

Independent lazy better-response dynamics on network games

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Game Theory



Altruism



Rational Behavior
(Selfish)

Game Theory



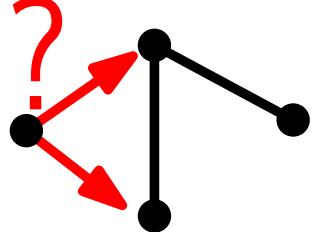
UNREALISTIC
Altruism



Rational Behavior
(Selfish)

The Internet

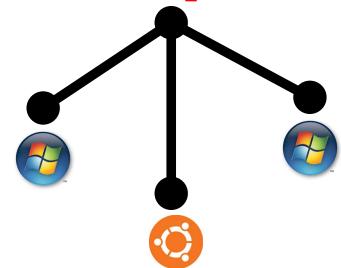
Build a Network:



anarchic
self-organized
open

```
graph TD; manyNodes --- manyNodes;
```

Play on a Network:

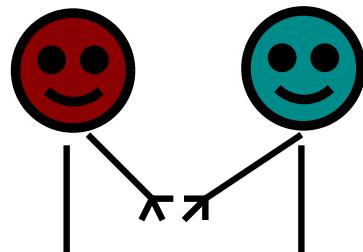


Share Resources:



Games

Examples...

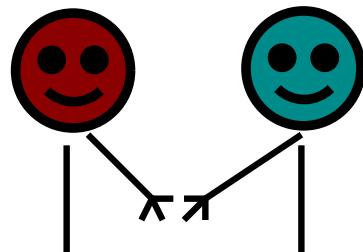


Dinner together

Same place, same time

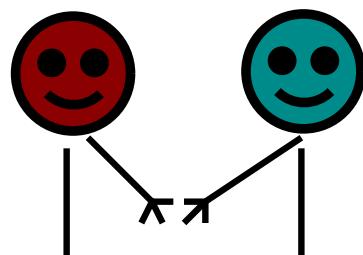
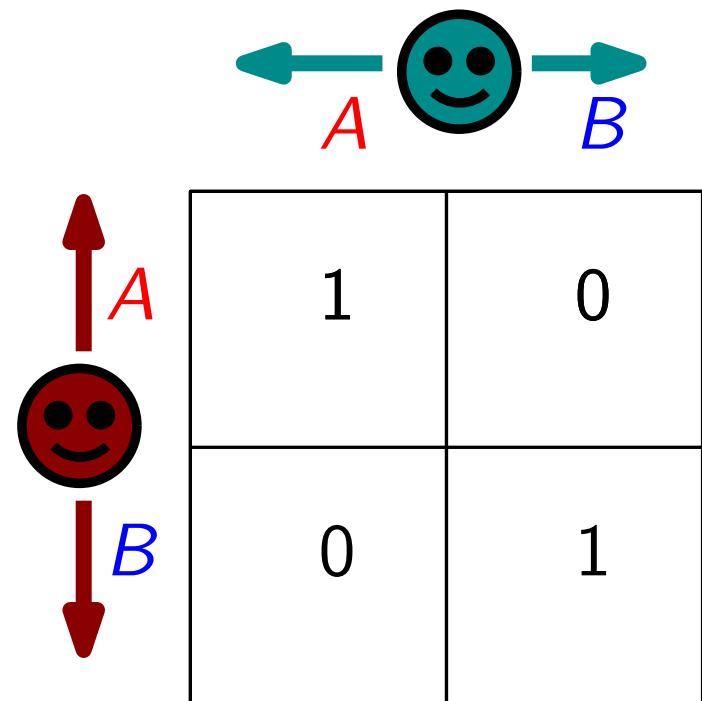
Examples...

