Appendix: Butcher Tableau of IMEX-RK

$$
\begin{array}{c|cc}
0 & 0 & 0 \\
1 & 1 & 0 \\
\hline
 & 1/2 & 1/2
\end{array}
\qquad
\begin{array}{c|cc}
\gamma & \gamma & 0 \\
1-\gamma & 1-2\gamma & \gamma \\
\hline
 & 1/2 & 1/2
\end{array}
\qquad
\gamma = 1 - \frac{1}{\sqrt{2}}
$$

Table A.1: Tableau for the explicit (left) implicit (right) IMEX-SSP2(2,2,2) L-stable scheme.

$$
\begin{array}{c|ccc}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 \\
\hline
 & 0 & 1/2 & 1/2
\end{array}
\qquad
\begin{array}{c|ccc}
1/2 & 1/2 & 0 & 0 \\
0 & -1/2 & 1/2 & 0 \\
1 & 0 & 1/2 & 1/2 \\
\hline
 & 0 & 1/2 & 1/2
\end{array}
$$

Table A.2: Tableau for the explicit (left) implicit (right) IMEX-SSP2(3,2,2) stiffly accurate scheme.

$$
\begin{array}{c|ccc}
0 & 0 & 0 & 0 \\
1/2 & 1/2 & 0 & 0 \\
1 & 1/2 & 1/2 & 0 \\
\hline
 & 1/3 & 1/3 & 1/3
\end{array}
\qquad
\begin{array}{c|ccc}
1/4 & 1/4 & 0 & 0 \\
1/4 & 0 & 1/4 & 0 \\
1 & 1/3 & 1/3 & 1/3 \\
\hline
 & 1/3 & 1/3 & 1/3
\end{array}
$$

Table A.3: Tableau for the explicit (left) implicit (right) IMEX-SSP2(3,3,2) stiffly accurate scheme.

$$
\begin{array}{c|ccc}
0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 \\
1/2 & 1/4 & 1/4 & 0 \\
\hline
 & 1/6 & 1/6 & 2/3
\end{array}
\qquad
\begin{array}{c|ccc}
\gamma & \gamma & 0 & 0 \\
1-\gamma & 1-2\gamma & \gamma & 0 \\
1/2 & 1/2-\gamma & 0 & \gamma \\
\hline
 & 1/6 & 1/6 & 2/3
\end{array}
\qquad
\gamma = 1 - \frac{1}{\sqrt{2}}
$$

Table A.4: Tableau for the explicit (left) implicit (right) IMEX-SSP3(3,3,2) L-stable scheme.

$$
\begin{array}{c|cccc}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 \\
1/2 & 0 & 1/4 & 1/4 & 0 \\
\hline
 & 0 & 1/6 & 1/6 & 2/3
\end{array}
\qquad
\begin{array}{c|cccc}
\alpha & \alpha & 0 & 0 & 0 \\
0 & -\alpha & \alpha & 0 & 0 \\
1 & 0 & 1-\alpha & \alpha & 0 \\
1/2 & \beta & \eta & 1/2-\beta-\eta-\alpha & \alpha \\
\hline
 & 0 & 1/6 & 1/6 & 2/3
\end{array}
$$

$$\alpha = 0.24169426078821, \quad \beta = 0.06042356519705 \quad \eta = 0.12915286960590$$

Table A.5: Tableau for the explicit (left) implicit (right) IMEX-SSP3(4,3,3) L-stable scheme.

Bibliography

[1] A.M. Anile and S. Pennisi, Thermodynamic derivation of the hydrodynamical model for charge transport in semiconductors. *Phys. Rev B* **46** (1992), 13186–13193.

[2] A.M. Anile and O. Muscato, Improved hydrodynamical model for carrier transport in semiconductors. *Phys. Rev B* **51** (1995), 16728–16740.

[3] M. Arora and P.L. Roe, Issues and strategies for hyperbolic problems with stiff source terms. In: *Barriers and challenges in computational fluid dynamics*, Hampton, VA, 1996, Kluwer Acad. Publ., Dordrecht, (1998), 139–154.

[4] U. Ascher and L. Petzold, *Computer Methods for Ordinary Differential Equations, and Differential Algebraic Equations.* SIAM, Philadelphia, 1998.

[5] U. Ascher, S. Ruuth and R.J. Spiteri, Implicit-explicit Runge-Kutta methods for time dependent Partial Differential Equations. *Appl. Numer. Math.* **25** (1997), 151–167.

[6] A.Aw and M. Rascle, Resurrection of second order models of traffic flow. *SIAM. J. Appl. Math.* **60** (2000), 916–938.

[7] A. Aw, A. Klar, T. Materne and M. Rascle, Derivation of continuum traffic flow models from microscopic follow the leader models. *SIAM J. Appl. Math.* **63** (2002), 259–278.

[8] F. Bianco, G. Puppo and G. Russo, High Order Central Schemes for Hyperbolic Systems of Conservation Laws. *SIAM J. Sci. Comp.* **21**, (1999), 294–322.

[9] S. Boscarino, Error analysis of IMEX Runge Kutta methods derived from Differential-Algebraic systems. *SIAM J. Numer. Anal.* **45**, (2007), 1600–1621.

