Homework 4

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Q1

(a)

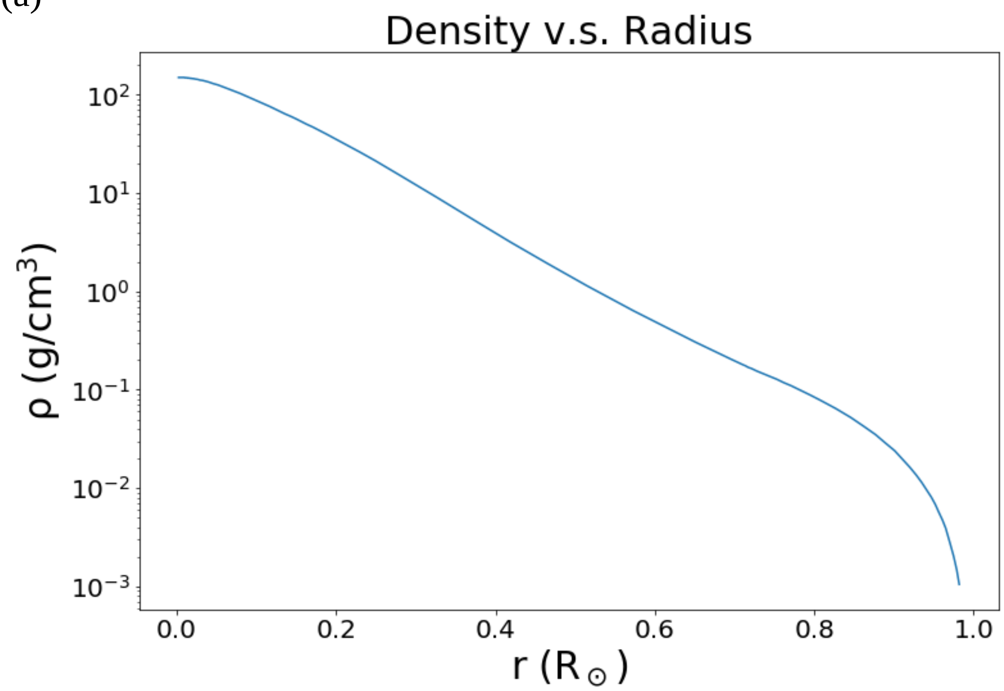

Figure 1

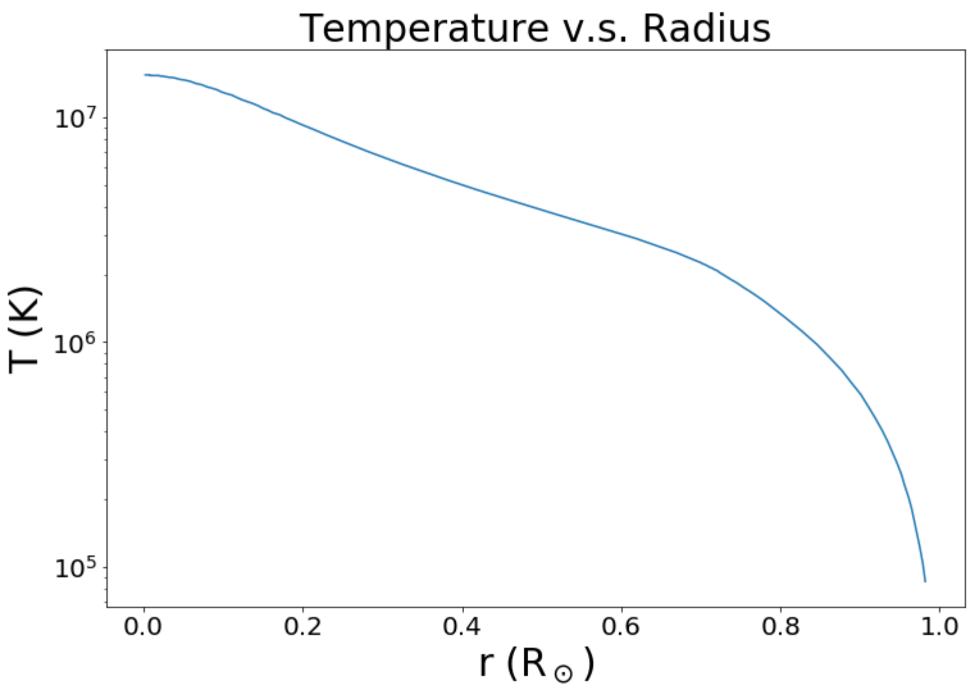

Figure 2

(b)