A	B	
A	1	0
B	0	1



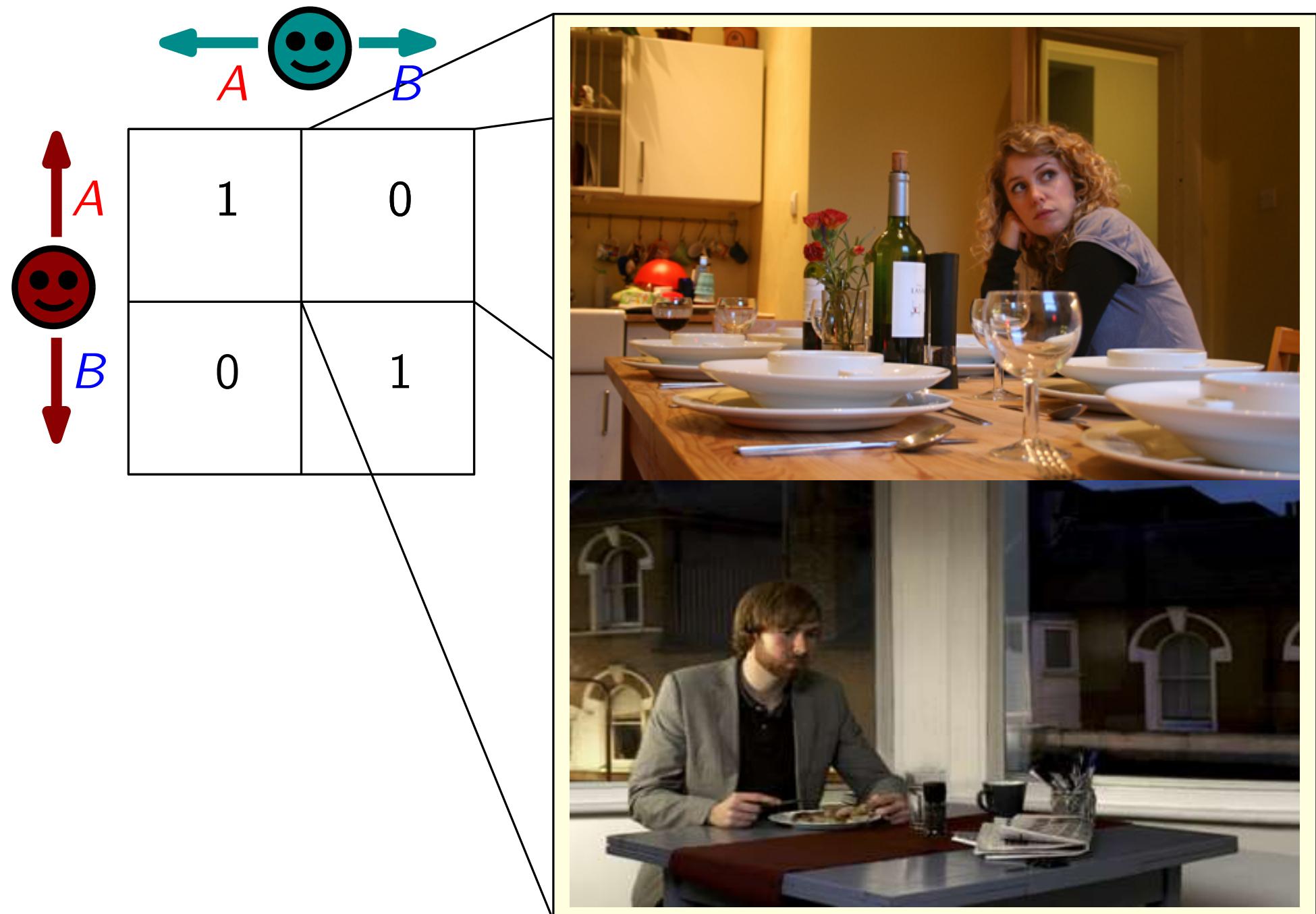
Dinner together

Examples...

