Pricing Financial Derivatives II

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To be done in groups of 2-3 students. To be submitted by email as a unique pdf file, and include the code in the pdf file. The answers need to be justified. Copied exercises between groups implies a 0 in the homework of both groups.

Deadline for submission : Tuesday March 16th 2021 at 10 :00 a.m.

Exercise 1 : Pricing Asian options

Consider a one-year Asian call option struck at K with daily setting dates assuming the underlying stock price satisfies Black Scholes with parameters S_0, σ and r.

1. Plot one realization of the stock price together with its arithmetic and geometric average.

2. For this realization, compute the value of the the Asian call at time $t = 0$ using both arithmetic and geometric average and compare it with the corresponding vanilla call.

3. For this realization, plot the value of the Asian call as a function of time using both arithmetic and geometric average together with the corresponding vanilla call. Comment the plot.

Exercise 2 : Bond prices with Vasicek model for interest rates

Consider the stochastic differential equation

$$dr_t = b\,(a - r_t)\,dt + \sigma dB_t \quad t \in [0, T]$$

where $b > 0, \sigma > 0$ and $a \geq 0$. This process tends to drift towards its long-term mean b : such a process is called mean-reverting.

1. Simulate several realizations of the process for different values of b, a, σ. Take $T = 1$ and $N = 1000$. Comment the plots. Do you observe the mean-reverting phenomena ?

2. For some of the realizations obtained in 1. plot the T-bond price $t \to p_t^T$. Comment the plots.

3. Plot some graphs of the yield to maturity $T \to y_0^T$. Comment the plots. Is the yield to maturity always a concave function ?