Homework 2

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Q1

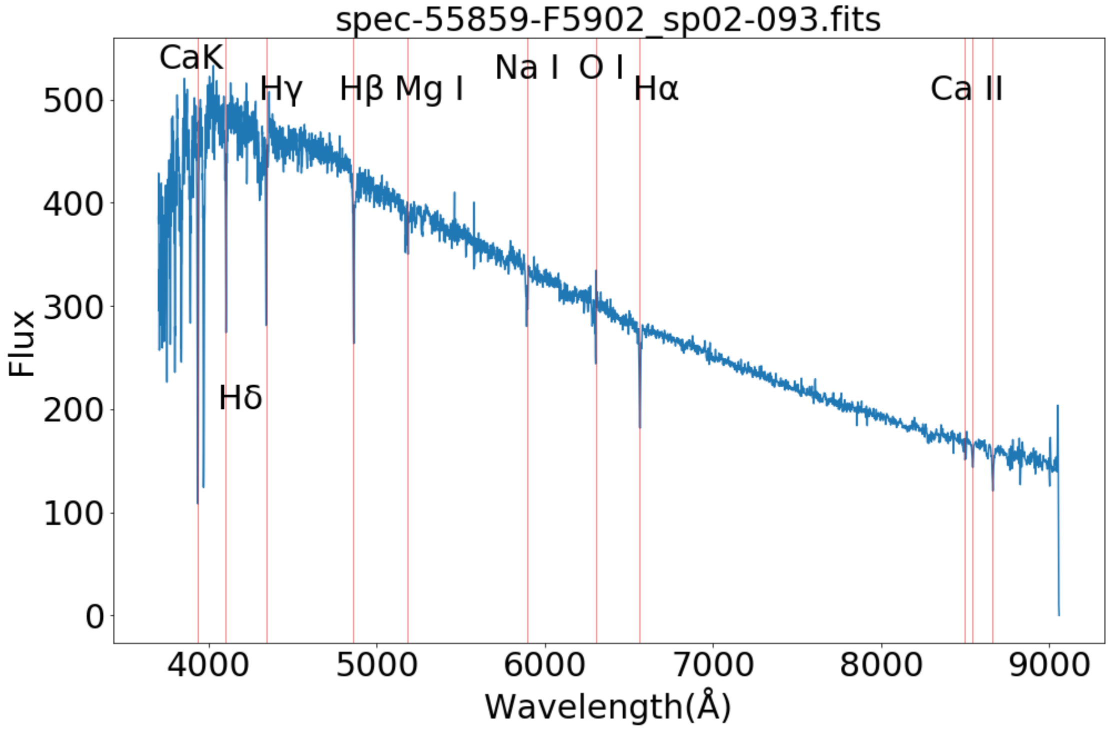

Figure 1

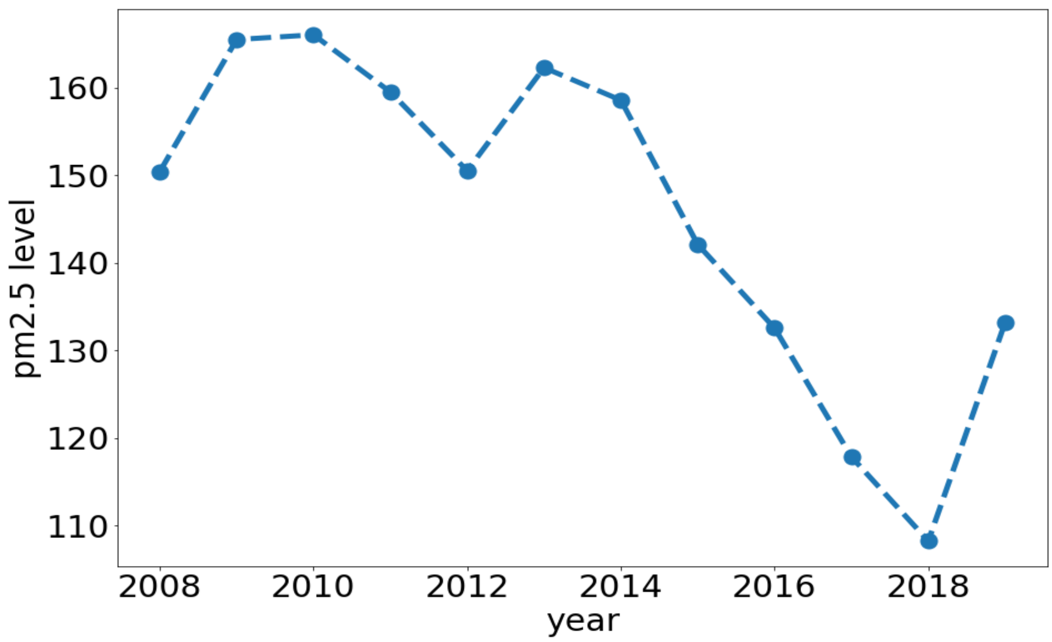

Figure 2

Q2

2.a & 2.b

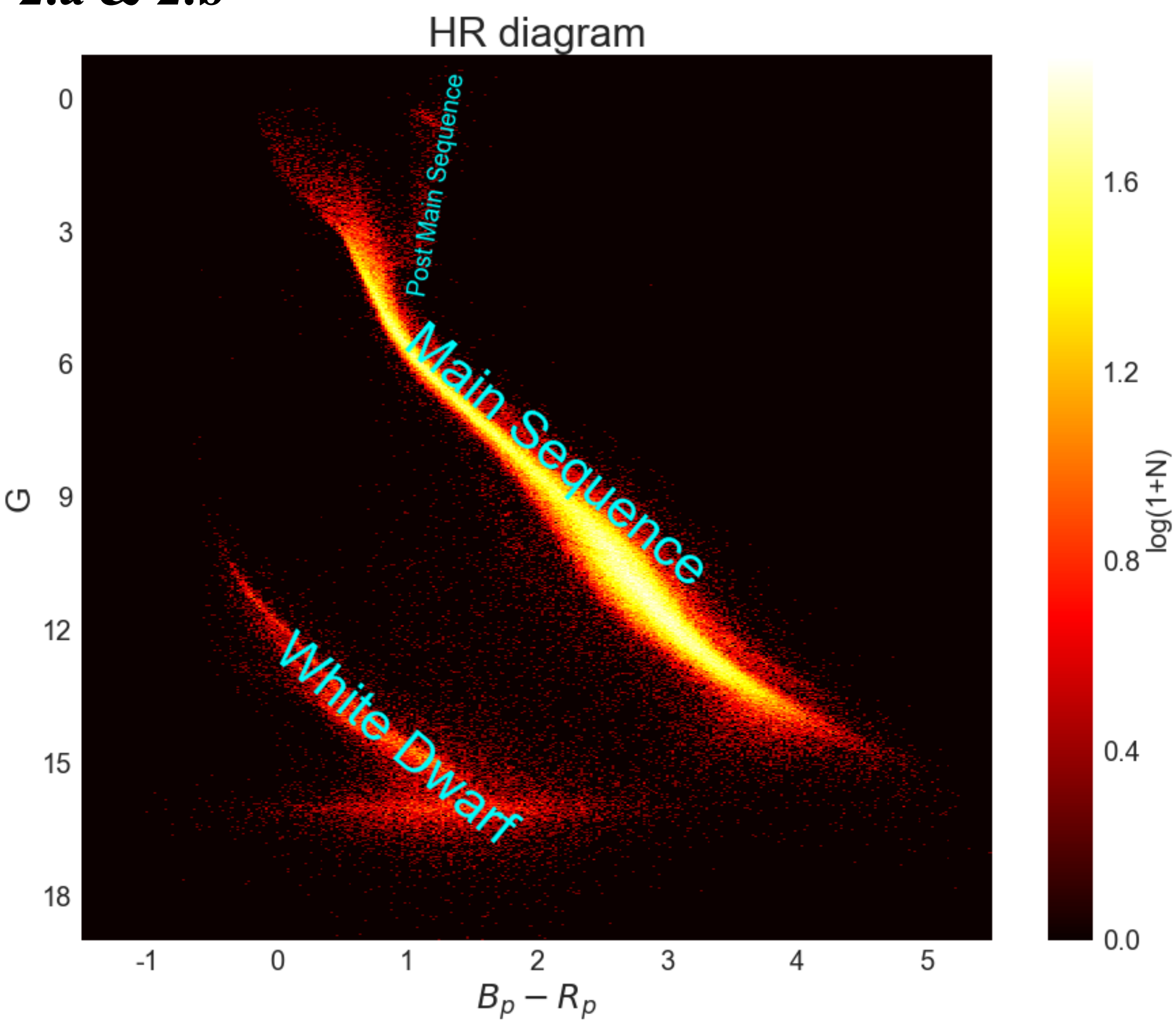

Figure 3

2.c

Because each original measurement such as flux has error, so the final physical properties such as $G_{absolute}, B_p, R_p$ have uncertainties and the point is that we should correctly calculate error propagation:

$$\sigma_{G_{apparent}} = \frac{2.5}{ln10} * \frac{\sigma_{flux}}{flux}$$

$$\sigma_{G_{absolute}} = \sqrt{\sigma_{G_{apparent}}{}^2 + (\frac{5}{ln10} * \frac{\sigma_{plx}}{plx})^2}$$

$$\sigma_{B_p - R_p} = \sqrt{{\sigma_{B_p}}^2 + {\sigma_{R_p}}^2}$$