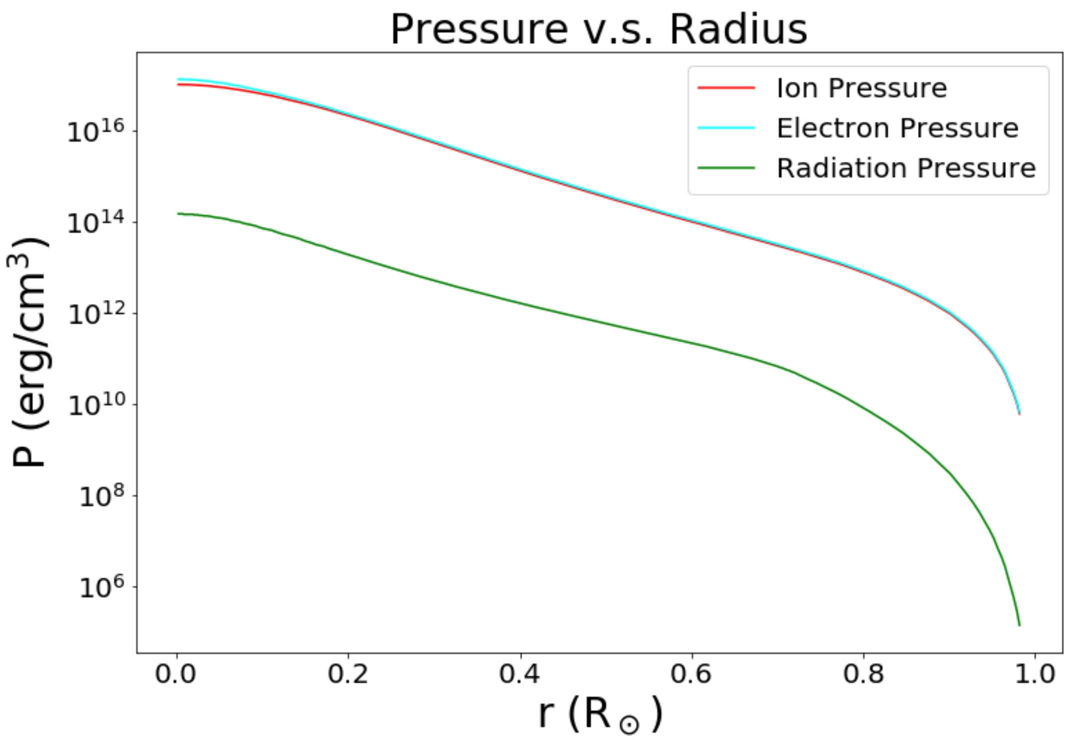

Figure 3

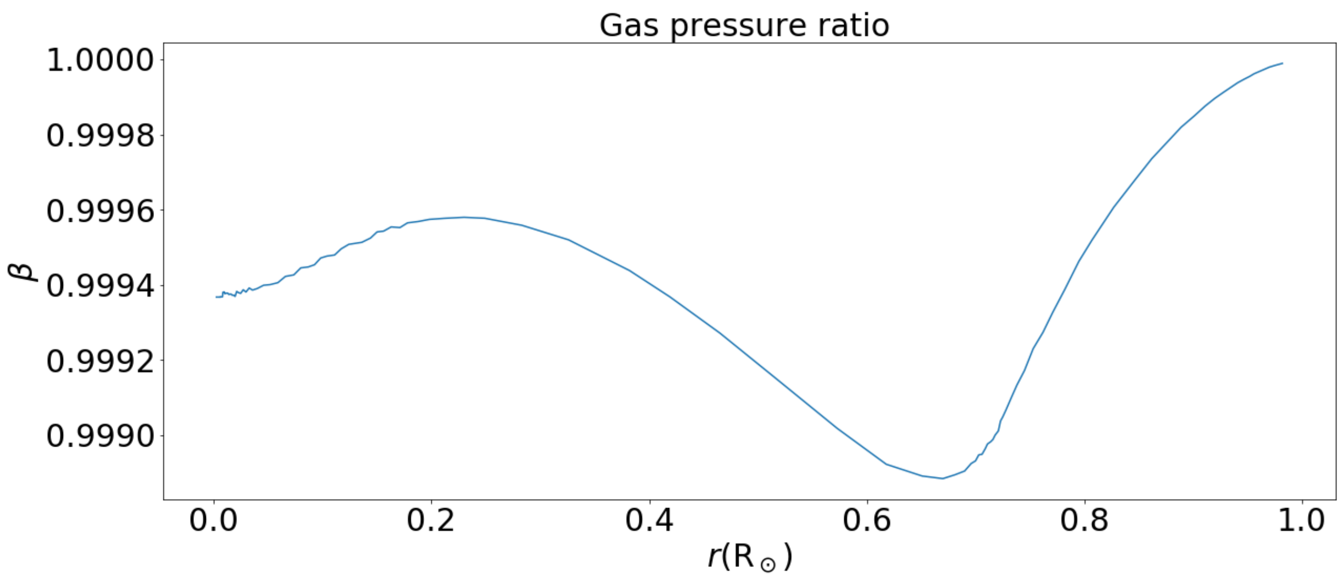

Figure 4

Assume ideal gas then we can calculate the pressure:

$$P_{ion} = \rho kT\left(\frac{X}{m_H} + \frac{Y}{m_{He}}\right)$$

$$P_e = \rho kT\left(\frac{X}{m_H} + \frac{2Y}{m_{He}}\right)$$

$$P_{rad} = \frac{1}{3}aT^4$$

$$P_{gas} = P_{ion} + P_e$$

$$P_{total} = P_{gas} + P_{rad}$$

Then we define: $$\beta = \frac{P_{gas}}{P_{total}}$$

Figure 3 and 4 show the pressure and β as a function of radius.

(c)

