Seat No. ☐

King Mongkut's University of Technology Thonburi

(International Program)

Midterm Examination, Semester 1/2015

Subject: **MTH 101 Mathematics I** For: Engineering and IT-CS Students

Date: Monday 21st September 2015 Time: 09:00 – 12:00

Instructions: 1.The exam consists of 2 parts with 9 problems on 12 pages (including this cover and a formula sheet): 80 points total.

2. Any calculators are NOT allowed in the exam room.
3. A dictionary is NOT permitted in the exam room.
4. Any notes or textbooks are NOT allowed in the exam room.
5. Write down your name and ID on every page before working on the exam.
6. Show your work and answer all questions neatly on this exam sheets.
7. You may write your answers with dark pencil.
8. Read each problem carefully before answering.

Dr. Anuwat Sae-Tang

Exam writers

Name.....ID.....Department.....

This exam paper had been approved by the board of department of mathematics.

(Asst. Prof. Dr. Thiradet Jiarasuksakun)

Head of Mathematics Department

Name.....I.D.....Department.....

Part A (Total 42 Points)

1. Determine whether each of the following limits exist or not. If yes, evaluate the limit. If no, please specify an appropriate reason. ***(12 Points, 3 each)***

a) $\lim_{x\to 2} \dfrac{\sqrt{6-x}-2}{\sqrt{3-x}-1}$

b) $\lim_{x\to 0} \left(\dfrac{|2x-1|-|2x+1|}{x} \right)$

c) $\lim_{x\to 1} \left(\dfrac{1}{\ln x} - \dfrac{1}{x-1} \right)$

d) $\lim_{x\to \infty} \left(e^x + x\right)^{\frac{1}{x}}$

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2. Let f be a function defined by

$$f(x)=\begin{cases} x\sqrt{x}\cos\left(\frac{1}{x^6}\right) & ; \quad x\neq 0 \\ 0 & ; \quad x=0 \end{cases}.$$

a) Show that f is continuous at $x=0$. *(4 Points)*

b) Determine whether $f'(0)$ exists or not. Justify your answer. *(4 Points)*

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3. Under certain circumstances, a rumor spreads according to the equation

$$p(t) = \frac{1}{1+\alpha e^{-kt}}$$

where $p(t)$ is the proportion of population that knows the rumor after t hours ($t \geq 0$) and α and k are some positive constants.

a) Find $\lim_{t \to \infty} p(t)$. What can we conclude from this result? ***(2 Points)***

b) Find the average rate of spread of the rumor during the first 24 hours (in terms of α and k).

(2 Points)

c) Find the rate of spread of the rumor at any time t hours (in terms of α and k). ***(3 Points)***

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4. Let $f(x) = x e^{-x}$. Find $f^{(n)}(x)$ for any positive integer n and $\left.\frac{d^{1000} f}{dx^{1000}}\right|_{x=2000}$. *(7 Points)*

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5. For the following functions, find $\frac{dy}{dx}$. *(8 Points, 4 each)*

a) $y = \log_3\left(\sqrt{\frac{3^{\sec x}}{\cot x}}\right)$

b) $y = x^{\tan x}\arctan\left(\sqrt{x}\right)$

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Part B (Total 38 Points)

6. Find the equation of the tangent line of the curve $x^2y^2 + xy = 2$ at the point $(-1,2)$. *(7 Points)*

7. Use differentials to find an appropriate linear approximation of $\ln(0.95)$. *(6 Points)*

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8. Two sides of triangle have fixed lengths 12 m. and 15 m. The angle between them is increasing at a rate of 0.5 rad / minute. How fast is the length of the third side increasing when the angle between the two fixed lengths is $\theta = \frac{\pi}{3} = 60^\circ$. (Here, it is given that $\sqrt{3} \approx 1.7$ and $\sqrt{189} = 17$.)

