

## Project2 Report

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### 1 PART 1

a).

if the number of packets in the system less than B, then the packet loss won't happens.

for markov chain, the maximum packet in the system is B, hence,

$$\lambda\pi_0 = \mu\pi_1 \quad (1.1)$$

$$\lambda\pi_1 = \mu\pi_2 \quad (1.2)$$

$$\dots \quad (1.3)$$

$$\lambda\pi_B = \mu\pi_{B+1} \quad (1.4)$$

$$(1.5)$$

therefore,

$$1 = \sum_{i=0}^B \pi_i \quad (1.6)$$

$$1 = \sum_{i=0}^B \pi_0 \left(\frac{\lambda}{\mu}\right)^i \quad (1.7)$$

$$\pi_0 = \frac{1 - \frac{\lambda}{\mu}}{1 - \frac{\lambda}{\mu}^{B+1}} \quad (1.8)$$

$$\pi_B = \frac{\frac{\lambda}{\mu}^B (1 - \frac{\lambda}{\mu})}{1 - \frac{\lambda}{\mu}^{B+1}} \quad (1.9)$$

$$= 1 - \frac{1 - \frac{\lambda}{\mu}^B}{1 - \frac{\lambda}{\mu}^{B+1}} \quad (1.10)$$

hence, the loss rate is

$$P_d = 1 - \frac{1 - \frac{\lambda^B}{\mu}}{1 - \frac{\lambda}{\mu}} \quad (1.11)$$

b).simulation code

```
# This is a simpy based simulation of a M/M/1 queue system
# Now the buffer packet size is limited to B

import random
import simpy
import math

RANDOM_SEED = 29
SIM_TIME = 1000000
MU = 1

""" Queue system """
class server_queue:
    def __init__(self, env, arrival_rate, Packet_Delay,
Server_Idle_Periods, Buffer_Size):
        self.server = simpy.Resource(env, capacity = 1)
        self.env = env
        self.queue_len = 0
        self.flag_processing = 0
        self.packet_number = 0
        self.sum_time_length = 0
        self.start_idle_time = 0
        self.drop = 0
        self.buffer_size = Buffer_Size
        self.arrival_rate = arrival_rate
        self.Packet_Delay = Packet_Delay
        self.Server_Idle_Periods = Server_Idle_Periods

    def drop_rate(self):#calculate the drop rate, drop/
total_packets
        return self.drop/self.packet_number

    def process_packet(self, env, packet):
        with self.server.request() as req:
            start = env.now
```

```

        yield req
        yield env.timeout(random.expovariate(MU)
        )
        latency = env.now - packet.arrival_time
        self.Packet_Delay.addNumber(latency)
        #print("Packet number {0} with arrival
            time {1} latency {2}".format(packet.
            identifier, packet.arrival_time,
            latency))
        self.queue_len -= 1
        if self.queue_len == 0:
            self.flag_processing = 0
            self.start_idle_time = env.now

def packets_arrival(self, env):
    # packet arrivals

    while True:
        # Infinite loop for generating packets
        yield env.timeout(random.expovariate(
            self.arrival_rate))
        # arrival time of one packet
        self.packet_number += 1
        # packet id
        arrival_time = env.now
        #print(self.num_pkt_total, "packet
            arrival")
        new_packet = Packet(self.packet_number,
            arrival_time)
        if self.flag_processing == 0:
            self.flag_processing = 1
            idle_period = env.now - self.
                start_idle_time
            self.Server_Idle_Periods.
                addNumber(idle_period)
            #print("Idle period of length
                {0} ended".format(
                idle_period))
        if self.queue_len < self.buffer_size:##
            #####drop packet
            self.queue_len += 1
            env.process(self.process_packet(
                env, new_packet))
        else:

```

```
self.drop += 1
```

```
""" Packet class """
```

```
class Packet:
```

```
    def __init__(self, identifier, arrival_time):
        self.identifier = identifier
        self.arrival_time = arrival_time
```

```
class StatObject:
```

```
    def __init__(self):
        self.dataset = []
```

```
    def addNumber(self, x):
        self.dataset.append(x)
```

```
    def sum(self):
        n = len(self.dataset)
        sum = 0
        for i in self.dataset:
            sum = sum + i
        return sum
```

```
    def mean(self):
        n = len(self.dataset)
        sum = 0
        for i in self.dataset:
            sum = sum + i
        return sum/n
```

```
    def maximum(self):
        return max(self.dataset)
```

```
    def minimum(self):
        return min(self.dataset)
```

```
    def count(self):
        return len(self.dataset)
```

```
    def median(self):
        self.dataset.sort()
        n = len(self.dataset)
        if n//2 != 0: # get the middle number
            return self.dataset[n//2]
        else: # find the average of the middle two numbers
            return ((self.dataset[n//2] + self.dataset[n//2 +
1])/2)
```

```
    def standarddeviation(self):
        temp = self.mean()
```

```

sum = 0
for i in self.dataset:
    sum = sum + (i - temp)**2
sum = sum/(len(self.dataset) - 1)
return math.sqrt(sum)

def cal_packet_loss(arrival_rate, buffer_size):#####the
theoretical loss rate
pd = 1 - (1 - (arrival_rate/MU)**buffer_size)/(1 - (
    arrival_rate/MU)**(buffer_size+1))
#print(Sum)#####each step print out the calculated
probability that the packet of system is <= buffer size

return pd

def main():
    random.seed(RANDOM_SEED)
    for buffer_size in [10, 50]:
        print("Simple_queue_system_model:mu={0},buffer_size={1}".format(MU, buffer_size))
        print("{0:<9}{1:<9}{2:<9}{3:<9}{4:<9}{5:<9}{6:<9}{7:<9}{8:<9}{9:<9}".format(
            "Lambda", "Count", "Min", "Max", "Mean", "Median", "Sd",
            "Utilization", "Loss_Rate", "Theoretical"))
        for arrival_rate in [0.2, 0.4, 0.6, 0.8, 0.9, 0.99]:
            env = simpy.Environment()
            Packet_Delay = StatObject()
            Server_Idle_Periods = StatObject()
            router = server_queue(env, arrival_rate,
                Packet_Delay, Server_Idle_Periods, buffer_size)
            env.process(router.packets_arrival(env))
            env.run(until=SIM_TIME)
            #expected_delay = 1/MU/(1-(arrival_rate/MU))
            print("{0:<9.3f}{1:<9}{2:<9.3f}{3:<9.3f}{4:<9.3f}{5:<9.3f}{6:<9.3f}{7:<9.3f}{8:<9.3f}{9:<9.3f}".format(
                round(arrival_rate, 3),
                int(Packet_Delay.count()),
                round(Packet_Delay.minimum(), 3),
                round(Packet_Delay.maximum(), 3),
                round(Packet_Delay.mean(), 3),
                round(Packet_Delay.median(), 3),
                round(Packet_Delay.standarddeviation(), 3),
                round(1-Server_Idle_Periods.sum()/SIM_TIME, 3),