I consider a Gaussian noise with such uncertainties on raw data and replot HR diagram. As figure 4 show, uncertainties have little effect on most regions compared with figure 3, which indicates that the quality of Gaia data is pretty high and Gaia photometry is good.

Besides, we can find that the faint region of white dwarf in HR diagram changed more diffused while in figure 3 there exits an apparent cutoff at G~16 mag. Then we can infer that the faint limit absolute magnitude of Gaia is 16 mag. Fortunately, this diffusion doesn't prevent us characterizing white dwarfs because the color uncertainties are relative small.

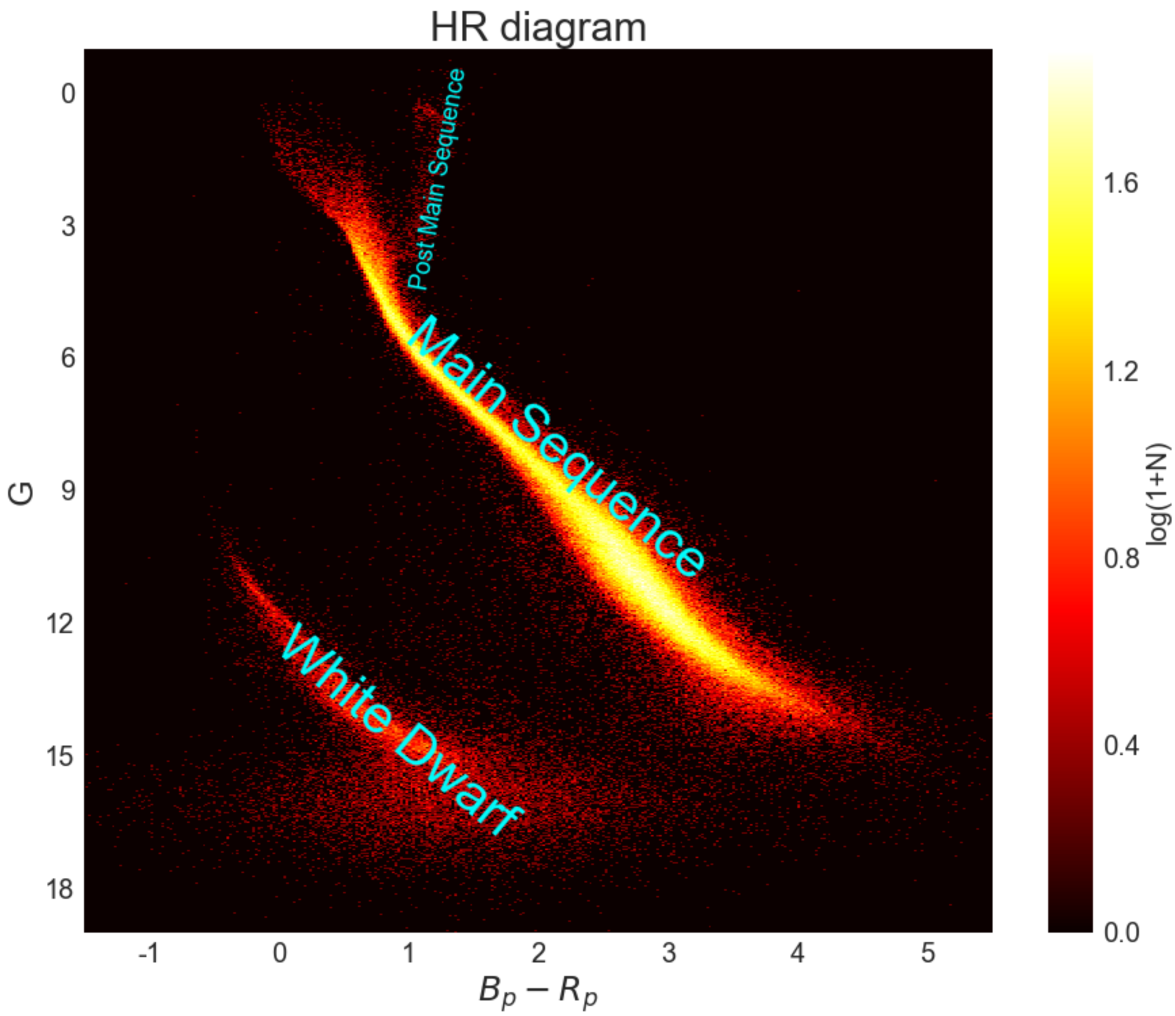

Figure 4

Q3

3.a

Pleiades membership

raw data from Vizier: Infrared observations of the Pleiades (Stauffer+, 2007)