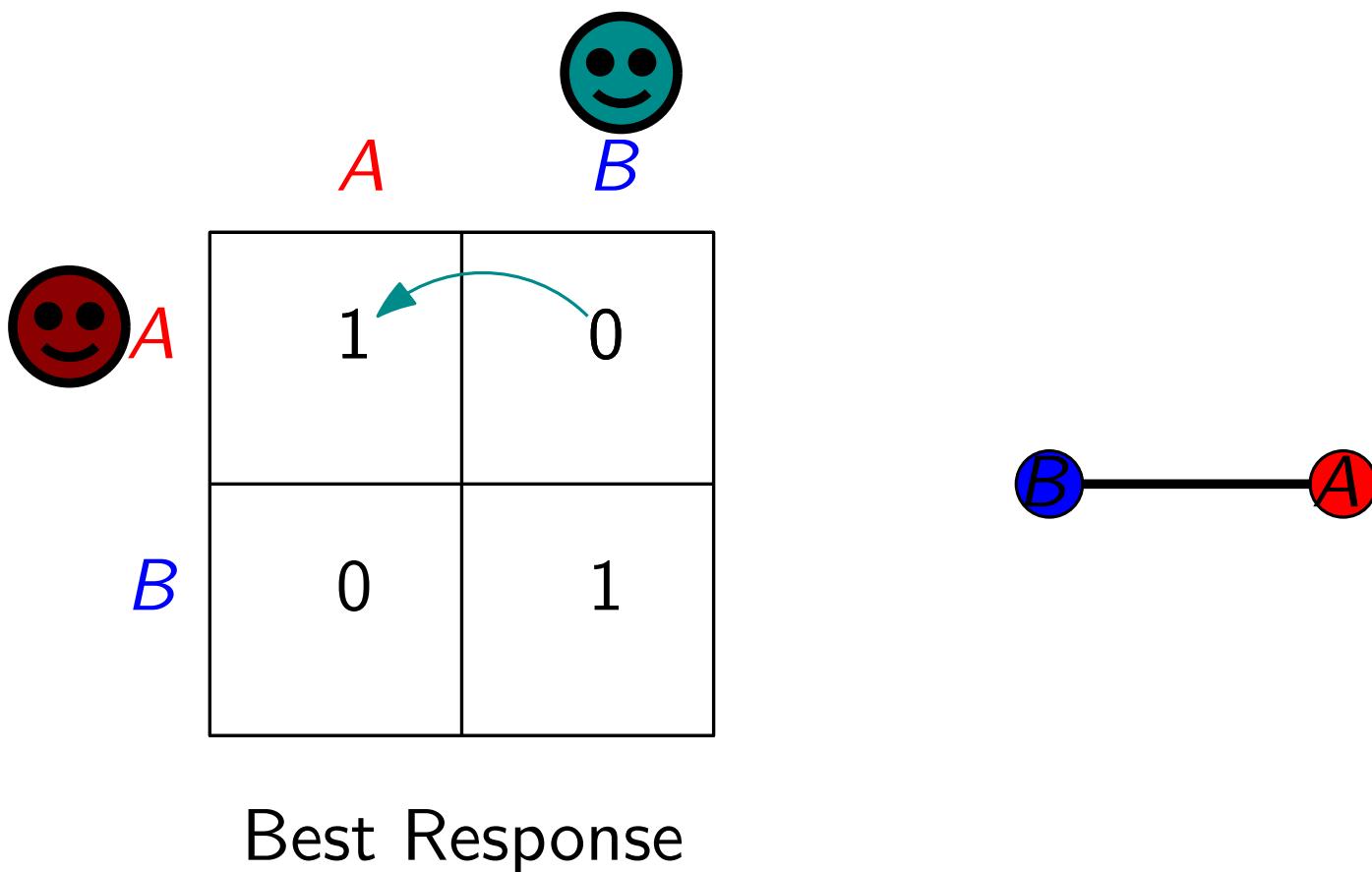


Dinner together

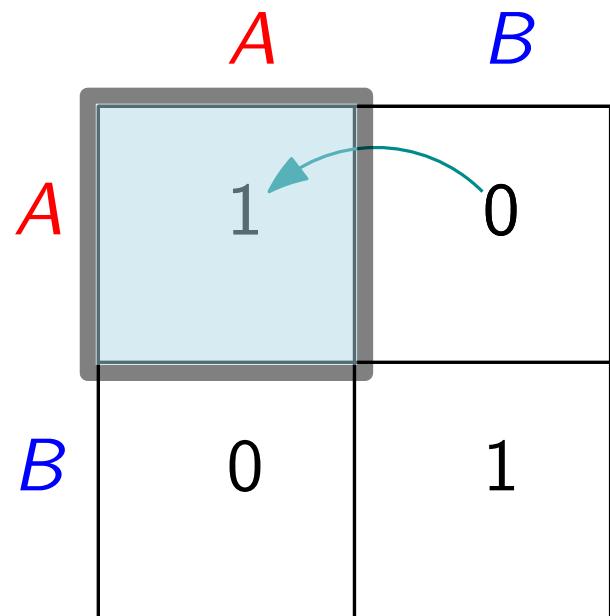
Examples...



Examples...



