

# **A Comparison of Numerical Methods of Pricing Asian Options on the JSE**

## **A Review of the Curran Approximation Method**

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by  
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## ABSTRACT

This document addresses the question of accuracy in the pricing of Asian options on the JSE. Current market practice is to use the Black-Scholes or Curran approximation for valuing these options. Without a liquid market with willing market participants, a fair value cannot be observed. The reliance on models thus grows.

Various numerical methods are explored with a view to increasing accuracy in pricing of such structures. Much like the conclusion of Wiklund, (2012), the Curran approximation performs well in pricing Asian options under certain conditions. The Curran approximation performs particularly well in pricing when in a low volatility environment. When the volatility of the market is higher, more mispricing occurs. Taking the Monte Carlo simulation with 20,000,000 simulation paths as the benchmark for the study, the Curran approximation always overprices Asian options in South Africa, with the degree of error increasing as the option moves out-of-the-money. When options are close to the position of at-the-money, there is greater volatility and so more mispricing.

With the introduction of stochastic volatility, the Heston model provided some key insights. For options deep-in-the-money, the Heston model showed that the Curran model overpriced options. As the option moves towards an at-the-money position, continues to overprice. When an option is at-the-money, the price difference is minimal. However, once the option moves out-of-the-money, the Curran approximation underprices the option. The further out-of-the-money the option becomes, the closer the prices then become with the two models. When the option is deep-out-of-the-money, both the Curran and Heston models converge on zero value.

In light of the above, the Monte Carlo simulation of the Heston model provides the most accurate prices for the South African market. Not only does it provide the best fit to the volatility implied by the options currently traded on the JSE but this model also captures the mean-reverting nature of the volatility in South Africa. The market for Asian options in South Africa does not require speed of calculation as yet. It is thus recommended to promote pricing accuracy while the market is stimulated.

## GLOSSARY OF TERMS

Asian Option	Asian options (or average options) are simple path-dependent options in which the payoff is dependent on the average level of the underlying asset over a period of time
Correlation	A measure of relationship between two variables
Dampening Parameter	A factor introduced to produce integrability
Geometric Brownian Motion	A continuous time stochastic process in which the logarithm of the randomly varying quantity follows a Brownian process or Weiner process with drift
Initial underlying price	The start price of the underlying stock
Initial variance	The volatility squared measure at the start of the process
Liquid Market	A market with willing buyers and sellers to assess a fair value
Lognormal	Denoting a set of data in which the logarithm of the variate is distributed according to a normal distribution
Mean reversion speed	The speed at which prices and returns eventually move back towards the mean or average.
Model Calibration	Process of adjustment of the model parameters and forcing within the margins of the uncertainties to obtain a model representation of the processes of interest that satisfies pre-agreed criteria.
Partial derivative	A derivative of a function of two or more variables with respect to one variable, the other(s) being treated as constant
Probability Density Function	Is a function whose value at any given point can be interpreted as providing a relative likelihood that the value of the random variable would equal that sample.
Risk free rate of return	The risk free rate of return is a theoretical rate of return of an investment with no risk of financial loss.
Risk Neutral	Neither risk averse nor risk seeking
Standard Deviation	A quantity expressing by how much the members of a group differ from the mean value of the group
Stochastic	Having a random probability distribution or pattern that may be analysed statistically but may not be predicted precisely.
Strike Price	The price at which a put or call option can be exercised
Vanilla Options	is a financial instrument that gives the holder the right, but not the obligation, to buy or sell an underlying asset, security or currency at a predetermined price within a given timeframe. A <b>vanilla option</b> is a normal call or put <b>option</b> that has no special or unusual features
Volatility	A statistical measure of dispersion of returns
Volatility Smile	A pattern of implied volatility when plotted against strike prices.

## LIST OF ABBREVIATIONS

Abbreviation	Meaning
ALSI	All Share Index
BRICS	Brazil, Russia, India, China, South Africa
CRR	Cox-Ross-Rubenstein
FFT	Fast Fourier Transform
JSE	Johannesburg Stock Exchange
Lsqnnonlin	Least squares non-linear (Matlab function)
MC	Monte Carlo
MCCV	Monte Carlo with Control Variate
SABR	Stochastic Alpha, Beta, Rho
VBA	Visual Basic for Applications (Microsoft)

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## 1. INTRODUCTION

*“A theory has the alternative of being right or wrong. A model has a third possibility: it may be right but irrelevant.”*

- MANFRED EIGEN

Financial derivatives have attracted a negative public opinion since the market crash of 2008 (Andrews, 2013). The connotation was further hampered when Warren Buffet referred to derivatives as “financial weapons of mass destruction” (Stulz, R, 2009). However, these structures are very often misunderstood. “Firms have to make sure that derivatives are used properly. This means that the risks of derivatives positions must be measured and understood” (Stulz, 2005, p31). These sentiments are echoed by Van Liedekerke & Cassimon, (2004) with, “Derivatives are not right or wrong, they are simply powerful. And power needs to be monitored and controlled.” (p74). As a result of their misuse, the use of derivatives is now heavily regulated. With the derivatives market being much larger than the stock market, one cannot ignore the importance of the derivative (J. Hull, 2014).

“A derivative security is a financial contract whose value is derived from the value of an underlying asset, hence the name.” (Jarrow & Turnbull, 2000, p.2). Derivatives are used to hedge or remove risk off the balance sheet, to speculate or for arbitrage (finding mismatches in prices to lock in a profit) (J. Hull, 2014). The underlying asset could also refer to a reference rate, multiple assets, an index or a commodity from which the derivative inherits its value (Kolb & Overdahl, 2003). With exchange traded derivatives, the market “generates publically observable prices containing the market’s assessment of the current and future economic value of certain assets.” (Kolb & Overdahl, 2003, p.19). Thus, derivatives eliminate the inefficiencies in the market for the underlying asset by removing pricing discrepancies (Investopedia, 2016). Kolb & Overdahl, (2003) refer to this as “price discovery” as the quantity and quality of price information increases in the underlying market. With a growing derivatives market and derivatives forming a substantial category of financial instruments, it is extremely important to price them accurately (Wang & Kao, 2016). Speed of price calculation has also become particularly important as technology has promoted high frequency trading, that is, having short holding periods and more frequent trades. High frequency traders improve the efficiency of

prices and thus the incorporation of information into each price. Hence the emphasis on the calculation speed cannot be undermined (Brogaard, Hendershott, & Riordan, 2014).

The Black-Scholes-Merton model made significant strides in improving the efficiency of pricing derivatives with a closed-form formula for pricing European puts and calls. With a straight forward formula that took account of various risk factors, the derivatives market began to grow rapidly (Ray, 2012). This led to the development of the over-the-counter derivatives markets where financial engineers have created non-standard derivatives (J. Hull, 2014). Unfortunately, many of these have no closed-form solutions and the values of these need to be approximated. These “exotics” are defined by both the dependency of the payoff on the price path of the underlying asset as well as whether early exercise of the option is allowed (Higham, 2004). The particular path-dependent option considered in this research is the Asian (or average) option. The details of these are discussed later in the research.

To be able to generate an accurate price for these non-standard derivatives, various approximations are used, however, to increase accuracy, repeated sampling such as in Monte Carlo simulations provide better pricing accuracy. Monte Carlo simulations are simple to use and can be easily adapted to the flexibility required for pricing exotic options. However, although accurate, these models are time consuming to run (Dave, 2008).

Through the examination of approximation methods (Binomial model, finite difference methods and the fast Fourier transform) and Monte Carlo simulation (including Monte Carlo with Control variate techniques), the key research question of accuracy can be answered. While the social importance of efficient pricing of these derivatives cannot be underestimated, the computations should promote speed. This is extremely important for traders who often need to make decisions on certain trades in a matter of seconds. Accuracy is just as important in the pricing of derivatives. Not only will the correct decisions be made with regard to trades but also with respect to hedging away risk (Tavella, 2003). Through this investigation more efficient methods of Monte Carlo simulation are compared with commonly used approximations to improve decision making.

This will be completed initially by comparing market prices for various Asian options with results for the same options obtained from the binomial model (the most commonly used market model), the finite difference model, the fast Fourier transform model and then by simulating price paths to assess the prices of these derivatives via Monte Carlo simulations. Additionally, stochastic volatility is introduced via the Heston and SABR models in simulating asset price paths. However, since the pricing models are based on future market scenarios or asset price paths, it is important that these models be calibrated correctly that is “with specifying the statistics of the sample paths in such a way that the model matches the prices of benchmark instruments traded in the market” (Avellaneda et al., 2001, p.2). This will involve the use of historical data to parameterise the models but may also involve other patterns to describe the movement of the variables in order to simulate a price path. “While the pricing problem is concerned with computing option values given the model parameters, the calibration problem is concerned with computing the model parameters given the option prices.” (Bu, 2007, p.15).

A challenge in the South African market is the availability of data. These non-standard derivatives are customized for customer specific requirements by the Johannesburg Stock Exchange (JSE) as part of the “Can-Do” platform. As a result, many contracts are either already expired or do not have sufficient trading volumes to ensure accurate market prices with all information reflected. As a result, various assumptions are made in order for valid comparisons to be made. The JSE has provided a detailed summary of its methods in calculating the market prices of Asian options (The Curran Approximation). The result of these methods will be assumed to be the correct and accurate market price for each of these options. Without a full and liquid exotic option market in South Africa, options will thus be created on a notional basis by comparing the results of the models with this calculated “market price”. In essence, this is referred to as “marking-to-model” whereby the investment price is based on the outputs of a financial model (Investopedia, n.d.).

### **1.1 WHY THE SOUTH AFRICAN MARKET?**

According to the paper by Strebel, (1977), the efficient market hypothesis applies to, at most, half of the stocks traded. Since then, with advances in financial regulation, markets are more efficient and transparent (South African Treasury, 2011), however, with a very immature derivatives market, efficiency and transparency remains a concern. As a member of BRICS,

South Africa has a vested interest in improving the efficiency and accuracy of the pricing of financial instruments. Through ensuring efficient pricing methods, policies and regulations can be better shaped to avoid exploitation (Gauteng Province Provincial Treasury, 2013).

With prescribed methods of calculating the prices of exotic options in South Africa, market values allow arbitrage and definite exploitation. True market prices are unavailable because of the lack of volumes traded and the low liquidity in this market. There are not enough willing buyers and sellers in the market to determine a fair price. With methods promoting more efficient pricing, the market is more likely to grow as investors increase in confidence in these complex products. This research seeks to examine various methodologies of pricing Asian options in an effort to close this gap.

In the sections that follow, a literature review will cover a theoretical background, followed by a section on the methodology to be used in the investigation. The calibration of the various models that will be simulated by Monte Carlo simulation for the asset price paths will be investigated along with an empirical analysis of the comparisons of the results of the model runs. Finally, conclusions are drawn based on the results of accuracy and appropriateness with recommendations for further and future developments of the investigation.

## 2. LITERATURE REVIEW

The social role of financial derivatives cannot be undermined in their provision of information to society. By improving the levels of information provision, “economic resources are allocated more efficiently than they would be if prices poorly reflected the economic value of the underlying assets.” (Kolb & Overdahl, 2003, p.19). This means that the quality of decision making is vastly improved because of the derivatives markets. Hence the importance of accurate and efficient pricing of derivatives cannot be underestimated. However, derivatives prove to be the most difficult to price (Marshall, 2008). As a result, many methods have been used to price derivatives. These methods can be classified into two main categories: Analytical and numerical methods.

By far the most famous analytical result has been the Black-Scholes-Merton model for option pricing (a class of derivative) which provides an elegant solution and precise price for simple European options (Brandimarte, 2007). Numerical methods on the other hand estimate the prices of more complex options. The first numerical method considered was presented by Cox, Ross, & Rubinstein, (1979), in their paper, Option Pricing: A simplified approach. The binomial asset-pricing model presented here “provides a powerful tool to understand arbitrage pricing theory and probability” in discrete time (Shreve, 2000, p.1). Another approximation technique is the use of finite difference methods to approximate the evolution of an option price over discrete time by iteratively solving a series of difference equations (Goddard, 2015). The last approximation method that warrants further research is that of the Fast Fourier Transform which represents a major advancement in computing and efficiency. Using this method a simple analytic expression is developed to which the Fourier transform is applied to value an option (Carr & Madan, Option valuation using the fast Fourier transform, 1999).

Finally, Monte Carlo simulation, “involves simulating paths of stochastic processes used to describe the evolution of underlying asset prices” (Glasserman, 2003, p.3) and “provides a probabilistic solution to the option pricing models” (Kwok, 2008). Due to the versatility of Monte Carlo simulations in pricing complex derivative structures, this method is the main focus of this research. Monte Carlo simulation is often the only method for pricing path-dependent options such as Asian, Barrier and Lookback options, especially if other sources of variability are introduced into the models such as stochastic volatility (Glasserman, Heidelberger, &

Shahabuddin, 1999). In particular, the incorporation of the stochastic volatility of the Heston and SABR models will be simulated and examined this research. The following sections provide some background as well as the advantages and disadvantages of each of the above models.

## 2.1 ASIAN OPTIONS

Asian options (or average options) are simple path-dependent options in which the payoff is dependent on the average level of the underlying asset over a period of time (Boyle, Broadie, & Glasserman, 1997). These options were first priced by David Spaughton and Mark Standish in 1987. The name of the option was derived from the fact that the two were working in Japan at the time (Lee, Kim, & Jang, 2014).

Due to their lower volatility, they are often cheaper than European options and are used to hedge cash flow risks (Lai & Yen, 2015). These characteristics “reduce the risk of market manipulation of the underlying risky asset at maturity” (Lee, Kim, & Jang, 2014, p. 1). The option can vary in terms of being an arithmetic or geometric average or whether it is the average of the underlying stock or for the value of the strike (Alexander, 2008). Thus far, only fixed strike Asian options which are European in nature have been traded on the JSE as a “Can-do” option. “To date, there is no known closed-form analytical solutions for options, because the lognormal assumptions generally collapse” (Lai & Yen, 2015, p.40). The pay-offs for Asian options can be defined as follows:

Average Price Asian Option:  $\max (\eta (S_A - X))$

Average Strike Asian Option:  $\max (\eta (S_T - S_A))$

Where  $\eta$  is a binary variable equal to 1 for a call and -1 for a put,  $S_A$  is the average price of the underlying asset,  $X$ , is the strike price.  $S_T$  is the price of the underlying at time  $T$ .

Not only can the method of averaging vary but also Asian options can be either European or American in nature (Lai & Yen, 2015).

## 2.2 THE BLACK-SCHOLES-MERTON MODEL

The difficulty with pricing options is that there is great uncertainty around the path of the underlying stock price as the price of the option is dependent on the stock price. Hence, a “continuous time stochastic model is developed for stocks” (Edwards, 2015, p.5). “Academics have studied stock prices observed by the market and decided that their probabilistic behaviour is well approximated by the lognormal distribution (i.e. logarithm of stock prices is approximately normally distributed)” (Krouglov, 2006, p.3). The following equation is most widely used to model stock price behaviour as a result:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \quad (1)$$

Where  $\mu$  is the constant drift rate or the stock's expected rate of return,  $\sigma$  is the constant volatility of the stock price and  $W_t$  is the standard Brownian Motion (J. Hull, 2014). Without arbitrage opportunities available in the market, the return on the portfolio must equal the risk free rate,  $r$ . The “volatility of the prices of stock shares can be explained by the variance of the logarithm of prices that is directly proportional to the length of investment period” (Krouglov, 2006, p.6).

By solving this differential equation explicitly for European options, one gains the formula for pricing these Vanilla options. This was the model developed by Black and Scholes in 1974 with an alternative approach by Merton in 1973 (Edwards, 2015). However, there are a number of assumptions that must be made in order to solve the Black-Scholes-Merton differential equation:

- 1) The stock price follows the differential equation described above.
- 2) Short selling of securities with the full use of proceeds is permitted.
- 3) There are no taxes or transaction costs. Securities are divisible perfectly.
- 4) No dividends are paid during the period of the derivative.
- 5) There are no arbitrage opportunities in existence.
- 6) Share and derivative trading is continuous.
- 7) The risk free rate of interest is  $r$ , and is constant across all maturities.



Some of these assumptions may be relaxed but are a requirement for the derivation of the initial formula (J. Hull, 2014).

The Black-Scholes-Merton model provides many advantages. The first of which is that it is able to model option prices in continuous time. The result is that the model offers “something almost unique at the output level: option prices in the model can be expressed in *closed-form* i.e., as particular explicit functions of the parameters.” (Sundaram & Das, 2010, p.308). As a result of this closed-form solution, the Black-Scholes-Merton model has the characteristic of speed. It allows the calculation of “a very large number of option prices in a very short time.” (Kilic, 2005, p.5). The model allows estimation of “market volatility of the underlying asset as a function of price and time” while also allowing a “self-replicating strategy or hedging” (Teneng, 2011, p.1). Analytical formulas help to gain insight into “how different factors affect option prices” (Brandimarte, 2007, p.6).

However, the Black-Scholes-Merton model is constrained by many assumptions as noted above. Implied volatility varies by maturity and by the variability in strike prices of options (Lai & Yen, 2015). Volatility is also never constant over the long-term while in the short-term this assumption may be appropriate (Teneng, 2011). This is not appropriate for predicting the asset’s price path.

With many factors affecting the price of the stock, the assumption of the random walk movement and hence the stock price moving up or down with equal probability is inappropriate. “The assumption that returns of log-normally distributed underlying stock prices are normally distributed” is also not appropriate when considering observed financial data (Ray, 2012, p.175). Weaknesses in other assumptions which are not true of the real world include the lack of commissions and transaction costs, perfectly liquid markets, constant interest rates and the lack of dividends from the underlying asset (Teneng, 2011).

In light of the research being conducted, a key weakness is that “most complex derivatives are not known to have closed-form formulas for their prices” (Wang & Kao, 2016, p.683). Many complex options cannot be priced analytically as by the Black-Scholes-Merton model, however, a formula for a simpler related option will help to validate the price arrived at through

numerical methods (to some extent) (Brandimarte, 2007). This is seen in the observed market option prices compared to the different prices that result from the Black Scholes formula (Backus, Foresi, & Wu, 2004).

### 2.2.1 IMPLIED VOLATILITY

Implied volatility is calculated from option prices observed in the market. This solves the Black-Scholes equation by equating this to the market price for the option to the Black-Scholes value. This assumes the same strike price and expiration time period (Bauer, 2012).

Franke & Stapleton (1999) state that options are continually underpriced by the Black-Scholes formula as the implied volatility “exceeds the historical volatility of the price of the underlying asset” (p.4). Implied volatility varies across strike prices and effectively, the degree of “moneyness” of the option in question. Volatility smiles and other patterns are generally observed and hence breaks down the assumption of normality in the underlying asset (Backus et al., 2004). From the empirical analysis of Fortune, (1996), higher implied volatilities are seen in off-the-money options than for at-the-money options. This results in over pricing of premiums for options that are far out-the-money and underpricing when options are in-the-money when using the Black-Scholes formula.

Modelling stochastic volatility has become more popular since the crash of 1987. The Black Scholes model has always assumed constant volatility regardless of strike prices whereas, volatility does vary with strike price (Fouque, Papanicolaou, & Sircar, 2000). In general, the following graph shows the pattern of implied volatility:

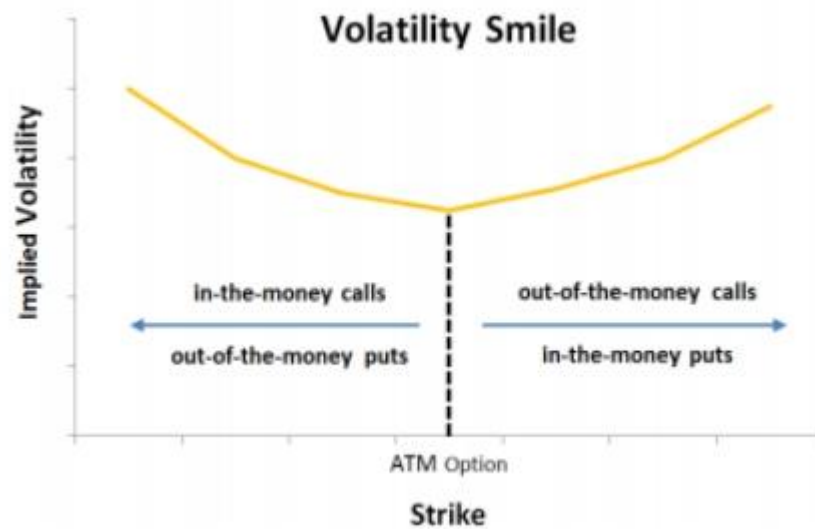


Figure 1: The Volatility Smile

(Source: (Nowak & Sibetz, 2012))

This has meant that to progress to a more realistic model of reality, volatility must be allowed to move stochastically. This is examined through the Heston and SABR models which allows volatility to vary according to its own function. However, “Despite its empirical shortcomings, the Black-Scholes model has continued to retain immense popularity and remains the benchmark model for pricing options” (Sundaram & Das, 2010, p.332).

## 2.2.2 THE CURRAN APPROXIMATION

Due to the lognormal distribution of stock prices, this property is shared with the geometric average method used to calculate some Asian options as described above. For this type of Asian option there therefore is a closed-form solution (Klassen, 2001). “The arithmetic mean does not follow a lognormal distribution and because of that it is not possible to obtain a closed form formula to price Arithmetic Asian options” (Wiklund, 2012, p. 8).

Due to its ease of implementation and adherence to the Black-Scholes principles, the Curran approximation is used by the JSE to price Asian options currently (The Johannesburg Stock Exchange, 2016). In his paper, “Valuing Asian and Portfolio Options by conditioning on the Geometric Mean Price”, Michael Curran presented an alternative method for valuing Asian options which he claimed was “more accurate than previous approaches.” (Curran, 1994, p.

