Epidemic Control with Learning & Optimization

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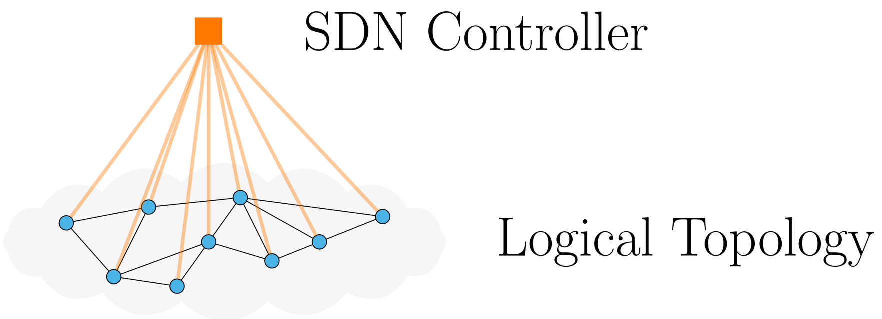
Network security

- Networked systems face propagation of malware, cascading hardware failures, DDoS.
- Software-defined networking enables full automated control over network topology.

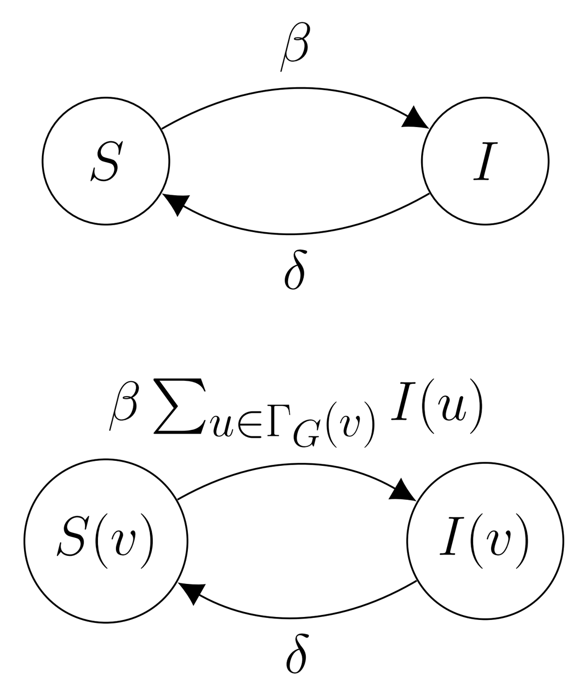


Model

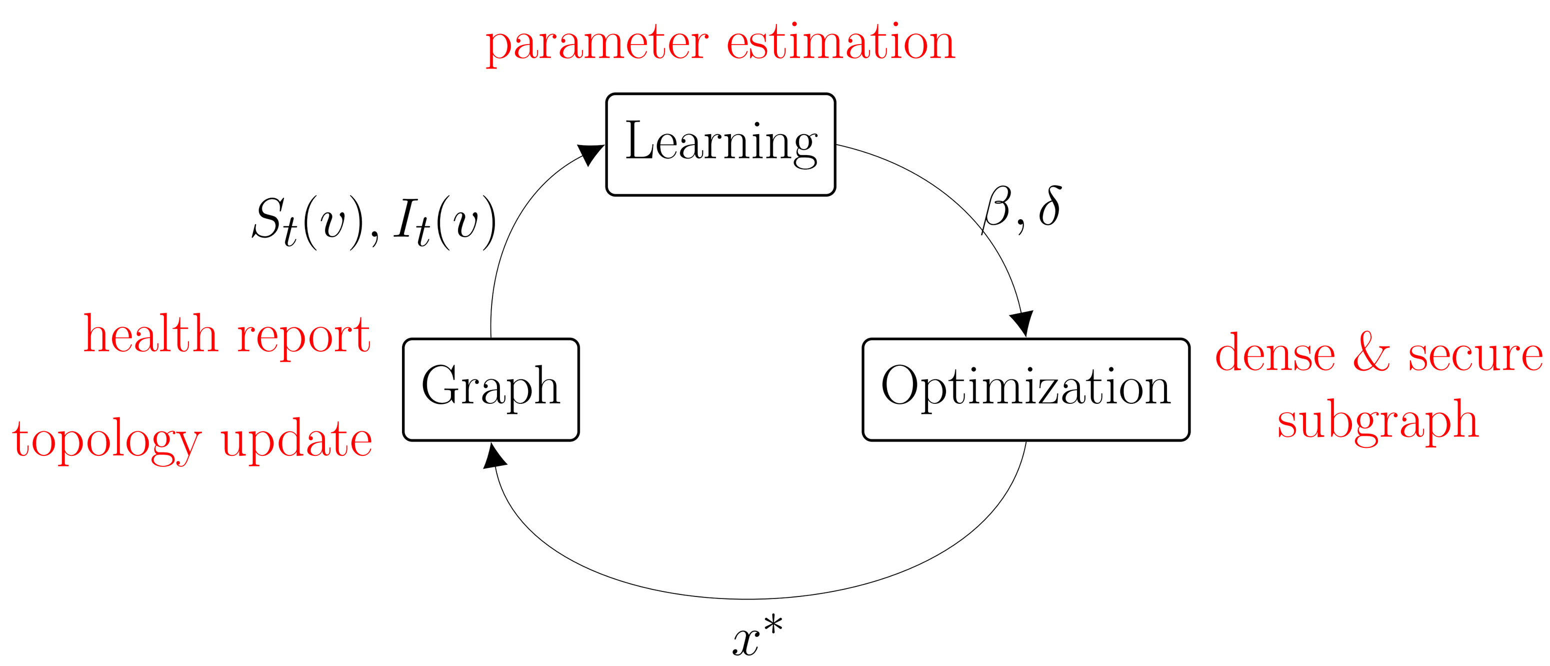
Base: Epidemic models can be used to represent propagating threats: each health status corresponds to a compartment (e.g.: S for susceptible, I for infected).