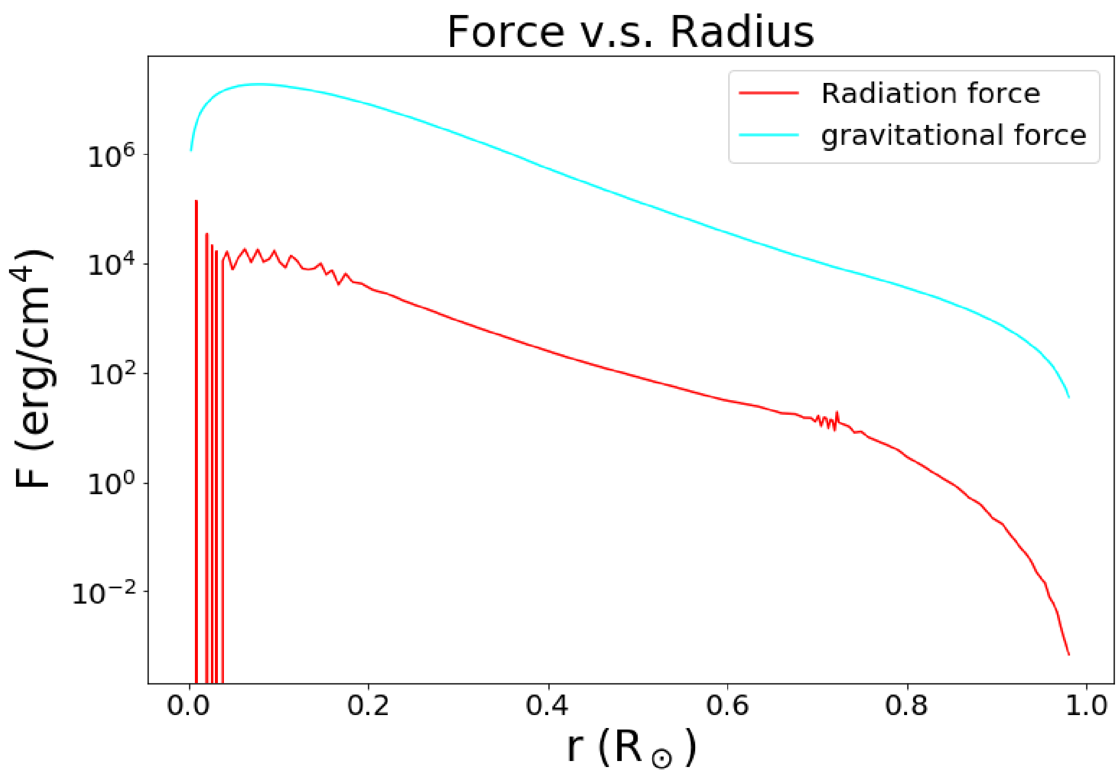

Figure 5

As figure 5 shows, radiation force is much smaller than the gravitational force so it's not important.

(d)

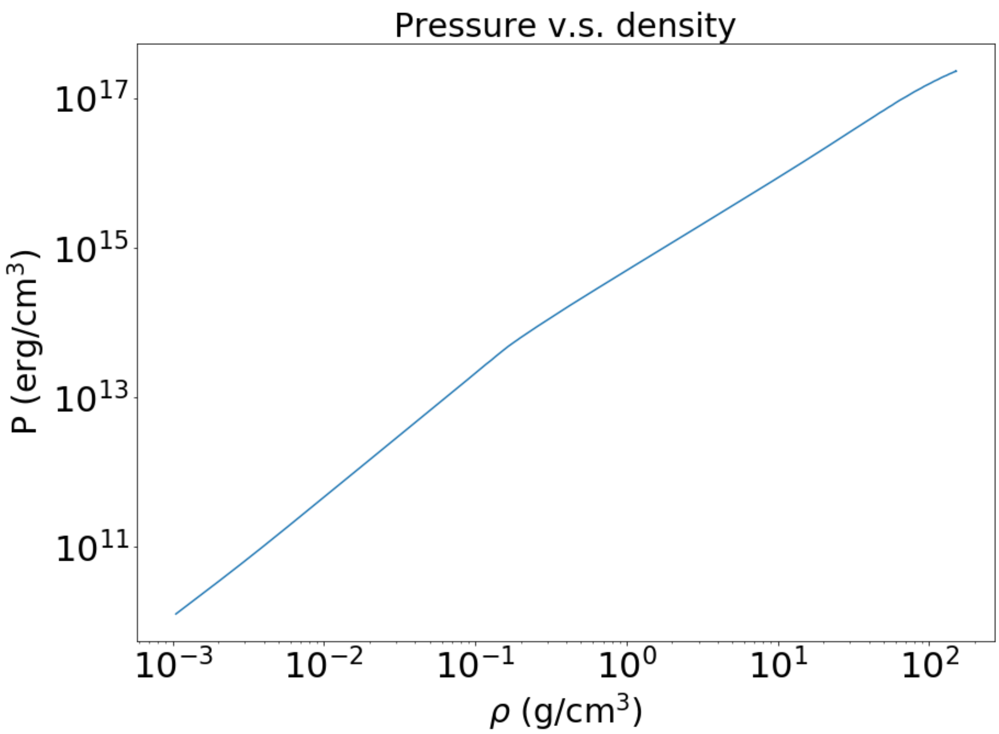

Figure 6

Figure 6 shows the relation between Pressure and density in log scale. We can find that it's very similar to linear relation, which means that: $P \sim \rho^{\gamma}$. Because of the similarity of such a kind of equation of state we can say that a polytrope model is very close to Standard Solar Model.