1710). In this research, results of the Curran approximation will be used as a benchmark price for each of the Asian options defined. This represents the mark-to-model analysis.

The following formula represents the price of an Asian call under the Curran approximation:

$$C = e^{-rT} \left[ \frac{1}{n} \sum_{i=1}^n e^{\mu + \sigma^2/2} N\left(\frac{\mu - \ln(\hat{x})}{\sigma_x} + \frac{\sigma_{xi}}{\sigma_x}\right) - X N\left(\frac{\mu - \ln(\hat{x})}{\sigma_x}\right) \right]$$

Where:

$$\mu_i = \ln(S) + (b - \frac{\sigma^2}{2})t_i$$

$$\sigma_i = \sqrt{\sigma^2[t_i + (i-1)\Delta t]}$$

$$\sigma_{xi} = \sigma^2\{t_1 + \Delta t[(i-1) - i(i-1)/2(n)]\}$$

$$\mu = \ln(S) + (b - \frac{\sigma^2}{2})[t_1 + (n-1)\Delta t/2]$$

$$\sigma_x = \sqrt{\sigma^2[t_1 + \Delta t(n-1)(2n-1)/6n]}$$

$$\hat{x} = 2X - \frac{1}{n} \sum_{i=1}^n \exp\left\{\mu_i + \frac{\sigma_{xi} [\ln(x) - \mu]}{\sigma_x^2} + \frac{\sigma_i^2 - \sigma_{xi}^2/\sigma_x^2}{2}\right\}$$

Where:

$S$  = Initial Stock Price

$X$  = Strike price of Option

$r$  = Risk Free rate of return

$b$  = Cost of carry

$T$  = Time to expiration of option in years

$t_i$  = Time to first averaging point

$\Delta t$  = Time between averaging points

$n$  = Number of averaging points

$\sigma$  = Volatility of the asset

$N(x)$  = The cumulative normal distribution function

(Gryshkevych & Tashbulatov, 2010)

While the formula above does seem complicated, implementation of this is simple and quick for market makers. The Curran approximation above also provides a price for each Asian option immediately compared to other techniques which take time to compute. As the number of averaging points increase, the model does tend towards the Black-Scholes results. However, the Curran model is not without its flaws. Volatility in this model remains static and thus provides a poor reflection of reality in faster moving markets such as commodities. The model also has a diverging effect at large strike prices. When the strike price tends to infinity, particularly at high volatilities or long maturity options, the approximation that the Curran approximation provides becomes far less accurate (Galabe, Franklin, Hayford, & Celestin, 2009).

In the analysis to follow, the JSE method of using the Curran approximation is compared to the resulting Asian option prices from other methods. Other advantages and disadvantages are revealed through the analysis.

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### 2.3 THE BINOMIAL MODEL

Another approach to option pricing is the binomial model introduced by Cox, Ross and Rubenstein in 1978. In this case, the underlying asset path is discretised and deterministic asset price paths are created (Brandimarte, 2007). The simplest binomial model considered consists of a single period. At time 0, the stock price is assumed to be  $S_0$ , which is known. It is assumed that at each step (in this case time 1), the price of the stock will go up ( $u$ ) or down ( $d$ ). This produces a tree of the underlying stock prices. At option expiration, the possible prices of the underlying and thus the value of the option is known (Kilic, 2005). “The movement of the asset is restricted to two successor nodes” that represent the up and down move of the asset (Glasserman, 2003, p.230).

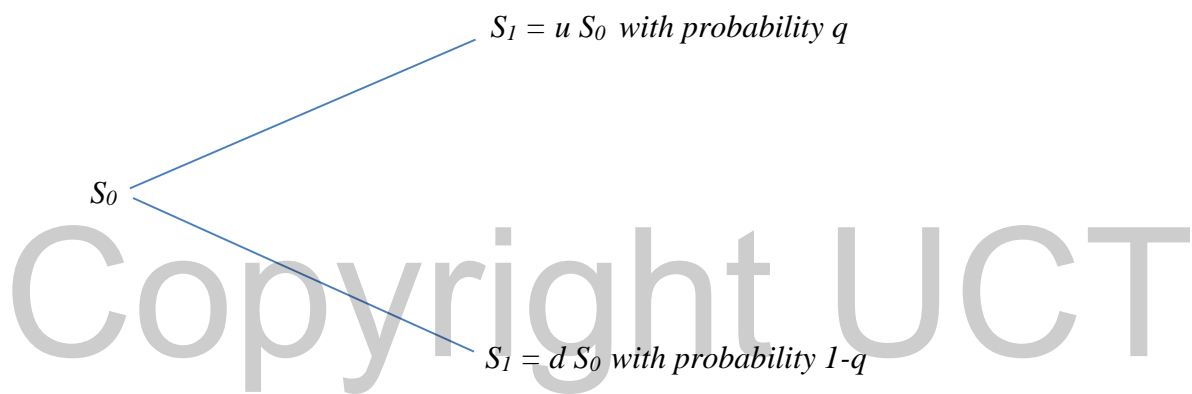


Figure 2: General one-period binomial model

(Source: (Shreve, 2000))

In Figure 2, the price at expiration or time 1 is either  $uS_0$  with probability  $q$  or  $d S_0$  with probability  $1-q$ . By setting  $r$  as the continuously compounded interest rate, we expect  $u > e^r > d$ . “If these inequalities did not hold, there would be profitable riskless arbitrage opportunities involving only the stock and riskless borrowing and lending” (Cox et al., 1979, p.6). In the above case, the corresponding option payoff would be  $f_u$  or  $f_d$  respectively. It is assumed that no arbitrage opportunities exist. A portfolio is set up so that there is no uncertainty and hence will earn the risk-free rate of return (J. Hull, 2014). The resulting option pricing formula is as follows:

$$f = e^{-r}(pf_u + (1 - p)f_d)$$

Where:

$$p = \frac{e^r - d}{u - d}$$

The option pricing formula of above does not have any probability relating to the chance of the stock price moving up or down ( $q$ ). These probabilities are already reflected in the price of the stock and do not need to be accounted for in the pricing of the option. Hence the parameter  $p$  is the risk neutral probability of an upward movement. “Assuming a risk-neutral world gives us the right option price for the world we live in as well as for a risk-neutral world” (Hull, 2014, p.257). The parameters  $u$  and  $d$  are expected to be “directly related to the volatility of the continuous diffusion process of the asset price” (Kwok, 2008, p.300). A method to maintain tree symmetry is assuming that  $u = \frac{1}{d}$ . Cox et al., (1979), suggest the following values of  $u$  and  $d$  to match the volatility of the stock price:

$$u = e^{\sigma\sqrt{\Delta t}} \text{ and } d = e^{-\sigma\sqrt{\Delta t}}.$$

This idea can be extended to a greater number of time steps and result in a recombining tree.

The following figure shows the evolution of the stock price over a three step binomial tree:

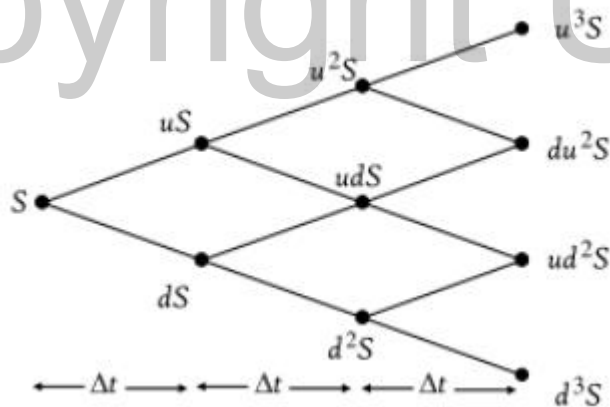


Figure 3: Three step Binomial model

(Source: (Alexander, 2008))

The one step binomial model has allowed one to observe the market price at two time points. With the transition probability assumed constant, the share price must grow at the risk-free rate of return. With the assumption that the underlying stock follows Geometric Brownian Motion, then as the number of time steps increase i.e ( $\Delta t$ ), the binomial model will converge to the Black-Scholes-Merton model (Alexander, 2008). As more time steps are added to the

tree, “the risk-neutral valuation principle continues to hold” (Hull, 2014, p.262). A similar process will be followed at each node with the constant risk neutral probabilities of up or downward movements. The following formulas generalise the process for the two step tree following the matched volatility movement factors defined above ( $u$  and  $d$ ):

$$p = \frac{e^{r\Delta t} - d}{u - d}$$

While generalising the equation above:

$$f = e^{-2r\Delta t}(p^2 f_{uu} + 2p(1 - p)f_{ud} + (1 - p)^2 f_{dd})$$

“When binomial trees are used in practice, the life of the option is typically divided into 30 or more time steps. In each time step there is a binomial stock price movement” (Hull, 2014, p.268).

The binomial model can be used to find the accurate price of American options, as the price is checked at each node of the tree for possible early exercise (Kilic, 2005). However, the calculations are simple yet tedious. With the use of software, this method is often used by analysts to assess option prices.

The binomial model is important in showing how to construct a risk free replicating portfolio. Dividends can be easily added to the model to value options on dividend paying securities. Lastly, the binomial model is useful in handling complex option structures (Chance & Brooks, 2010).

However, the binomial model does have the flaw of slow speed and tediousness of the calculation (Kilic, 2005), not to mention an increased requirement for computing power. The model is limited in carrying out thousands of option pricing calculations in terms of time. Other limitations include the discrete nature of the model while stocks and options are traded continuously as well as the possibility of only two stock prices at each node. Thus the assumption of the binomial distribution is also flawed. While limiting accuracy, the binomial model does provide analysts with a quick method of approximating option prices. Although faster, the accuracy of these prices become questionable using fewer time steps.



### 2.3.1 CONTROL VARIATE TECHNIQUE

One method to improve the efficiency of the binomial or “lattice” approach suggested by J. Hull & White, (1988), is the use of the control variate technique. As cited by J. Hull & White, (1988), Boyle suggested that this method could also be used with Monte Carlo simulations. Control variate techniques are variance reduction methods that reduce simulation run times. This method makes use of the correlation between a random variable of known expected value with that of a variable under investigation (Lidebrandt, 2007). Hence, if we wish to value an option, but have an accurate price for a second but similar option, the numerical process to value both options should be the same. This means that the same binomial structure can be used to price both the known option as well as the option under investigation (J. Hull & White, 1988). The error when the binomial lattice is used to price the known option is assumed to be equal to the error when the lattice is used to price the option under consideration (J.Hull, 2014). At the same time, this method reduces the standard error in approximating the value of the option, provided that:

$$\rho > \frac{\sigma_B}{2\sigma_A}$$

Where  $\rho$  is the correlation coefficient between option A and B. B is the known option and A is the option under consideration.  $\sigma_A$  is the standard deviation of Option A and  $\sigma_B$  is the standard deviation of Option B (J. Hull & White, 1988). The reduced variance means greater numerical efficiency in option pricing.

This method, as per Boyle’s suggestion above, is used as part of the Monte Carlo simulations in this research. This is purely due to the accuracy that can be achieved with Monte Carlo simulations and the need for time reduction in obtaining option prices.

## 2.4 FINITE DIFFERENCE METHODS

Finite difference methods have been used for a long period of time to approximate the solutions to partial differential equations (Duffy, 2006). Partial differential equations form a vital component in the pricing of derivatives (in particular, this has led to the Black-Scholes-Merton formula), but also forms a “framework for pricing rather complex derivatives” (Brandimarte, 2007, p.289). Finite difference methods solve differential equations through difference equations that are solved iteratively (J.Hull, 2014). The most widely used finite difference methods are the implicit, explicit and Crank-Nicolson methods (Edwards, 2015). The premise of finite difference methods lies in the approximation of the slope of a curve:

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

As  $h$  approaches 0, the above approximation becomes more accurate for the first derivative of the function. “Applying a first order difference twice gives a central finite difference approximation to the second order derivative of the function:

$$f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

The function  $f$  represents the price of an option to be valued and much like the binomial lattice approach described above, the price  $f$  can evolve in a two dimensional space (stock price and time). The following figure shows how this is visualized:

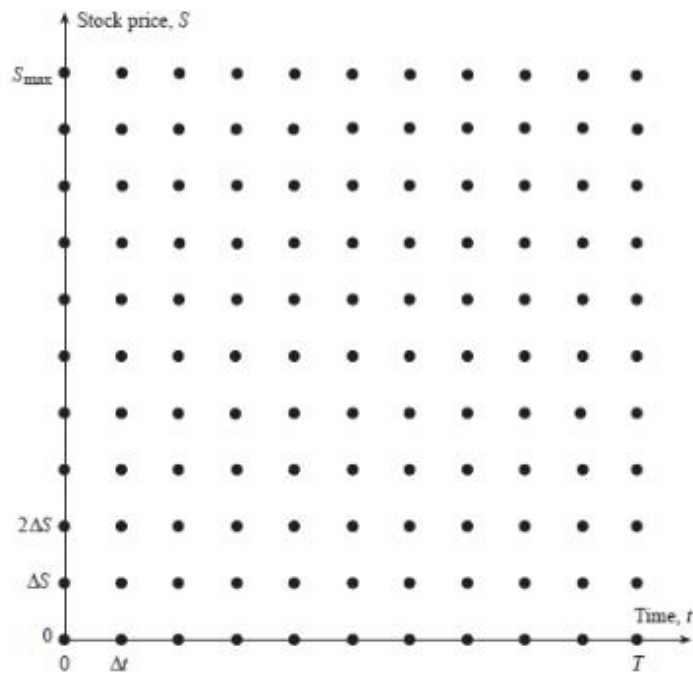


Figure 4: Grid for finite difference methods

(Source: (Alexander, 2008))

Each point in the grid represents a future value of the underlying asset. Each node within the grid can thus result in three future stock values. This is depicted in the following figure:

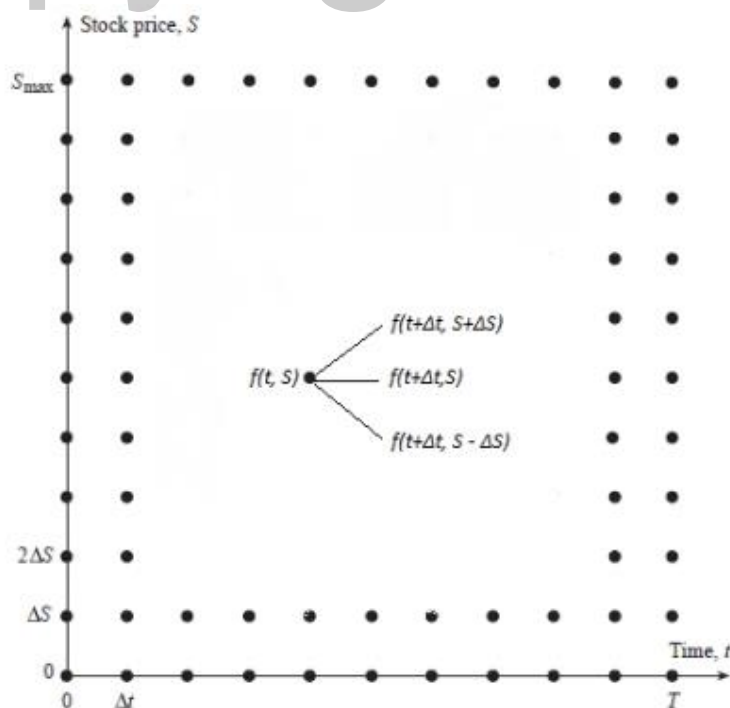


Figure 4: Evolution of stock price with time

(Source: (Alexander, 2008))

Finite differences are used to approximate the values of the partial derivatives in the Partial differential equation at each of the prior nodes. The process is repeated until arriving at the initial node at time 0. Currently the most stable method is the Crank-Nicolson method for finite differences (Alexander, 2008).

All methods have the following similar steps:

- The partial differential equation should be discretised
- The grid will be specified on the future price of the underlying asset
- The option payoff is calculated at specified boundaries on the grid
- The option price is calculated at all other grid points iteratively. The iteration method is either implicit, explicit or Crank-Nicolson. (Goddard, 2015, para.6)

“An explicit scheme predicts all quantities at each new advanced time step from known values at the current time step or steps, whereas implicit schemes difference the partial differential equations in such a manner as to require that a system of equations including the boundary condition equations must be solved to determine quantities at each new advanced time step” (Jeppson, 1972, p.7).

As  $\Delta S$  and  $\Delta t$  approach 0, the implicit finite difference method always converges to the solution for the differential equation and is therefore very robust. However, this method does present many simultaneous equations that need to be solved and can be computationally very intensive and slow (J. Hull, 2014). The implicit scheme is also only first order accurate with regard to the time space but this can be corrected with extrapolation (Duffy, 2004). The explicit scheme on the other hand, is far simpler and faster to set up but is less stable and would require the change in time to be split in a more granular manner (J.Hull, 2014). The computational time of the finite difference methods can be further enhanced by the use of control variate techniques as discussed under the Binomial Model (J. Hull & White, 1988).

The Crank-Nicholson method “may be regarded as a hybrid between the explicit and the fully implicit approach” (Weitian & Xi, n.d, p.8). Essentially this method is similar to the implicit method implementation but averages the implicit and explicit equations (J.Hull, 2014). The

result is greater accuracy in the second order in both time and space and hence, this scheme is unconditionally stable (Kwok, 2008). However, the Crank-Nicholson scheme does not always produce accurate results and while it is easy to implement and programme, the method does not hold in all circumstances (Duffy, 2004). As quoted by Kwok, (2008), Pooley et al. showed that if there are discontinuous payouts at the terminal nodes, convergence using Crank-Nicholson cannot be achieved. When hedging using the Crank-Nicholson method, one runs the risk of inaccuracy where the pay-off function is not smooth (Duffy, 2004).

Finite difference methods “can handle American-style as well as European-style derivatives but cannot easily be used in situations where the payoff from a derivative depends on the past history of the underlying variable” (Hull, 2014, p.465). This makes the method difficult to use for path dependent options such as Asian, Barrier and Lookback options. Hence, the Finite Difference method will not be considered further in this research but remains a possibility for further research.

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## 2.5 MONTE CARLO SIMULATION

Monte Carlo simulation is the most versatile method for the analysis of behaviour or processes that are uncertain in nature (Castellano & Cedrola, 2015). With derivative structures increasing in their complexity, the uncertainty increases and closed-form solutions for pricing these structures become unavailable (Sundram & Das, 2012). The increased complexity of new derivative structures has meant financial engineers have had to evaluate many dimensions of the pricing process (Boyle et al., 1997). Valuing a derivative is essentially valuing an expectation, which if written as an integral would be of a large dimension for complex structures. This is what makes Monte Carlo simulation an attractive choice in the valuation of complex options (Glasserman, 2003).

Monte Carlo simulation involves generating sample paths that the underlying asset may take according to some distribution. For each path, the option payoff is calculated and then discounted at the risk-free rate of return. The average of all payoffs from all paths shows the value of the derivative (J. Hull, 2014). This is a form of random sampling from the set of all possible outcomes and then examining a proportion of these random draws “that fall in a given set as an estimate of the set’s volume. The law of large numbers ensures that this estimate converges to the correct value as the number of draws increases” (Glasserman, 2003, p.1).

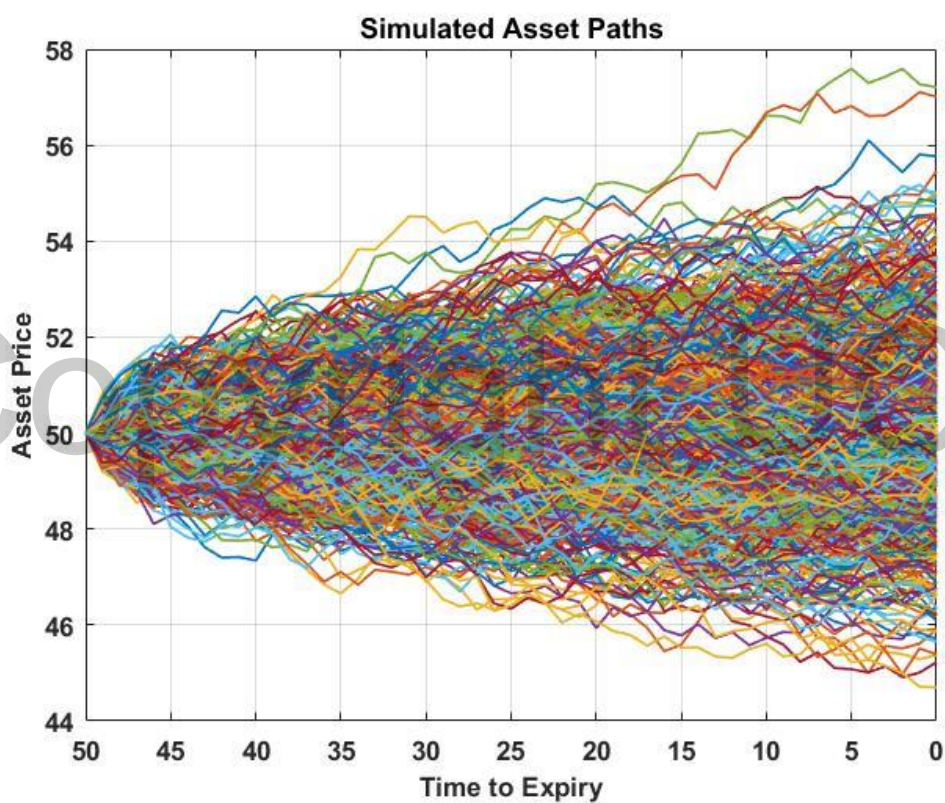
The Monte Carlo process involves the following processes:

- Simulating the asset price paths with the relevant Probability Density functions over the life of the derivative
- For each sample path, calculate the derivative payoff and discount this to the present
- Over all sample paths calculate the average of the present value of derivative payoffs (Kwok, 2008).