```

```

round(router.drop_rate(),3),
round(cal_packet_loss(arrival_rate, buffer_size)
,3)))

```

```

if __name__ == '__main__': main()

```

c).tables

| Simple queue system model:mu = 1 |        |       |         |        |        |        |             |             |  |
|----------------------------------|--------|-------|---------|--------|--------|--------|-------------|-------------|--|
| Lambda                           | Count  | Min   | Max     | Mean   | Median | Sd     | Utilization | Theoretical |  |
| 0.200                            | 200377 | 0.000 | 15.023  | 1.251  | 0.867  | 1.254  | 0.200       | 1.250       |  |
| 0.400                            | 400070 | 0.000 | 18.180  | 1.658  | 1.146  | 1.660  | 0.399       | 1.667       |  |
| 0.600                            | 601173 | 0.000 | 30.204  | 2.529  | 1.749  | 2.539  | 0.603       | 2.500       |  |
| 0.800                            | 799713 | 0.000 | 54.270  | 4.996  | 3.452  | 4.988  | 0.800       | 5.000       |  |
| 0.900                            | 898679 | 0.000 | 90.886  | 9.558  | 6.698  | 9.370  | 0.897       | 10.000      |  |
| 0.990                            | 991142 | 0.000 | 419.359 | 86.938 | 64.409 | 76.626 | 0.990       | 100.000     |  |

  

| Simple queue system model:mu = 1, buffer size = 10 |        |       |        |       |        |       |             |           |             |
|--|--------|-------|--------|-------|--------|-------|-------------|-----------|-------------|
| Lambda   | Count  | Min   | Max    | Mean  | Median | Sd    | Utilization | Loss Rate | Theoretical |
| 0.200  | 200377 | 0.000 | 15.023 | 1.251 | 0.867  | 1.254 | 0.200       | 0.000     | 0.000       |
| 0.400  | 401172 | 0.000 | 23.096 | 1.664 | 1.154  | 1.660 | 0.402       | 0.000     | 0.000       |
| 0.600  | 599482 | 0.000 | 24.461 | 2.455 | 1.730  | 2.375 | 0.601       | 0.003     | 0.002       |
| 0.800  | 781331 | 0.000 | 30.045 | 3.790 | 2.954  | 3.200 | 0.781       | 0.023     | 0.023       |
| 0.900  | 854259 | 0.000 | 32.460 | 4.640 | 3.931  | 3.528 | 0.854       | 0.051     | 0.051       |
| 0.990  | 905912 | 0.000 | 27.556 | 5.389 | 4.887  | 3.687 | 0.904       | 0.085     | 0.086       |

  

| Simple queue system model:mu = 1, buffer size = 50 |        |       |        |        |        |        |             |           |             |
|--|--------|-------|--------|--------|--------|--------|-------------|-----------|-------------|
| Lambda   | Count  | Min   | Max    | Mean   | Median | Sd     | Utilization | Loss Rate | Theoretical |
| 0.200  | 199916 | 0.000 | 14.758 | 1.241  | 0.859  | 1.244  | 0.199       | 0.000     | 0.000       |
| 0.400  | 400067 | 0.000 | 26.712 | 1.664  | 1.152  | 1.666  | 0.399       | 0.000     | 0.000       |
| 0.600  | 599600 | 0.000 | 30.476 | 2.491  | 1.733  | 2.477  | 0.599       | 0.000     | 0.000       |
| 0.800  | 801180 | 0.000 | 48.397 | 4.976  | 3.456  | 4.933  | 0.801       | 0.000     | 0.000       |
| 0.900  | 898139 | 0.000 | 68.222 | 9.539  | 6.721  | 9.105  | 0.897       | 0.000     | 0.001       |
| 0.990  | 974218 | 0.000 | 79.242 | 23.823 | 22.600 | 15.271 | 0.975       | 0.016     | 0.015       |

## 2 PART 2