

Stellar Structure and Evolution Homework 5

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1. TIDES

1.1. *a*

We define these parameters:

mass of Earth: $M_e = 5.972 \times 10^{27}$ g, mass of Moon: $M_m = 7.349 \times 10^{25}$ g

distance between Earth and mass center: d_e

distance between Moon and mass center: d_m

radius of Earth: $R_e = 6.378 \times 10^8$ cm, radius of Moon: $R_m = 1.738 \times 10^8$ cm

orbital angular velocity: $\omega_e = \omega_m = \omega$

spin period of Earth: $P_e = 1$ day, spin period of Moon: $P_m = 27.32$ days

spin angular velocity of Earth: $\Omega_e = \frac{2\pi}{P_e}$, spin angular velocity of Moon: $\Omega_m = \frac{2\pi}{P_m}$

$q = M_m/M_e \approx 0.012$, $M = M_e + M_m = (1 + q)M_e$, $d = d_e + d_m = 3.844 \times 10^{10}$ cm.

According to Newton's second law, we have:

$$\begin{cases} \frac{GM_e M_m}{(d_e + d_m)^2} = M_e \omega^2 d_e \\ \frac{GM_e M_m}{(d_e + d_m)^2} = M_m \omega^2 d_m \end{cases} \quad (1)$$

$$\therefore \begin{cases} M_e = \frac{1}{1+q} M, \quad M_m = \frac{q}{1+q} M \\ d_e = \frac{q}{1+q} d, \quad d_m = \frac{1}{1+q} d \\ \omega = \sqrt{\frac{GM}{d^3}} \end{cases} \quad (2)$$

Then we can calculate the angular momentum of orbit and spin by assuming $\Omega = \omega$:

$$\therefore \begin{cases} J_{e,o} = M_e d_e^2 \omega = \frac{q^2}{(1+q)^3} \sqrt{GM^3 d} \\ J_{m,o} = M_m d_m^2 \omega = \frac{q}{(1+q)^3} \sqrt{GM^3 d} \\ J_{e,s} = \frac{2}{5} M_e R_e^2 \Omega_e \\ J_{m,s} = \frac{2}{5} M_m R_m^2 \Omega_m \end{cases} \quad (3)$$

We can find that: $\frac{J_{e,o}}{J_{m,o}} = q = 0.012$, $\frac{J_{m,s}}{J_{e,s}} = q(\frac{R_m}{R_e})^2 \frac{P_e}{P_m} \approx 3.34 \times 10^{-5}$, which means that the orbital angular momentum of the Earth and the spin angular of the Moon are small compared to the other terms.

So the total angular momentum is: $J_{total} \approx J_{m,o} + J_{e,s} \approx (2.82 + 0.71) \times 10^{41} = 3.53 \times 10^{41}$ erg·s

1.2. *b*

$$J_{total} = J_{m,o} + J_{e,s} = \frac{q}{(1+q)^{5/2}} \sqrt{GM^3 d_m} + \frac{2}{5} M_e R_e^2 \Omega_e \quad (4)$$

From conservation of angular momentum: $\frac{dJ_{total}}{dt} = 0$, then we get:

$$\frac{q}{2(1+q)^{5/2}} \sqrt{\frac{GM^3}{d_m}} \frac{d_m}{dt} + \frac{2}{5} M_e R_e^2 \frac{d\Omega_e}{dt} = 0 \quad (5)$$

$$\therefore \frac{d\Omega_e}{dt} = A \frac{d_m}{dt} < 0 \quad (6)$$

1.3. *c*

$$\begin{aligned} \frac{dP_e}{dt} &= \frac{-2\pi}{\Omega_e^2} \frac{d\Omega_e}{dt} = \frac{5\pi q}{2(1+q)^3 M_e R_e^2 \Omega_e^2} \sqrt{\frac{GM^3}{d}} \frac{d_m}{dt} \\ &\approx 1.7 \times 10^{-5} \text{ s} = 17 \mu\text{s} \end{aligned} \quad (7)$$

2. THE PRE-MAIN SEQUENCE

2.1. *a*2.2. *b*

We assume the function of radius:

$$R(t) = \frac{R_0}{\lambda(t)} \quad (8)$$

The gravitational potential energy is:

$$\Omega(t) = \Omega(R_0) \frac{R_0}{R(t)} = \lambda(t) \Omega(R_0) \quad (9)$$

By assuming no nuclear sources, the luminosity is:

$$L = 0.5 \frac{d\Omega}{dt} = 0.5 \Omega(R_0) \frac{d\lambda(t)}{dt} \quad (10)$$

Then we have:

$$\lambda(t) = \frac{L}{0.5 \Omega(R_0)} t + 1 \quad (11)$$

Therefore, the radius function with time is:

$$R(t) = \frac{R_0}{1 + \frac{t}{\tau}}, \text{ where } \tau = \frac{0.5 \Omega(R_0)}{L} \quad (12)$$

2.3. *c*

2.4. *d*

2.5. *e*

3. SUPERNOVAE AND NEUTRON STARS

3.1. *a*

From conservation of momentum:

$$M_{neuron} V_{kick} = \frac{1\% \times E_{supernova}}{c} \quad (13)$$

We can estimate that $V_{kick} = \frac{1\% \times E_{supernova}}{1.4M_{\odot} \cdot c} \approx 1.2 \times 10^7 \text{ cm s}^{-1} = 120 \text{ km s}^{-1}$

3.2. *b*

$$E = \frac{1}{2} I \omega^2 = \frac{2\pi^2 I}{P^2} \quad (14)$$

$$|\frac{dE}{dt}| = \frac{4\pi^2 I}{P^3} \dot{P} \quad (15)$$

$$t \sim E / |\frac{dE}{dt}| = \frac{P}{2\dot{P}} \approx 1.7 \times 10^{14} \text{ yr} \quad (16)$$

3.3. *c*

I use these value:

Lethal dose: 5 Sv, which means $E_{lethal} = 250 \text{ J}$ for a man with 50 kg.