Using Monte Carlo methods for simple vanilla options may not be necessary, however, for the more complex derivative structures, it is easier to generate the random values of the asset price process than to fit the exact distribution of the underlying (Takahashi & Yoshida, 2005). Monte Carlo simulation is particularly useful for path-dependent options. The pricing of such options “involves only keeping track of the stock prices” during the period of simulation (Sundram & Das, 2012, p.935). “The whole underlying asset path is required to be considered” for the

valuation of path-dependent options (Au-yeung, 2010, p.2). This makes Monte Carlo simulation ideal for pricing Asian and Barrier options.

The main disadvantage of the Monte Carlo approach is that it can be slow due to the number of sample paths that need to be generated (J.Hull, 2014). A large number of simulations are required to achieve a greater level of accuracy. To improve on this however, variance reduction techniques have been developed to increase the accuracy of results (Boyle et al., 1997) while partial simulation approaches have been developed to improve the speed of the simulations (J. Hull, 2014). The figure below shows an example of simulated asset price paths.



*Figure 5: Simulated Asset Price Paths with Monte Carlo Simulation*

The control variate technique for variance reduction within the Monte Carlo simulation is demonstrated in this research. This was described in the Binomial Model section. Matlab code for the Monte Carlo by Control Variate technique is provided in the Appendix A.

### 2.5.1 QUASI-MONTE CARLO

A particular technique for improving the efficiency of the traditional Monte Carlo method is known as the quasi-Monte Carlo or low-discrepancy method. Numbers used in this method are deterministic rather than random. The result is that there is faster convergence in the result (Joy, Boyle, & Tan, 1996). The focus of this method is to approximate the integral and not to simulate the behaviour of random variables (Takahashi & Yoshida, 2005). Quasi-Monte Carlo generates more accurate estimates than traditional Monte Carlo methods (Lemieux & L'Ecuyer, 2001). This is particularly the case where the dimension is small. However, if the dimension is large but few of the variables are responsible for the variability in the simulation, quasi-Monte Carlo performs just as well (Takahashi & Yoshida, 2005).

As mentioned above, deterministic sequences of numbers are used instead of random numbers for “faster convergence with known error bounds” (Joy et al., 1996, p.2). Various sequences can be used depending on the dimensionality of the problem such as the Van der Corput sequence for single dimension problems, the Halton sequence (a multivariate alternative) and the Faure sequence (similar to the Halton sequence) (Takahashi & Yoshida, 2005).

While Monte Carlo methods are “flexible and easy to implement and modify” (Boyle et al., 1997, p.1269), further aspects of uncertainty can be examined. Thus, in addition to the underlying stock price process being stochastic, one can consider further areas of random movement such as volatility. Further models considered in this research include the Heston and the SABR model. These are discussed further below:

### 2.5.2 THE HESTON MODEL

With evidence that volatility was stochastic and that risky asset returns displayed a distribution with longer tails than those of the normal distribution, Steven Heston created a stochastic volatility model (Poon, 2011). The Heston model “allows volatility to vary stochastically but still to be correlated with the stock” (McDonald, 2013, p.741). The following stock price and volatility processes resulted:

$$\begin{aligned}dS_t &= \mu S_t dt + S_t \sqrt{v_t} dZ_1 \\dv_t &= \kappa (\theta - v_t) dt + \eta \sqrt{v_t} dZ_2\end{aligned}$$



Where  $\kappa$ ,  $\theta$  and  $\eta$  are greater than 0 and are constant parameters. The Brownian motion processes,  $Z_1$  and  $Z_2$  are correlated such that  $\text{Corr}[dZ_1, dZ_2] = \rho dt$ .  $S_t$  is a Geometric Brownian motion with time varying volatility while  $v_t$ , the variance process, is a square root, mean reverting process with the long-run mean of the variance process,  $\theta$ .  $\kappa$  then controls the speed of the mean reversion of  $v_t$  to its long run mean  $\theta$  (Poon, 2011). Heston assumes in his model that the asset has a variance that follows a mean reverting Cox-Ingersoll-Ross process (Crisostomo, 2014). The parameter  $\rho$  changes the skewness of the distribution of asset returns. If  $\rho > 0$  then as asset returns increase, volatility will increase. However, if  $\rho < 0$ , the volatility will increase with a decrease in asset returns (Moodley, 2005). Thus the underlying asset price is derived from a stochastic process “with a constant drift and stochastic volatility. Stochastic volatility is another process but has a correlation with the first process” (Lai & Yen, 2015, p.48). This means that market players need to hedge both stock price movements as well as volatility risk (McDonald, 2013).

The Heston model has been widely used to price equity and commodity options (Lai & Yen, 2015). This is because the modelling framework used allows for characteristics typically observed empirically. Mean reversion is an important characteristic in financial markets, without which assets would be extremely volatile or have no variance at all. Typically seen in the markets are the correlations between assets and their volatilities. This is another important characteristic captured by the model (Crisostomo, 2014). The Heston model has made it possible to examine certain biases in option pricing as well as provide a measure of effectiveness of other option pricing models (Heston, 1993).

With these beneficial characteristics, the research will examine the impact on pricing efficiency using the Heston model for the simulation of asset price paths. Calibration of the Heston model is thus a key challenge as model fitness is highly dependent on the complexity of the calibration. The model also does not produce reliable results for short maturities. The model will thus need to be extended further to account for the short term skew produced by the market. This emphasises the importance of the Heston model calibration (Bauer, 2012). Model calibration will be introduced further in the methodology section and in more detail in the chapter on calibration.

### 2.5.3 THE SABR MODEL

Another stochastic volatility model was introduced by Pat Hagan called the Stochastic alpha, beta rho model (SABR model). The SABR model allows volatility to vary with time and strike price. However, the SABR model further allows volatility to vary over time. The extra randomness is allowed for through the scaling factor  $\kappa$  (volatility of volatility) (Hensen, 2011). The following stock price and volatility dynamics result:

$$\begin{aligned}dS_t &= \sigma_t S_t^\beta dZ_1 \\d\sigma_t &= \kappa \sigma_t dZ_2\end{aligned}$$

Where  $\text{Corr}[dZ_1, dZ_2] = \rho dt$ ,  $\beta$  is the skewness parameter and is constant.  $0 \leq \beta \leq 1$  and  $\kappa \geq 0$ . By setting  $\beta = 1$  and  $\kappa = 0$ , the original Black-Scholes differential equation remains. The empirical analysis of Hensen, (2011), showed that  $\rho$  and  $\beta$  determined the curvature of the volatility smile, while  $\beta$  and  $\kappa$  shifted the smile up or down.

Unlike the Heston model, the volatility does not mean revert. This means its particular use is over short durations (Gatheral, 2006). It has a particular usefulness in that although a closed-form solution does not result, “it has an accurate asymptotic solution” (Hagan, Lesniewski, & Woodward, 2005, p.3). Nguyen & Weigardh, (2014), showed that the SABR model accurately captured the volatility smiles in the market particularly for shorter durations. Over the longer duration terms, effects were minimal particularly when volatility curves are flat. The asymptotic solution to the SABR model thus is useful for the “valuation and risk management of large portfolios of options in real time” (Hagan & Lesniewski, 2008, p. 7).

The SABR model has the advantage of being easy to parameterise with great accuracy (Nowak & Sibetz, 2012). With previous models making use of local or constant volatility, this has led to unstable hedges. The SABR model, with its improved accuracy in modelling volatility, succeeds in overcoming this problem and results in more stable hedges (Zhang, 2011). The SABR model accurately predicts the dynamics of the implied volatility curves, making it effective in managing smile risk (P. S. Hagan, Kumar, Lesniewski, & Woodward, 2002).

Without the mean reversion characteristic of the Heston model, the SABR model shows reduced accuracy with longer maturities and small strike prices (Nowak & Sibetz, 2012). The mean-reverting feature limits volatilities within boundaries that are more realistic. This is because volatility in the SABR model follows a log-normal distribution. The SABR model also has an absorbing boundary at the zero interest rate barrier which means that the model will stay at a zero interest rate forever upon reaching this. This is unrealistic as even monetary authorities will not allow this to remain for long (“Approximate smiles in an extended SABR model,” 2012). In addition to these shortcomings, there is no time parameter in the model to show the dependency of volatility on time (Nowak & Sibetz, 2012). However, the time component and variability thereof will not be considered in this study.

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## 2.6 THE FAST FOURIER TRANSFORM (FFT)

Carr & Madan, (1999), proposed using the fast Fourier transform in the pricing of options. They found “that the use of the FFT is considerably faster than most available methods and, furthermore, that the traditional method described by Heston (1993), Bates (1996), Bakshi and Madan (1999), and Scott (1997) can be both slow and inaccurate.” (p.71). As per Carr & Madan, Chung & Wong, (2014), suggest that this method only be used “when the characteristic function for the return is known analytically.” (p.133).

As per Carr & Madan, (1999), consider the Fourier transform and inverse Fourier transform as described by Moodley, (2005):

$$\mathcal{F}\{f(x)\} = \int_{-\infty}^{\infty} e^{i\varphi x} f(x) dx = F(\varphi)$$

$$\mathcal{F}^{-1}\{F(\varphi)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\varphi x} F(\varphi) d\varphi = f(x)$$

Where  $f(x)$  is the risk neutral density function of the log-return of the underlying asset.  $F(\varphi)$  represents the characteristic function of  $f(x)$ . The FFT approximates the Fourier transform discretely with the following summation:

$$w(k) = \sum_{j=1}^N e^{-i\frac{2\pi}{N}(j-1)(k-1)} x(j)$$

If  $x_t = \ln S_t$  and  $k = \ln K$ , where  $K$  is the strike price of an option, Carr & Madan, (1999), show that the price of a European call option at maturity  $T$  is represented by:

$$C_T(k) = e^{-rT} \int_k^{\infty} (e^{x_T} - e^k) f_T(x_T) dx_T$$

where  $f_T(x)$  is the risk neutral density function of the underlying as defined above (Moodley, 2005). For integrability a dampening parameter ( $\alpha$ ) is introduced in the form of a modified price function:

$$c_T(k) = e^{\alpha k} C_T(k) \quad \alpha > 0$$

Hence for a fixed  $T$ ,  $c_T(k)$  is square integrable. Thus for a European call the following integral results:

$$\begin{aligned} F_{C_T}(\varphi) &= \int_{-\infty}^{\infty} e^{i\varphi k} e^{\alpha k} e^{-rT} \int_k^{\infty} (e^{x_T} - e^k) f_T(x_T) dx_T dk \\ &= \frac{e^{-rT} F_{C_T}(\varphi - (\alpha + 1)i)}{\alpha^2 + \alpha - \varphi^2 + i(2\alpha + 1)\varphi} \end{aligned}$$

This represents the characteristic function of  $x_T$ . Carr & Madan, (1999), then show that:

$$C(k_u) = \frac{e^{-\alpha k_u}}{\pi} \sum_{j=1}^N e^{-i\frac{2\pi}{N}(j-1)(u-1)} e^{ibv_j} F_{c_T(v_j)} \frac{\eta}{3} (3 + (-1)^j - \delta_{j-1})$$

Where  $\delta_n$  is the “Kronecker delta function that is unity for  $n = 0$  and zero otherwise (Carr & Madan, 1999, p. 68). “The advantages of the FFT are generic to the widely known improvements in computation speed attained by this algorithm and is not connected to the particular characteristic function or process” that was analysed (Carr & Madan, 1999, p. 72). Chung & Wong, (2014), suggest the following form for the characteristic function for an Arithmetic Asian option:

$$C_{0,S_0}^{T,w}(k) = \frac{e^{-ak-rT}}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-ivk}}{(iv+a)^2} E_0[e^{(iv+a)wY_T}]$$

The derivation of this function is beyond the scope of this research paper. However, Chung & Wong show that the analytical solution that results is “more accurate and efficient than the Monte Carlo simulation.” (p.137).

The main advantage of the FFT method of pricing options is that of its generality. This leads to faster calibrations and hence faster pricing results (Matsuda, 2004). However, characteristic functions are not easily found for non-standard options as experienced in this research investigation. The FFT however has the disadvantage of convergence and in particular for that of discontinuous functions. This means discontinuous options show difficulty in pricing with the FFT.

## 2.7 CONCLUSION

In conclusion to this literature review, while there have been significant contributions in approximating accurate prices for derivatives, and in particular, path dependent options such as Asian, Barrier and Lookback options, the question of appropriateness and accuracy of the Curran approximation has not been answered. The benefits of more efficient pricing goes beyond the making of accurate decisions within a smaller time frame but also the social benefit of bringing more information into the market cannot be understated. With the exotic derivative market in its infancy in South Africa, more efficient methods of pricing these structures will also help to stimulate the market for such products. “A fair price for a derivative product only exists if the market is complete” (Youmbi, 2013, p.1).

With the evolution of pricing models and their increased complexity, so too has the market grown with more complex structures. Each model attempts to mimic an aspect of real world behaviour with the consequence of more accurate pricing. However, in doing so, the time taken for the calculation and more importantly the calibration to be completed, increases. This research attempts to investigate the appropriateness of the use of the Black-Scholes approximation or the Curran Approximation that is currently being used by the JSE. The appropriateness is defined as both accuracy and speed of calculation, through the testing and comparison of various methodologies used in the market in an effort to improve information contained within market prices as well as produce better decision making capabilities. Not only will this research help in better hedging in the market and thus improvement in investor protection, but this will also stimulate the market in Asian options, increasing the levels of liquidity in the exotic derivatives market in South Africa and hence better, more accurate pricing in the future.

While each of the pricing methods have shown advantages and disadvantages, a key problem within the South African market remains the lack of data. Approximations and assumptions are often used to “fill in the blanks” but there remain large gaps. With a greater focus on big data analytics and the value to be obtained from past data, it is hoped that more blanks can be filled in future with better technology. This research attempts to “fill in” some of those gaps to better focus future research.

### 3. METHODOLOGY

The aim of this research is to examine various methodologies of pricing Asian options in the South African market to determine the accuracy and appropriateness of alternate methods that can be employed relative to the Curran approximation that is currently employed by the JSE and market makers. The investigation focuses on pricing accuracy and to a lesser extent, speed of calculation. This chapter describes the methodology used in determining these results.

#### 3.1 PHILOSOPHY

A positivist philosophy is followed in the completion of this research. The measurable properties under investigation in this research are the accuracy and speed of calculations using various methodologies. Both measures are quantitative in nature. This will increase the understanding of each of the calculation methodologies available which subscribes itself to a positivist philosophy (Myers, 1997). Through this exploration there will be an enhancement in the understanding of the pricing of Asian and Barrier options to allow more information to be passed between parties through more efficient pricing.

#### 3.2 APPROACH

The approach followed in this research is completely quantitative in nature. As cited by Sukamolson, (2007), Creswell (1994) defined quantitative research as “explaining phenomena by collecting numerical data that are analysed using mathematically based methods” (p.2). This definition sums up the method of the investigation as the numerical data collected is in the form of popular option and stock prices available from the JSE website. The analysis will then begin using various pricing methodologies to approximate the prices of the same instruments in order to compare the results with the market values based on various numerical metrics.

The research is based on the South African exotic derivatives market or the “Can-Do” market as it is referred to by the JSE. Unfortunately, the size of this over-the-counter exotic derivative market cannot compare to that of a more developed market (Hassan, 2013). With a low volume of trade and even lower volume for the particular structures (Asian and Barrier options) examined, accuracy of market prices based on trade is questionable. The JSE thus employs standardised approaches to calculate the market prices for these derivatives and it is assumed

that these methodologies are accurate and will be used to assess notional structures where required.

### 3.3 DATA COLLECTION AND ANALYSIS METHODS

Data (historical derivative and equity prices) was collected from the JSE historical database located online (<https://www.jse.co.za/downloadable-files?REQUESTNODE=/SAFEX>). This data was then used to calibrate the various pricing models that are commonly used in the market (the Binomial model, finite difference models as well as Monte-Carlo simulations) (Tavella, 2003). These models were adapted to produce both faster calculation times and more accurate prices. Thus the comparison with the Monte-Carlo models were only with the most efficient forms of the popular pricing models.

A comparison of results was completed using accuracy as the primary measure of appropriateness in order to be able to answer the research question. Accuracy was measured with the mean squared error (Lebanon, 2010) while speed of calculation was merely calculated as the time to finish the calculation. All data storage, model calibration, implied volatility calculations and model runs were carried out using MathWorks MATLAB version 2016. While Microsoft Excel was a viable option, the random number generator does have its difficulties (Rotz, Falk, & Joshee, 2004). Random number generation is vital in creating the asset price paths required for Monte-Carlo simulation, and while a generator is difficult to find (Hellekalek, 1998), MATLAB performs adequately providing suitable results (Risse, 2000). Provided methods of calibration are followed as described in this research, the research results produced are reproducible. Microsoft Excel VBA was used where common comparison was deemed appropriate rather than full simulation.

### 3.4 MATERIALS

A wide array of internet resources was used to learn about the Matlab programming language and coding examples. In particular, the calibration methods used were based on the option pricing code from Alexander (2008).

The models were run on a laptop with an Intel® Core™ i7-6700 processor, with 32 Gigabytes of RAM and a 512 Gigabyte Solid State Hard drive. Model times that are calculated in the analysis that follows should be compared against a machine of similar specifications should the



investigation be replicated. As mentioned above, Matlab R2016a was used as the software medium for the running of such quantitative models as is widely used within the quantitative finance industry.

### **3.5 ETHICS, RELIABILITY, VALIDITY, GENERALISABILITY AND LIMITATIONS**

#### ***3.5.1 Ethics***

No human subjects were involved in this study. Data was publically available and no confidentiality or data protection was required.

#### ***3.5.2 Reliability***

The results from this investigation can be reproduced provided the methodology is followed as outlined. Further to this, data for the same time period should be used along with the same calibration methods. Accuracy of pricing that results from the Monte-Carlo simulation runs are dependent on the number of asset price paths that are simulated. Results for each number of simulations are defined in the analysis.

#### ***3.5.3 Validity***

The study questions the appropriateness of the various popular pricing methodologies used in the markets today relative to the current measurement regime of the Curran approximation. Each of the methods are compared to assess this important measurement criterion in order to answer the research question. The answer is objective and subject to pricing methodology used.

#### ***3.5.4 Generalisability and Limitations***

The study was limited to the South African “Can-do” over-the-counter derivatives market. As mentioned above, the low trading volume means that accuracy of price based on the market trade and what market players feel is a fair price for each instrument, may not be accurate. However, the methods used in this study, along with the comparison of the methodologies can be generalised to any market. The application to the South African market applies in the measurement of accuracy as the market values are currently calculated using a formulaic approach (Johannesburg Stock Exchange, 2014).

## 4. NUMERICAL ANALYSIS

### 4.1 DATA

As mentioned above, data was obtained from the JSE website on ALSI index options. Data was obtained as at 28 September 2016 for call options on the ALSI index. The corresponding futures prices was also obtained. The expiration date for the options used for the analysis was 15 December 2016 and thus the time to expiry of the options considered was 78 days. Daily ALSI data was taken for the prior 78 days in order to obtain various parameters of the ALSI index. The following table summarises these findings over the 78 days prior to 28 September 2016:

Symbol	Parameter	Value
$S_0$	Initial underlying price	45,234
X	Strike Price	Varies with study
$\mu$	Expected Return	6,97%
r	Risk free rate of return	7,00%
dt	Time steps	$\frac{1}{365}$
time	Time to expiry	78 days

*Table 1: Data Used for modelling process*

Since Asian options are not freely available on the JSE or are not actively traded, Asian options had to be created on a notional basis to analyse the performance of the valuation methodology used by the JSE (The Curran Approximation). Through the use of the JSE's Can-Do platform, this is a possibility as options are created based on investor needs. The following set of fixed strike, arithmetic Asian options were chosen with daily averaging to ensure practicality and a higher level of accuracy in pricing. These options are valued as call options as the associated put prices can be evaluated through put-call parity. As per the above table, the current price of the ALSI index is 45,234:

<b>Strike Price</b>	<b>“Moneyness”</b>
31,000	Deep In-the-money
44,900	In-the-money
45,000	In-the-money
45,050	In-the-money
45,234	At-the-money
45,450	Out-of-the-money
45,500	Out-of-the-money
45,600	Out-of-the-money
45,700	Out-of-the-money
49,500	Deep Out-of-the-money

*Table 2: List of Notional Asian Call Options for Evaluation*

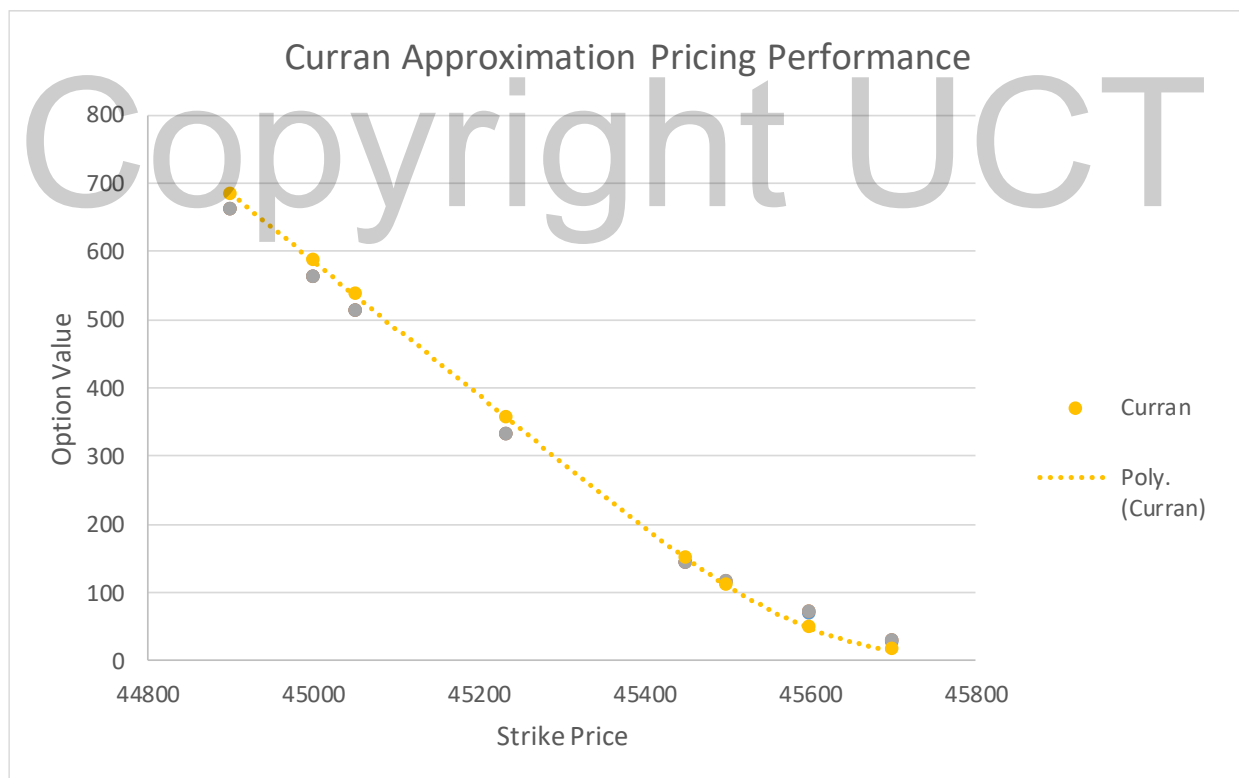
In the sections that follow, the prices of these Asian options are calculated and where possible the time taken to obtain these prices is recorded. The accuracy and time taken to arrive at these values will then be compared.

