering time = 80s
Elapsed ordering time = 85s
Elapsed ordering time = 90s
Elapsed ordering time = 95s
Elapsed ordering time = 100s
Elapsed ordering time = 105s
Elapsed ordering time = 110s
Elapsed ordering time = 115s
Elapsed ordering time = 120s
Elapsed ordering time = 125s
Elapsed ordering time = 130s
Elapsed ordering time = 135s
Elapsed ordering time = 140s
Elapsed ordering time = 145s
Elapsed ordering time = 150s
Elapsed ordering time = 155s
Elapsed ordering time = 160s
Elapsed ordering time = 165s
Elapsed ordering time = 170s
Elapsed ordering time = 175s
Ordering time: 177.71s

Barrier statistics:
 AA' NZ : 1.052e+07
 Factor NZ : 6.736e+07 (roughly 5.0 GB of memory)
 Factor Ops : 3.121e+11 (roughly 12 seconds per iteration)
 Threads : 2

Barrier performed 0 iterations in 383.80 seconds (147.93 work units)
Barrier solve interrupted - model solved by another algorithm

Concurrent spin time: 64.10s (can be avoided by choosing Method=3)

Solved with primal simplex

Root relaxation: objective 1.299339e+07, 111622 iterations, 322.04 seconds (70.18 work units)

Nodes		Current Node		Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
* 0	0		0		1.299339e+07	1.2993e+07	0.00%	-	410s

Explored 1 nodes (111622 simplex iterations) in 412.42 seconds (147.59 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 1.29934e+07 8.10415e+06 7.62323e+06

Optimal solution found (tolerance 1.00e-04)
Best objective 1.299338600000e+07, best bound 1.299338600000e+07, gap 0.0000%
========== FIN ==========
========== INICIO ==========
Instância = 900 1524 5 7 6
D => (900, 6)
r => (7, 6, 5)
R => (7, 1524)
C => (5, 1524)
C_p => (6, 5, 1524)
C_t => (6, 1524, 900)

Gurobi Optimizer version 9.5.1 build v9.5.1rc2 (win64)
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 32832 rows, 8275320 columns and 16870680 nonzeros
Model fingerprint: 0x8cb1c60d
Variable types: 0 continuous, 8275320 integer (0 binary)
Coefficient statistics:
  Matrix range [1e+00, 1e+00]
  Objective range [1e+01, 1e+02]
  Bounds range [0e+00, 0e+00]
  RHS range [1e+01, 1e+03]
Found heuristic solution: objective 5633760.0000
Presolve removed 0 rows and 0 columns (presolve time = 6s) ...
Presolve removed 9144 rows and 0 columns (presolve time = 10s) ...
Presolve removed 10668 rows and 0 columns (presolve time = 15s) ...
Presolve removed 10668 rows and 0 columns (presolve time = 20s) ...
Presolve removed 10668 rows and 0 columns (presolve time = 25s) ...
Presolve removed 10668 rows and 0 columns (presolve time = 30s) ...
Presolve removed 10668 rows and 0 columns

Presolve time: 30.04s
Presolved: 22164 rows, 8275320 columns, 16550640 nonzeros
Variable types: 0 continuous, 8275320 integer (0 binary)
Found heuristic solution: objective 5860726.0000

Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...

Root barrier log...

Ordering time: 0.09s

Barrier statistics:
 AA' NZ : 8.275e+06
 Factor NZ : 8.579e+07 (roughly 4.0 GB of memory)
 Factor Ops : 3.892e+11 (roughly 17 seconds per iteration)
 Threads : 2

	Objective		Residual			
Iter	Primal	Dual	Primal	Dual	Compl	Time
0	2.17047079e+08	5.59380735e+06	4.34e+02	5.05e+02	8.45e+01	150s
1	2.47169058e+07	1.51893113e+07	3.33e+01	1.48e+02	7.06e+00	173s
2	9.13809164e+06	1.51962327e+07	9.05e-01	4.45e+01	6.01e-01	204s
3	8.71513252e+06	1.06444539e+07	2.66e-03	1.02e+01	1.30e-01	227s
4	9.04323956e+06	9.85142054e+06	8.24e-04	3.77e+00	5.38e-02	259s
5	9.25098580e+06	9.76144158e+06	3.24e-03	1.55e+00	3.29e-02	288s
6	9.29880341e+06	9.74966001e+06	1.19e-02	1.42e+00	2.91e-02	314s
7	9.35117884e+06	9.73386051e+06	6.39e-02	1.20e+00	2.47e-02	338s
8	9.40016313e+06	9.72630339e+06	1.02e-01	1.03e+00	2.11e-02	359s
9	9.45433115e+06	9.71435516e+06	1.03e-01	8.59e-01	1.69e-02	381s
10	9.51275122e+06	9.71141456e+06	8.11e-02	2.72e-01	1.24e-02	407s
11	9.57809930e+06	9.71183147e+06	5.52e-02	1.70e-01	8.32e-03	430s
12	9.63961935e+06	9.71275901e+06	2.41e-02	4.94e-02	4.50e-03	452s
13	9.67986522e+06	9.71241714e+06	3.66e-01	2.16e-02	2.54e-03	472s

Barrier performed 13 iterations in 484.53 seconds (187.87 work units)
Barrier solve interrupted - model solved by another algorithm

Concurrent spin time: 1.58s

Solved with dual simplex

Root relaxation: objective 9.711961e+06, 937668 iterations, 420.92 seconds (65.17 work units)