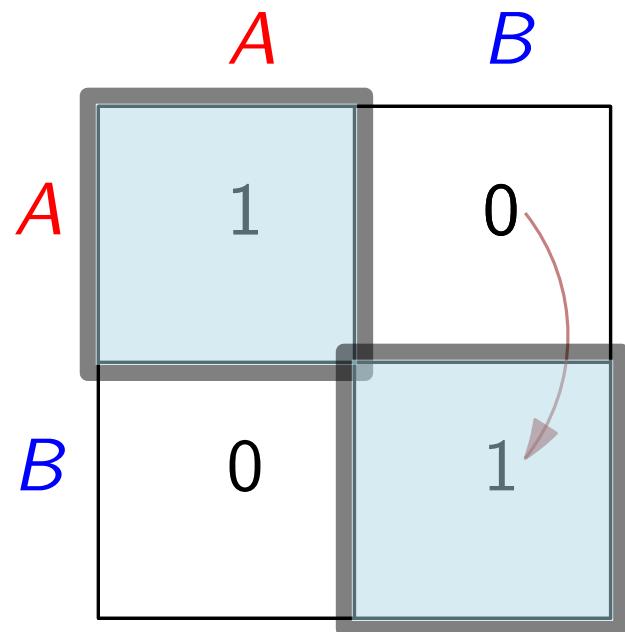
Examples...



Best Response



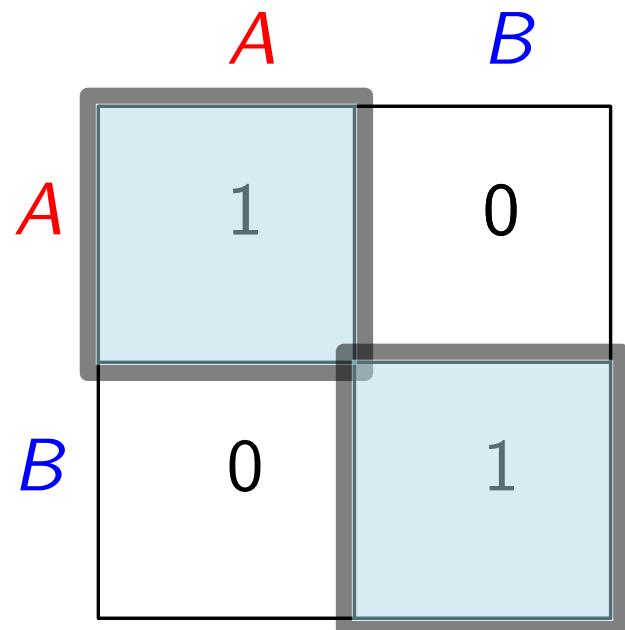
Examples...



Best Response



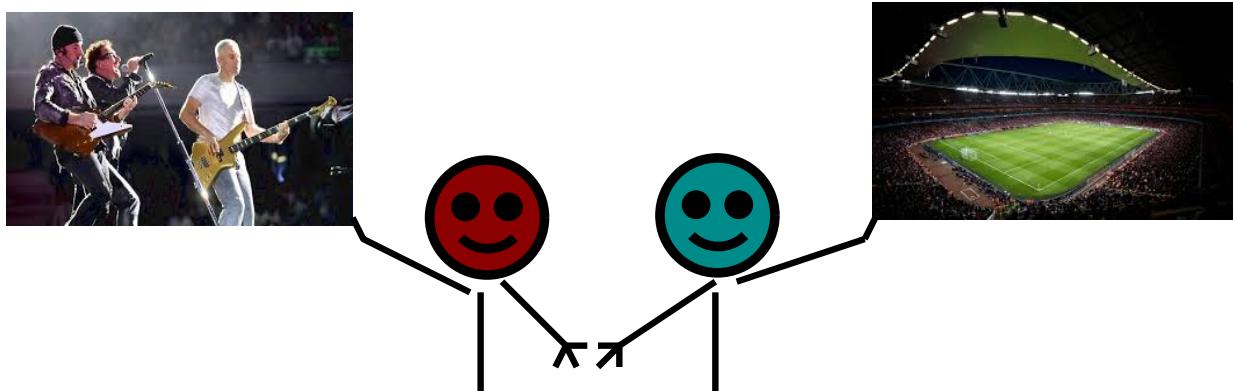
Examples...