## **4.2 THE CURRAN APPROXIMATION**

The Curran Approximation “computes the option payoff conditional on the geometric mean of the relevant prices and integrates with respect to the (known) distribution of the geometric mean price” (Curran, 1994, p. 1705). This satisfies many of the Black-Scholes assumptions and hence produces immediate prices. The following table shows the calculated prices for the above Asian options using the Microsoft Excel VBA code found in Appendix B. VBA code was used due to its simplicity and immediate response time for the Curran model:

Strike Price	Curran Approximation Price
31,000	14,377.7416
44,900	684.1229
45,000	585.6077
45,050	536.3503
45,234	355.1733
45,450	150.5502
45,500	110.0676
45,600	47.8635
45,700	14.7331
49,500	0.0000

*Table 3: Curran Approximation Prices for selected Asian Options*



*Figure 1: The Curran Approximation Price Performance*

The Curran Approximation provides an almost linear price process for the arithmetic average Asian options in question with a zero value for options that far are out-of-the money. While

these far out-of-the-money options may not have any intrinsic value, there should remain some time value in the option itself with 78 days to expiry (Folger, n.d.). The expectation at this stage of the analysis is therefore that the Curran Approximation may undervalue options that are out-of-the money.

#### 4.3 PRICING COMPARISONS

As described above, various numerical methods were used to obtain the prices of the Asian options defined above. Methods used include the Binomial or Cox-Ross-Rubenstein (CRR) model, the full Monte Carlo simulation and the Monte Carlo simulation including a control variate to increase the speed of convergence. The Matlab codes used for these methods can be found in Appendix C, D and A respectively. The following tables summarise the pricing results for some of the model runs across strike prices:

<b>Strike Price: 31,000 - Deep-in-the-money</b>	
<b>Model</b>	<b>Price Result</b>
Curran Approximation	14,377.7416
CRR model – 100 time steps	14,459.9032
CRR model – 10,000 time steps	14,459.8952
Monte Carlo 10,000 simulations	14,357.2241
Monte Carlo 1,000,000 simulations	14,356.0933
<b>Monte Carlo 20,000,000 simulations</b>	<b>14,356.2297</b>
Monte Carlo (Control Variate) 10,000 simulations	14,357.5818
Monte Carlo (Control Variate) 1,000,000 simulations	14,357.0651
Monte Carlo (Control Variate) 20,000,000 simulations	14,357.1601

*Table 4: Model results with Strike of 31,000*

Given the robustness of Monte Carlo simulations in dealing with various forms of variation (Maya, 2004), the accuracy of simulations can be increased with an increasing number of paths (Jabbour & Liu, 2005). Hence, the highlighted simulation can be regarded as the most accurate with regard to price. As can be seen from the above, the CRR model, as well as the Curran approximation tends to overprice Asian options that are deep in the money (relative to the prices obtained from Monte Carlo simulations). Given that the CRR model is commonly used in practice, pricing Asian options with this method can possibly lead to a much larger overpricing error. The comparison between the Monte Carlo simulation and the Curran approximation shows a small error in pricing of 0.15%.

<b>Strike Price: 45,234 - At-the-money</b>	
<b>Model</b>	<b>Price Result</b>
Curran Approximation	355.1733
CRR model – 100 time steps	329.9776
CRR model – 10,000 time steps	330.0593
Monte Carlo 10,000 simulations	334.0457
Monte Carlo 1,000,000 simulations	333.8637
<b>Monte Carlo 20,000,000 simulations</b>	<b>333.6081</b>
Monte Carlo (Control Variate) 10,000 simulations	333.1656
Monte Carlo (Control Variate) 1,000,000 simulations	333.9051
Monte Carlo (Control Variate) 20,000,000 simulations	333.6682

*Table 5: Model results with Strike of 45,234*

The table 5 above shows how the pricing differs when the option is at-the-money. In this instance, all of the models price the option at less than the Curran approximation. The CRR model now shows a closer price to that of the Monte Carlo simulations. The Monte Carlo Control Variate method also provided a faster convergence at no decimal places in just 10,000 simulations. The degree of error between the Monte Carlo simulation and the Curran approximation now increases to 6.1%.

<b>Strike Price: 45,600 - Out-of-the-money</b>	
<b>Model</b>	<b>Price Result</b>
Curran Approximation	47.8635
CRR model – 100 time steps	68.5785
CRR model – 10,000 time steps	70.0332
Monte Carlo 10,000 simulations	35.5567
Monte Carlo 1,000,000 simulations	35.5221
<b>Monte Carlo 20,000,000 simulations</b>	<b>35.4415</b>
Monte Carlo (Control Variate) 10,000 simulations	35.8975
Monte Carlo (Control Variate) 1,000,000 simulations	35.2881
Monte Carlo (Control Variate) 20,000,000 simulations	35.4475

*Table 6: Model results with Strike of 45,600*

Table 6 above shows very varied results. The Curran approximation shows a value between that of the CRR model and the traditional Monte Carlo model, along with the Monte Carlo control variate method. The traditional Monte Carlo simulation however, shows that there should still be value associated with this option while the CRR model tends to price at almost double the Monte Carlo convergence price. For out-of-the-money Asian options, the Curran's approximation marginally over prices instruments in this position.

Strike Price: 49,500 - Deep-out-of-the-money	
Model	Price Result
Curran Approximation	0.0000
CRR model – 100 time steps	0.0000
CRR model – 10,000 time steps	0.0000
Monte Carlo 10,000 simulations	0.0000
Monte Carlo 1,000,000 simulations	0.0000
<b>Monte Carlo 20,000,000 simulations</b>	<b>0.0000</b>
Monte Carlo (Control Variate) 10,000 simulations	0.0000
Monte Carlo (Control Variate) 1,000,000 simulations	0.0000
Monte Carlo (Control Variate) 20,000,000 simulations	0.0000

Table 7: Model results with Strike of 49,500

At the extreme of an Asian option with a strike of 49,500 or very deep-out-of-the-money, all valuation models produced the same result of zero value. With these options still having 78 days until expiry however, time value presented a non-substantial component of the price. The following figures show the performance of each of the pricing models over a range of strike prices relative to the Curran Approximation:

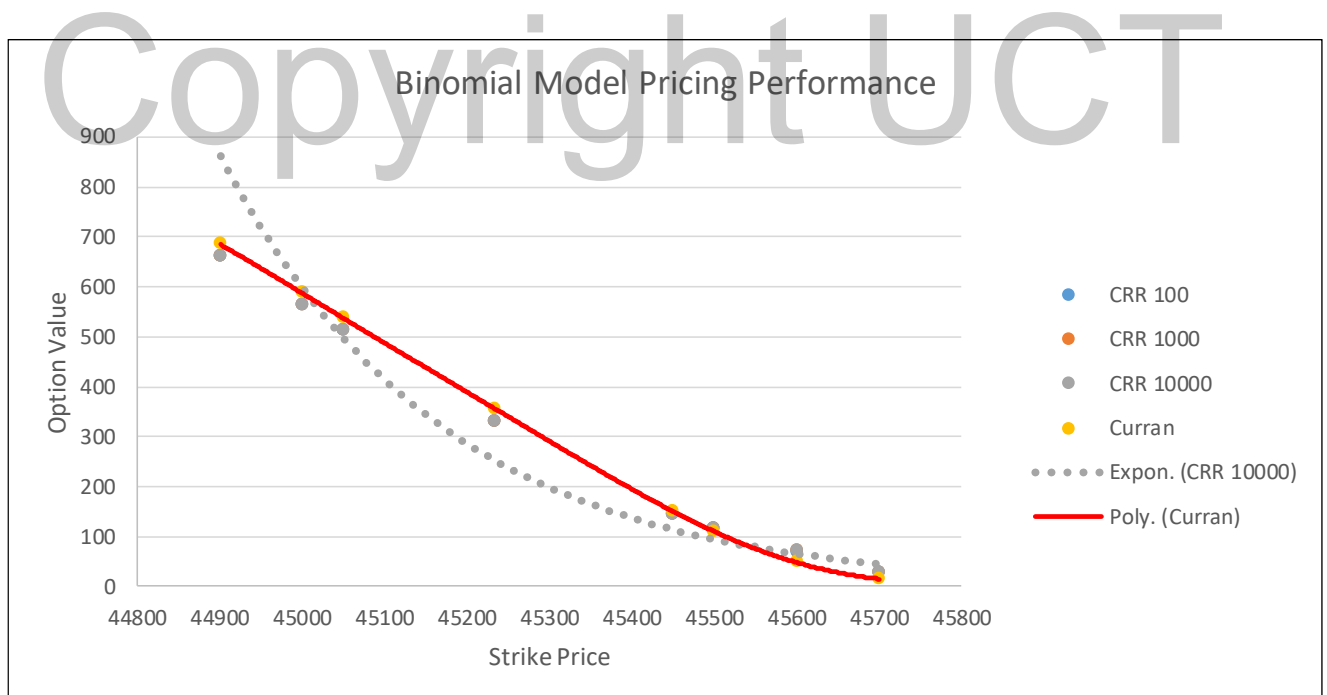


Figure 2: Relative pricing performance of the CRR model

In Figure 1 above, the CRR or binomial model tends to price above the Curran approximation when the Asian option is deep-in-the-money or deep-out-of-the-money. As the option moves

towards an at-the-money position, the Curran approximation prices above the CRR pricing model. With the CRR model being used commonly in the market, this leaves arbitrage opportunities open over the entire strike price horizon. At high strike prices, the Curran Approximation under prices options relative to the CRR model.

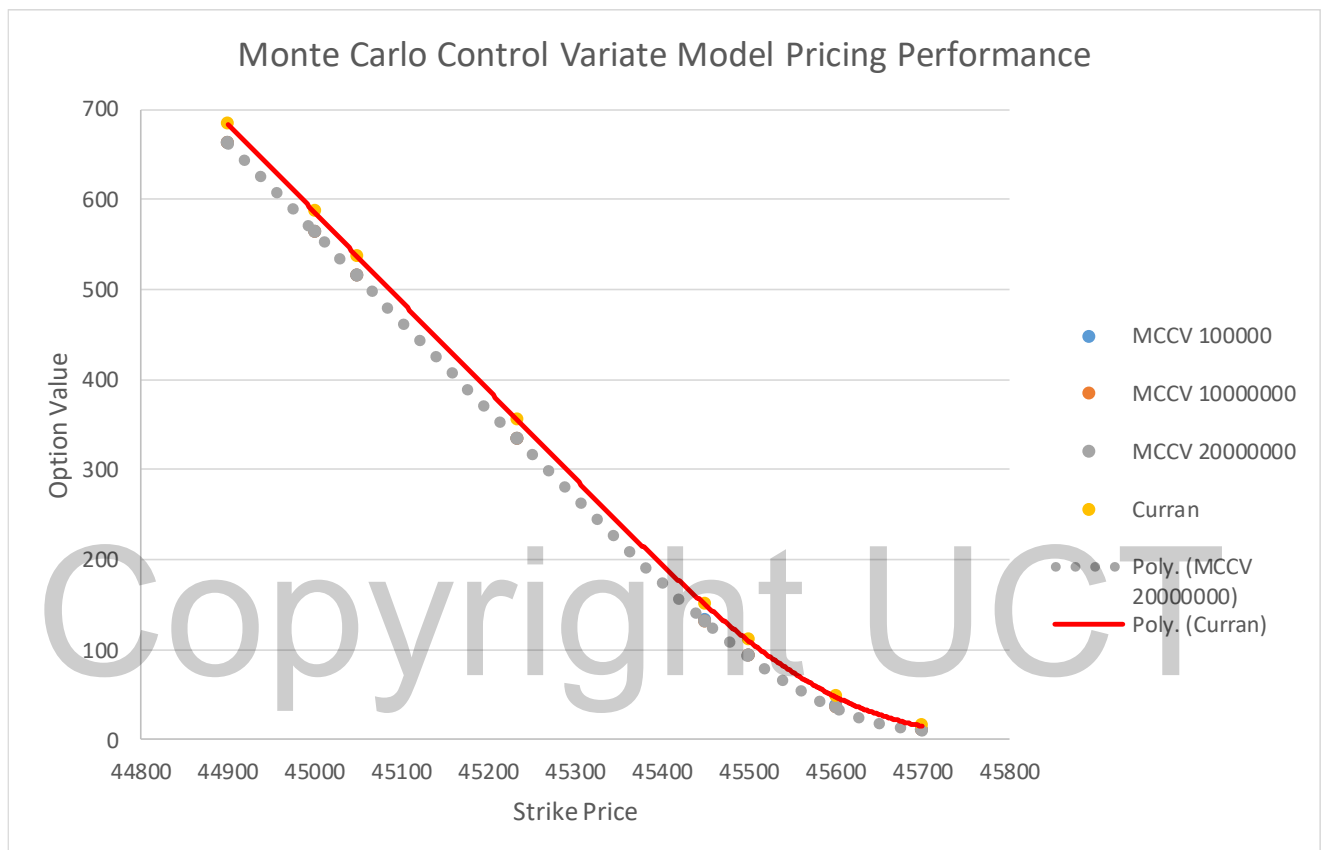


Figure 3: Relative pricing performance of the Monte Carlo Control Variate model

In the figure 2, the Curran approximation always prices above the Monte Carlo simulation with Control variate. As the option moves further out-of-the-money, both the Curran model and the Monte Carlo with control variate model converge on zero at around the same strike price.



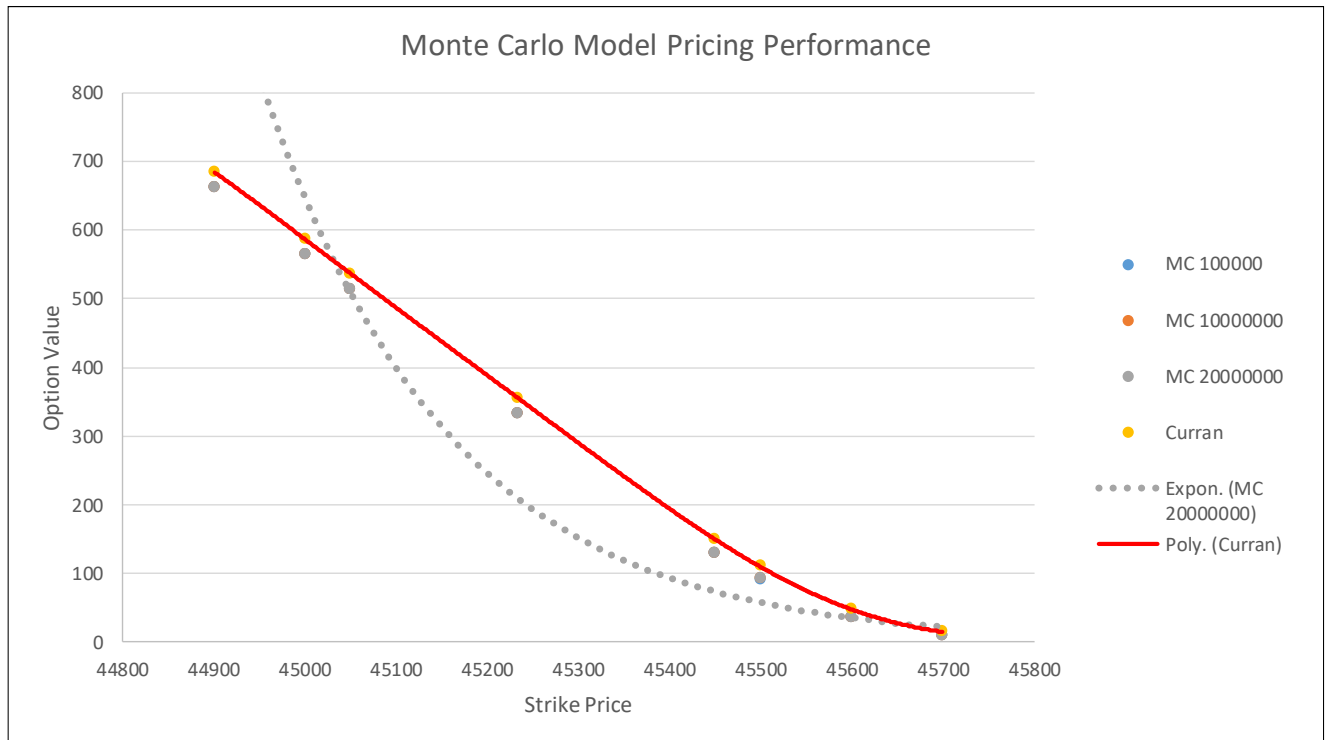


Figure 4: Relative pricing performance of the traditional Monte Carlo model

From Figure 3 above, it can be seen that the Curran approximation, while showing a similar pricing pattern as that of the Monte Carlo simulation, tends to overprice Asian options that are in-the-money. The fitted exponential distribution shows that the Monte Carlo simulation should actually price above that of the Curran model. The degree of error relative to the Monte Carlo simulation with 20,000,000 paths increases as the option moves closer to the at-the-money position. The following table shows the degree of error at various strike prices:

Strike Prices	31,000	44,900	45,000	45,050	45,234	45,450	45,500	45,600	45,700	49,500
Error	0.15%	3.22%	3.75%	4.10%	6.20%	13.33%	16.36%	27.08%	40.00%	0%

Table 8: Degree of Error relative to MC 20,000,000

Table 8 shows that at very deep-in-the-money Asian options, the accuracy of the Curran approximation is high. As the option moves into an at-the-money position, the degree of error increases. However, the Curran approximation then shows a lagged effect in that, the error continues to increase. The errors calculated are slightly misleading as the option prices obtained start to converge at higher strike prices, eventually converging to zero value at the very deep-

out-of-the-money options. Due to the differences being of smaller prices, the error rates calculated above depict a larger proportion but are small in absolute terms.

#### 4.4 SENSITIVITY IN PRICING ACCURACY

Following the methodology of Wiklund, (2012), the number of paths used in the Monte Carlo simulations offer key insight into the accuracy of the price estimates. Taken directly from the Wiklund paper:

$$f_{mean} = \frac{1}{n} \sum_{i=1}^n f_i$$

Where  $f_i$  represents the result of each path and  $n$  is the number of paths simulated. Since each path is statistically independent:

$$Var(f_{mean}) = \frac{1}{n^2} \sum_{i=1}^n Var(f_i) = \frac{Var(f)}{n}$$

Hence:

$$SE(f_{mean}) = \sqrt{\frac{Var(f)}{n}}$$

“Since the standard error of the estimate decreases proportionally to the square root of the number of simulations, the accuracy improves with a larger number of simulations” (Wiklund, 2012, p. 11). The following tables assess the Monte Carlo simulations with changes not only in the number of paths simulated, but also with changes in volatility and in time to maturity. These are all assessed for at-the-money arithmetic average Asian options.

