Admissibility of Safety Margin Aggregation Under Conservative Load Composition

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Abstract

Engineering design frameworks routinely rely on conservative aggregation of loads, factors, and safety margins to ensure robustness under uncertainty. These frameworks are typically evaluated through verification, validation, and probabilistic uncertainty quantification. However, such evaluations implicitly assume that the underlying constraint structure admits at least one realizable state. This note examines that assumption.

We introduce *admissibility* as the minimal pre-simulation criterion: whether a system's explicitly stated constraints admit at least one realizable state under declared composition rules. We argue that admissibility is logically prior to verification and validation, and cannot be deferred to downstream analysis when constraints are enforced simultaneously and without semantic relaxation.

As a concrete case study, we apply an admissibility evaluation to a representative safety margin aggregation problem in load and resistance factor design (LRFD). Using a purely syntactic, constraint-level analysis, we demonstrate that commonly assumed worst-case compositions can fail to admit any realizable state. The result is invariant, reproducible, and independent of probabilistic interpretation.

The purpose of this note is not to assess the empirical success or historical value of LRFD methodologies, but to illustrate the necessity of explicit admissibility analysis as an upstream requirement in conservative engineering systems.

1. Introduction

Modern engineering systems are increasingly characterized by conservative design philosophies. Load factors, resistance factors, safety margins, and worst-case assumptions are combined to ensure robustness against uncertainty, modeling error, and unforeseen operating conditions. These practices are well motivated and have proven effective across a wide range of domains.

Evaluation of such systems typically proceeds through established processes: verification to ensure correct implementation, validation to assess fidelity to physical reality, and uncertainty quantification to characterize variability and risk. Implicit in all of these processes is an assumption that the system specification itself is internally consistent – that its declared constraints can, in principle, be satisfied.

This assumption is rarely examined explicitly.

In many cases, admissibility is tacitly resolved through engineering judgment, hierarchical reasoning, or staged application of constraints. However, as systems grow more complex and conservative aggregation becomes more explicit and simultaneous, the question of whether all stated commitments can coexist becomes nontrivial. When constraints are enforced concurrently, without prioritization or relaxation, internal consistency can no longer be taken for granted.

This note isolates that upstream question.

We focus on *admissibility*: whether a system specification, taken at face value, admits at least one realizable state. Admissibility is not concerned with likelihood, performance, optimality, or realism. It asks a simpler and more fundamental question: **does the system described by these constraints exist at all?**

To ground the discussion, we examine a representative example drawn from load and resistance factor design (LRFD), specifically the aggregation of safety margins under conservative load composition. This is structurally critical as it exposes a class of failures that are not detectable through simulation, probabilistic analysis, or validation, because those processes presuppose the very existence that is in question.

2. Admissibility

2.1 Definition of Admissibility

In this work, *admissibility* is defined as a minimal structural property of a system specification:

A system is admissible if its explicit constraints admit at least one realizable state under the declared rules of composition.

Admissibility concerns existence only. It does not evaluate whether a realizable state is desirable, likely, efficient, safe, or representative of reality. A system may be admissible yet impractical; conversely, a system that is inadmissible cannot be meaningfully analyzed, as no state satisfies its own stated commitments.

The admissibility criterion adopted here is deliberately conservative. Only constraints explicitly stated in the specification are considered. All constraints are enforced simultaneously. No semantic interpretation, prioritization, relaxation, or domain-specific judgment is introduced to restore coherence.

Under this criterion, coherence is not inferred; it is demonstrated.

2.2 Relation to Verification and Validation

Verification and validation presuppose admissibility.

Verification establishes whether a model has been correctly implemented relative to its specification. Validation assesses whether the model adequately represents a target physical or operational system. Both processes implicitly assume that the specification describes at least one internally consistent state.

Admissibility addresses a logically prior question: **whether the specification itself admits any realizable state at all**. If it does not, verification and validation become ill-posed, as there is no coherent system to verify or validate.

In practice, admissibility is often assumed, deferred, or resolved informally. This is typically unproblematic when constraints are loosely coupled or applied sequentially. However, in systems characterized by conservative aggregation, worst-case composition, and simultaneous enforcement of safety margins, admissibility becomes a nontrivial precondition that must be evaluated explicitly.

2.3 Admissibility Evaluation Procedure

The admissibility analysis performed in this note operates strictly upstream of simulation, optimization, or probabilistic uncertainty analysis. It evaluates whether the full set of declared constraints can be satisfied concurrently under the stated rules of composition.

Several principles govern the evaluation:

1. **Syntax over semantics**
Constraints are interpreted exactly as stated. Semantic intent does not modify syntactic commitments.

2. **Simultaneity**
All constraints are enforced at once. No ordering, staging, or conditional relaxation is assumed unless explicitly specified.

3. **No resolution in favor of coherence**
Ambiguity is not resolved to preserve consistency. A system either admits a realizable state or it does not.

4. **Reviewer invariance**
Given the same explicit constraints and composition rules, independent reviewers must obtain the same admissibility result.

2.4 Scope and Intent

The purpose of this analysis is not to challenge the historical effectiveness, regulatory role, or empirical calibration of LRFD methodologies. Rather, it is to demonstrate that admissibility is a distinct and necessary precondition that may fail even in well-established frameworks when conservative assumptions are compounded without explicit admissibility checks.

The LRFD example that follows is treated as a case study. Its role is to make admissibility – often implicit – explicit and testable.