The neutrino-nucleon cross section: $\sigma \approx 10^{-47} \text{ m}^2$

Atom numbers of human body: $N = 7 \times 10^{27}$

Then we can calculate the effective area: $S = \sigma N = 7 \times 10^{-16} \text{ cm}^2$, and the energy flux: $F_{lethal} = \frac{E_{lethal}}{S} \approx 4 \times 10^{24} \text{ erg cm}^{-2}$

Therefore, the distance is:

$$d_{lethal} = \sqrt{\frac{L}{4\pi F_{lethal}}} \sim 10^{14} \text{ cm} \sim 10 \text{ AU} \quad (17)$$

4. HALF-LIFE OF NI

4.1. *a*

$$\begin{cases} \frac{dN_{Ni}}{dt} = -\lambda_{Ni} n_{Ni}, & \text{where } \lambda_{Ni} = \frac{\ln 2}{\tau_{1/2,Ni}} \\ \frac{dN_{Co}}{dt} = -\lambda_{Co} n_{Co} + \lambda_{Ni} n_{Ni}, & \text{where } \lambda_{Co} = \frac{\ln 2}{\tau_{1/2,Co}} \end{cases} \quad (18)$$

4.2. b

Assume that the initial number density of pure Ni is n_0 , then the solution of Ni is:

$$n_{Ni} = n_0 e^{-\lambda_{Ni} t} \quad (19)$$

Then we take this into equation of Co and multiples $e^{\lambda_{Co} t}$ to each side:

$$e^{\lambda_{Co} t} \frac{dN_{Co}}{dt} = -e^{\lambda_{Co} t} \lambda_{Co} n_{Co} + e^{\lambda_{Co} t - \lambda_{Ni} t} \lambda_{Ni} n_0 \quad (20)$$

$$\frac{d(e^{\lambda_{Co} t} n_{Co})}{dt} = e^{\lambda_{Co} t - \lambda_{Ni} t} \lambda_{Ni} n_0 \quad (21)$$

Therefore, the jnumber density of Co is:

$$n_{Co} = \frac{\lambda_{Ni} n_0}{\lambda_{Ni} - \lambda_{Co}} (e^{-\lambda_{Co} t} - e^{-\lambda_{Ni} t}) \quad (22)$$

4.3. c

$$L(t) = Q_{Ni} \lambda_{Ni} N_{Ni} + Q_{Co} \lambda_{Co} N_{Co} \quad (23)$$

$$= Q_{Ni} \lambda_{Ni} N_0 e^{-\lambda_{Ni} t} + \frac{Q_{Co} \lambda_{Co} \lambda_{Ni} N_0}{\lambda_{Ni} - \lambda_{Co}} (e^{-\lambda_{Co} t} - e^{-\lambda_{Ni} t}) \quad (24)$$

4.4. d

see Figure 1

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5. REFERENCES

Neutrino cross section: <http://cupp.oulu.fi/neutrino/nd-cross.html>

Lethal dose of radiation: http://www.radioactivity.eu.com/site/pages/Doses_Classification.html

Atom numbers of human body: https://education.jlab.org/qa/mathatom_04.html

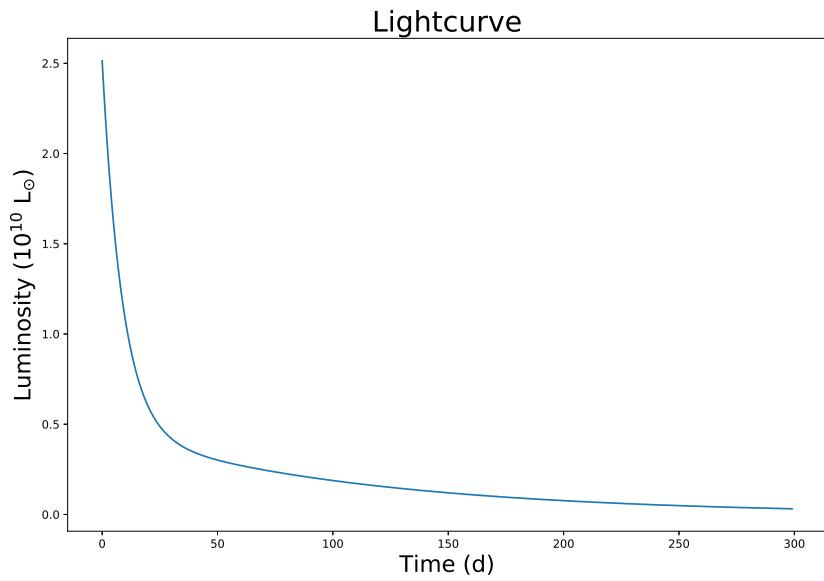


Figure 1. This is the lightcurve for an explosion that produces $1 M_\odot$ of ^{56}Ni