Asian Option, At-the-money, K=45,234, T=78days, r=7%								
Simulations	Vol	MC Price	MCCV Price	Curran Approximation	Deviation (MC/Curran)	Deviation (MCCV/Curran)	SE MC	SE MCCV
1000	0,01	336	333	355,17	5,63%	6,2%	0,18%	0,20%
	0,05	446	462	468,54	4,81%	1.40%	0,15%	0,04%
	0,10	700	670	701,76	0.25%	4.53%	0,01%	0,14%

<b>10 000</b>	0,01	334	334	355,07	5,93%	5,93%	0,06%	0,06%
	0,05	440	440	468,54	6,09%	6,09%	0,06%	0,06%
	0,10	650	683	701,76	7,38%	2,67%	0,07%	0,03%
<b>100 000</b>	0,01	334	334	355,07	5,93%	5,93%	0,02%	0,02%
	0,05	438	443	468,54	6,52%	5,45%	0,02%	0,02%
	0,10	665	667	701,76	5,24%	4,95%	0,02%	0,02%
<b>1 000 000</b>	0,01	334	334	355,07	5,93%	5,93%	0,01%	0,01%
	0,05	440	440	468,54	6,09%	6,09%	0,01%	0,01%
	0,10	660	659	701,76	5,95%	6,09%	0,01%	0,01%

Table 9: Analysis of change in Volatility and simulations with time to maturity of 78 days

Asian Option, At-the-money, K=45,234, T=30days, r=7%								
Simulations	Vol	MC Price	MCCV Price	Curran Approximation	Deviation (MC/Curran)	Deviation (MCCV/Curran)	SE MC	SE MCCV
<b>1000</b>	0,01	128	128	150,92	15,19%	15,19%	0,48%	0,48%
	0,05	214	228	255,36	16,20%	10,71%	0,51%	0,34%
	0,10	355	371	420,45	15,57%	11,76%	0,49%	0,37%
<b>10 000</b>	0,01	131	129	150,92	13,20%	14,52%	0,13%	0,15%
	0,05	225	216	255,36	11,89%	15,41%	0,12%	0,15%
	0,10	369	367	420,45	12,24%	12,71%	0,12%	0,13%
<b>100 000</b>	0,01	130	130	150,92	13,86%	13,86%	0,04%	0,04%
	0,05	221	220	255,36	13,46%	13,85%	0,04%	0,04%
	0,10	363	364	420,45	13,66%	13,43%	0,04%	0,04%
<b>1 000 000</b>	0,01	130	130	150,92	13,86%	13,86%	0,01%	0,01%
	0,05	221	221	255,36	13,46%	13,46%	0,01%	0,01%
	0,10	364	363	420,45	13,43%	13,66%	0,01%	0,01%

Table 10: Analysis of change in Volatility and simulations with time to maturity of 30 days

Asian Option, At-the-money, K=45,234, T=100days, r=7%								
Simulations	Vol	MC Price	MCCV Price	Curran Approximation	Deviation (MC/Curran)	Deviation (MCCV/Curran)	SE MC	SE MCCV
<b>1000</b>	0,01	427	421	448,36	4,76%	6,10%	0,15%	0,19%
	0,05	531	544	560,60	5,28%	2,96%	0,17%	0,09%
	0,10	806	911	815,15	1,12%	11,76%	0,04%	0,37%
<b>10 000</b>	0,01	427	426	448,36	4,76%	4,99%	0,05%	0,05%
	0,05	528	542	560,60	5,82%	3,32%	0,06%	0,03%
	0,10	769	788	815,15	5,66%	3,33%	0,06%	0,03%
<b>100 000</b>	0,01	427	425	448,36	4,76%	5,21%	0,02%	0,02%
	0,05	534	534	560,60	4,74%	4,74%	0,02%	0,02%
	0,10	780	776	815,15	4,31%	4,80%	0,01%	0,02%
<b>1 000 000</b>	0,01	426	426	448,36	4,99%	4,99%	0,00%	0,00%
	0,05	532	532	560,60	5,10%	5,10%	0,01%	0,01%
	0,10	776	778	815,15	4,80%	4,56%	0,00%	0,00%

Table 11: Analysis of change in Volatility and simulations with time to maturity of 100 days

Tables 9, 10 and 11 calculate the deviations of the Curran's approximation method from the traditional Monte Carlo method as well as the Monte Carlo method using the Control Variate technique. These tables also show the standard errors calculated for each of the options. Table 9 refers to the 78-day maturity which the actual data presented with, while table 10 referred to notional 30 day options and table 11 referred in a similar way to 100 day options.

A few relationships can be established from the tables above. Firstly, with a larger number of simulations, there is greater stability of pricing in both the Monte Carlo and Monte Carlo Control Variate methods. As per the conclusion of Wiklund (2012), at least 100,000 simulations need to be carried out in order for the standard errors to fall below 0.05%.

It is also observed that many of the Black-Scholes properties holds. With an increase in volatility, there is an increase in price across all models considered. This also increased the amount of deviation between methods as the volatility was increased. As the time to maturity increased prices increased across all models as there is a greater chance that the option will end

in-the-money. An increased time to maturity was also observed to reduce the deviations between the considered methods. At the shortest time to maturity of 30 days in table 10, the deviations were never below 10%, while at the 100-day time to maturity (table 11), the deviations only once breached 6%. Thus it can be concluded that long duration Asian options with low volatility show the most stable prices, however, with a greater number of simulated paths, the accuracy can be increased in proportion to the square root of the number of paths simulated.

<b>Asian Option, At-the-money, K=45,234, T=78days, r=7%</b>				
<b>Time steps</b>	<b>Vol</b>	<b>CRR Price</b>	<b>Curran Approximation</b>	<b>Deviation (MC/Curran)</b>
<b>100</b>	0,01	330	355,17	7,09%
<b>1 000</b>	0,01	330	355,17	7,09%
<b>10 000</b>	0,01	330	355,17	7,09%
<b>Asian Option, At-the-money, K=45,234, T=30days, r=7%</b>				
<b>100</b>	0,01	130	151,21	14,03%
<b>1 000</b>	0,01	130	151,22	14,03%
<b>10 000</b>	0,01	130	151,22	14,03%
<b>Asian Option, At-the-money, K=45,234, T=100days, r=7%</b>				
<b>100</b>	0,01	420	448,33	6,32%
<b>1 000</b>	0,01	420	448,32	6,32%
<b>10 000</b>	0,01	420	448,32	6,32%

*Table 12: Analysis of CRR model performance over various times to maturity*

Table 12 above was completed for the same volatility as increases in volatility resulted in the binomial tree expanding to much larger proportions and hence possibilities. Convergence occurs quickly under the CRR model, at 100 time steps. However, a similar pattern as that of the Monte Carlo simulations, is observed with regard to the changes in times to maturity. The larger the time to maturity, the higher the value of the option as there is a greater amount of time for the option to move into, or further into the money.

Under normal market conditions, with liquid trading, one would assess these models and their accuracy against the actual market prices determined by willing buyers and sellers. Monte

Carlo simulation is clearly the best option for pricing Asian options due to its flexibility in incorporating many of the market factors into its modelling. The accuracy can be increased by considering more paths for simulation. However, as mentioned previously, as with other models that have been considered, the time the model takes to produce viable results is also an important factor to consider. This is considered in the next section.

#### 4.5 THE ISSUE OF TIMING

While the accuracy remains of high concern, the time it takes to obtain prices pose a different challenge to different types of investors. High frequency traders for example, would require prices within milliseconds in order to make transactional decisions. Time taken to obtain a price for these Asian options should therefore be considered in terms of cost and benefit. High frequency traders may be willing to sacrifice some accuracy in the price of an option, should the price be obtained in a short amount of time in order to take advantage of short-term price variances.

Monte Carlo simulation provides very accurate results with a greater number of simulation paths. This is because many more possibilities are considered and become part of the average price calculated at the end of each simulation. However, each path that is added takes additional time. Each path also requires additional computing power to consider the variation in certain variables that are considered. This of course means a greater expense to the user.

This major drawback is counteracted with the use of the control variate technique which reduces the variance that is used in the traditional Monte Carlo technique. This then reduces the number of paths to achieve a similar level of accuracy in the pricing. This technique used the traditional price of a vanilla European call option to limit the variances. However, with higher variances, more paths still need to be generated and this still requires computing power and time.

The CRR model, while simpler, increases in accuracy as the time steps considered become smaller. This means that if a greater number of time steps are used between the valuation date and the expiry date of the option in question, the price becomes more accurate given that it is sampled more regularly.

This is where the Curran Approximation gains an advantage. The price for the Asian option is produced immediately without any delay, allowing split second decisions to be made. The following tables show the time taken to produce the prices used in this research:

No. of Simulations	Monte Carlo	Monte Carlo with Control Variate
	Time in Seconds	Time in Seconds
1 000	0,00638	0,00670
2 500	0,00678	0,03652
5 000	0,06831	0,01739
10 000	0,02703	0,03410
25 000	0,04721	0,08280
50 000	0,11480	0,10805
100 000	0,29182	0,33395
250 000	0,51218	0,51908
500 000	0,93749	1,02943
1 000 000	2,42507	2,80276
10 000 000	21,36206	22,00188
20 000 000	40,91696	47,43719

Table 13: Time comparison for MC and MCCV methods

No. of Time steps	CRR Model (Binomial)
	Time in Seconds
10	0,05500
20	0,05327
40	0,09170
50	0,09269
75	0,12029
100	0,15393
200	0,31541
500	1,07761
1 000	3,49281
5 000	233,27284
10 000	1219,97182

Table 14: Timing for CRR model runs

Traders thus need to sacrifice accuracy in order to obtain prices faster. An area mentioned earlier and that warrants further research, is the use of the Fast Fourier Transform. This is said to have both accuracy and speed (Lee, Kim, & Jang, 2014).

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## 5. THE INTRODUCTION OF STOCHASTIC VOLATILITY

Thus far, all the models have assumed that volatility remains constant throughout the time horizon of the option in question. As shown in the section on implied volatility, volatility varies with time. Stock market volatility varies with changes in economic activity, financial leverage, stock trading activity and sentiment. In particular, it is noted that stock market variability is unusually high during periods of market turmoil such as during the Great Depression (Schwert, 1989). Hence, the Black-Scholes assumption of constant volatility is incorrect. However, traders still use the model in an unintended way. Traders “allow the volatility used to price an option to depend on its strike price and time to maturity” (Hull, 2006, p. 375). This has led to the use of the implied volatility. The following figure shows the implied volatility for call options that were traded on the JSE, on the ALSI index. Data for this is available in Appendix E and was labelled OptionData4.txt when referring to the Matlab code:

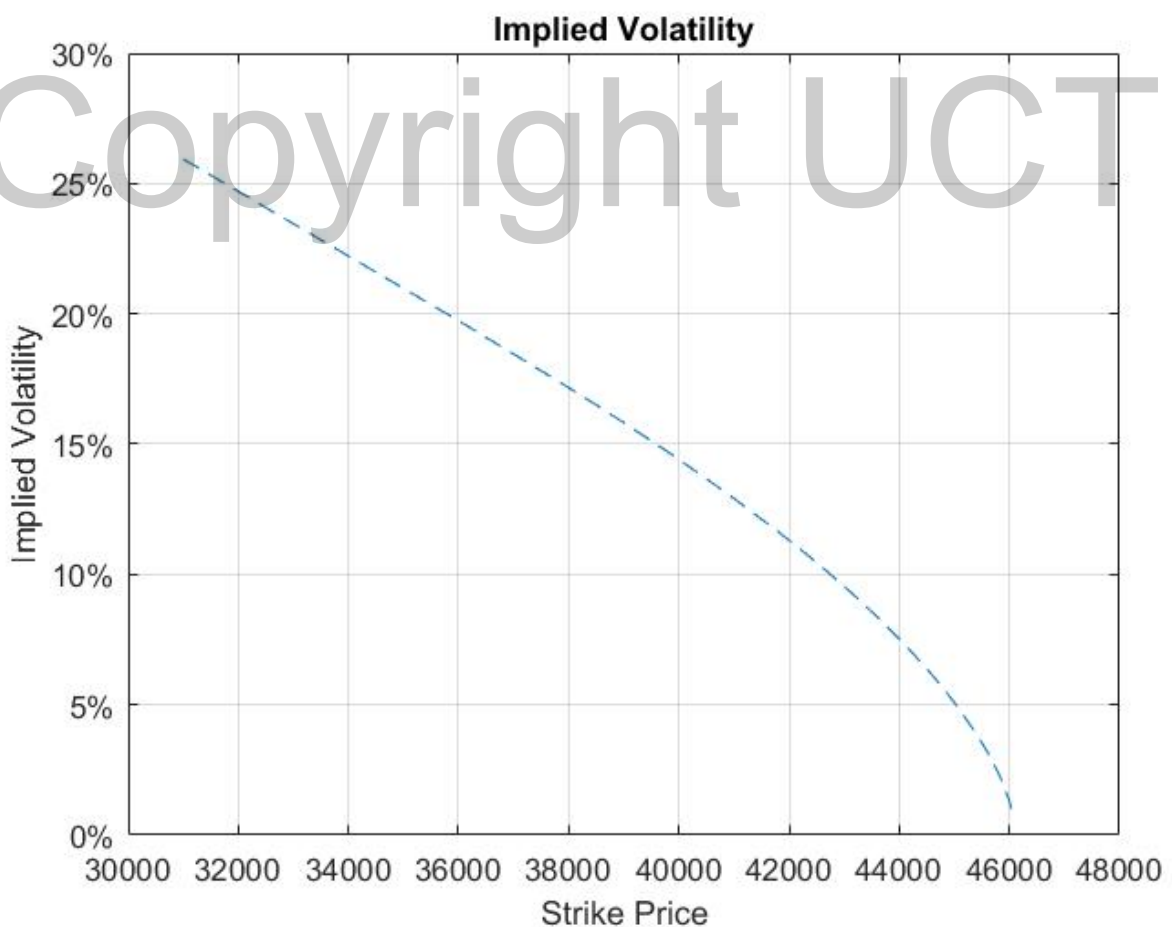


Figure 5: Implied Volatility of the ALSI index options

This resembles a classic volatility smirk or skew for equities. Volatility decreases with an increase in strike price and thus the volatility that is used to price a deep-in-the-money call option is much higher than that used to price a deep-out-of-the-money call (Hull, 2006) and does not remain constant as assumed in the Black-Scholes-Merton model. Hull, (2006), explains that a possible cause for this shape is due to company leverage. With a decrease in equity, a company's leverage increases and thus this increases volatility through greater risk.

To improve the accuracy of the models used previously, with a constant volatility factor, a stochastic volatility factor is introduced. This means that the volatility is allowed to vary by a certain distribution but within certain parameters that are dictated by market indicators and information. In order to calculate these parameters, a process called calibration is followed whereby the parameters of each stochastic volatility model are fit to market conditions. As mentioned previously, the two stochastic volatility models investigated in this research is the Heston model and the SABR model.

## 5.1 CALIBRATION

As mentioned previously, the development of these advanced models is to better mimic reality (Kovachev, 2014). However, for more realistic outcomes from each of these models, the greater the complexity and time involved in the calibration process (Moodley, 2005). “The better the calibration that can be achieved, the higher the predictive validity of a model is and the more valuable it becomes as a tool for risk management and portfolio optimisation” (Kovachev, 2014, p. 16). Identifying these parameters from observed data is referred to as the “calibration problem” (Fatone, Mariani, Recchioni, & Zirilli, 2014). As cited by Moodley, (2005), Bakshi, Cao & Chen (1997) have shown that implied parameters and their associated time-series estimates are different. Hence empirical estimates cannot be used and parameters should be solved for indirectly through an implied structure. Both Moodley, (2005) and di Laurea Magistrale, (2013), refer to the most popular approach as being the minimisation of the error between model and market prices. Hence this simplifies to a non-linear least squares optimisation problem i.e:

$$\min_{\Omega} S(\Omega) = \min_{\Omega} \sum_{i=1}^N w_i [C_i^{\Omega}(K_i T_i) - C_i^M(K_i T_i)]^2$$

Where  $\Omega$  is a vector of parameters,  $C_i^{\Omega}(K_i T_i)$  is the  $i^{th}$  option price from the model while  $C_i^M(K_i T_i)$  is the  $i^{th}$  market price with strike price of  $K_i$  and maturity of  $T_i$ . The problem however is that  $S(\Omega)$  does not have an implied structure as was premised by the calibration process. Three problems have been highlighted:

- Parameters cannot be found by optimisation of gradient functions
- One may not be able to find a global minimum
- A unique solution may not be possible and thus only local minima may be found

(Moodley, 2005)

The method used in this research to deal with the intricacies of the above calibration problem is the Matlab function `lsqnonlin`. This function is discussed further in the next section. These results were based on the option pricing code from Alexander (2008).

## 5.2 MATLAB'S `LSQNONLIN` FUNCTION

The Matlab function `lsqnonlin` is an inbuilt nonlinear least-squares solver. This solves curve fitting problems of the form:

$$\min_x f(x) = \min_x (f_1(x)^2 + f_2(x)^2 + \dots f_n(x)^2)$$

(MathWorks, 2016)

Which broadly represents the function above to be minimised as part of the calibration problem. The syntax used in the calibration is `lsqnonlin(fun, x0, lb, ub)` which minimises the function (`fun`) with the starting point `x0` and specifying lower bounds (`lb`) and upper bounds (`ub`) within which the parameters are “allowed” to be.

The trial and error it was confirmed that the parameter result is highly “dependent on the choice of `x0`, the initial estimate” as well as the vectors defining the boundaries, `lb` and `ub`. This would represent a local optimisation (Moodley, 2005, p. 24). In the sections that follow, the results of the calibration process are shown for both the Heston and the SABR stochastic volatility models. Matlab code is provided in the appendices for reference.

### 5.3 THE HESTON CALIBRATION

Recall the Heston equations from earlier in the research which defines the stock price process as well as the volatility process:

$$dS_t = \mu S_t dt + S_t \sqrt{v_t} dZ_1$$
$$dv_t = \kappa (\theta - v_t) dt + \eta \sqrt{v_t} dZ_2$$

The Heston model has five parameters that require estimation. This includes  $\kappa$ , the variance mean-reversion speed,  $\theta$ , the long-term variance,  $v_t$ , the variance at time  $t$ ,  $\eta$ , the volatility of the variance process and  $\rho$ , the correlation between the stock price and volatility processes. With Matlab's `lsqnonlin`, local optimisation was run, however, the following bounds were used to maintain both mathematically feasible solutions as well as parameter estimates that are acceptable economically. The following boundary conditions were used:

- **Long-term variance and initial variance ( $\theta$  and  $v_t$ )**

With the mean-reversion parameter, volatility would rarely move beyond a 100% level thus bounds used were between 0 and 1.

- **Correlation ( $\rho$ )**

Correlation, from a statistical point of view, can only take on values between -1 and 1. While correlation between volatility and stock prices have traditionally been negative, there are circumstances where this can be positive, hence the entire range was taken as bounds for this parameter.

- **Volatility of variance ( $\eta$ )**

This parameter should exhibit only positive values. However, the volatility within the stock market can also change drastically and thus larger positive boundaries were used for this parameter of between 0 and 5.

- **Mean-reversion speed ( $\kappa$ )**

$\kappa$  should only take on positive values to ensure mean-reversion in the volatility parameter. The boundaries were more flexible with this parameter and so values of between 0 and 20 were used as the conditions.

(Crisostomo, 2014)

With Vanilla calls on the ALSI index listed on the JSE as market data (which can be found in the Appendix E in the format in which it was used in Matlab), the `lsqnonlin` function was run and yielded the following results:

Parameters	$\kappa$	$\Theta$	$v_t$	$\eta$	$\rho$
Estimates	13.0478	0.0093	0.0934	0.0527	-0.4555

*Table 15: Parameter estimation results for Heston model*

Next, the Feller condition had to be assessed. This condition needs to be observed so that the variance process remains strictly positive:

Feller Condition:

$$2\kappa\theta > \eta^2$$

(Xiong, 2015)

$$2\kappa\theta = 2 \times 13.0478 \times 0.0093 = 0.242689$$

$$\eta^2 = 0.0527^2 = 0.00278$$

Hence the Feller condition holds and the variance process does not reach zero or negative values.

The following figure shows the fit of these parameters with the implied volatility that was calculated above:

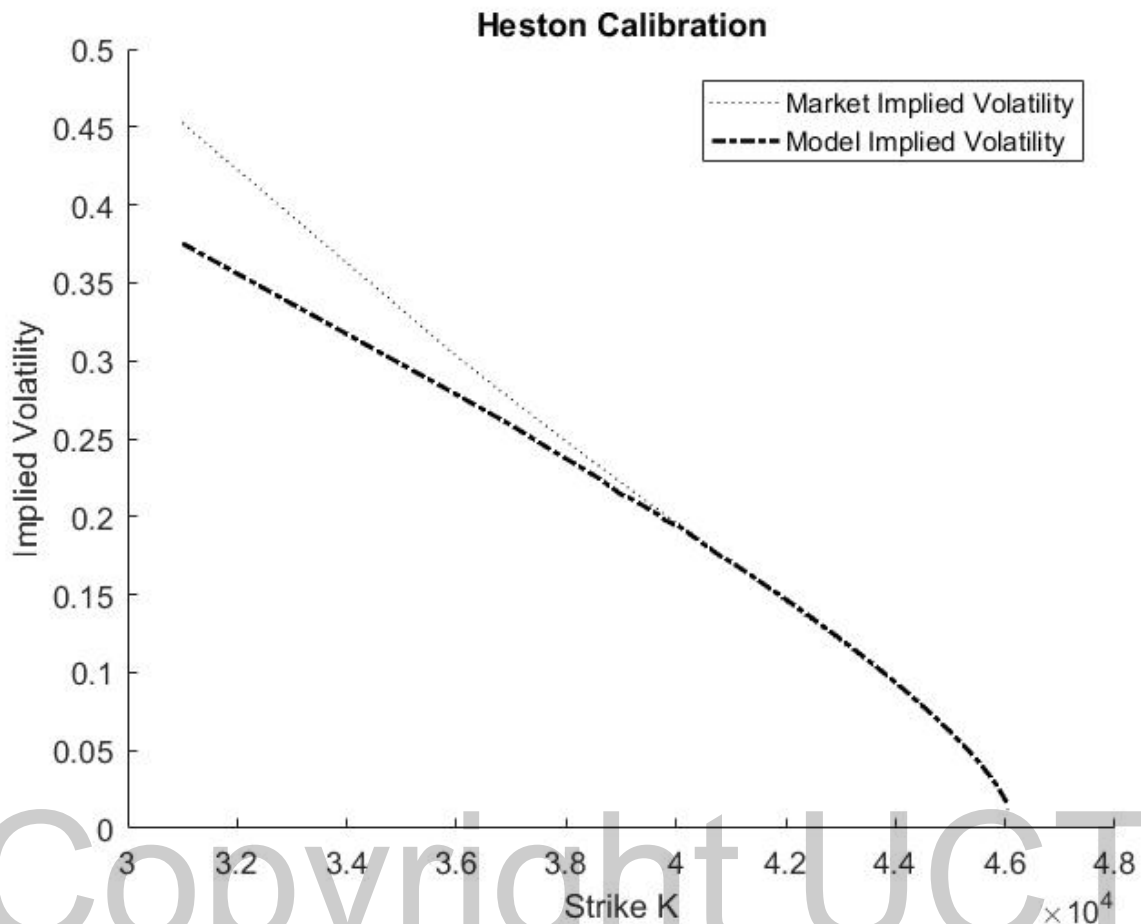


Figure 6: The fit of the Heston model Volatility relative to the calculated Implied Volatility

As can be seen from the figure 6, the fit of the Heston model is extremely close particularly at higher strike prices. The Heston model predicts lower volatility than the volatility implied by the market at lower strike prices. To demonstrate the closeness of the fit, the mean-square error was calculated and showed a value of 13.74079. Relative to an at the money strike price this shows an error of 0%, hence showing the closeness of the fit.