(**Hint:** Use the law of cosine stating that $a^2 = b^2 + c^2 - 2bc\cos\theta$ where a, b, c are three sides of the triangle and θ is the angle between b and c) ***(8 Points)***

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9. Let $g(x)=\dfrac{x^2}{(x-2)^2}$.

a) Find all vertical and horizontal asymptotes of this function g. ***(3 Points)***

b) Determine the interval of x for which g is increasing and the interval for which g is decreasing. ***(3 Points)***

c) Find the interval of concave up and concave down and specify all inflection points of this function g. ***(4 Points)***

d) Find all critical points of this function g and check if each of these critical points gives a relative (local) maximum, relative (local) minimum or neither. ***(4 Points)***

e) Use the information from parts a) - d) to sketch the graph of $g(x)=\frac{x^2}{(x-2)^2}$. *(3 Points)*

(**Hint:** Evaluating the function at some point such as the relative (local) extremum and the inflection point may be helpful.)

TABLE OF DERIVATIVES

1. $\frac{d}{dx}(c) = 0$, c is a constant
2. $\frac{d}{dx}(cu) = c\frac{du}{dx}$, $u = u(x)$
3. $\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$
4. $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$
5. $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$, $v = v(x)$
6. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$, $v \neq 0$
7. $\frac{d}{dx}(\sin u) = \cos u\frac{du}{dx}$
8. $\frac{d}{dx}(\cos u) = -\sin u\frac{du}{dx}$
9. $\frac{d}{dx}(\tan u) = \sec^2 u\frac{du}{dx}$
10. $\frac{d}{dx}(\cot u) = -\csc^2 u\frac{du}{dx}$
11. $\frac{d}{dx}(\sec u) = \sec u\tan u\frac{du}{dx}$
12. $\frac{d}{dx}(\csc u) = -\csc u\cot u\frac{du}{dx}$
13. $\frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$, $-1 < u < 1$
14. $\frac{d}{dx}(\cos^{-1} u) = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$, $-1 < u < 1$
15. $\frac{d}{dx}(\tan^{-1} u) = \frac{1}{1+u^2}\frac{du}{dx}$
16. $\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$
17. $\frac{d}{dx}(\log_a u) = \frac{1}{u\ln a}\frac{du}{dx}$, $a \neq 0, 1$
18. $\frac{d}{dx}e^u = e^u\frac{du}{dx}$
19. $\frac{d}{dx}a^u = a^u \ln a\frac{du}{dx}$

TABLE OF INTEGRALS

1. $\int du = u + C$
2. $\int adu = au + C$, a is a constant
3. $\int [f(u) + g(u)]du = \int f(u)du + \int g(u)du$
4. $\int u^n du = \frac{u^{n+1}}{n+1} + C$, $n \neq -1$
5. $\int \frac{1}{u}du = \ln|u| + C$
6. $\int e^u du = e^u + C$
7. $\int a^u du = \frac{a^u}{\ln a} + C$, $a > 0$, $a \neq 1$
8. $\int \sin u du = -\cos u + C$
9. $\int \cos u du = \sin u + C$
10. $\int \tan u du = \ln|\sec u| + C$
11. $\int \cot u du = \ln|\sin u| + C$
12. $\int \csc u du = \ln|\csc u - \cot u| + C$
13. $\int \sec u du = \ln|\sec u + \tan u| + C$
14. $\int \sec^2 u du = \tan u + C$
15. $\int \csc^2 u du = -\cot u + C$
16. $\int \tan^2 u du = \tan u - u + C$
17. $\int \cot^2 u du = -\cot u - u + C$
18. $\int \sin^2 u du = \frac{u}{2} - \frac{1}{4}\sin 2u + C$
19. $\int \cos^2 u du = \frac{u}{2} + \frac{1}{4}\sin 2u + C$
20. $\int \sec u\tan u du = \sec u + C$
21. $\int \csc u\cot u du = -\csc u + C$

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$
$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$
$2\sin A\cos B = \sin(A + B) + \sin(A - B)$
$2\sin A\sin B = \cos(A - B) - \cos(A + B)$
$2\cos A\cos B = \cos(A + B) + \cos(A - B)$
$\sin 2A = 2\sin A\cos A$
$\cos 2A = \cos^2 A - \sin^2 A$
$\cos 2A = 2\cos^2 A - 1$
$\sin 2A = 2\sin A\cos A$
$\sin^2 A + \cos^2 A = 1$
$\sec^2 A - \tan^2 A = 1$
$\csc^2 A - \cot^2 A = 1$
$\sin^2 A = \frac{1}{2} - \frac{1}{2}\cos 2A$
$\cos^2 A = \frac{1}{2} + \frac{1}{2}\cos 2A$