01TXCSM: Assignment Linear Programming Model Problem Formulation

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Sets and Matrices

E	set of exams
N	adjacency matrix of conflicting exams graph
A	unweighted adjacency matrix of conflicting exams graph
P	vector containing exponential contributes relative to penalty

$$n_{ij} \in N = \begin{cases} \text{n} & \text{if } i\text{th and } j\text{th exams are in conflict with weight n and } i \neq j \\ 0 & \text{otherwise} \end{cases}$$

$$a_{ij} \in A = \begin{cases} 1 & \text{if } i\text{th and } j\text{th exams are in conflict and } i \neq j \\ 0 & \text{otherwise} \end{cases}$$

$$p_d \in P = \begin{cases} 2^{5-d} & \text{if } 1 \leq d \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

Parameters and Indexes

e	cardinality of set E
d	distance between time slots of conflicting exams
t_{max}	number of time slots available

$1 \leq i \leq e$	$i \in \mathbb{N}$
$1 \leq j \leq e$	$j \in \mathbb{N}$
$1 \leq k \leq t_{max}$	$k \in \mathbb{N}$
$1 \leq d \leq 5$	$d \in \mathbb{N}$

Variables

$$X_{ik} = \begin{cases} 1 & \text{if the } i\text{th exam is scheduled in } k\text{th time slot} \\ 0 & \text{otherwise} \end{cases}$$

$$Y_{ijd} = \begin{cases} 1 & \text{if } \text{X}_{ik} = 1 \text{ and } X_{jk+d} = 1 \\ 0 & \text{otherwise} \end{cases}$$

Constraints

$$a_{ij}(X_{ik} + X_{jk}) \leq 1 \qquad \forall i, j, k \tag{1}$$

$$\sum_{k=1}^{t_{max}} X_{ik} = 1 \qquad \forall i \tag{2}$$

$$X_{ik} + X_{jk+d} \leq Y_{ijd} + 1 \qquad \forall i, j, k, d \text{ s.t. } a_{ij} \neq 0 \wedge k + d \leq t_{max} \tag{3}$$

Objective Function

$$\min \quad \sum_{i=1}^{e} \sum_{j=1}^{e} \sum_{d=1}^{5} Y_{ijd} p_d n_{ij} \tag{4}$$

Domains

$t_{max} \in \mathbb{N}$
$n_{ij} \in \mathbb{N}$
$a_{ij} \in \{0,1\}$
$p_d \in \mathbb{N}$
$X_{ik} \in \{0,1\}$
$Y_{ijd} \in \{0,1\}$

Model Explanation

$\boldsymbol{X}$: matrix encoding an exam for every line and a time slot for every column, elements take value 1 ('True') whenever our model predict the exam to be in the time slot related to that column.
$\boldsymbol{Y}$: auxiliary matrix useful to linearize constraint (6), which allows us to take into account the correct distance between exam's time slots.
(1): Conflicting exams cannot be scheduled in the same time slot.
(2): Every exam must be in one and one only time slot.
(3): Through this constraint Indicator $\boldsymbol{Y}$ is connected to the solutions. This is a logical constraint of the form $X_{ik} + X_{jk+d} \implies \mathrm{Y}_{ijd}$ assuming variable $\boldsymbol{Y}$ is initialized at 0 from the compiler by means of MIN objective function the opposite Y_{ijd} $\implies \mathrm{X}_{ik} + X_{jk+d}$ is assumed verified.
(4): Objective function only minimize penalty P_d only when two conflicting exams are closer than 5 or less time slots (i.e $Y_{ijd} = 1$).