## 5.4 HESTON MODEL RESULTS

The Heston model was run as a Monte Carlo simulation with 100 000 paths to stabilise results. The following table summarises the results obtained from the model with a calculated deviation:

Strike Price	Curran Approximation	Heston Model	Deviation
31,000	14,378	12,830	10,77%
44,900	684	569	16,81%
45,000	587	535	8,86%
45,050	536	427	20,34%
45,234	355	352	0,85%
45,450	151	248	-64,24%
45,500	110	160	-45,45%
45,600	48	55	-14,58%
45,700	15	0	100,00%
49,500	0	0	0,00%

Table 16: Comparison in prices between the Curran Approximation and the Heston Model

Figure 7 below better illustrates the pricing differences between the constant volatility of the Curran approximation compared to the changing volatility of the Heston model:

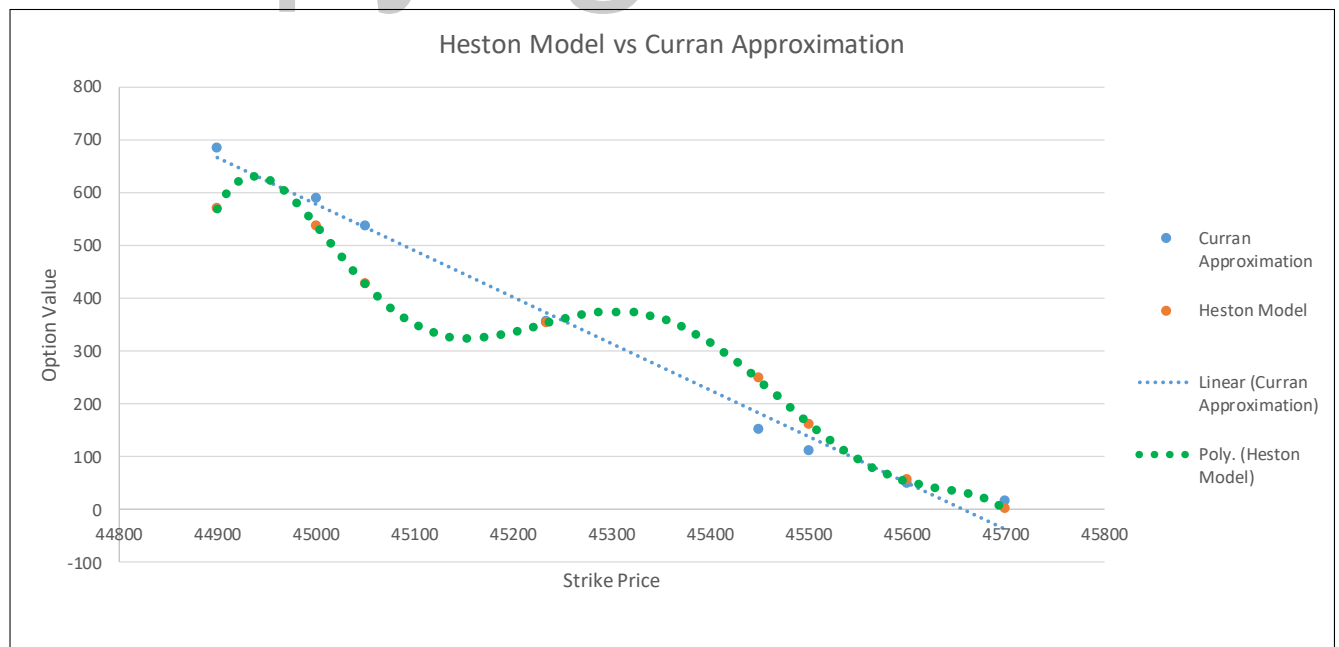


Figure 7: Heston Model Pricing compared to the Curran Approximation Pricing

A few observations are very evident from both Table 16 and Figure 7:

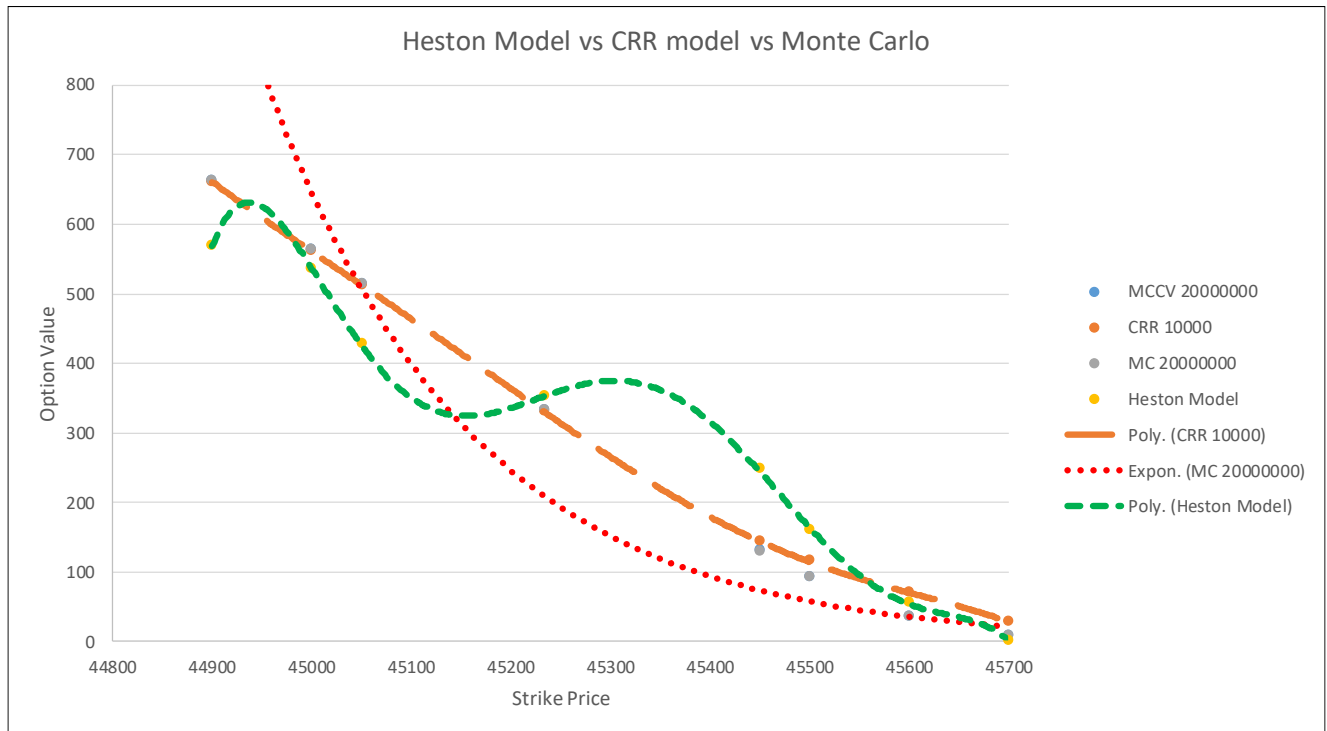
- The Heston model shows the mean reversion in volatility through a sinusoidal type graph.
- The Curran approximation seems to depict this mean that the Heston model is reverting to.
- At low strike prices, when the Asian call options considered are in-the-money, the Curran approximation prices above the Heston model.
- When the Asian option is at-the-money, the price from the Curran approximation and that of the Heston model are very similar with a deviation of less than 1%.
- As these Asian call options move out-of-the-money, at higher strike prices, the Heston model now prices above the Curran approximation.
- At extreme strike prices, (far out-of-the-money options), the prices obtained from both models converge at zero.

The following table summarises the pricing differences between the other constant volatility models and shows the deviations using the Heston model as the base:

Strike Price	CRR (Binomial Model)	Monte Carlo Simulation	Heston Model	Deviation (CRR/Heston)	Deviation (MC/Heston)
31,000	14,460	14,356	12,830	-12,63%	-11,89%
44,900	661	662	569	-16,17%	-16,34%
45,000	561	564	535	-4,86%	-5,42%
45,050	512	514	427	-19,91%	-20,37%
45,234	330	333	352	6,25%	5,40%
45,450	143	130	248	42,34%	47,58%
45,500	116	92	160	27,50%	42,50%
45,600	70	35	55	-27,27%	36,36%
45,700	28	9	0	0%	0%
49,500	0	0	0	0%	0%

*Table 17: Price Comparison between Heston Model, CRR Model and Monte Carlo Simulations*





*Figure 8: Comparison between the Heston model, CRR model and the Monte Carlo simulations*

In figure 8 above compares the Heston pricing model to the Monte Carlo simulation as well as the Binomial model. A few more additional insights can be derived from Figure 8 and Table 17:

- Without any mean reversion parameters in both the CRR (Binomial) and the Monte Carlo simulations, prices can become extreme under these models at the low strike prices or when the Asian call options move deeper into-the-money.
- When comparing the Heston model prices with that of the Monte Carlo simulation, the Monte Carlo model also tends to price above the Heston model at low strike prices with almost equivalent prices when the option is at-the-money.
- As the strike prices increase, the Monte Carlo simulation then prices below the Heston model simulations.
- Once again convergence occurs with all three models at extreme strike prices (far out-of-the-money options).
- When comparing the Heston model with the CRR model, the prices are relatively close when the option in question is deep-in-the-money.

- The CRR model thereafter, as strike prices increase, always produces a higher option price than the Heston model simulations.

No. of Simulations	Heston Model
	Time in Seconds
100	0,01214
250	0,01642
500	0,02248
1,000	0,03659
5,000	0,14089
10,000	0,26773
50,000	1,24089
100,000	2,59114
500,000	12,54996
1,000,000	26,55011

*Table 18: Heston model run times*

Table 18 above shows the run times for the Monte Carlo Heston model for a various number of simulations. The stochastic element that is introduced in the variance process significantly slows down the simulation time. The Monte Carlo simulations with constant volatility showed times of 2.42 seconds for 1,000,000 paths while 10,000,000 paths were run in 21.36 seconds. Once again, the price to pay for greater accuracy is a sacrifice in the time to obtain such prices.

## 5.5 THE SABR MODEL CALIBRATION

Recall the SABR model equations from earlier in the research which defines the stock price process as well as the volatility process:

$$dS_t = \sigma_t S_t^\beta dZ_1$$

$$d\sigma_t = \kappa \sigma_t dZ_2$$

The SABR model has four parameters that require estimation. This includes  $\kappa$ , in this instance represents the volatility of the volatility,  $\sigma_t$ , represents the at-the-money forward volatility. This can be observed directly in the market. The correlation between the stock price and volatility processes like in the Heston model, is represented by  $\rho$ . Lastly,  $\beta$  “mainly affects the slope of the backbone of the volatility smile” (Zhang, 2011, p. 30). This is illustrated with the following figures:

- $\beta = 0$

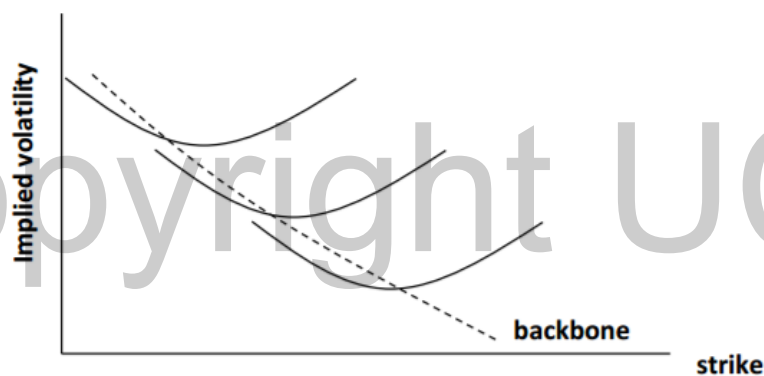


Figure 9: Volatility smile when Beta = 0

- $\beta = 1$

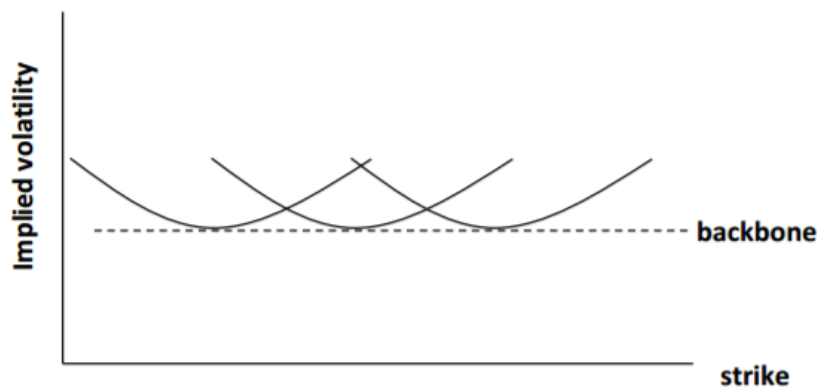


Figure 10: Volatility smile when Beta = 1

(Source: Zhang, 2011)

When  $\beta = 0$  the backbone is steeper when compared to when  $\beta = 1$  in which case, the backbone is flat.

As with the Heston model, local optimisation was run with Matlab's `lsqnonlin`, however, the following bounds were used to maintain both mathematically feasible solutions as well as parameter estimates that are acceptable economically. The following boundary conditions were used:

- **Correlation ( $\rho$ )**

As with the Heston model, correlation, from a statistical point of view, can only take on values between -1 and 1. While correlation between volatility and stock prices have traditionally been negative, there are circumstances where this can be positive, hence the entire range was taken as bounds for this parameter. As  $\rho$  becomes negative, the smile becomes more negatively sloped. However, decreases in  $\rho$  also changes the steepness of the smile hence playing against the  $\beta$  parameter described below (Rebonato, McKay, & White, 2009).

- **Skewness Parameter or Backbone ( $\beta$ )**

This is a constant parameter and has boundaries of  $0 \leq \beta \leq 1$ . It is said that there is redundancy in estimating  $\beta$  and  $\rho$ . Thus  $\beta$  is usually kept fixed while the other parameters are estimated (Hagan, Lesniewski, & Woodward, Probability distribution in the SABR model of stochastic volatility, 2015). However, this parameter was also estimated in this research. "In practice, the choice of  $\beta$  has little effect on the resulting shape of the volatility curve produced by the SABR model, so the choice of  $\beta$  is not crucial." (Rouah, n.d, p.3). However, as mentioned above, as  $\beta$  moves from 1 to 0, there is a steepening of the smile. The smile also increases in curvature as  $\beta$  moves from 1 to 0 (Rebonato, McKay, & White, 2009).

- **Volatility of variance ( $\kappa$ )**

As with the Heston model above, this parameter should exhibit only positive values. However, the volatility within the stock market can also change drastically and thus larger positive boundaries were used for this parameter of between 0 and 5. As  $\kappa$  increases, the curvature of the smile increases (Rebonato, McKay, & White, 2009).

- **At-the-money forward volatility ( $\sigma_t$ )**

The key difference from the Heston model, in this instance, is that the volatility does not mean revert. This means that boundaries for this parameter are set larger but should still remain positive. With a shift in this volatility, the main result is a shift in the volatility smile. With an increase in the initial volatility, the smile is shifted upwards (Rebonato, McKay, & White, 2009).

The same dataset of Vanilla calls on the ALSI index listed on the JSE was used to calibrate the SABR model. Once again, the Matlab `lsqnonlin` function was run. As with the Heston model SABR calibration requires the minimisation of the differences between the market prices and the model produced prices. The following table shows the results of the parameters estimated:

Parameters	$\alpha$	$\beta$	$\sigma_t$	$\rho$
Estimates	2.9991	0.6009	3.000	-0.7923

*Table 19: Parameter estimation results for SABR model*

Without a mean-reversion factor for the long-term variance, the Feller condition cannot be tested. However, in order to maintain positivity in the derived volatility, full truncation was used (only absolute derived volatilities were used as inputs in the stock price process).

The following figure shows the fit of these parameters with the implied volatility that was calculated above:

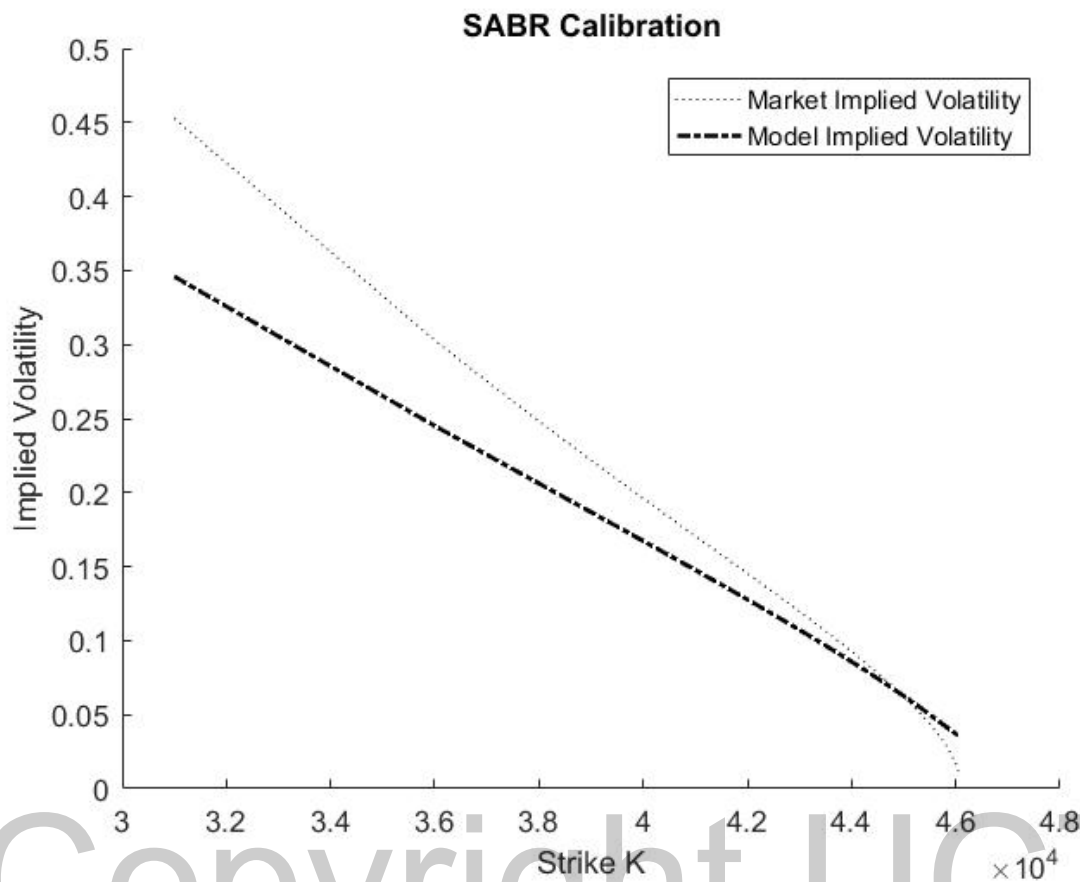


Figure 11: The fit of the SABR model Volatility relative to the calculated Implied Volatility

As can be seen from the figure above, the fit of the SABR model is close but increases in deviation as the option moves further into the money. The SABR model, like the Heston model, predicts lower volatility than the volatility implied by the market at lower strike prices. To demonstrate the closeness of the fit, the mean-square error was calculated and showed a value of 62.86143. Relative to an at the money strike price this shows an error of 0.14%, hence showing the closeness of the fit. However, the Heston model thus showed a closer fit to the market data.