(e)(f)

For a polytrope of $n = 3, \beta = 0.9995853$:

$$r = \alpha z \quad with \quad \alpha = \frac{R_\odot}{z_3}$$

$$\rho = \rho_c w^3$$

$$\rho_c \approx 54.1825\overline{\rho} = 54.1825\frac{3M}{4\pi R^2}$$

$$K = N_3 G M^{\frac{2}{3}} \quad with \quad N_n = \frac{(4\pi)^{\frac{1}{3}}}{4}\Theta_n^{-\frac{2}{3}}$$

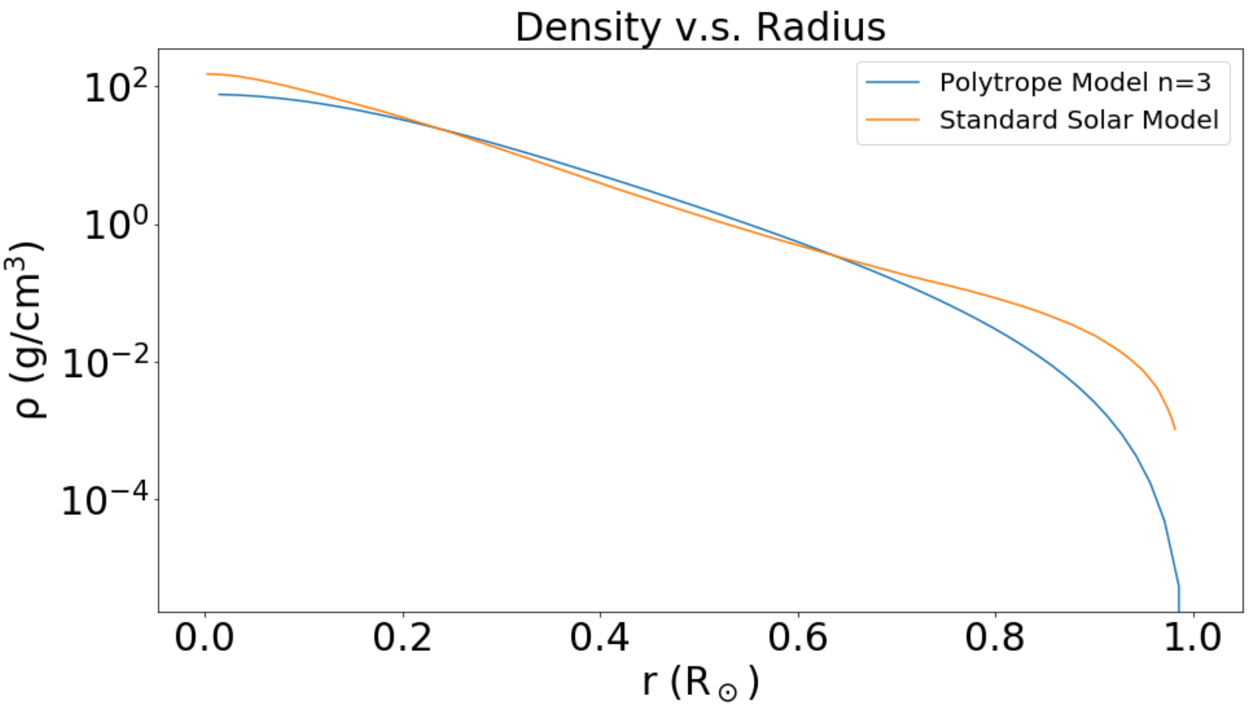