[10] J.E. Broadwell, Shock structure in a simple discrete velocity gas. *Phys. Fluids* **7** (1964), 1013–1037.

[11] S. Chapman and T.G. Cowling, *The mathematical theory of nonuniform gases.* Cambridge University Press, London, 1960.

[12] R.E. Caflisch, S. Jin and G. Russo, Uniformly accurate schemes for hyperbolic systems with relaxation, *SIAM J. Numer. Anal.* **34** (1997), 246–281.

[13] G.Q. Chen, D. Levermore and T.P. Liu, Hyperbolic conservations laws with stiff relaxation terms and entropy, *Comm. Pure Appl. Math.* **47** (1994), 787–830.

[14] B. Cockburn, C. Johnson, C.-W. Shu and E. Tadmor, *Advanced Numerical Approximation of Nonlinear Hyperbolic Equations*. Lecture Notes in Mathematics (editor: A. Quarteroni), Springer, Berlin, 1998.

[15] R. Courant and K.O. Friedrichs, *Supersonic flow and shock waves*. Applied Mathematical Sciences, 21, New York, 1976.

[16] C. Dafermos, *Hyperbolic Conservation Laws in Continuum Physics*, Second Edition. Springer, Berlin–Heidelberg, 2005.

[17] B.O. Dia and M. Schatzman, Commutateur de certains semi-groupes holomorphes et applications aux directions alternées. *Mathematical Modelling and Numerical Analysis* **30** (1996), 343–383.

[18] B. Engquist and S. Osher, One sided difference approximations for nonlinear conservation laws. *Math. Comp.* **36** (1981), 321–351.

[19] K.O. Friedrichs and P.D. Lax, System of Conservation Laws With a Convex extension. *Proc. Nat. Acad. Sci. USA* **68** (1971), 1686–1688.

[20] R. Gatignol, *Théorie cinétique des gaz à répartition discrète de vitesses*. Lecture Notes in Physics, vol. 36, Springer, Berlin–New York, 1975.

[21] E. Godlewski, and P.-A. Raviart, *Numerical approximation of hyperbolic systems of conservation laws*. Applied Mathematical Sciences, 118, Springer, New York, 1996.

[22] S. Gottlieb and C.-W. Shu, Total Variation Diminishing Runge-Kutta schemes. *Math. Comp.* **67** (1998), 73–85.

[23] S. Gottlieb, C.-W. Shu and E. Tadmor, Strong-stability-preserving high order time discretization methods. *SIAM Reviews* **43** (2001), 89–112.

[24] E. Hairer and G. Wanner, *Solving ordinary differential equations*, Vol. 2: *Stiff and differential-algebraic problems*. Springer, New York, 1987.

[25] A. Harten, B. Engquist, S. Osher and S. Chakravarthy, Uniformly High Order Accurate Essentially Non-oscillatory Schemes III. *J. Comput. Phys.* **71** (1987), 231–303.

[26] A. Harten, P.D. Lax and B. van Leer, On upstream differencing and Godunov-type schemes for hyperbolic conservation laws. *SIAM Rev.* **25** (1983), 35–61.

[27] T. Jahnke and C. Lubich, *Error bounds for exponential operator splitting.* BIT, 2000, 735–744.

[28] G.-S. Jiang and C.-W. Shu, Efficient Implementation of Weighted ENO Schemes. *JCP* **126** (1996), 202–228.

[29] S. Jin, Runge-Kutta methods for hyperbolic systems with stiff relaxation terms. *J. Comp. Phys.* **122** (1995), 51–67.

[30] S. Jin and Z.P. Xin, The relaxation schemes for systems of conservation laws in arbitrary space dimensions. *Comm. Pure Appl. Math.* **48** no. 3 (1995), 235–276.

[31] C.A. Kennedy and M.H. Carpenter, Additive Runge-Kutta schemes for convection-diffusion-reaction equations. *Appl. Numer. Math.* **44** (2003), 139–181.

[32] R. Kupferman, A numerical study of the axisymmetric Couette-Taylor problem using a fast high-resolution second-order central scheme. *SIAM Journal on Scientific Computing* **20** (1998), 858–877.

[33] A. Kurganov and E. Tadmor, New High-Resolution Central Schemes for Nonlinear Conservation Laws and Convection-Diffusion Equations. *J. Comput. Phys.* **160** (2000), 214–282.

[34] A. Kurganov, S. Noelle and G.Petrova, Semidiscrete central-upwind schemes for hyperbolic conservation laws and Hamilton-Jacobi equations. *SIAM J. Sci. Comput.* **23** (2001), 707–740 .

[35] P.D. Lax, Weak Solutions of Non-Linear Hyperbolic Equations and Their Numerical Computation. *CPAM* **7** (1954), 159–193.

[36] P.D.Lax, *Hyperbolic systems of conservation laws and the mathematical theory of shock waves.* SIAM, Philadelphia, 1973.

[37] P.D. Lax and B. Wendroff, *Communications on Pure and Applied Mathematics*, 1960.