## 5.6 SABR MODEL RESULTS

As with the Heston model, the SABR model was run as a Monte Carlo simulation with 100 000 paths to stabilise results. The following table summarises the results obtained from the model with a calculated deviation:

Strike Price	Curran Approximation	SABR Model	Deviation
31,000	14,378	14,347	0,22%
44,900	684	644	5,85%
45,000	587	543	7,50%
45,050	536	524	2,24%
45,234	355	341	3,94%
45,450	151	144	4,64%
45,500	110	104	5,45%
45,600	48	16	66,67%
45,700	15	2	86,67%
49,500	0	0	0%

Table 20: Comparison in prices between the Curran Approximation and the SABR Model

The figure below better illustrates the pricing differences between the Curran approximation and the SABR model:

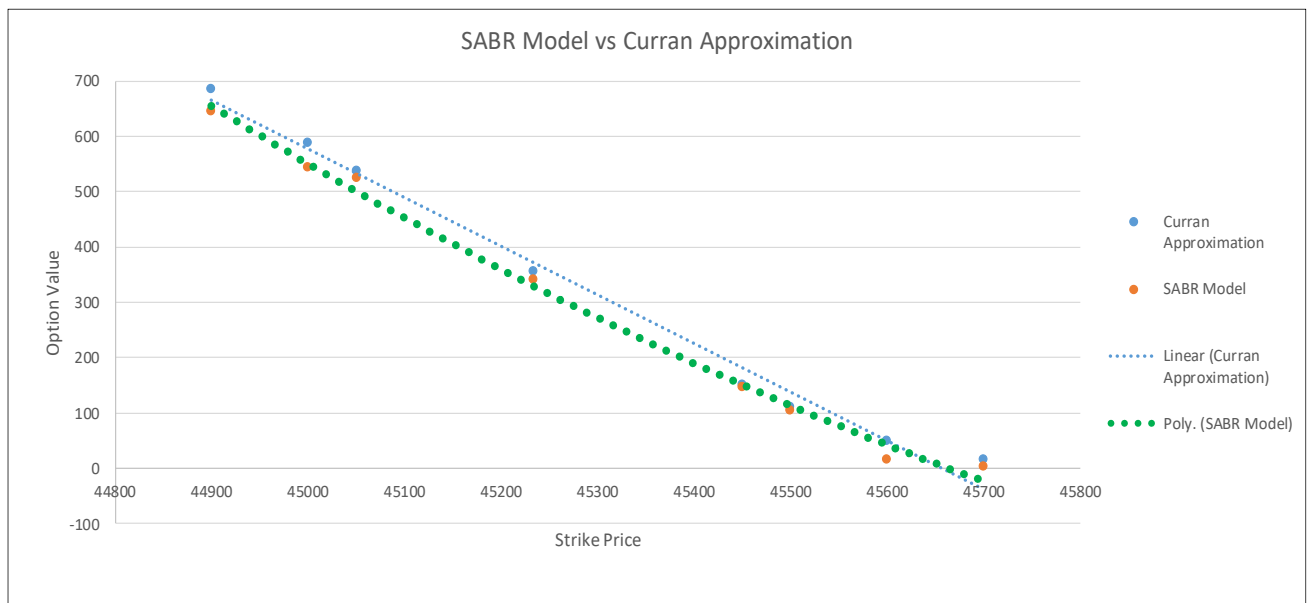


Figure 12: SABR Model Pricing compared to the Curran Approximation Pricing

A few observations are very evident from both Table 20 and Figure 12:

- The SABR model shows an almost constant volatility. However, this is probably the effect of the full truncation and the use of absolute volatility figures into the stock price process.
- Although a polynomial is fit to the SABR prices, the pricing function is almost linear in nature.
- At low strike prices, when the Asian call options considered are in-the-money, the Curran approximation prices marginally above the SABR model.
- When the Asian option is at-the-money, the price deviation between the Curran and SABR models represents a local maximum at 3.94%. The figure also shows a wider gap at this point.
- As these Asian call options move out-of-the-money, at higher strike prices, the SABR model continues to price below the Curran approximation.
- At extreme strike prices, (far out-of-the-money options), the prices obtained from both models converge at zero.

The following table summarises the pricing differences between the other constant volatility models and shows the deviations using the SABR model as the base:

Strike Price	CRR (Binomial Model)	Monte Carlo Simulation	SABR Model	Deviation (CRR/Heston)	Deviation (MC/Heston)
31,000	14,460	14,356	14,347	-0,72%	-0,06%
44,900	661	662	644	-2,64%	-2,80%
45,000	561	564	543	-3,31%	-3,87%
45,050	512	514	524	2,29%	1,91%
45,234	330	333	341	3,23%	2,35%
45,450	143	130	144	0,69%	9,72%
45,500	116	92	104	-11,54%	11,54%
45,600	70	35	16	-337,50%	-118,75%
45,700	28	9	2	-1300,00%	-350,00%
49,500	0	0	0	0%	0%

*Table 21: Price Comparison between SABR Model, CRR Model and Monte Carlo Simulations*



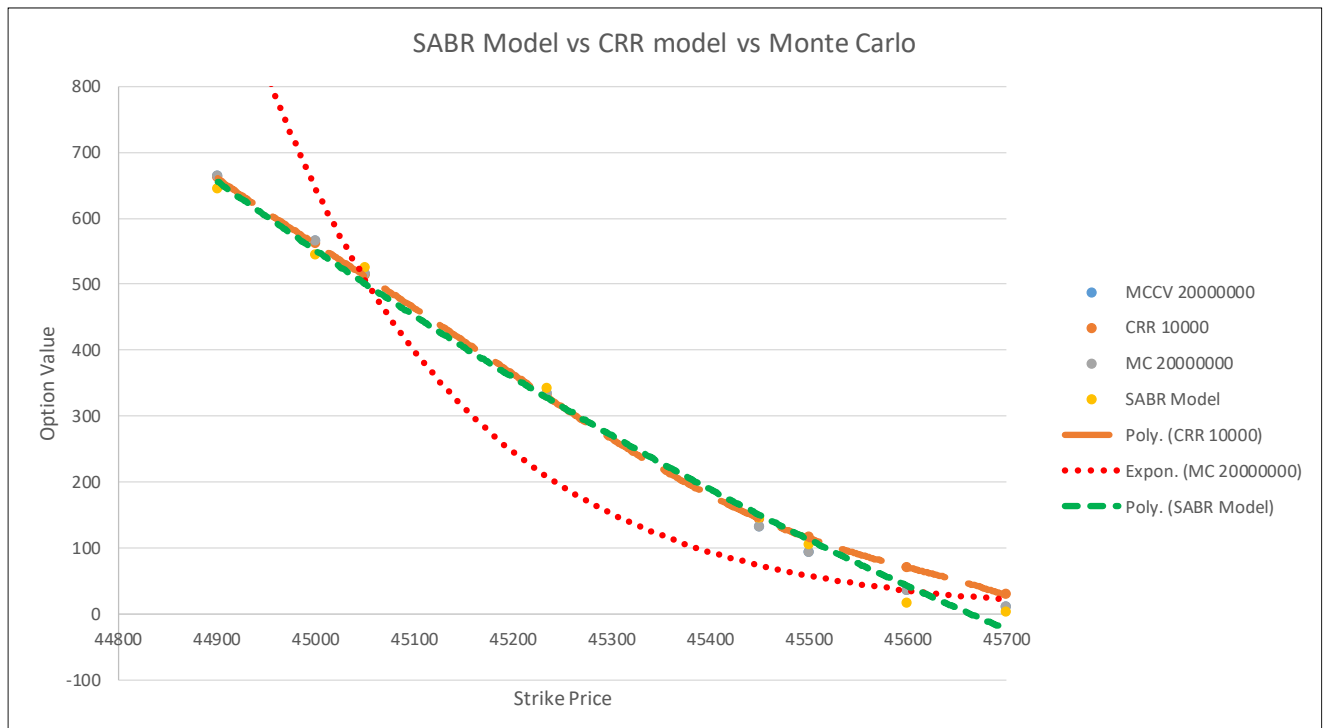


Figure 13: Comparison between the SABR model, CRR model and the Monte Carlo simulations

Figure 13 above compares the SABR pricing model to the Monte Carlo simulation as well as the Binomial model. A few more additional insights can be derived from Figure 13 and Table 21:

- When comparing the SABR model prices with that of the Monte Carlo simulation, the Monte Carlo model also tends to price below the SABR model at low strike prices while the SABR model then prices below the Monte Carlo simulation when the Asian call is at-the-money.
- As the strike prices increase, the Monte Carlo simulation then again prices below the SABR model simulations.
- Once again convergence occurs with all three models at extreme strike prices (far out-of-the-money options).
- When comparing the SABR model with the CRR model, the prices are relatively close when the option in question is deep-in-the-money as well as close to the at-the-money position.

- The CRR model thereafter, as strike prices increase, always produces a higher option price than the SABR model simulations.

No. of Simulations	SABR Model
	Time in Seconds
100	0,05173
250	0,13128
500	0,28623
1,000	0,44334
5,000	1,94281
10,000	3,91531
50,000	19,30120
100,000	37,94696
500,000	239,74269
1,000,000	637,03452

*Table 22: SABR model run times*

Table 22 above shows the run times for the Monte Carlo SABR model for a various number of simulations. Not only does the stochastic element that is introduced in the variance process significantly slows down the simulation time, but the structure in which this stochastic element is introduced also has a significant impact. While the Monte Carlo simulations were shown to be much faster, the SABR model runs much slower than the Heston model. With 1,000,000 simulation paths of the Heston model taking just over 26 seconds, the SABR model took just less than 240 seconds for 500,000 paths and more than 637 seconds for 1,000,000 paths.

## 5.7 THE IMPORTANCE OF MEAN-REVERTING VOLATILITY

In the paper by Engle & Patton, (2001), the main criteria for a volatility model is that it “should be able to forecast volatility” (p. 238). This essentially means that a good model should capture the mean-reverting feature of volatility. Without a mean-reversion factor, this can allow for the “possibility that volatility in stock markets will increase in the long run without bound” (Goudarzi, 2013, p.1689). In a study by Kotze & Joseph, (2009), on the South African market, mean reversion in volatility is particularly evident. This makes the feature of volatility mean reversion particularly important in the modelling of volatility. The shocks to prices tend to be temporary thus increasing predictability with a reversion factor (Goudarzi, 2013) thus favouring the Heston model more.

## 6. CONCLUSION, RECOMMENDATIONS AND FURTHER STUDY

This study embarked on answering the question of pricing accuracy of Asian options on the JSE “Can-Do” platform. In exploring this issue, improvements in pricing can not only lead to growth in the very infant-like market of exotic options in South Africa, better hedging and decision making, but also can lead to social benefits such as market transparency and improved investor confidence. The journey began with a review of pricing models that could be used to price these options including the Black-Scholes-Merton formula, the binomial model, finite difference methods, Monte Carlo simulation and the Fast Fourier transform.

A few of the common methods used in industry were then explored further, by running pricing models on South African data. In particular, the Binomial or CRR model, Monte Carlo simulations and Monte Carlo simulations with control variate methods were run. These were then compared to the Curran approximation currently used by the JSE to price Asian options. It was found that each model had various levels of complication, pricing accuracy and time to run to obtain a price for each option. Accuracy was the main focus area when comparing these results with the Curran Approximation prices.

It was found that for deep-in-the-money options the Curran approximation priced above the prices obtained from both Monte Carlo simulation as well as the Binomial models. However, the degree of difference is at its smallest at this point over the strike price spectrum. As the option moves to a position of at-the-money, the Curran approximation remains above the price obtained from other models tested. However, the difference between the Monte Carlo simulation with 20,000,000 simulation paths is much larger. This difference increases as the strike price increase and the call option moves further out-of-the-money. However, for far out-of-the-money options, all models converge at a zero value.

While these models made the comparison and assessed under- and over-pricing, the key assumption still remained that volatility remained constant. The research then went on to improve on the models by including a stochastic volatility factor. Two stochastic volatility models were investigated. This included the Heston model and the SABR model. These models were calibrated for their various parameters according to market data and then the fit was assessed through a mean-square error calculation. Once this was completed, the stochastic

volatility models were then simulated with Monte Carlo models to assess differences in prices. The Heston model showed large deviations from the Curran approximation due to changes in the volatility calculated but captures the volatility of option price movement adequately. The SABR model captures prices that are far closer to the Curran approximation but may not appear to be realistic.

Much like the conclusion of Wiklund, (2012), the Curran approximation performs well in pricing Asian options under certain conditions. The Curran approximation performs particularly well in pricing when in a low volatility environment. When the volatility of the market is higher, then more mispricing occurs. Taking the Monte Carlo simulation with 20,000,000 simulation paths as the benchmark for the study, the Curran approximation always overprices Asian options in South Africa, with the degree of error increasing as the option moves out-of-the-money. When options are close to the position of at-the-money, there is greater volatility and so more mispricing.

When assessing the Curran approximation against the stochastic volatility models, the SABR model was always outpriced by the Curran model. However, the Heston model provided some key insights. For options deep-in-the-money, the Heston model showed that the Curran model overpriced options. As the option moves towards an at-the-money position, the Curran approximation continues to overprice. When an option is at-the-money, the price difference is minimal. However, once the option moves out-of-the-money, the Curran approximation underprices the option. The further out-of-the-money the option becomes, the closer the prices then become with the two models. When the option is deep-out-of-the-money, both the Curran and Heston models converge on zero value.

In light of the above, the Monte Carlo simulation of the Heston model provides the most accurate prices for the South African market. Not only does it provide the best fit to the volatility implied by the options currently traded on the JSE as depicted by Figure 6, but this model also captures the mean-reverting nature of the volatility in South Africa. With the volatility experienced in the financial markets in South Africa with changes in finance ministers, allegations of corruption, questionable political decisions (Mavee, Perrelli, & Schimmelpfennig, 2016), the Curran model is not well suited to the environment. Wiklund

(2012), on the Curran approximation, “As for the Asian approximation formula, these over-and underestimates are very large for OTM options and decreases for ATM and ITM options.” (Wiklund, 2012, p. i). This has been confirmed in this study. It remains though that volatility is a major factor to be considered in the South African environment.

As mentioned previously, the study was limited by the availability of Asian options trading on the JSE. The “Can-do” platform on the JSE allows for trade of exotic options, however, volumes traded and frequency of trades are small. Notional options were created in this research which may have hampered the true price process that is used on the JSE i.e transactional costs, frictional costs and taxes were not taken into account in the modelling of the price of such options.

While times were recorded, all methods of calculating option prices were not investigated. An example of this is the investigation of the Fast Fourier Transform which has the reputation of being a very fast and accurate method of pricing options. Hence in order to answer the question of efficiency, not all possibilities were taken into account. Efficiency in Matlab programming was not dealt with as the main focus was accuracy of the pricing results in this research. Code programmed into Matlab could have been optimised for faster running.

It is hoped that this research will help in improving the pricing methodologies used by the JSE and by practitioners alike. The social benefit of improved pricing cannot be stressed enough. However, through improved pricing, there would arise improved investor confidence, market transparency and a stimulated market. The cascading effect would mean even further improvements in pricing of these exotics as a substantial market would be created to a point at which a fair value can be observed in the market place.

It is therefore a key recommendation that, given the current situation of the market, with companies and individuals approaching the JSE to create Asian options, that more accurate pricing methods be used that are suitable for the volatile South African market. Trades of the Asian options are not frequent and thus time can be used to obtain a more accurate price. Monte Carlo methods and stochastic volatility methods like the Heston model should be incorporated into the pricing of such options. The case of South Africa allows for time to be taken in the

pricing of Asian options as there is no immediate need for instantaneous prices given the infancy of the market.

Future research in this area is warranted for South Africa. The South African market has its own nuances which need to be explored further. Examples of these include increased financial regulation such as the introduction of Twin Peaks as well as Basel 3 in the banking sector and Solvency Assessment and Management in the Insurance sector (PwC, 2015). The increasing regulation has increased the focus on risk management within the industry and the proper protection of the investor. However, the issue of efficiency still needs to be answered. The FFT is one of those areas that needs to be expanded upon for its processing speed and accuracy. Timing and accuracy of pricing can then be fully interrogated and advised upon. In addition to this, Matlab was chosen to run simulations due to its power and its use in the industry. However, alternate programming languages should also be explored for simplicity, speed and accuracy in the future.

It is hoped that this research will now provide a basis for future research in pricing accuracy and efficiency. It is hoped that practitioners will use the methods recommended to obtain prices that reflect greater information in the market and help to drive this within the industry. With these recommended methods comes a new area of research and the risk management of using such methods. Model governance, control processes and general risk management should also be explored in this regard so that manipulation, mistakes and errors are limited. Decision making models such as those recommended, need to maintain their integrity in order for proper thought processes and the correct decisions to be made. Market participants should be protected in order to grow investor confidence and stimulate this market.

With the adoption of the recommended methods into the market place, arbitrage will be a reality as early adopters make use of the models. However, arbitrage drives more efficient markets as more participants start to adopt these methodologies. However, a model can only be as good as its calibration. Over-calibration can also lead to spurious results. Models are depictions of reality but not all of reality can be captured. With additional factors being added to produce more accurate results, the risk is that the model can become irrelevant as per the opening

statement of this dissertation. Simpler models are not only easier to understand, but can be adapted to various situations. Models still help us make sense of the world.

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## APPENDIX A

The code below generates and plots equity paths based on a set of inputs based on the Monte Carlo by Control Variate method. This code is adapted from Goddard consulting (<http://www.goddardconsulting.ca/matlab-monte-carlo-assetpaths.html>)

```
% Script to price an Put and Call on an Asian option using
% the Monte-Carlo approach with the control variate variance
% reduction technique

% Define the underlying parameters
tic
S0 =45234;
X = 45234;
r = 0.07-0.0003;
sig = 0.01;
dt = 1/365;
etime = 100; % days to expiry
T = dt*etime;

nruns = 1000;

% Calculate the Black-Scholes price for the European Call and Put
% This assumes that the Financial Toolbox is available
[blsEC,blsEP] = blsprice(S0,X,r,T,sig)
% Price the above vanilla Put and Call using Monte-Carlo
```



```

% The European option is not path dependent so only one step is required
S = AssetPaths(S0,r,sig,T,1,nruns);
Sf = S(end,:); % The simulated prices at expiry
% Calculate the Monte-Carlo European Put and Call prices
mcEC = mean(max(Sf-X,0))*exp(-r*T)
S = AssetPaths(S0,r,sig,dt,etime,nruns);
CallPayoffT = max(mean(S)-X,0);
callPrice0 = mean(CallPayoffT)*exp(-r*T);
callPrice = callPrice0 + (blsEC - mcEC);
ElapsedTime= toc;
table(callPrice, ElapsedTime)

```

## APPENDIX B

The code below calculates the Asian option price via a user interface (Gryshkevych & Tashbulatov, 2010)

VBA code

```

'Calculates option price

Public Function curran(cp As Integer, S As Double, avs As Double, k As Double, t1 As Double, T As Double, n As Double, m As Double, r As Double, b As Double, v As Double)

'Function arguments:

'cp = call/put flag

'S      = asset price
'avs = historical average
'k      = strike
't1 = time between averaging points
'T      = time to maturity in years
'n      = number of averaging points
'm      = number of fixings
'r      = risk-free rate
'b      = cost of carry

```



```

'v      = volatility

Dim dt As Double, my As Double, myi As Double

Dim vxi As Double, vi As Double, vx As Double
Dim Km As Double, sum1 As Double, sum2 As Double Dim ti As Double, EA As
Double Dim i As Long

On Error Resume Next

'Time in days or years

If Sheets("asia").ComboBox3.Value = "Days" Then
t1 = (t1 / Sheets("asia").ComboBox4.Value) * T
T = T / Sheets("asia").ComboBox4.Value
End If

dt = (T - t1) / (n - 1)

If b = 0 Then

EA = S
Else
EA = S / n * Exp(b * t1) * (1 - Exp(b * dt * n)) / (1 - Exp(b * dt))
End If

If m > 0 Then
If avs > n / m * k Then
'put alue is 0
If cp = -1 Then
'put alue is 0
curran = 0
ElseIf cp = 1 Then
'exercise is certain for a call
avs = avs * m / n + EA * (n - m) / n
curran = (avs - k) * Exp(-r * T)
End If
GoTo Finish
'only one fixings left
ElseIf m = n - 1 Then
k = n * k - (n - 1) * avs

curran = GBlackScholes(cp, S, k, T, r, b, v) * 1 / n
GoTo Finish
End If

End If

If m > 0 Then

k = n / (n - m) * k - m / (n - m) * avs
End If

vx = v * Sqr(t1 + dt * (n - 1) * (2 * n - 1) / (6 * n))

my = Log(S) + (b - v * v * 0.5) * (t1 + (n - 1) * dt / 2)
sum1 = 0

```

```

'Calculating second term of a sum
For i = 1 To n
ti = dt * i + t1 - dt
vi = v * Sqr(t1 + (i - 1) * dt)
vxi = v * v * (t1 + dt * ((i - 1) - i * (i - 1) / (2 * n)))
myi = Log(S) + (b - v * v * 0.5) * ti
sum1 = sum1 + Exp(myi + vxi / (vx * vx) * (Log(k) - my) + (vi * vi - vxi *
vxi / (vx * vx)) * 0.5) Next i

Km = 2 * k - 1 / n * sum1

sum2 = 0
'Calculating second term of the sum
For i = 1 To n
ti = dt * i + t1 - dt
vi = v * Sqr(t1 + (i - 1) * dt)
vxi = v * v * (t1 + dt * ((i - 1) - i * (i - 1) / (2 * n)))
myi = Log(S) + (b - v * v * 0.5) * ti
sum2 = sum2 + Exp(myi + vi * vi * 0.5) * NormProb(cp * ((my - Log(Km)) / vx
+ vxi / vx)) Next i

'returning the value of the function (option price)

curran = Exp(-r * T) * cp * (1 / n * sum2 - k * NormProb(cp * (my -
Log(Km)) / vx)) * (n - m) / n

Finish:
End Function

'Abromowitz and Stegun approximation for the cumulative normal distribution
function Public Function NormProb(X As Double) As Double

Dim T As Double

Const b1 = 0.31938153
Const b2 = -0.356563782
Const b3 = 1.781477937
Const b4 = -1.821255978
Const b5 = 1.330274429
Const p = 0.2316419
Const c = 0.39894228

If X >= 0 Then

T = 1# / (1# + p * X)
NormProb = (1# - c * Exp(-X * X / 2#) * T * (T * (T * (T * (T * b5 + b4) +
b3) + b2) + b1)) Else

T = 1# / (1# - p * X)
NormProb = (c * Exp(-X * X / 2#) * T * (T * (T * (T * (T * b5 + b4) + b3) +
b2) + b1))
End If
End Function

'Generalized BlackScholes formula for call option

Public Function GBlackScholes(cp As Integer, S As Double, k As Double, T As
Double, r As Double, b As Double, v As Double) As Double

