Supplemental Material for Nodal Frequency Dynamics Modeling

MODELING

A. Nodal Frequency Dynamics Modeling

The unified nodal frequency response model incorporates the governor dynamics of SGs, the power–frequency control loops of converter-interfaced resources (CIRs), as well as the (virtual) inertia and damping characteristics of both synchronous generators (SGs) and CIRs. Since this paper focuses on nodal frequency response, generators at each node are aggregated into a single-machine model [18].

The angle and frequency dynamics for SG bus i are:

$$\Delta\dot{\delta}_i^{SG} = \Delta\omega_i^{SG} \tag{1}$$

$$M_i^{SG}\Delta\dot{\omega}_i^{SG} = \Delta P_i^M - \Delta P_i^E + \Delta P_i^{PFR} - D_i^{SG}\Delta\omega_i^{SG} \tag{2}$$

$$\Delta P_i^{PFR} = -\frac{(1+F_i^{SG}T_i^{SG}s)}{R_i^{SG}(1+T_i^{SG}s)}\Delta\omega_i^{SG}. \tag{3}$$

Similarly, the angle and frequency dynamics for CIR bus j are as follows:

$$\Delta\dot{\delta}_j^{CIR} = \Delta\omega_j^{CIR} \tag{4}$$

$$M_j^{CIR}\Delta\dot{\omega}_j^{CIR} = -\Delta P_j^E + \Delta P_j^{PFR} \tag{5}$$

$$\Delta P_j^{PFR} = -\frac{D_j^{CIR}}{1+T_j^{CIR}s}\Delta\omega_j^{CIR}. \tag{6}$$

Let N^G represents the set of generations buses, which consists of SGs and CIRs and N^L denotes the set of load buses. In a lossless power grid, we can separate the susceptance matrix B into four blocks:

$$B = \begin{bmatrix} B_{GG} & B_{GL} \\ B_{LG} & B_{LL} \end{bmatrix}. \tag{7}$$

To derive the mathematical description of the nodal electrical power variation $\Delta P^{E,G/L}$, by linearizing around the power flow operating point (e.g., δ_{io} and δ_{jo}) [38], we can obtain:

$$\begin{bmatrix} \Delta\boldsymbol{P}^{E,G} \\ \Delta\boldsymbol{P}^{E,L} \end{bmatrix} = \begin{bmatrix} P_{G,G}^S & P_{G,L}^S \\ P_{L,G}^S & P_{L,L}^S \end{bmatrix} \cdot \begin{bmatrix} \Delta\boldsymbol{\delta}^G \\ \Delta\boldsymbol{\delta}^L \end{bmatrix} \tag{8}$$

$$P_{G/L,G/L}^S(i,j) = \begin{cases} -V_{i0}V_{j0}B_{ij}\cos(\delta_{io}-\delta_{jo}), \ i \neq j \\ \sum V_{i0}V_{j0}B_{ij}\cos(\delta_{io}-\delta_{jo}), \ i = j \end{cases}, \tag{9}$$

where $P_{G/L,G/L}^S(i,j)$ denotes the synchronization power coefficients.

Using Kron reduction, the electrical power variation of bus N^G is formulated by:

$$\Delta\boldsymbol{P}^{E,G} = K_S\Delta\boldsymbol{\delta}^G + K_D\Delta\boldsymbol{P}^{E,L}, \tag{10}$$

where $K_S = P_{G,G}^S - P_{G,L}^S(P_{L,L}^S)^{-1}P_{L,G}^S$ and $K_D = (P_{L,L}^S)^{-1}P_{L,G}^S$.

By substituting (89) into (80)-(85), the nodal frequency dynamic equation can be expressed below:

$$\Delta\dot{\boldsymbol{X}}^{SYS} = A_{SYS}\Delta\boldsymbol{X}^{SYS} + B_{SYS}\Delta\boldsymbol{P}^D \tag{11}$$

$$\Delta\boldsymbol{X}^{SYS} = [\Delta\boldsymbol{\delta}^G \ \ \Delta\boldsymbol{\omega}^G \ \ \Delta\boldsymbol{P}^{PFR}]^T \tag{12}$$

$$\Delta\boldsymbol{P}^D = [\Delta\boldsymbol{P}^M \ \ \Delta\boldsymbol{P}^{E,L}]^T \tag{13}$$

$$A_{SYS} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} \\ -A_S & -(\boldsymbol{M}^G)^{-1}\boldsymbol{D}^G & (\boldsymbol{M}^G)^{-1} \\ \Gamma A_S & \Upsilon\boldsymbol{D}^G - (\boldsymbol{T}^G)^{-1}\boldsymbol{D}^{PFR} & -\Upsilon - (\boldsymbol{T}^G)^{-1} \end{bmatrix} \tag{14}$$

$$B_{SYS} = \begin{bmatrix} \boldsymbol{0} & (\boldsymbol{M}^G)^{-1} & -\Upsilon \\ \boldsymbol{0} & -A_D & \Gamma A_D \end{bmatrix}^T, \tag{15}$$

where

$$\Gamma = \boldsymbol{D}^{PFR}\boldsymbol{F}^{G}, \Upsilon = \Gamma(\boldsymbol{M}^{G})^{-1} \tag{16}$$

$$A_S = (\boldsymbol{M}^{G})^{-1}K_S, A_D = (\boldsymbol{M}^{G})^{-1}K_D \tag{17}$$

$$\boldsymbol{M}^{G} = \mathrm{diag}[\boldsymbol{M}^{SG}\ \ \boldsymbol{M}^{CIG}], \boldsymbol{D}^{G} = \mathrm{diag}[\boldsymbol{D}^{SG}\ \ \boldsymbol{0}] \tag{18}$$

$$\boldsymbol{D}^{PFR} = \mathrm{diag}[(\boldsymbol{R}^{SG})^{-1}\ \ \boldsymbol{D}^{CIG}], \boldsymbol{T}^{G} = \mathrm{diag}[\boldsymbol{T}^{SG}\ \ \boldsymbol{T}^{CIG}] \tag{19}$$

$$\boldsymbol{F}^{G} = \mathrm{diag}[\boldsymbol{F}^{SG}\ \ \boldsymbol{0}]. \tag{20}$$

By calculating the nodal frequency at generation nodes $\Delta\boldsymbol{\omega}^{G}$, the nodal frequency characteristic of load bus *j* can be formulated as follows [22]:

$$\omega_j^L = \sum_{N^G} \frac{V_i}{V_j} \bar{B}_{GL}(i,j)\cos(\delta_i^G - \delta_j^L)\omega_i^G, \tag{21}$$

where $\bar{B}_{GL}(i,j)$ is the element in the *j*th row and *i*th column of $\bar{B}_{GL} = -B_{LL}^{-1}B_{LG}$.

Linearizing (21), we can obtain $\Delta\boldsymbol{\omega}^{L}$ as the simple matrix form:

$$\Delta\boldsymbol{\omega}^{L} = FF\Delta\boldsymbol{\omega}^{G}, \tag{22}$$

where $FF(i,j) = (V_i/V_j)\bar{B}_{GL}(i,j)\cos(\delta_{i0}^G - \delta_{j0}^L)$.

Reference

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