HOMEWORK # 2

Pricing Financial Derivatives I

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To be done in groups of 2-3 students. To be submitted in a unique pdf file, and include the code in the pdf file. The answers need to be justified. Copied exercises between groups implies a 0 in the homework of both groups.

Deadline for submission : Tuesday 16th February 2021 at 10 :00 a.m. by email

Exercise 1 : Time-dependent volatility

Consider a 6-months European call struck at K. Assume that the underlying stock price follows a Black-Scholes model with S_0 and interest rate r. Assume that a trader has estimated a time-dependent volatility of

$$\sigma_t = \begin{cases} 17\% & \text{if} \quad 6-t < 1 \text{ month}, \\ 15\% & \text{if} \quad 1 \leq 6-t < 3 \text{ months}, \\ 13\% & \text{if} \quad 3 \leq 6-t < 5 \text{ months}, \\ 12\% & \text{if} \quad 6-t \geq 5 \text{ months}. \end{cases}$$

1. Simulate several paths of the stock price using the fact the the increments of Brownian motion are iid Gaussian. Take $N = 1000$.
2. Plot the value of the call option as a function of time using one of the paths obtained in 1.
3. Plot the value of the hedging portfolio assuming continuous hedging and compare it with a daily rebalanced one. Plot the hedging error.

Exercise 2 : Implied volatilities

Consider the data on the excel file. For each value of the call and for each of the 3 stock prices compute the implied volatility. Plot the implied volatility as a function of the strike and time to maturity (volatility surface). Explain if you observe any smile or skew.