3. Symbolic Safety Margin Aggregation Under LRFD

3.1 Symbol Definitions

Let the structural capacity (resistance) be denoted by

$R > 0$

Let applied loads be denoted by

$L_i \geq 0$ for $i = 1, \ldots, n$

Let resistance and load factors be given by

$\varphi \in (0, 1), \ \gamma\_i > 1$

where φ is a resistance reduction factor and γ_i are load amplification factors, as defined in standard LRFD formulations.

3.2 LRFD Design Inequality

The governing LRFD design condition is written as:

$\varphi R \geq \Sigma\_(i = \ ) \text{ (upper } n) \ \ \gamma\_i \, L\_i$

This inequality is taken as an explicit constraint on admissible system states.

3.3 Conservative Composition Assumption

Under conservative aggregation, the following assumptions are made explicitly:

1. All load effects L_i may occur simultaneously
2. Each load is amplified independently by its corresponding factor γ_i
3. No probabilistic correlation, sequencing, or conditional relaxation is assumed.

These assumptions define the composition rules under which admissibility is evaluated.

3.4 Admissibility Test

Rewriting the LRFD inequality:

$R \geq (1 / \varphi) \ \Sigma\_(i = \ ) \text{ (upper } n) \ \ \gamma\_i \, L\_i$

For admissibility, there must exist at least one realizable tuple

$(R, L\_1, \ldots, L\_n)$

satisfying all stated constraints simultaneously.

However, under conservative aggregation, each L_i is itself defined as a maximum credible load. When all maxima are enforced concurrently and amplified, the right-hand side grows without bound relative to any finite R, unless additional limiting structure is imposed.

Absent such structure, the constraint set admits no realizable state.

Thus, under the stated assumptions, the admissible solution set is empty.

4. Concrete Numerical Example

Consider a simplified two-load case:

$L\_1 = 100, L\_2 = 100$

$\gamma_1 = \gamma_2 = 1.5$, $\varphi = 0.9$

The LRFD inequality becomes:

$0.9R \geq 1.5(100) + 1.5(100) = 300$

or equivalently

$R \geq 333.333...$

If R itself is defined as a factored or bounded resistance, consistent with conservative material assumptions, the inequality cannot be satisfied. Increasing R merely shifts the contradiction unless resistance bounds are removed entirely.

The result mirrors the symbolic case: under simultaneous worst-case aggregation, no realizable state satisfies all constraints.

5. Discussion

The failure demonstrated above is not probabilistic, empirical, or statistical in nature. It arises purely from the simultaneous enforcement of explicitly stated constraints under conservative composition rules.

The result does not imply that LRFD methodologies are ineffective, unsafe, or historically invalid. Rather, it highlights that admissibility is a distinct structural property that may fail independently of verification, validation, or uncertainty quantification.

In practice, admissibility is often restored implicitly through engineering judgment, staged application of loads, or semantic interpretation of safety factors. However, when constraints are taken literally and enforced simultaneously, admissibility cannot be assumed.

The key implication is narrow but fundamental: **existence must be established before downstream analysis is meaningful**. Admissibility is therefore an upstream requirement, orthogonal to and prior to conventional verification and validation processes.

Appendix A: Formal Admissibility Evaluation

A.1 Explicit Constraint Set

For clarity and reproducibility, the admissibility evaluation considers only the following explicitly stated constraints:

1. **Resistance constraint**

$R > 0$

2. **Load constraints**

$L_i \geq 0$ for $i = 1, \ldots, n$

3. **Factor constraints**

$\varphi \in (0, 1)$, $\gamma_i > 1$

4. **LRFD inequality**

$\varphi R \geq \Sigma_{(i =)}$ (upper n) $\gamma_i L_i$

5. **Composition rule**

All load effects L_i are permitted to occur simultaneously and are amplified independently by their corresponding factors.

No additional constraints, correlations, sequencing rules, or interpretive assumptions are introduced.

A.2 Admissibility Gate

The admissibility gate applied in this analysis is defined as follows:

A specification passes the admissibility gate if and only if there exists at least one assignment ($R, L_1, \ldots, L_n$) **satisfying all explicit constraints simultaneously under the declared composition rules.**

Failure to satisfy this condition results in an inadmissible specification.

This gate is evaluated prior to any simulation, optimization, probabilistic sampling, or uncertainty quantification.

A.3 Evaluation Principles

The admissibility evaluation enforces the following principles:

1. **Syntax precedence**

Semantic intent does not modify syntactic commitments.

2. **Simultaneity**
All constraints are enforced concurrently unless explicitly staged.

3. **No coherence repair**
Ambiguity is not resolved in favor of consistency.

4. **Existence only**
The evaluation concerns realizability, not likelihood or desirability.

These principles ensure that admissibility results are invariant with respect to interpretation.

A.4 Reviewer Invariance

Given:

- the same explicit constraint set,
- the same composition rules,
- and the same admissibility gate,

independent reviewers must obtain the same admissibility result.

No subjective judgment, calibration, or domain-specific expertise is required to reproduce the outcome. Disagreement can arise only from modification of the stated constraints or composition rules.

A.5 Implementation Note

The admissibility evaluation described above is implemented using a constraint-based admissibility analysis kernel (MidnightSun). The kernel performs a direct existence test over the explicit constraint set, without introducing auxiliary assumptions or semantic interpretation.

Its role in this note is strictly procedural: to demonstrate that admissibility can be evaluated deterministically and upstream of verification and validation.