Best Response

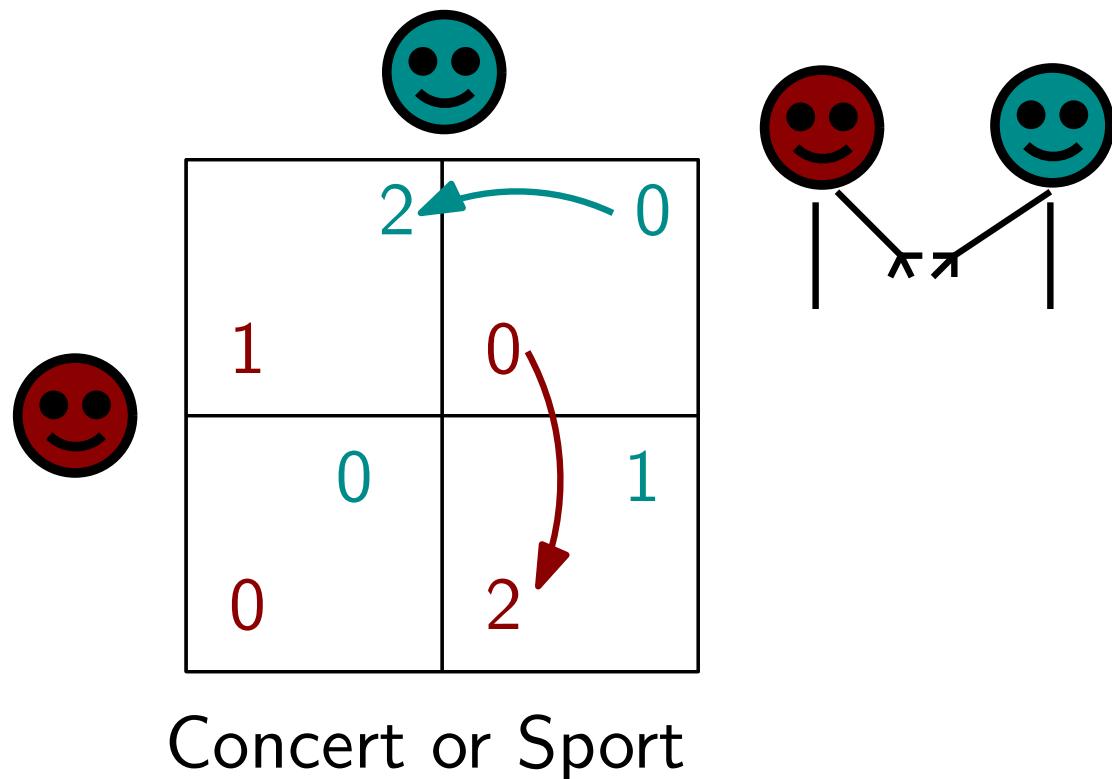
Nash equilibrium: no player wants to move

Examples...

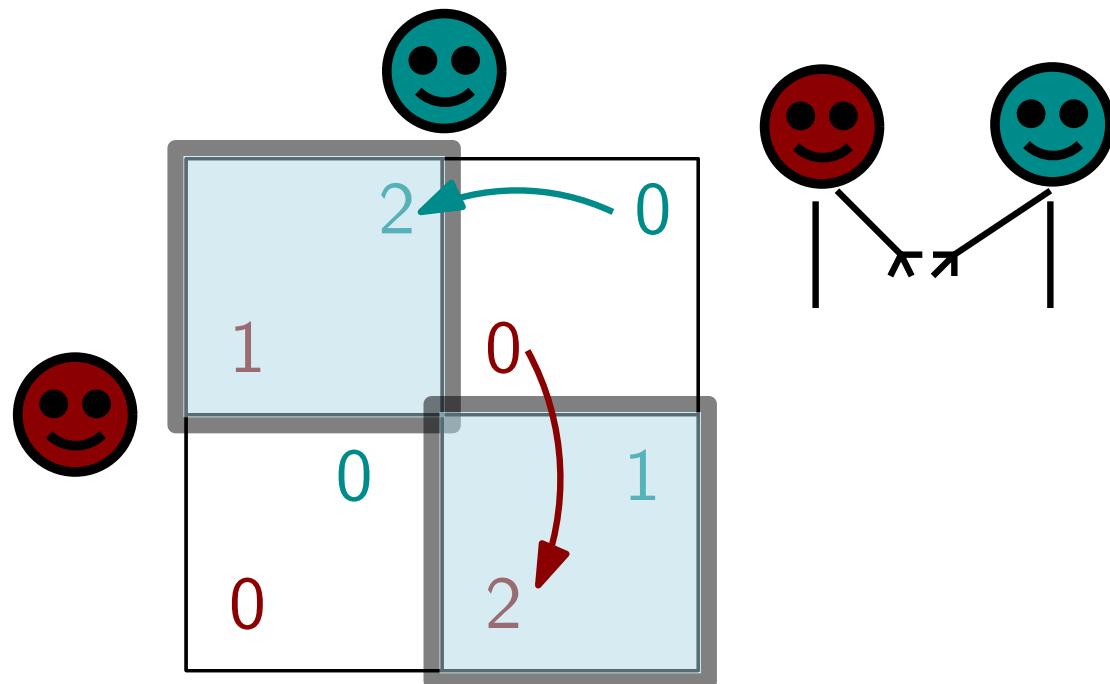


Different preferences

Examples...