Figure 7

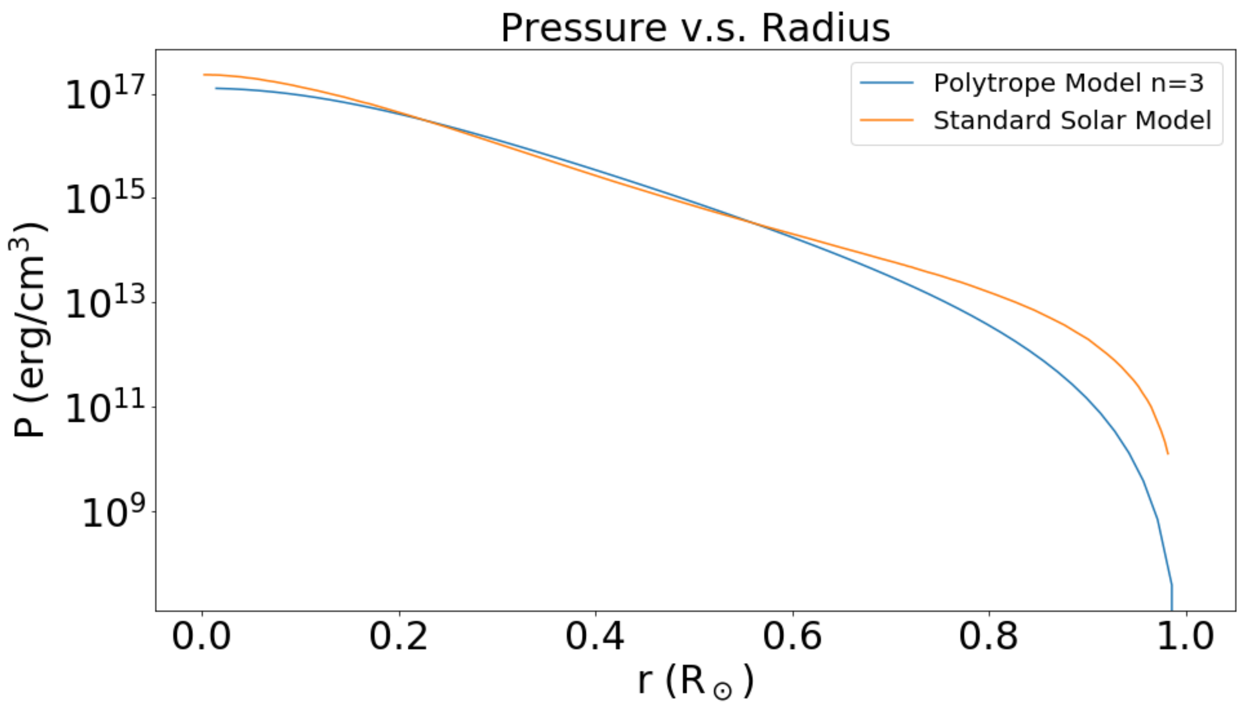

Figure 8

Figure 7 and 8 show the density and pressure distributions of these two models. Figure 9 demonstrates the pressure versus density. We can see that they are very similar so we can conclude that this polytrope model with n=3 is very accurate compared with Standard Solar Model.