Hyades membership

Vizier: Pan-STARRS1 (PS1) observations of the Hyades (Goldman+, 2013)

NGC752 membership

Vizier: Photometry and radial velocities in NGC752 (Daniel+, 1994)

3.b

Take Pleiades for example:

Step 1:

I use CDS X-match service to crossmatch Pleiades(Stauffer+, 2007, hereafter called Table P) with Gaia DR2 in 5 arcsec radius. Then I find that the number of matched table(hereafter called Table M) is larger than number of raw catalogue. The reason may be that CDX X-match service will return all the object within the match radius.

Step 2:

In order to secure that all the objects are from Table P, I use Topcat to crossmatch the Table M with Table P and get Table Final. Because Topcat returns the best match(the nearest match), I can make sure that each of object in Table Final is in Table P. The number of Table P is 1367.

Step 3:

I choose 1280 data whose relative plx error <10% and then draw 3 distributions in figure 5, figure 6, figure 7. Figure5 shows the distance distribution of Table Final and I use FWHM as the radius criteria of a cluster, which may underestimate the number of a cluster. In this case it’s about 132 and 140 pc. Figure 6 and Figure 7 show the proper motion distribution and I set a very loose criteria as 15, 25 and -50, -40. Using these three criteria, I get 682 cluster members.

I use the same method and choose 329 Hyades members from 742 raw data(Goldman+, 2013) and 83 NGC752 members from 247 raw data(Daniel+, 1994)