[38] R.J. LeVeque, *Numerical Methods for Conservation Laws.* Second edition. Lectures in Mathematics ETH Zürich, Birkhäuser, Basel, 1992.

[39] R.J. LeVeque, *Finite Volume methods for Hyperbolic Problems.* Cambridge University Press, 2002.

[40] D. Levy, G. Puppo and G. Russo, Central WENO Schemes for Hyperbolic Systems of Conservation Laws. *Math. Model. and Numer. Anal.* **33** no. 3 (1999), 547–571.

[41] D. Levy, G. Puppo and G. Russo, Compact Central WENO Schemes for Multidimensional Conservation Laws. *SIAM J. Sci. Comp.* **22** (2000), 656–672.

[42] S.F. Liotta, V. Romano and G. Russo, Central schemes for balance laws of relaxation type. *SIAM J. Numer. Anal.* **38** no. 4 (2000), 1337–1356.

[43] A. Marquina, Local piecewise hyperbolic reconstructions for nonlinear scalar conservation laws. *SIAM J. Sci. Comput.* **15** (1994), 892–915.

[44] I. Müller and T. Ruggeri, *Rational extended thermodynamics.* Springer, Berlin, 1998.

[45] H. Nessyahu and E. Tadmor, Non-oscillatory Central Differencing for Hyperbolic Conservation Laws. *J. Comput. Phys.* **87** no. 2 (1990), 408–463.

[46] L. Pareschi, Central differencing based numerical schemes for hyperbolic conservation laws with stiff relaxation terms. *SIAM J. Num. Anal.* **39** (2001), 1395–1417.

[47] L. Pareschi and G. Russo, Implicit-Explicit Runge-Kutta Schemes for Stiff Systems of Differential Equations. In: *Recent Trends in Numerical Analysis* (D. Trigiante, Ed.). Nova Science Publ., 2000, 269–288.

[48] L. Pareschi and G. Russo, High order asymptotically strong-stability-preserving methods for hyperbolic systems with stiff relaxation. In: *Proceedings HYP2002.* Pasadena USA, Springer, 2003, 241–255.

[49] L. Pareschi and G. Russo, Implicit-Explicit Runge-Kutta schemes and applications to hyperbolic systems with relaxation. *J. Sci. Comput.* **25** no. 1–2 (2005), 129–155.

[50] L. Pareschi, G. Puppo and G. Russo, Central Runge-Kutta schemes for conservation laws. *SIAM J. Sci. Comput.* **26** no. 3 (2005), 979–999.

[51] G. Puppo, Adaptive application of characteristic projection for central schemes. To appear in the proceedings of the HYP2002 conference.

[52] J. Qiu and C.-W. Shu, On the construction, comparison, and local characteristic decomposition for high-order central WENO schemes. *J. Comput. Phys.* **183** no. 1 (2002), 187–209.

[53] P.L. Roe, Approximate Riemann Solvers, Parameter Vectors, and Difference Schemes. *JCP* **43** (1981), 357–372.

[54] G. Russo, Central schemes and systems of balance laws. In: *Hyperbolic Partial Differential Equations, Theory, Numerics and Applications.* Vieweg, 2002.

[55] R. Sanders and W. Weiser, High resolution staggered mesh approach for nonlinear hyperbolic systems of conservation laws. *J. Comput. Phys.* **10** (1992), 314.

[56] C.-W. Shu, Essentially Non-Oscillatory and Weighted Essentially Non-Oscillatory Schemes for Hyperbolic Conservation Laws. In: *Advanced Numerical Approximation of Nonlinear Hyperbolic Equations.* Lecture Notes in Mathematics 1697 (editor: A. Quarteroni), Springer, Berlin, 1998.

[57] C.-W. Shu, Total variation diminishing time discretizations. *SIAM J. Sci. Stat. Comput.* **9** (1988), 1073–1084.

[58] C.-W. Shu and S. Osher, Efficient implementation of essentially nonoscillatory shock-capturing schemes. *J. Comput. Phys.* **77** no. 2 (1988), 439–471.

[59] C.-W. Shu and S. Osher, Efficient Implementation of Essentially Non-Oscillatory Shock-Capturing Schemes, II. *JCP* **83** (1989), 32–78.

[60] J. Shi, C. Hu and C.-W. Shu, A technique of treating negative weights in WENO schemes, *Journal of Computational Physics* **175** (2002) 108–127.

[61] R.J. Spiteri and S.J. Ruuth, A new class of optimal strong-stability-preserving time discretization methods. *SIAM. J. Num. Anal.* **40** no. 2 (2002), 469–491.

[62] G. Strang, On the construction and comparison of difference schemes. *SIAM J. Numer. Anal.* **5** (1968), 505–517.

[63] E. Tadmor, Approximate Solutions of Nonlinear Conservation Laws. In: Cockburn, Johnson, Shu and Tadmor Eds., Lecture Notes in Mathematics 1697, Springer, 1998.

[64] G.B. Whitham, *Linear and Nonlinear Waves.* Wiley, 1974.

[65] X. Zhong, Additive Semi-Implicit Runge-Kutta methods for computing high speed nonequilibrium reactive flows. *J. Comp. Phys.* **128** (1996), 19–31.