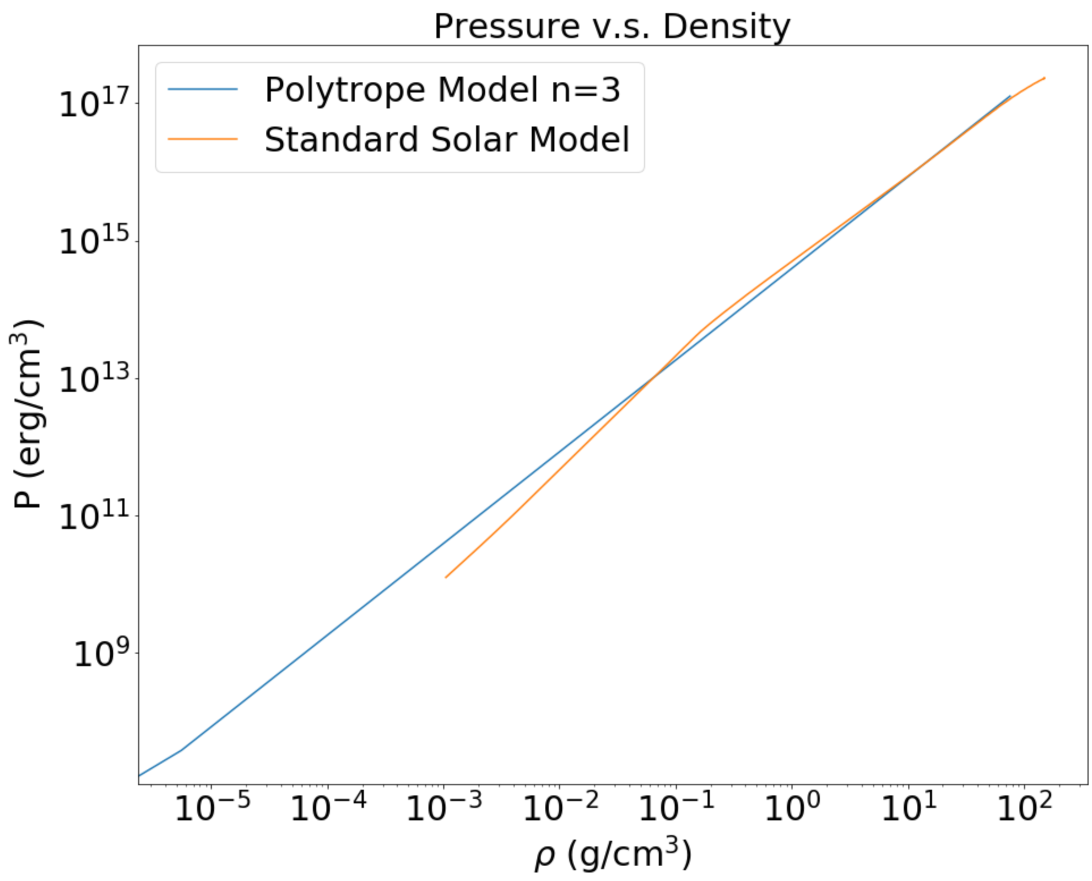

Figure 9

Q2

(a)
$$F_{conv} = \rho c_P T \left(\frac{l_c}{H_P}\right)^2 \sqrt{\frac{1}{2} g H_P} \left(\nabla - \nabla_{ad}\right)^{\frac{3}{2}}$$

And we have:
$$F_{conv} = \frac{F(r)}{4\pi r^2} = \frac{L_\odot}{\pi R_\odot^2}$$

$$g = G\frac{m(r)}{r^2} = \frac{2GM_\odot}{R_\odot^2}$$

$$H_P = \frac{P}{\rho g} = \frac{kT}{\mu m_H g}$$

$$T = 10^7 K, \rho = 1 g/cm^3, X = 0.733, Y = 0.253, Z = 0.014$$

$$\therefore \frac{\delta T}{T} = \frac{l_c}{H_P}\left(\nabla - \nabla_{ad}\right) \approx 3.8\times10^{-8}$$

(b)

$$v_c \approx \sqrt{\frac{1}{2} l_c g \frac{\delta T}{T}} = 1.91\times10^{3}\ cm/s$$

$$c_s = \left(\frac{kT}{\mu m_H}\right)^{0.5} \approx 3.7\times10^{7}\ cm/s$$

$$\frac{v_c}{c_s} \approx 5\times10^{-5}$$

So the velocity of the convective element is much smaller than sound speed.

(c) $$\beta = \frac{\rho v_c^2}{P} \sim \frac{\rho v_c^2}{nkT} \sim \left(\frac{v_c}{c_s}\right)^2 \sim 10^{-9}$$

So the convection doesn't alter significantly the hydrostatic structure of the region.

(d)

The crossing time is:

$$t_c = \frac{l_c}{v_c} \approx 1.8\times10^{6}\, s \approx 5.8\times10^{-2}\, yr$$

The thermal timescale is:

$$t_{KH} = \frac{GM_\odot^2}{2R_\odot L_\odot} \sim 10^{7}\, yr$$

The nuclear timescale is:

$$t_{nuc} = \phi f_{nuc} \frac{M_\odot c^2}{L_\odot} \sim 10^{10}\, yr$$

So the times are:

$$\frac{t_{th}}{t_c} \sim 10^{8}, \frac{t_{nuc}}{t_c} \sim 10^{11}$$

So that over a thermal timescale, and certainly over a nuclear timescale, convective elements will cross the region many times and makes convective region inside a star mixed homogeneously.

