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ECONOMETRICS

Assignment 2: The second on multiple regression - Elasticity and nonlinear models

Submission deadline: November 12, 7 pm. This is an individual or group assignment: submit your answers to daniel.dejuan@barcelonagse.eu.

1) The Stata file **gasoline.dta** has data on the US gasoline market (36 yearly observations, 1960 to 1995). The variables are:
G – total US gasoline consumption (total expenditure divided by price index)
Pg – price index for gasoline
Y – per capita disposable income
Pnc – price index for new cars
Puc – price index for used cars
Ppt – price index for public transportation
Pd – aggregate price index for consumer durables
Pn – aggregate price index for consumer nondurables
Ps – aggregate price index for consumer services
Pop – US total population in millions
Year – year of the data

a) Compute the regression of per capita consumption of gasoline (G/pop) on all other explanatory variables, including the time trend (you can use either Year directly or (Year-1960)). Comment on whether the signs of the estimates agree with your expectations (i.e. comment on your expectations, too!).

b) Test the hypothesis that consumers do not differentiate between changes in the prices of new and used cars.

c) Find the value of the elasticity of gasoline demand to its own price, to income and the cross-price elasticity with respect to changes in the price of public transportation from the results of this linear model (you will have to do a bit of thinking here, and remember the concept of elasticity).

d) Reestimate the above regression in logarithms (except for the time trend). All the coefficients are now directly elasticities. How do the estimates compare with the results in the previous analysis?

e) Do the two regression models imply the same regarding the elasticities? Which one do you think is more correct: the linear model or the log-log model? Why?

2) The Stata file **Production.dta** has a famous dataset (Mizon, 1977, *Econometrica*) with data for 24 productive sectors in the UK during three yerars, for a total of 72 observations. The variables are

valueadd – value added (a measure of output)

labor – labor input
capital – capital stock

A micro/macroeconomics professor may (or may not) have told you about Cobb-Douglas production functions such as: $Y = AL^{\beta}K^{\gamma}$, where A is a composite factor ("technology"), L is labor and K is capital. The parameters β and γ measure the elasticity of output to labor and capital. This function can be estimated by nonlinear least squares, which will find the values of A, β and γ that minimize:

$$\sum_{i=1}^{N} e_i^2 = \sum_{i=1}^{N} \left(Y_i - AL_i^{\beta}K_i^{\gamma}\right)^2$$

a) Use nonlinear least squares to get estimates of the elasticities of output to labor and capital (see below how to do this with Stata and R).

b) Transform now this nonlinear model by taking logs of value added, labor and capital. Notice that the model now is perfectly linear, so it can be estimated by OLS. Do it, and compare the estimates of A, β and γ. Have these changed much?

c) Test the following hypotheses:

a. Labor elasticity is equal to 0.6.
b. Constant returns to scale (β + γ = 1).
c. Labor elasticity is equal to 0.6 *and* the production function has constant returns to scale.

The Cobb-Douglas function may be a little too simplistic, so more flexible forms have been suggested. One of them is the translog production function, which can be interpreted as a Cobb-Douglas function that relaxes the assumption of unitary elasticity of substitution or as a second-order approximation (in a Taylor-series sense) to a general production function:

$$\ln y = \beta_0 + \beta_1 \ln L + \beta_2 \ln K + \beta_3 (1/2 \ln^2 L) + \beta_4 (1/2 \ln^2 K) + \beta_5 \ln L \ln K + e$$

The Cobb-Douglas function can be interpreted as a first-order approximation or as a "restricted form" of the translog function.

d) Test whether the translog function is more appropriate than the Cobb-Douglas function (so, in a sense, test the linearity of ln*y* or that the Cobb-Douglas function is enough to account for output). Are the coefficients of the translog function as easy to interpret as those of the Cobb-Douglas function?

Stata commands

In order to do nonlinear least squares with Stata, you just have to write the expression you want to estimate using "alpha", "beta", etc… as your parameter names:

nl (valueadd = {alpha} * labor^{beta}* capital^{gama})

R commands

In order to do nonlinear least squares with R, you use the **nls** command and give names to your parameters (R will know which are variables and which are parameters from the dataset). You must provide starting values for the optimization procedure:

nls (valueadd ~(alpha*labor^beta*capital^gama), data=production, start=list(alpha=1.5, beta=0.5,gama=0.5))