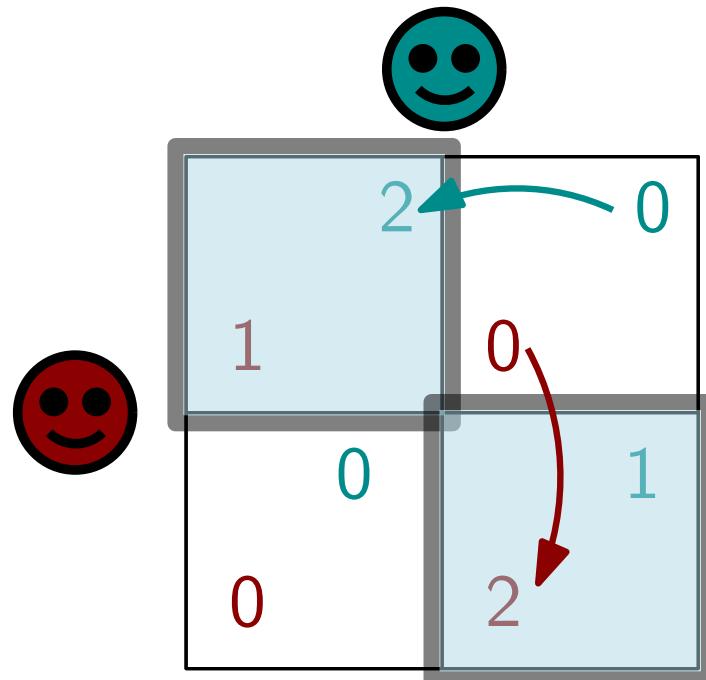
Examples...



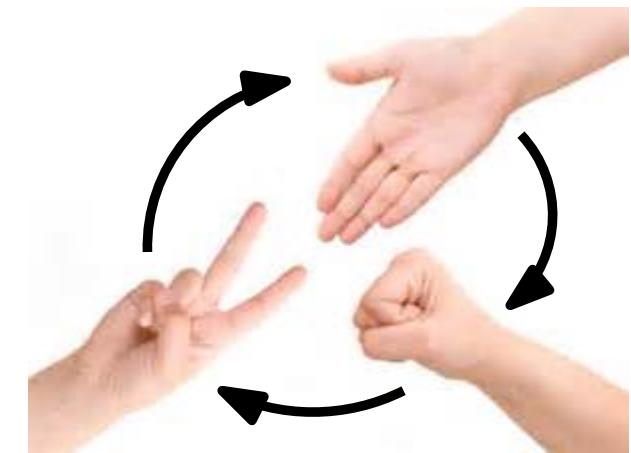
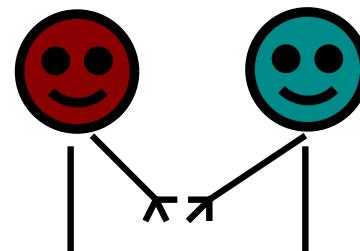
Concert or Sport

Nash equilibrium: no player wants to move

Examples...