Refinement: Standard compartmental models may be refined with network structure: underlying topology is given by an undirected graph $G = (V, E)$.

Result: The Markov process has $|\{S, I\}|^{|V|}$ states and the transition rates for a node depend on the state of its neighbours. The model parameters are β and δ.



Turning a theorem into a control system



Definition (Spectral radius)
The spectral radius of a graph is the largest eigenvalue of its adjacency matrix and satisfies:

$$\frac{1}{n} \sum_{v \in V} \deg_G(v) \leq \lambda_{\max}(G) \leq \max_{v \in V} \deg_G(v).$$

Theorem (Ganesh et al., 2005)
Given a SIS epidemic with parameters β and δ on a graph G:

$$\lambda_{\max}(G) < \frac{\delta}{\beta}$$

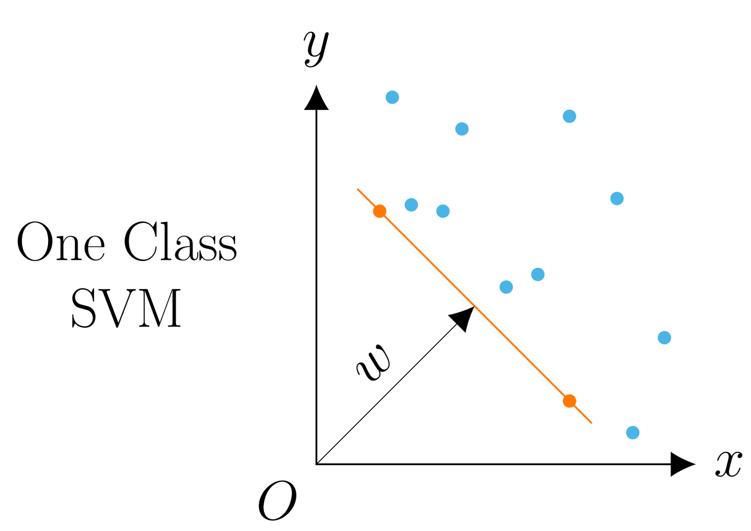
implies that the epidemic dies out in time $O(\log n)$.

Know your enemy: learning epidemic parameters

Anomaly detection

- Each node determines its health status by learning.
- One Class SVMs are a family of classifiers used for anomaly detection.
- A OC-SVM is trained on "healthy" data only: the system does not require prior experience of the epidemic to come.

Maximizing the margin w.r.t. the origin



Parameter estimation

Input: Time series of node health data.

Model: SIS model with unknown parameters β and δ.

Estimate: β and δ.

Approximation algorithms for the secure subgraph problem

Closed walks

- Norm inequalities give:

$$\lambda_{\max}(A) = O(||A||_{\log n}).$$

- The number of closed walks of length k is $||A||_k^k$.
- Find subgraph with few closed walks of length $\log n$.

Mathematical program

$$\max \sum_{e \in E} x_e$$
$$\sum_{e \in E} x_e A_e \preceq \delta / \beta I$$
$$x \in \{0,1\}^m$$

- A_e is the adjacency matrix of edge e.

SDP and random matrices

- Continuous relaxation of the mathematical program gives a SDP.
- Optimal solution x^* used as a distribution.
- Leverage concentration of measure for symmetric random matrices.

Interlacing polynomials

- Polynomial-valued r.v.s related to the characteristic polynomial of a graph.
- Undirected graphs have real roots: is it a rare property?
- Bounding the spectral radius by bounding roots.