HOMEWORK # 1

Pricing Financial Derivatives I

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To be done in groups of 2-3 students. To be submitted in a UNIQUE PDE FILE, and include the code in the pdf file. The answers need to be justified. Copied exercises between groups implies a 0 in the homework of both groups.

Deadline for submission : Tuesday 2nd February 2021 at 10 :00 a.m. by email : eulalia.nualart@upf.edu

Exercise 1 : Pricing and hedging using the tree approach (Excel)

Consider a stock that is worth S_0 today and at each month increases a $u\%$ of its value or decreases a $d\%$ of its value. We want to price a four-month European call struck at K. Assume that the compounded annual interest rate is r.

1. Give the condition on the parameters implied by the no-arbitrage assumption and give the risk neutral probability.
2. Choose some particular values of S_0, u, d, K and r satisfying 1. and construct a tree for the different values of S_n, $n = 0, 1, 2, 3, 4$, n =months.
3. Compute the value of the call C_0 using the formula $e^{-rT}E(\max(S_4 - K, 0))$.
4. Using the backward recursion determine the different values of the call C_4, C_3, C_2, C_1.
5. Compute the Delta-hedging strategy Δ_n, $n = 0, 1, 2, 3$ assuming a particular path for S_n and make a table explaining the steps $0, 0+, 1, 1+, 2, 2+, 3, 3+, 4$.

Exercise 2 : Simulation of log-normal prices (Matlab, R, Python)

Simulate several trajectories of stock prices assuming the log-normal model (slide 11) under the risk-neutral probability (slide 12) for different values of the drift μ, the volatility σ, and the interest rate r, for $T = 1$, and $N = 10$, $N = 100$, and $N = 1000$. Comment and compare your plots. Check that the condition on the parameters implied by the no-arbitrage assumption (slide 12) is satisfied. Do you observe that the drift plays no role in the risk-neutral limiting model ?

Exercise 3 : Simulation of call prices (Matlab, R, Python)

Consider one of the trajectories of exercise 2 with the parameters of Exercise 1. Using the fact that the Black and Scholes models is time-homogeneous plot the value of the call and the delta. For the delta use the discrete time formula and the formula in slide 17 and compare both plots.