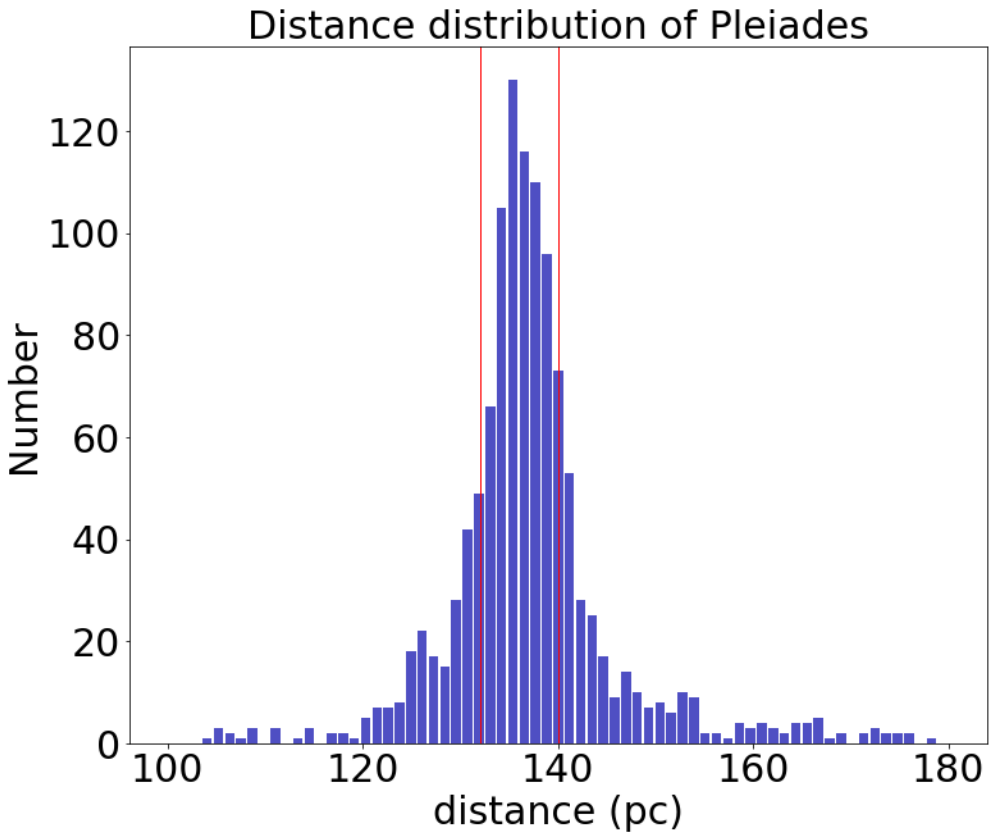

Figure 5

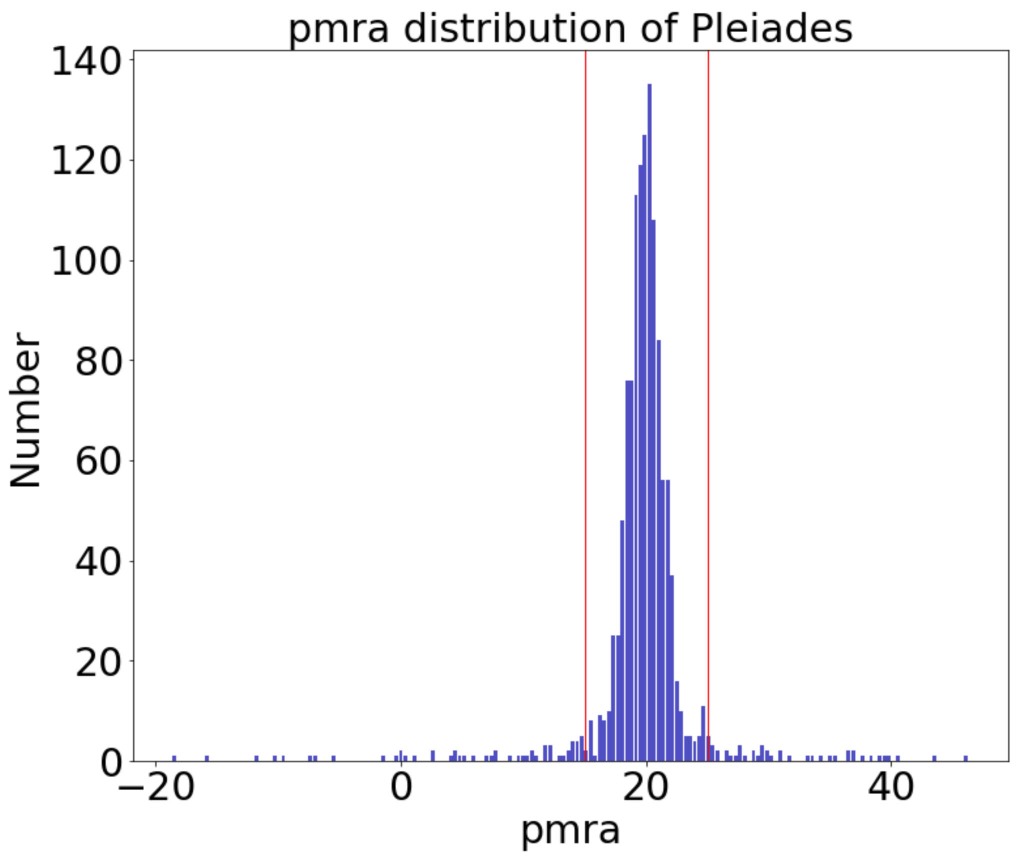

Figure 6

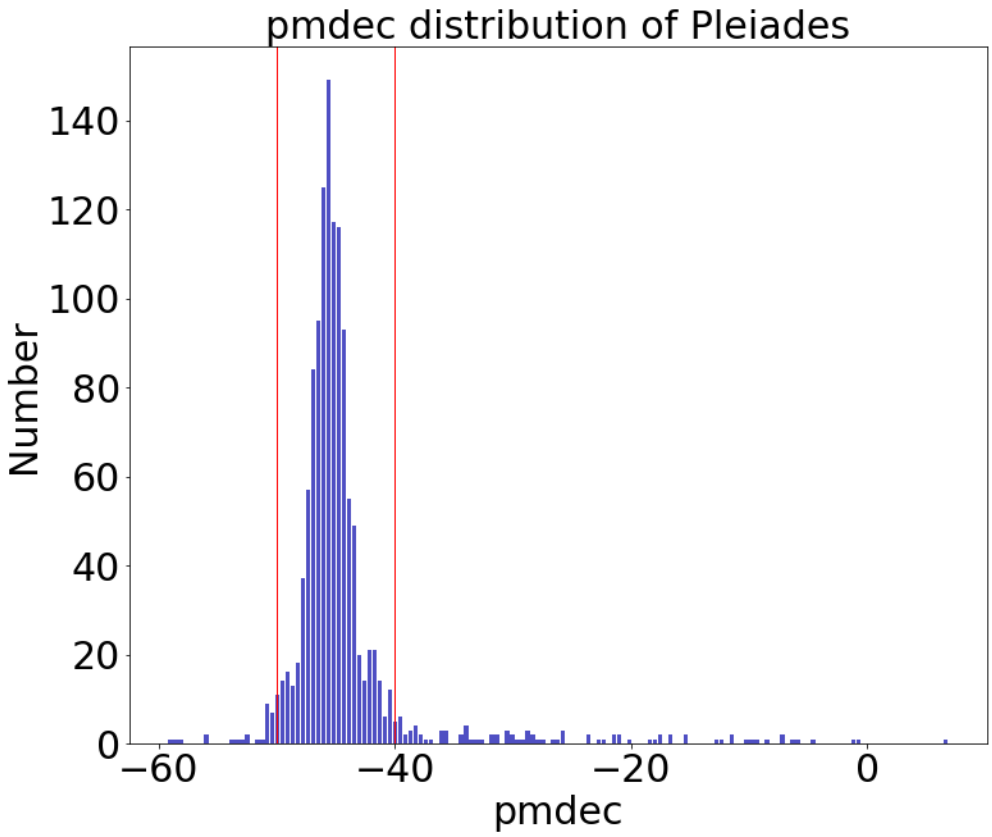

Figure 7

3.c

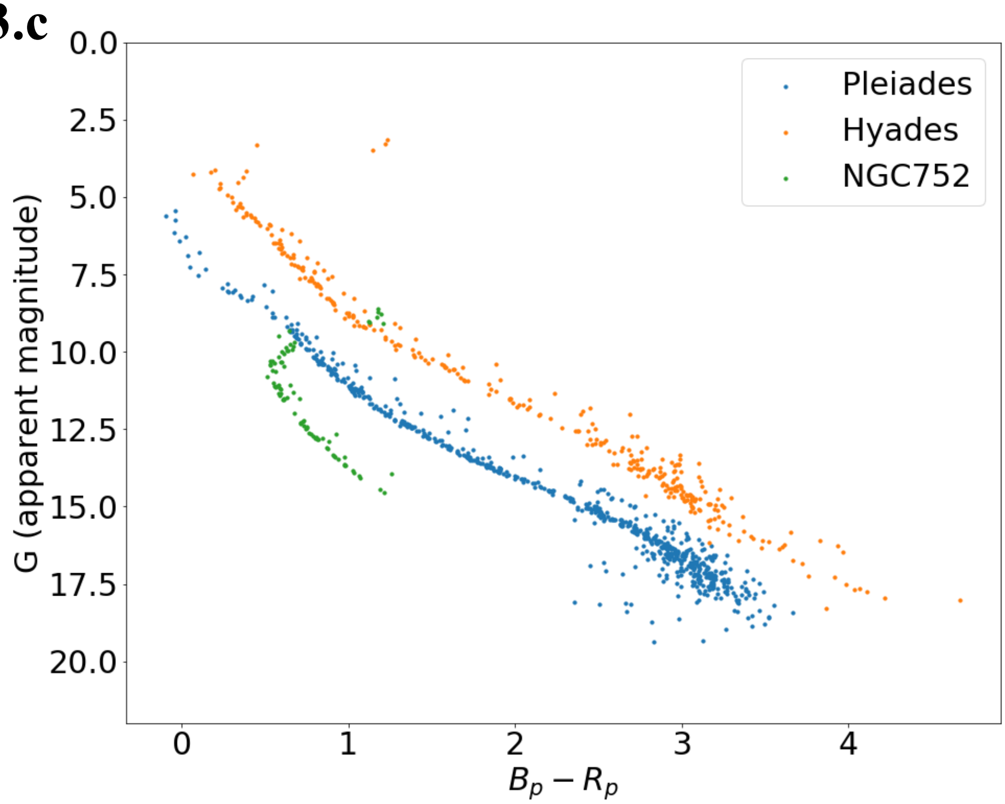

Figure 8

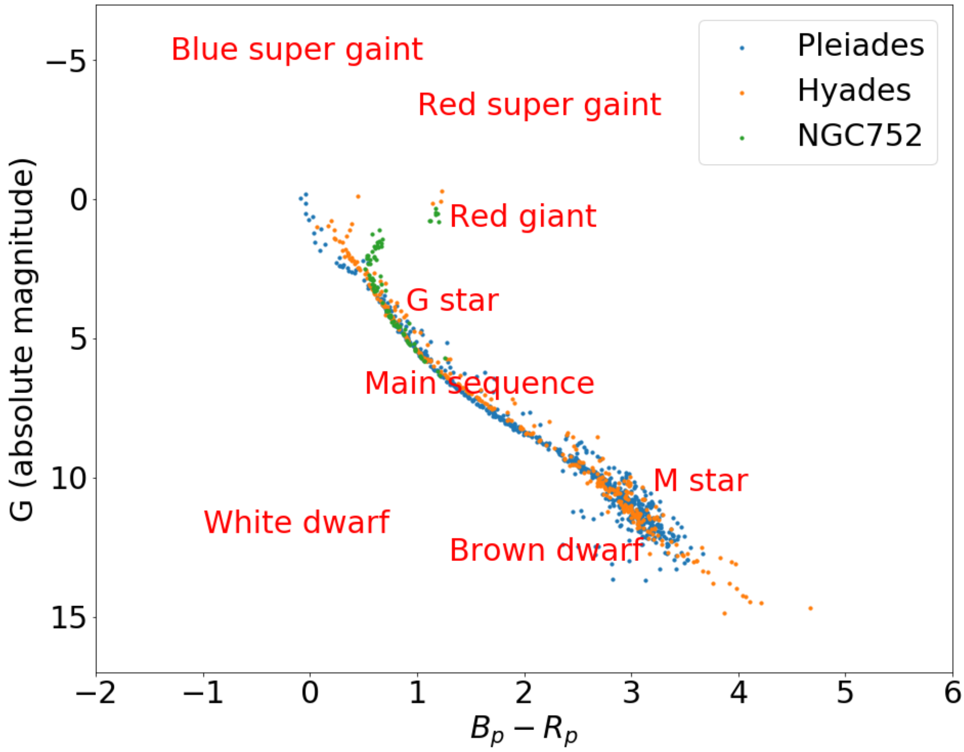

Figure 9

The y-axis of figure 8 is apparent magnitude and that of figure 9 is absolute magnitude.

Brown dwarf is radiating gravitational energy.

White dwarf is purely radiating thermal energy without nuclear reaction.

Main sequence star is burning hydrogen in the core.

Red giant is burning hydrogen in the hydrogen shell around the helium core.

Super giant is burning metal elements such as carbon.

3.d

In figure 8, clusters have difference in apparent magnitude, which indicates that Hyades is the closest cluster and NGC752 is the furthest cluster. In figure 9, we can see that Pleiades doesn't have red giants, which indicates that it is the youngest cluster.