Concert or Sport

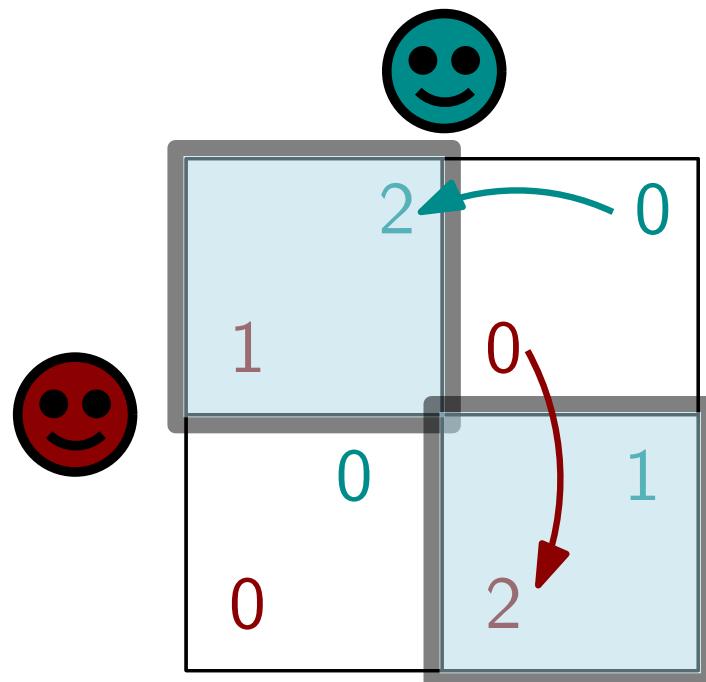


Rock-Paper-Scissor

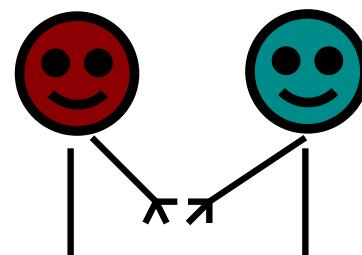
Nash equilibrium: no player wants to move

Not always like that!!

Examples...



Concert or Sport

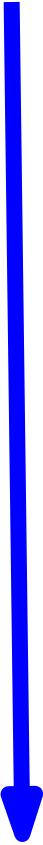


Rock-Paper-Scissor

Nash equilibrium: no player wants to move

Not always like that!!