Q3

(a)

The value I use is: $m_e = 0.511\ MeV$

$$m_{^1H} = 938.272\ MeV, m_{^2H} = 1875.613\ MeV$$

$$m_{^3He} = 2808.392\ MeV, m_{^4He} = 3727.378\ MeV$$

Then I calculate the energy released in each reaction:

Table 1:PP1 energy released

Reaction of PPI	Energy released (MeV)
$^1H + {}^1H \rightarrow {}^2H + e^+ + \nu$	E1=1.442
$^2H + {}^1H \rightarrow {}^3He + \gamma$	E2=5.493
$^3He + {}^3He \rightarrow {}^4He + 2\ {}^1H$	E3=12.861

(b)

The total energy released in the PPI reaction is:

$$Q_{PP1} = 2E_1 + 2E_2 + 2E_3 + E_4 = 26.73MeV$$

(c)

The average energy of neutrino is:

$$\langle E_{\nu 1} \rangle = 0.265MeV,\ \langle E_{\nu 2} \rangle = 0.814MeV,\ \langle E_{\nu 3} \rangle = 6.71MeV$$

Then the energy of the three pp-chain release is:

$$Q_{PP1} = 26.2MeV, Q_{PP2} = 25.66MeV, Q_{PP3} = 19.76MeV$$

So PP1 release the most energy.

The energy released by unit mass is:

$$\frac{Q_{PP1}}{m_{He}} = 6.30\times10^{18}\ erg/g$$
$$\frac{Q_{PP2}}{m_{He}} = 6.18\times10^{18}\ erg/g$$
$$\frac{Q_{PP3}}{m_{He}} = 4.76\times10^{18}\ erg/g$$

(d)

The triple-α reaction is:

$$3\,^4He -> \,^{12}C + \gamma$$

The energy released is:

$$Q_{3\alpha} = \left(3m_{He} - m_C\right)c^2 = 7.275MeV$$

The energy release per unit mass is:

$$\frac{Q_{3\alpha}}{m_c} = 5.9\times10^{17}\ erg/g$$

(e)

The mass of the core is:

$$m = 0.2M_\odot = 3.978\times10^{32}\,g$$

The total energy release is:

$$E_{PP1} = 2.51\times10^{51} erg\ , E_{3\alpha} = 2.35\times10^{50} erg$$

So the time is:\

$$t_{PP1} = \frac{E_{PP1}}{L_\odot} = 6.54\times10^{17}\,s = 2.07\times10^{10}\,yr$$

$$t_{3\alpha} = \frac{E_{3\alpha}}{L_\odot} = 6.13\times10^{16}\,s = 1.94\times10^{9}\,yr$$

Q4

(a)

the condition is :

$$F_{rot} = m\omega^2 R \ll F_{gra} = G\frac{Mm}{R^2}$$

$$\Leftrightarrow \delta = \frac{\omega^2 R^3}{GM} \ll 1$$

(b)

Table 2: rotation parameters

Spectral type	$M/M_{\odot}$	$R/R_{\odot}$	V_{rot}	δ
O5	60	12	190	0.038
B2.5	7	3.23	200	0.097
B5	4	2.35	210	0.136
A0	2.5	1.81	190	0.137
A7	1.8	1.55	160	0.116
F0	1.6	1.48	95	0.044
F5	1.4	1.36	25	3.2×10^{-3}
G0	1.2	1.13	12	7.1×10^{-4}
G2	1	1	2.3	2.8×10^{-5}

As **table 2** shows, δ of A0 is the largest so the largest departure for spherical symmetry occurs in early A stars.

Acknowledgement:

I thank my roommate Yuxuan Pang, for we did so many discussions. I also thank Jiayi Tang and Tai Zhou.

Reference:

Onno pols 《STELLAR STRUCTURE AND EVOLUTION》
Wiki