The turn-off point of NGC752 is fainter than that of Hyades, which indicates that relative long lifetime stars are dying. Therefore, NGC752 is the oldest cluster among these three clusters.

I also see that there are no white dwarfs or super giants in three clusters.

3.e

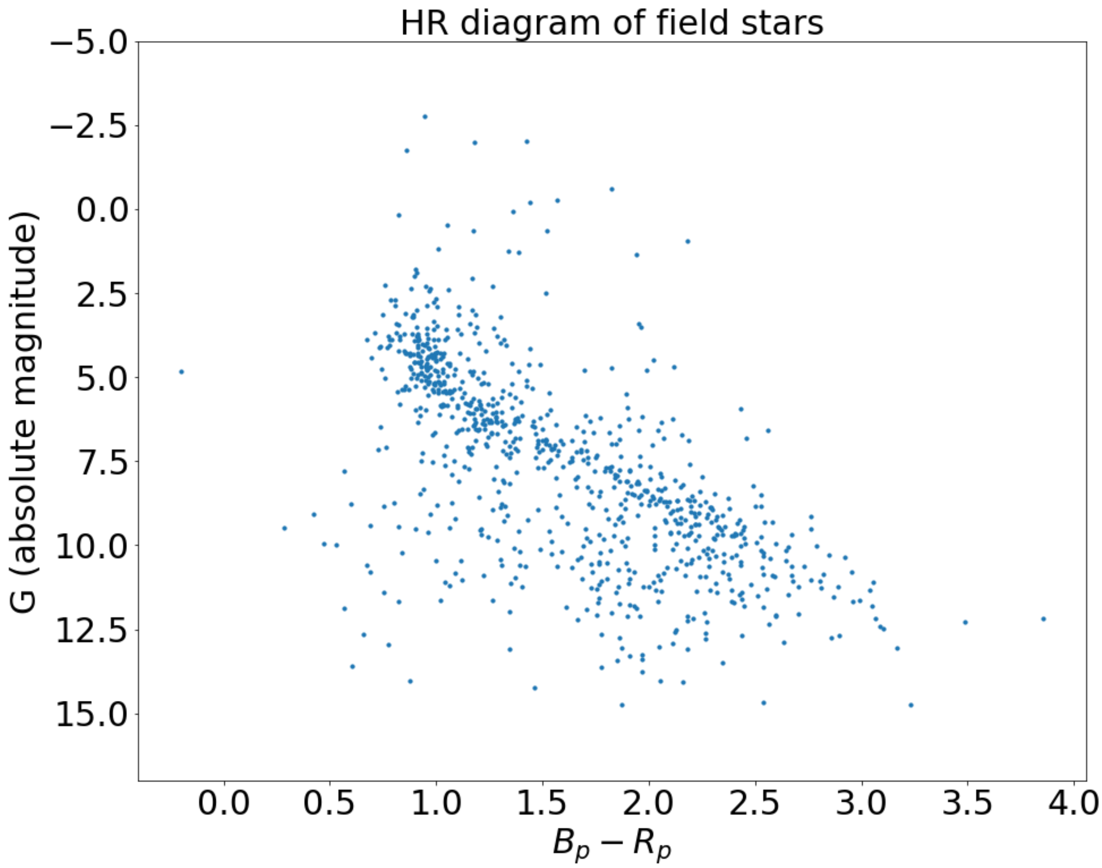

Figure 10

I use online Vizier query to get field stars in regions where 50<ra<50.5, 18<dec<18.5. As figure 10 shows, the distribution of HR diagram is very diffused and there exit white dwarfs compared with clusters above. So we can infer that each different field star may have very different burn time and physical properties while stars in a cluster can be a stellar population.