```

```

Dim d1 As Double, d2 As Double

d1 = (Log(S / X) + (b + v ^ 2 / 2) * T) / (v * Sqr(T))

d2 = d1 - v * Sqr(T)

If cp = 1 Then

GBlackScholes = S * Exp((b - r) * T) * NormProb(d1) - k * Exp(-r * T) *
NormProb(d2) ElseIf cp = -1 Then

GBlackScholes = k * Exp(-r * T) * NormProb(-d2) - S * Exp((b - r) * T) *
NormProb(-d1) End If

End Function

'Construction of graph

Sub graph()

Dim cp As Integer, S As Double, avs As Double, k As Double, t1 As Double, T
As Double, b As Double, r As Double, v As Double, n As Double, m As Double

'Initial input parameters

S = Sheets("asia").Cells(2, 2)
avs = Sheets("asia").Cells(3, 2)
k = Sheets("asia").Cells(4, 2) t1 = Sheets("asia").Cells(5, 2) T =
Sheets("asia").Cells(6, 2) n = Sheets("asia").Cells(7, 2) m =
Sheets("asia").Cells(8, 2) r = Sheets("asia").Cells(9, 2) b =
Sheets("asia").Cells(10, 2) v = Sheets("asia").Cells(11, 2) 'call or put

If Sheets("asia").ComboBox1.Value = "Call" Then cp = 1
Else cp = -1 End If

Sheets("didata").Cells.Clear

'initial values for diagram data

If Sheets("asia").Cells(29, 2) = "" Or Sheets("asia").Cells(30, 2) = "" Or
Sheets("asia").Cells(31, 2) = "" Then
MsgBox "Input all data, please"
GoTo Finish
End If

Start = Sheets("asia").Cells(29, 2)

endd = Sheets("asia").Cells(30, 2)
steps = Sheets("asia").Cells(31, 2)
buf = Start
ds = (endd - Start) / steps
For i = 1 To steps + 1
Sheets("didata").Cells(i, 1) = buf
buf = buf + ds
Next i

'select independent variable

```

```
Select Case Sheets("asia").ComboBox2.Value
```

```
Case "Risk-free rate"
```

```
For i = 1 To steps + 1
r = Sheets("didata").Cells(i, 1)
If Sheets("asia").CheckBox1.Value Then
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 2) = curran(1, S, avs, k, t1, T, n, m, r, b, v)
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 3) = curran(-1, S, avs, k, t1, T, n, m, r, b, v)
ElseIf cp = 1 Then
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 2) = curran(cp, S, avs, k, t1, T, n, m, r, b, v)
Else
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 3) = curran(cp, S, avs, k, t1, T, n, m, r, b, v)
End If
Next i
```

```
Case "Strike"
```

```
For i = 1 To steps + 1
k = Sheets("didata").Cells(i, 1)
If Sheets("asia").CheckBox1.Value Then
Sheets("didata").Cells(i, 2) = curran(1, S, avs, k, t1, T, n, m, r, b, v)
Sheets("didata").Cells(i, 3) = curran(-1, S, avs, k, t1, T, n, m, r, b, v)
ElseIf cp = 1 Then
Sheets("didata").Cells(i, 2) = curran(cp, S, avs, k, t1, T, n, m, r, b, v)
Else
Sheets("didata").Cells(i, 3) = curran(cp, S, avs, k, t1, T, n, m, r, b, v)
End If
Next i
```

```
Case "Cost of carry"
```

```
For i = 1 To steps + 1
b = Sheets("didata").Cells(i, 1)
If Sheets("asia").CheckBox1.Value Then
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 2) = curran(1, S, avs, k, t1, T, n, m, r, b, v)
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 3) = curran(-1, S, avs, k, t1, T, n, m, r, b, v)
ElseIf cp = 1 Then
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 2) = curran(cp, S, avs, k, t1, T, n, m, r, b, v)
Else
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 3) = curran(cp, S, avs, k, t1, T, n, m, r, b, v)
End If
Next i
```

```
Case "Volatility"
```

```
For i = 1 To steps + 1
v = Sheets("didata").Cells(i, 1)
If Sheets("asia").CheckBox1.Value Then
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 2) = curran(1, S, avs, k, t1, T, n, m, r, b, v)
```

```

k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 3) = curran(-1, S, avs, k, t1, T, n, m, r, b, v)
ElseIf cp = 1 Then
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 2) = curran(cp, S, avs, k, t1, T, n, m, r, b, v)
Else
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 3) = curran(cp, S, avs, k, t1, T, n, m, r, b, v)

End If
Next i

```

Case "Historical average"

```

For i = 1 To steps + 1
avs = Sheets("didata").Cells(i, 1)
If Sheets("asia").CheckBox1.Value Then
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 2) = curran(1, S, avs, k, t1, T, n, m, r, b, v)
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 3) = curran(-1, S, avs, k, t1, T, n, m, r, b, v)
ElseIf cp = 1 Then
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 2) = curran(cp, S, avs, k, t1, T, n, m, r, b, v)
Else
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 3) = curran(cp, S, avs, k, t1, T, n, m, r, b, v)
End If
Next i

```

Case "Asset price"

```

For i = 1 To steps + 1
S = Sheets("didata").Cells(i, 1)
If Sheets("asia").CheckBox1.Value Then
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 2) = curran(1, S, avs, k, t1, T, n, m, r, b, v)
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 3) = curran(-1, S, avs, k, t1, T, n, m, r, b, v)
ElseIf cp = 1 Then
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 2) = curran(cp, S, avs, k, t1, T, n, m, r, b, v)
Else
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 3) = curran(cp, S, avs, k, t1, T, n, m, r, b, v)
End If
Next i

```

Case "Number of m fixings"

```

For i = 1 To steps + 1
m = Sheets("didata").Cells(i, 1)
If Sheets("asia").CheckBox1.Value Then
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 2) = curran(1, S, avs, k, t1, T, n, m, r, b, v)
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 3) = curran(-1, S, avs, k, t1, T, n, m, r, b, v)
ElseIf cp = 1 Then
k = Sheets("asia").Cells(4, 2)

```

```

Sheets("didata").Cells(i, 2) = curran(cp, S, avs, k, t1, T, n, m, r, b, v)
Else
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 3) = curran(cp, S, avs, k, t1, T, n, m, r, b, v)
End If
Next i

```

Case "Number of n fixings"

```

For i = 1 To steps + 1
n = Sheets("didata").Cells(i, 1)
If Sheets("asia").CheckBox1.Value Then
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 2) = curran(1, S, avs, k, t1, T, n, m, r, b, v)
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 3) = curran(-1, S, avs, k, t1, T, n, m, r, b, v)

```

```

ElseIf cp = 1 Then
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 2) = curran(cp, S, avs, k, t1, T, n, m, r, b, v)
Else
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 3) = curran(cp, S, avs, k, t1, T, n, m, r, b, v)
End If
Next i

```

Case "Time to maturity"

```

For i = 1 To steps + 1
'T = Sheets("didata").Cells(i, 1)
If Sheets("asia").CheckBox1.Value Then
T = Sheets("didata").Cells(i, 1)
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 2) = curran(1, S, avs, k, t1, T, n, m, r, b, v)
T = Sheets("didata").Cells(i, 1)
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 3) = curran(-1, S, avs, k, t1, T, n, m, r, b, v)
ElseIf cp = 1 Then
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 2) = curran(cp, S, avs, k, t1, T, n, m, r, b, v)
Else
k = Sheets("asia").Cells(4, 2)
Sheets("didata").Cells(i, 3) = curran(cp, S, avs, k, t1, T, n, m, r, b, v)
End If

```

Next i

End Select

'estimate min & max for diagram scaling

```

If Sheets("asia").CheckBox1.Value Then
Maxc = Sheets("didata").Cells(1, 2)
minc = Sheets("didata").Cells(1, 2)
maxp = Sheets("didata").Cells(1, 2)
minp = Sheets("didata").Cells(1, 2)
For i = 1 To steps + 1
If Sheets("didata").Cells(i, 2) > Maxc Then
Maxc = Sheets("didata").Cells(i, 2)
ElseIf Sheets("didata").Cells(i, 2) < Min Then

```

```

minc = Sheets("didata").Cells(i, 2)
End If

If Sheets("didata").Cells(i, 3) > maxp Then

maxp = Sheets("didata").Cells(i, 3)
ElseIf Sheets("didata").Cells(i, 3) < minp Then
minp = Sheets("didata").Cells(i, 3)
End If
Next i

If maxp > Maxc Then

Max = maxp
Else
Max = Maxc
End If

If minp < minc Then

Min = minp
Else
Min = minc
End If
Else
If cp = 1 Then
Max = Sheets("didata").Cells(1, 2)
Min = Sheets("didata").Cells(1, 2)
For i = 1 To steps + 1
If Sheets("didata").Cells(i, 2) >
Max Then
Max = Sheets("didata").Cells(i, 2)
ElseIf Sheets("didata").Cells(i, 2)
< Min Then
Min = Sheets("didata").Cells(i, 2)
End If
Next i
Else
Max = Sheets("didata").Cells(1, 3)
Min = Sheets("didata").Cells(1, 3)
For i = 1 To steps + 1
If Sheets("didata").Cells(i, 3) >
Max Then
Max = Sheets("didata").Cells(i, 3)
ElseIf Sheets("didata").Cells(i, 3)
< Min Then
Min = Sheets("didata").Cells(i, 3)
End If
Next i
End If
End If
'create the diagram
Sheets("Diagram").Select
ActiveChart.ChartArea.Select
Selection.Clear

```

```

ActiveChart.ChartType = xlLine
ActiveChart.SeriesCollection.NewSeries
ActiveChart.SeriesCollection.NewSeries
ActiveChart.SeriesCollection(1).XValues = Range(Sheets("didata").Cells(1, Sheets("didata").Cells(steps + 1, 1)))
If Sheets("asia").CheckBox1.Value Then
ActiveChart.SeriesCollection(1).Values = Range(Sheets("didata").Cells(1, Sheets("didata").Cells(steps + 1, 2)))
ActiveChart.SeriesCollection(2).Values = Range(Sheets("didata").Cells(1, Sheets("didata").Cells(steps + 1, 3)))
'add legend

ActiveChart.SeriesCollection(1).Name = ""call""
ActiveChart.SeriesCollection(2).Name = ""put""
ActiveChart.HasLegend = True
ActiveChart.Legend.Select
Selection.Position = xlRight
Else
If cp = 1 Then
ActiveChart.SeriesCollection(1).Values = Range(Sheets("didata").Cells(1, 2), Sheets("didata").Cells(steps + 1, 2))
Else
ActiveChart.SeriesCollection(1).Values = Range(Sheets("didata").Cells(1, 3), Sheets("didata").Cells(steps + 1, 3))

End If

End If

With ActiveChart
.HasTitle = True
.ChartTitle.Characters.Text = "Price dynamic"
.Axes(xlCategory, xlPrimary).HasTitle = True
.Axes(xlCategory, xlPrimary).AxisTitle.Characters.Text = Sheets("asia").ComboBox2.Value
.Axes(xlValue, xlPrimary).HasTitle = True

.Axes(xlValue, xlPrimary).AxisTitle.Characters.Text = "Price"
With ActiveChart.Axes(xlValue)
.MinimumScale = Min
.MaximumScale = Max
.MinorUnitIsAuto = True
.MajorUnitIsAuto = True
.Crosses = xlAutomatic
.ReversePlotOrder = False
.ScaleType = xlLinear
.DisplayUnit = xlNone
End With

```



```

End With
'formatting chart
ActiveChart.Axes(xlValue).Select
With Selection.Border
.ColorIndex = 57
.Weight = xlMedium
.LineStyle = xlContinuous
End With
With Selection
.MajorTickMark = xlOutside
.MinorTickMark = xlNone
.TickLabelPosition = xlNextToAxis
End With
ActiveChart.Axes(xlCategory).Select
With Selection.Border
.ColorIndex = 57
.Weight = xlMedium
.LineStyle = xlContinuous
End With
With Selection
.MajorTickMark = xlOutside
.MinorTickMark = xlNone
.TickLabelPosition = xlNextToAxis
End With
ActiveChart.SeriesCollection(1).Select
With Selection.Border
.ColorIndex = 57
.Weight = xlThick
.LineStyle = xlContinuous
End With
With Selection
.MarkerBackgroundColorIndex = xlNone
.MarkerForegroundColorIndex = xlNone
.MarkerStyle = xlNone
.Smooth = False
.MarkerSize = 3
.Shadow = False
End With

```

```

If Sheets("asia").CheckBox1.Value Then

```

```

ActiveChart.SeriesCollection(2).Select
With Selection.Border
.ColorIndex = 57
.Weight = xlThick
.LineStyle = xlContinuous
End With
With Selection
.MarkerBackgroundColorIndex = xlNone
.MarkerForegroundColorIndex = xlNone
.MarkerStyle = xlNone
.Smooth = False

.MarkerSize = 3
.Shadow = False
End With
End If

```

```

ActiveChart.ChartArea.Select

With ActiveChart.Axes(xlCategory)
.HasMajorGridlines = True
.HasMinorGridlines = False
End With
With ActiveChart.Axes(xlValue)
.HasMajorGridlines = True
.HasMinorGridlines = False
End With
ActiveChart.Axes(xlCategory).MajorGridlines.Select
With Selection.Border
.ColorIndex = 57
.Weight = xlHairline
.LineStyle = xlDot
End With
ActiveChart.Axes(xlValue).MajorGridlines.Select
With Selection.Border
.ColorIndex = 57
.Weight = xlHairline
.LineStyle = xlDot
End With
ActiveChart.Axes(xlValue).Select
Selection.TickLabels.NumberFormat = "0.0000"
ActiveChart.Axes(xlCategory).Select
Selection.TickLabels.NumberFormat = "0.00"

```

```

Finish:
End Sub

```

## APPENDIX C

The code below generates a binomial tree and prices an Asian option on this. This code is adapted from Goddard consulting (<http://www.goddardconsulting.ca/matlab-monte-carlo-assetpaths.html>)

```

tic
RateSpec = intenvset('Rates', 0.07-0.0003, 'StartDates',...
'28-Sep-2016', 'EndDates', '28-Oct-2016')

% Define StockSpec with the underlying asset information
Sigma = 0.01;
AssetPrice = 45234;

StockSpec = stockspec(Sigma, AssetPrice);

% Define the Asian option
Settle = '28-September-2016';
ExerciseDates = '28-October-2016';
Strike = 45234;
OptSpec = 'call';

% Create the time specification of the tree
NPeriods = 100;
TreeValuationDate = '28-September-2016';

```

```

TreeMaturity = '28-October-2016';
TimeSpec = crrtimespec(TreeValuationDate, TreeMaturity, NPeriods);

% Build the tree
CRRTree = crrtree(StockSpec, RateSpec, TimeSpec);

% Price the European Asian option using the CRR lattice model.
% The function 'asianbycrr' computes prices of arithmetic and geometric
% Asian options.
AvgType = {'arithmetic'};
AmericanOpt = 0;
PriceCRR20 = asianbycrr(CRRTree, OptSpec, Strike, Settle, ExerciseDates,...
                        AmericanOpt, AvgType);
Elapsedtime= toc;
table(PriceCRR20, Elapsedtime)

```

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## APPENDIX D

The code below generates a Monte Carlo simulation and prices an Asian option on this. This code is adapted from Goddard consulting (<http://www.goddardconsulting.ca/matlab-monte-carlo-assetpaths.html>)

```
% Script to price an Asian option using a monte-carlo approach.
tic
S0 = 45234;          % Price of underlying today
X = 45234;          % Strike at expiry
mu = 0.07 - 0.0003;  % expected return
sig = 0.1;          % expected vol.
r = 0.07;           % Risk free rate
dt = 1/365;         % time steps
etime = 100;        % days to expiry
T = dt*etime;       % years to expiry

nruns = 1000000;    % Number of simulated paths

% Generate potential future asset paths
S = AssetPaths(S0, mu, sig, dt, etime, nruns);

% calculate the payoff for each path for a Put
PutPayoffT = max(X - mean(S), 0);

% calculate the payoff for each path for a Call
CallPayoffT = max(mean(S) - X, 0);

% discount back
putPrice = mean(PutPayoffT) * exp(-r*T)
callPrice = mean(CallPayoffT) * exp(-r*T);
ElapsedTime = toc;
table(callPrice, ElapsedTime)
```

## APPENDIX E

Options on ALSI index on the JSE on 28 September 2016:

Put or Call	Spot Price	Futures Price	Option Value	Strike Price	Days to Maturity	Rate
1	45234	45822	14694.895	31000	78	0.070
1	45234	45822	9772.823	36000	78	0.070
1	45234	45822	8789.660	37000	78	0.070
1	45234	45822	8151.131	37650	78	0.070
1	45234	45822	7807.498	38000	78	0.070
1	45234	45822	7611.207	38200	78	0.070
1	45234	45822	7218.806	38600	78	0.070
1	45234	45822	6826.645	39000	78	0.070
1	45234	45822	6679.663	39150	78	0.070
1	45234	45822	6141.089	39700	78	0.070
1	45234	45822	6043.254	39800	78	0.070
1	45234	45822	5945.398	39900	78	0.070
1	45234	45822	5896.521	39950	78	0.070
1	45234	45822	5847.613	40000	78	0.070
1	45234	45822	5066.181	40800	78	0.070
1	45234	45822	4871.121	41000	78	0.070
1	45234	45822	4822.363	41050	78	0.070
1	45234	45822	4481.510	41400	78	0.070
1	45234	45822	4384.204	41500	78	0.070
1	45234	45822	4092.639	41800	78	0.070
1	45234	45822	3898.468	42000	78	0.070
1	45234	45822	3655.990	42250	78	0.070
1	45234	45822	3510.727	42400	78	0.070
1	45234	45822	3413.892	42500	78	0.070
1	45234	45822	3317.107	42600	78	0.070
1	45234	45822	3220.441	42700	78	0.070
1	45234	45822	3123.776	42800	78	0.070
1	45234	45822	2978.983	42950	78	0.070
1	45234	45822	2930.686	43000	78	0.070
1	45234	45822	2834.271	43100	78	0.070
1	45234	45822	2545.275	43400	78	0.070
1	45234	45822	2449.120	43500	78	0.070
1	45234	45822	2256.959	43700	78	0.070
1	45234	45822	2208.991	43750	78	0.070
1	45234	45822	2161.034	43800	78	0.070
1	45234	45822	1969.473	44000	78	0.070
1	45234	45822	1873.778	44100	78	0.070
1	45234	45822	1778.163	44200	78	0.070
1	45234	45822	1396.542	44600	78	0.070
1	45234	45822	1348.944	44650	78	0.070

1	45234	45822	1301.357	44700	78	0.070
1	45234	45822	1111.266	44900	78	0.070
1	45234	45822	1016.351	45000	78	0.070
1	45234	45822	968.933	45050	78	0.070
1	45234	45822	684.938	45350	78	0.070
1	45234	45822	590.552	45450	78	0.070
1	45234	45822	543.365	45500	78	0.070
1	45234	45822	449.049	45600	78	0.070
1	45234	45822	354.834	45700	78	0.070
1	45234	45822	260.729	45800	78	0.070
1	45234	45822	213.721	45850	78	0.070
1	45234	45822	72.918	46000	78	0.070
1	45234	45822	26.021	46050	78	0.070

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## APPENDIX F

Code for Heston Model simulation by Monte Carlo methods.

```
%Code for Heston simulations
%Reference Fabrice : Heston and extensions book
tic
S(1)=45234;
X=45234;
r=0.07-0.0003;
dt=1/365;
etime = 100;
T = dt*etime;
k=13.0478;
theta=0.0093;
sigSig=0.0527;
timesteps=78;
v(1)=0.0934;
rho=-0.4555;
sims=1000000;
for j=1:sims
    Z1=randn;
    Z2=randn;
    Zv=Z1;
    Zs=rho*Z1+sqrt(1-rho^2)*Z2;
    for t=1:timesteps
        v(t+1)=abs(v(t))+k*(theta-abs(v(t)))*dt+sigSig*sqrt(abs(v(t))*dt)*Zv;
    %Euler discretisation scheme
        S(t+1)=S(t)+r*S(t)*dt+sqrt(abs(v(t+1))*dt)*S(t)*Zs;
    %Full truncation used to avoid negative variances
    end
    CallPayoffT = max(mean(S)-X,0);
    Callprice=mean(CallPayoffT)*exp(-r*T);
end
Elapsedtime = toc;
table(Callprice,Elapsedtime)
```

## APPENDIX G

Code for SABR Model simulation by Monte Carlo methods.

```
%Code for Heston simulations
%Reference Fabrice : Heston and extensions book
tic
S(1)=45234;
X=45234;
r=0.07-0.0003;
dt=1/365;
etime = 78;
T = dt*etime;
a(1)=2.9991;
beta=0.6009;
sigSig=2;
timesteps=78;
v=3.000;
rho=-0.7923;
sims=1000000;
for j=1:sims
    Z1=randn;
    Z2=randn;
    Zv=Z1;
    Zs=rho*Z1+sqrt(1-rho^2)*Z2;
    for t=1:timesteps
        a(t+1)=a(t)+v*a(t)*Zs*sqrt(dt); %Milstein discretisation scheme
        S(t+1)=S(t)+r*S(t)*dt+a(t)*exp(r*(beta-1)*T)*(S(t)^beta)*sqrt(dt)*Z1;
    %Full truncation used to avoid negative variances
        CallPayoffT(j) = max(mean(S)-X,0);
    end
    Callprice=mean(CallPayoffT)*exp(-r*T);
end
Elapsedtime = toc;
table(Callprice,Elapsedtime)
```