Nodes		Current Node		Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time

```
*        0        0                   0        9711961.0000 9711961.00  0.00%    -  488s

Explored 1 nodes (937668 simplex iterations) in 490.73 seconds (124.90 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 9.71196e+06 5.86073e+06 5.63376e+06

Optimal solution found (tolerance 1.00e-04)
Best objective 9.711961000000e+06, best bound 9.711961000000e+06, gap 0.0000%
======================  FIN  ======================
====================  INICIO  ====================
Instância =  1000 1154 5 5 8
D =>  (1000, 8)
r =>  (5, 8, 5)
R =>  (5, 1154)
C =>  (5, 1154)
C_p =>  (8, 5, 1154)
C_t =>  (8, 1154, 1000)
---------------------------------------------------
Gurobi Optimizer version 9.5.1 build v9.5.1rc2 (win64)
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 28772 rows, 9278160 columns and 18787120 nonzeros
Model fingerprint: 0xbac6002e
Variable types: 0 continuous, 9278160 integer (0 binary)
Coefficient statistics:
  Matrix range  [1e+00, 1e+00]
  Objective range  [1e+01, 1e+02]
  Bounds range          [0e+00, 0e+00]
  RHS range             [1e+01, 1e+03]
Found heuristic solution: objective 8389227.0000
Presolve removed 0 rows and 0 columns (presolve time = 5s) ...
Presolve removed 0 rows and 0 columns (presolve time = 10s) ...
Presolve removed 4616 rows and 0 columns (presolve time = 15s) ...
Presolve removed 5770 rows and 0 columns (presolve time = 21s) ...
Presolve removed 5770 rows and 0 columns (presolve time = 25s) ...
Presolve removed 5770 rows and 0 columns (presolve time = 32s) ...
Presolve removed 5770 rows and 0 columns
Presolve time: 33.35s
Presolved: 23002 rows, 9278160 columns, 18556320 nonzeros
Variable types: 0 continuous, 9278160 integer (0 binary)
Found heuristic solution: objective 8674864.0000

Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...

Root barrier log...

Elapsed ordering time = 5s
```

Elapsed ordering time = 10s
Elapsed ordering time = 15s
Elapsed ordering time = 20s
Elapsed ordering time = 26s
Elapsed ordering time = 30s
Elapsed ordering time = 35s
Elapsed ordering time = 40s
Elapsed ordering time = 45s
Elapsed ordering time = 50s
Elapsed ordering time = 55s
Elapsed ordering time = 60s
Elapsed ordering time = 65s
Elapsed ordering time = 70s
Elapsed ordering time = 75s
Elapsed ordering time = 80s
Elapsed ordering time = 85s
Elapsed ordering time = 90s
Elapsed ordering time = 95s
Elapsed ordering time = 100s
Elapsed ordering time = 105s
Elapsed ordering time = 110s
Elapsed ordering time = 115s
Elapsed ordering time = 120s
Elapsed ordering time = 125s
Elapsed ordering time = 130s
Elapsed ordering time = 135s
Elapsed ordering time = 140s
Elapsed ordering time = 145s
Elapsed ordering time = 150s
Elapsed ordering time = 155s
Ordering time: 156.01s

Barrier statistics:
 AA' NZ : 9.278e+06
 Factor NZ : 4.256e+07 (roughly 4.0 GB of memory)
 Factor Ops : 1.381e+11 (roughly 8 seconds per iteration)
 Threads : 2

	Objective		Residual			
Iter	Primal	Dual	Primal	Dual	Compl	Time
0	2.11316935e+08	8.35431646e+06	2.83e+02	5.19e+02	7.45e+01	326s
1	7.10973223e+07	1.79227374e+07	8.23e+01	2.98e+02	2.21e+01	335s
2	1.89969427e+07	2.29426041e+07	8.18e+00	9.19e+01	2.74e+00	344s
3	1.32137847e+07	2.13784712e+07	5.47e-01	4.24e+01	6.50e-01	353s
4	1.28539778e+07	1.74452097e+07	4.57e-04	2.23e+01	2.87e-01	364s
5	1.31228700e+07	1.45927547e+07	3.97e-04	6.03e+00	8.96e-02	376s
6	1.32986512e+07	1.43928719e+07	2.88e-04	4.13e+00	6.61e-02	387s
7	1.33913167e+07	1.43474401e+07	4.55e-05	3.45e+00	5.73e-02	397s

```
   8   1.34208576e+07  1.43420610e+07  7.21e-04 3.19e+00  5.50e-02   406s
   9   1.34356345e+07  1.43237590e+07  7.08e-03 2.98e+00  5.29e-02   414s
  10   1.35861855e+07  1.43118820e+07  1.88e-01 2.60e+00  4.35e-02   423s
  11   1.37020013e+07  1.43015584e+07  2.63e-01 2.23e+00  3.61e-02   438s
  12   1.37713203e+07  1.42943787e+07  2.49e-01 1.81e+00  3.12e-02   459s

Barrier performed 12 iterations in 465.66 seconds (155.96 work units)
Barrier solve interrupted - model solved by another algorithm

Concurrent spin time: 1.38s

Solved with dual simplex

Root relaxation: objective 1.429082e+07, 1046619 iterations, 391.31 seconds (67.00 work
units)

            Nodes |          Current Node  |          Objective Bounds       |          Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent BestBd   Gap | It/Node Time

*        0        0                0         1.429082e+07 1.4291e+07  0.00%   -  472s

Explored 1 nodes (1046619 simplex iterations) in 475.86 seconds (113.67 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 1.42908e+07 8.67486e+06 8.38923e+06

Optimal solution found (tolerance 1.00e-04)
Best objective 1.429082200000e+07, best bound 1.429082200000e+07, gap 0.0000%
======================  FIN  ======================
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