Q4

4.a

Planck function: $$(\lambda, T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda k_B T}}-1}$$

4.b

$$\frac{dB_\lambda}{d\lambda} = 0$$

$$\Leftrightarrow \quad \frac{-5}{\lambda} B_\lambda + B_\lambda \frac{\frac{hc}{\lambda^2 k_B T} e^{\frac{hc}{\lambda k_B T}}}{e^{\frac{hc}{\lambda k_B T}} - 1} = 0$$

$$\Leftrightarrow \quad (-5) \cdot \left(e^{\frac{hc}{\lambda k_B T}} - 1 \right) + \frac{hc}{\lambda k_B T} e^{\frac{hc}{k_B(\lambda T)}} = 0$$

make $x = \frac{hc}{\lambda k_B T}$, we have $\frac{x e^x}{e^x - 1} - 5 = 0$

solve this equation and we can get:

$$\lambda_{peak} * T = const$$

4.c

λ_{peak}, the wavelength of the black-body radiation peak flux, is inversely proportional to temperature.

4.d

We define L: luminosity of Sun,

D: distance between Earth and Sun

R:radius of Earth

Earth's absorption term: $0.6 \times \frac{L}{4\pi D^2} \times \pi R^2$

Earth's emission term: $4\pi R^2 \times \sigma T^4$

Because we regard Earth as a blackbody, then we have:

$$0.6 \times \frac{L}{4\pi D^2} \times \pi R^2 = 4\pi R^2 \times \sigma T^4$$

$$\therefore T = (0.6 \cdot \frac{L}{16\pi\sigma D^2})^{1/4}$$

we use $L = 3.845 \times 10^{26}\ W$, $D = 1AU$ and get $T = 245K$

Q5

5.a

$$\rho(r) = \rho_c[1 - (\frac{r}{R_*})^2]$$

$$\therefore M_* = \int_0^{R_*} \rho(r) 4\pi r^2\, dr = 4\pi\rho_c \left(\frac{r^3}{3} - \frac{r^5}{5R_*^2}\right)\Big|_0^{R_*}$$

$$= \frac{8}{15}\pi\rho_c R_*^3$$

$$\Longrightarrow \rho_c = \frac{15M_*}{8\pi R_*^3}$$

5.b

$$\frac{dP}{dr} = -\frac{Gm}{r^2}\rho = -4\pi G\rho_c(\frac{r^3}{3} - \frac{r^5}{5R_*^2})\frac{\rho_c}{r^2}[1 - (\frac{r}{R_*})^2]$$

$$= -4\pi G{\rho_c}^2(\frac{r}{3} - \frac{8r^3}{15R_*^2} + \frac{r^5}{5R_*^4})$$

$$\therefore P(r) = -4\pi G\rho_c^2(\frac{r^2}{6} - \frac{2r^4}{15R_*^2} + \frac{r^6}{30R_*^4}) + constant$$

$$\because P(R_*)=0$$

$$\therefore P_c = constant = \frac{4}{15}\pi G\rho_c^2 R_*^2 = \frac{15}{16}\frac{GM_*^2}{\pi R_*^4}$$

$$\Longrightarrow P(r) = P_c[1 - \frac{5}{2}(\frac{r}{R_*})^2 + 2(\frac{r}{R_*})^4 - \frac{1}{2}(\frac{r}{R_*})^6]$$

if we take $M_\odot = 1.989 \times 10^{30} kg, R_\odot = 6.955 \times 10^8 m$:

$$P_c = 3.37 * 10^{14}(\frac{M}{M_\odot})^2(\frac{R}{R_\odot})^{-4} \ \ Pa$$

5.c

For ideal gas: $P = nkT = \frac{\rho}{\mu m_\mu}kT$

$$\therefore T_c = \frac{\mu m_\mu}{k}\frac{P_c}{\rho_c} = \frac{\mu m_\mu G M_*}{2kR_*}$$

for pure ionized H: $\mu = \frac{1}{2}, T_c = \frac{m_\mu G M_*}{4kR_*}$

for pure ionized He: $\mu = \frac{4}{3}, T_c = \frac{2m_\mu G M_*}{3kR_*}$

5.d

$$-3\int PdV = -3\int_0^{R_*} P(r)4\pi r^2 dr = -\frac{16}{21}\pi P_c R_*^3$$

$$= -\frac{5}{7}\frac{GM_*^2}{R_*}$$

$$\Omega = \int_0^{M_*} -\frac{Gm(r)dm}{r}$$

$$= \int_0^{R_*} -\frac{4\pi\rho_c G}{r}\left(\frac{r^3}{3} - \frac{r^5}{5R_*^2}\right)4\pi\rho_c\left(r^2 - \frac{r^4}{R_*^2}\right) dr$$

$$= -\frac{5}{7}\frac{GM_*^2}{R_*} = -3\int PdV$$

so the virial theorem is satisfied.

Q6

a. ideal gas：lots of mass points without interactions and their collisions are perfectly elastic. The ideal gas follows the equation: $PV = nRT$

b. virial theorem: in a gravitational bound stable system, the relationship of average kinetic energy <T> and average potential energy <V> follows the equation:

$$2 < T >= -< V >$$

c. blackbody: an ideal object that can absorb radiation completely at every wavelength.

d. energy transport by convection: energy transport from high temperature region to low temperature region because of the temperature gradient.

e. Kelvin-Helmholz timescale: It's a theoretical estimate of how long a star would shine with current luminosity if the only power source were the conversion of gravitational potential energy.

f. HR diagram: It's a 2-d figure with luminosity/magnitude versus effective temperature/color index, which can show different properties of different objects.

g. Jeans mass: If the mass of a gaseous cloud exceeds the critical mass called Jeans mass, the cloud will collapse.

h. stellar effective temperature: the all-wavelength radiation emitted by a star equals the total energy emitted by a blackbody at a certain temperature called stellar effective temperature.

i. hydrostatic equilibrium: In this state, downward force exerted by gravity of the fluid is balanced by an upward force exerted by the pressure.

Acknowledgement:

I thank my roommate Pang Yuxuan, for we did so many discussions.

Reference:

Wiki

Vizier: http://vizier.u-strasbg.fr/

Gherczeg: http://kiaa.pku.edu.cn/~gherczeg/stellar/

Appendix: python code for Q3

```
from astropy.io import fits
import numpy as np
import matplotlib.pyplot as plt
import math
import pandas as pd

font=30


#import fits data

Pleiades_fits=fits.open('Greg_HW2_3_data-master/Pleiades_Gaia.fits')
Hyades_fits=fits.open('Greg_HW2_3_data-master/Hyades_Gaia.fits')
NGC752_fits=fits.open('Greg_HW2_3_data-master/NGC752_Gaia.fits')
Pleiades=Pleiades_fits[1].data
Hyades=Hyades_fits[1].data
NGC752=NGC752_fits[1].data


### raw Pleiades data

bprp_Pleiades=Pleiades['bp_rp']
Gmag_Pleiades=Pleiades['phot_g_mean_mag']
plx_Pleiades=Pleiades['parallax']
plx_err_Pleiades=Pleiades['parallax_error']
pmra_Pleiades=Pleiades['pmra']
pmdec_Pleiades=Pleiades['pmdec']
print('Pleiades:')
print('raw data:',len(Gmag_Pleiades))


### delete data whose plx relative error>0.1

index_Pleiades=(plx_err_Pleiades/plx_Pleiades<0.1)
plx_Pleiades=plx_Pleiades[index_Pleiades]
Gmag_Pleiades=Gmag_Pleiades[index_Pleiades]
bprp_Pleiades=bprp_Pleiades[index_Pleiades]
pmra_Pleiades=pmra_Pleiades[index_Pleiades]
pmdec_Pleiades=pmdec_Pleiades[index_Pleiades]
print('data within 0.1 error:',len(Gmag_Pleiades))


### choose cluster mumbers

distance_Pleiades=1000/plx_Pleiades
```

```python
# plot distance distribution of Pleiades

plt.figure(figsize=(12,10))
n, bins, patches = plt.hist(x=distance_Pleiades, bins='auto',range=[100,180],\
                            color='#0504aa',alpha=0.7, rwidth=0.85)
plt.title('Distance distribution of Pleiades',fontsize=font)
plt.axvline(132, color='r', linewidth=1)
plt.axvline(140, color='r', linewidth=1)
plt.xlabel('distance (pc)',fontsize=font)
plt.ylabel('Number',fontsize=font)
plt.tick_params(labelsize=font)
plt.savefig('Distance_Pleiades.png',format='png')
plt.show()

# plot pmra distribution of Pleiades

plt.figure(figsize=(12,10))
n, bins, patches = plt.hist(x=pmra_Pleiades,
bins='auto',color='#0504aa',alpha=0.7, rwidth=0.85)
plt.title('pmra distribution of Pleiades',fontsize=font)
plt.axvline(15, color='r', linewidth=1)
plt.axvline(25, color='r', linewidth=1)
plt.xlabel('pmra',fontsize=font)
plt.ylabel('Number',fontsize=font)
plt.tick_params(labelsize=font)
plt.savefig('pmra_Pleiades.png',format='png')
plt.show()

# plot pmdec distribution of Pleiades

plt.figure(figsize=(12,10))
n, bins, patches = plt.hist(x=pmdec_Pleiades,
bins='auto',color='#0504aa',alpha=0.7, rwidth=0.85)
plt.title('pmdec distribution of Pleiades',fontsize=font)
plt.axvline(-50, color='r', linewidth=1)
plt.axvline(-40, color='r', linewidth=1)
plt.xlabel('pmdec',fontsize=font)
plt.ylabel('Number',fontsize=font)
plt.tick_params(labelsize=font)
plt.savefig('pmdec_Pleiades.png',format='png')
plt.show()

# choose the radius of cluster according to FWHM

index_Pleiades=((distance_Pleiades>132) & (distance_Pleiades<140) &\
                 (pmra_Pleiades>15) & (pmra_Pleiades<25) &
(pmdec_Pleiades>-50) & (pmdec_Pleiades<-40) )
```

```
distance_Pleiades_cluster=distance_Pleiades[index_Pleiades]
Gmag_Pleiades_cluster=Gmag_Pleiades[index_Pleiades]
bprp_Pleiades_cluster=bprp_Pleiades[index_Pleiades]
pmra_Pleiades_cluster=pmra_Pleiades[index_Pleiades]
pmdec_Pleiades_cluster=pmdec_Pleiades[index_Pleiades]
G_abs_Pleiades_cluster=Gmag_Pleiades_cluster+5-5*np.log10(distance_Pleiades_
cluster)
print('cluster member:',len(Gmag_Pleiades_cluster),'\n')



### raw Hyades data
bprp_Hyades=Hyades['bp_rp']
Gmag_Hyades=Hyades['phot_g_mean_mag']
plx_Hyades=Hyades['parallax']
plx_err_Hyades=Hyades['parallax_error']
pmra_Hyades=Hyades['pmra_2a']
pmdec_Hyades=Hyades['pmdec']
print('Hyades:')
print('raw data:',len(Gmag_Hyades))

### delete data whose plx relative error>0.1
index_Hyades=(plx_err_Hyades/plx_Hyades<0.1)
plx_Hyades=plx_Hyades[index_Hyades]
Gmag_Hyades=Gmag_Hyades[index_Hyades]
bprp_Hyades=bprp_Hyades[index_Hyades]
pmra_Hyades=pmra_Hyades[index_Hyades]
pmdec_Hyades=pmdec_Hyades[index_Hyades]
print('data within 0.1 error:',len(Gmag_Hyades))

### choose the radius of cluster according to FWHM
distance_Hyades=1000/plx_Hyades
# this process is similar with former so I make them comments
'''
# plot distance distribution of Hyades
plt.figure(figsize=(12,10))
n, bins, patches = plt.hist(x=distance_Hyades,
bins='auto',range=[20,80],color='#0504aa',alpha=0.7, rwidth=0.85)
plt.title('Distance distribution of Hyades',fontsize=font)
plt.xlabel('distance (pc)',fontsize=font)
plt.ylabel('Number',fontsize=font)
```

```
plt.tick_params(labelsize=font)
plt.show()
# plot pmra distribution of Hyades
plt.figure(figsize=(12,10))
n, bins, patches = plt.hist(x=pmra_Hyades,
bins='auto',color='#0504aa',alpha=0.7, rwidth=0.85)
plt.title('pmra distribution of Hyades',fontsize=font)
plt.xlabel('pmra',fontsize=font)
plt.ylabel('Number',fontsize=font)
plt.tick_params(labelsize=font)
plt.show()
# plot pmdec distribution of Hyades
plt.figure(figsize=(12,10))
n, bins, patches = plt.hist(x=pmdec_Hyades,
bins='auto',color='#0504aa',alpha=0.7, rwidth=0.85)
plt.title('pmdec distribution of Hyades',fontsize=font)
plt.xlabel('pmdec',fontsize=font)
plt.ylabel('Number',fontsize=font)
plt.tick_params(labelsize=font)
plt.show()
'''
index_Hyades=((distance_Hyades>44) & (distance_Hyades<52) &\
                   (pmra_Hyades>0) & (pmra_Hyades<200) & (pmdec_Hyades>-100)
& (pmdec_Hyades<100))
distance_Hyades_cluster=distance_Hyades[index_Hyades]
Gmag_Hyades_cluster=Gmag_Hyades[index_Hyades]
bprp_Hyades_cluster=bprp_Hyades[index_Hyades]
G_abs_Hyades_cluster=Gmag_Hyades_cluster+5-5*np.log10(distance_Hyades_clu
ster)
print('data in cluster:',len(Gmag_Hyades_cluster),'\n')


### raw NGC752 data

bprp_NGC752=NGC752['bp_rp']
Gmag_NGC752=NGC752['phot_g_mean_mag']
plx_NGC752=NGC752['parallax']
plx_err_NGC752=NGC752['parallax_error']
pmra_NGC752=NGC752['pmra']
pmdec_NGC752=NGC752['pmdec']
print('NGC752:')
print('raw data:',len(Gmag_NGC752))


### delete data whose relative error>0.1
```

```
index_NGC752=(plx_err_NGC752/plx_NGC752<0.1)
plx_NGC752=plx_NGC752[index_NGC752]
Gmag_NGC752=Gmag_NGC752[index_NGC752]
bprp_NGC752=bprp_NGC752[index_NGC752]
pmra_NGC752=pmra_NGC752[index_NGC752]
pmdec_NGC752=pmdec_NGC752[index_NGC752]
print('data within 0.1 error:',len(Gmag_NGC752))

### choose the members of cluster according to FWHM
distance_NGC752=1000/plx_NGC752
'''
# plot distance distribution of NGC752
plt.figure(figsize=(12,10))
n, bins, patches = plt.hist(x=distance_NGC752,
bins='auto',range=[400,500],color='#0504aa',alpha=0.7, rwidth=0.85)
plt.title('Distance distribution of NGC752',fontsize=font)
plt.xlabel('distance (pc)',fontsize=font)
plt.ylabel('Number',fontsize=font)
plt.tick_params(labelsize=font)
plt.show()
# plot pmra distribution of NGC752
plt.figure(figsize=(12,10))
n, bins, patches = plt.hist(x=pmra_NGC752,
bins='auto',color='#0504aa',alpha=0.7, rwidth=0.85)
plt.title('pmra distribution of NGC752',fontsize=font)
plt.xlabel('pmra',fontsize=font)
plt.ylabel('Number',fontsize=font)
plt.tick_params(labelsize=font)
plt.show()
# plot pmdec distribution of NGC752
plt.figure(figsize=(12,10))
n, bins, patches = plt.hist(x=pmdec_NGC752,
bins='auto',color='#0504aa',alpha=0.7, rwidth=0.85)
plt.title('pmdec distribution of NGC752',fontsize=font)
plt.xlabel('pmdec',fontsize=font)
plt.ylabel('Number',fontsize=font)
plt.tick_params(labelsize=font)
plt.show()
'''
index_NGC752=((distance_NGC752>435) & (distance_NGC752<460) &\
                    (pmra_NGC752>8) & (pmra_NGC752<12) & (pmdec_NGC752>-13) &
(pmdec_NGC752<-10))
distance_NGC752_cluster=distance_NGC752[index_NGC752]
```

```
Gmag_NGC752_cluster=Gmag_NGC752[index_NGC752]
bprp_NGC752_cluster=bprp_NGC752[index_NGC752]
G_abs_NGC752_cluster=Gmag_NGC752_cluster+5-5*np.log10(distance_NGC752_cl
uster)
pmra_NGC752_cluster=pmra_NGC752[index_NGC752]
pmdec_NGC752_cluster=pmdec_NGC752[index_NGC752]
print('data in cluster:',len(Gmag_NGC752_cluster),'\n')


####plot HR diagram
plt.figure(figsize=(15,12))
plt.scatter(bprp_Pleiades_cluster,Gmag_Pleiades_cluster,s=10,label='Pleiades')
plt.scatter(bprp_Hyades_cluster,Gmag_Hyades_cluster,s=10,label='Hyades')
plt.scatter(bprp_NGC752_cluster,Gmag_NGC752_cluster,s=10,label='NGC752')
plt.ylim(22,0)
plt.legend(fontsize=font)
plt.tick_params(labelsize=font)
plt.xlabel('$B_{p}-R_{p}$',fontsize=font)
plt.ylabel('G (apparent magnitude)',fontsize=font)
plt.savefig('HR_cluster_apparent.png',format='png')
plt.show()

plt.figure(figsize=(15,12))
plt.scatter(bprp_Pleiades_cluster,G_abs_Pleiades_cluster,s=10,label='Pleiades')
plt.scatter(bprp_Hyades_cluster,G_abs_Hyades_cluster,s=10,label='Hyades')
plt.scatter(bprp_NGC752_cluster,G_abs_NGC752_cluster,s=10,label='NGC752')
plt.text(-1,12,'White dwarf',fontsize=font,color='red')
plt.text(-1.3,-5,'Blue super gaint',fontsize=font,color='red')
plt.text(1,-3,'Red super gaint',fontsize=font,color='red')
plt.text(1.3,13,'Brown dwarf',fontsize=font,color='red')
plt.text(0.5,7,'Main sequence',fontsize=font,color='red')
plt.text(0.9,4,'G star',fontsize=font,color='red')
plt.text(3.2,10.5,'M star',fontsize=font,color='red')
plt.text(1.3,1,'Red giant',fontsize=font,color='red')
plt.xlim(-2,6)
plt.ylim(17,-7)
plt.legend(fontsize=font)
plt.tick_params(labelsize=font)
plt.xlabel('$B_{p}-R_{p}$',fontsize=font)
plt.ylabel('G (absolute magnitude)',fontsize=font)
plt.savefig('HR_cluster_absolute.png',format='png')
plt.show()
```

```
#import field fits data and plot HR diagram
field_fits=fits.open('Greg_HW2_3_data-master/50_505_18_185.fits')
field=field_fits[1].data

bprp_field=field['bp_rp']
Gmag_field=field['phot_g_mean_mag']
plx_field=field['parallax']
index=plx_field>0
bprp_field=bprp_field[index]
Gmag_field=Gmag_field[index]
plx_field=plx_field[index]
G_abs_field=Gmag_field-10+5*np.log10(plx_field)

plt.figure(figsize=(15,12))
plt.scatter(bprp_field,G_abs_field,s=10)
plt.ylim(17,-5)
plt.legend(fontsize=font)
plt.tick_params(labelsize=font)
plt.xlabel('$B_{p}-R_{p}$',fontsize=font)
plt.ylabel('G (absolute magnitude)',fontsize=font)
plt.title('HR diagram of field stars',fontsize=font)
plt.savefig('HR_diagram_field.png',format='png')
plt.show()
```