Potential Games

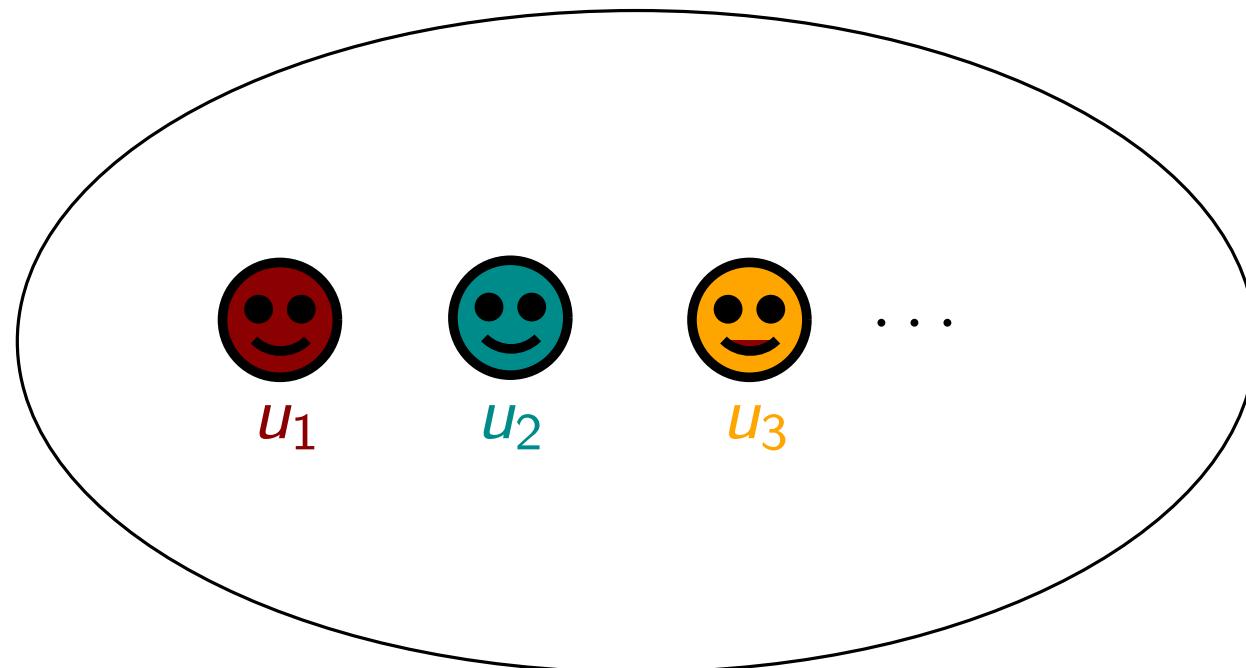


Always exist

Nash equilibrium: no player wants to move

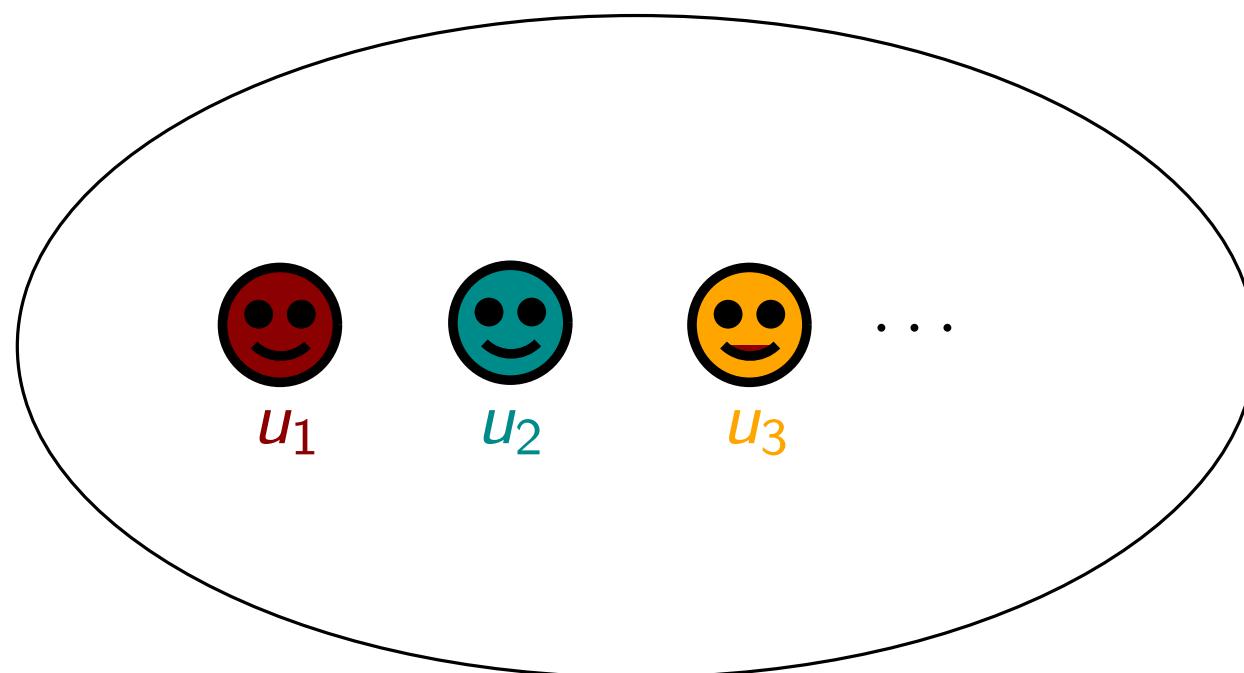
Potential Games

$$u_i(s) - u_i(s') = pot(s) - pot(s')$$



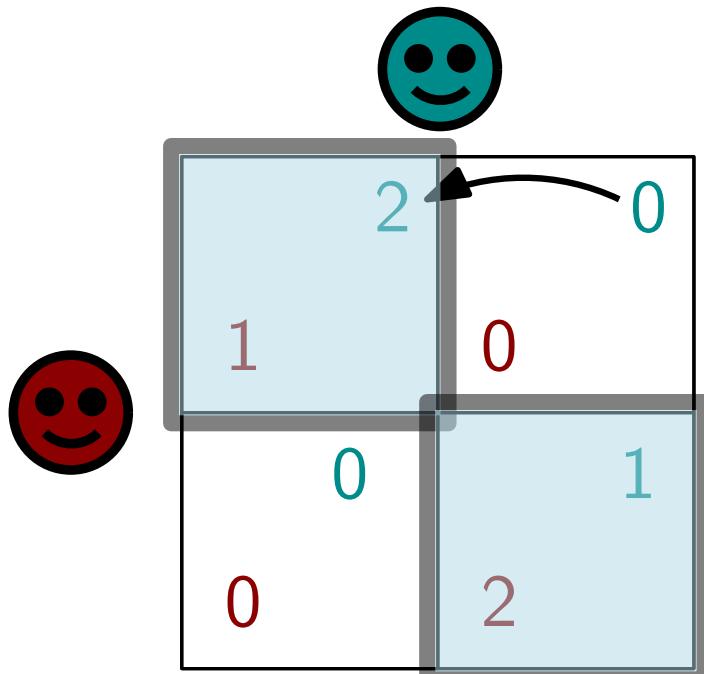
Potential Games

$$\underbrace{u_i(s) - u_i(s')}_{\text{player improvement}} = pot(s) - pot(s')$$



Potential Games

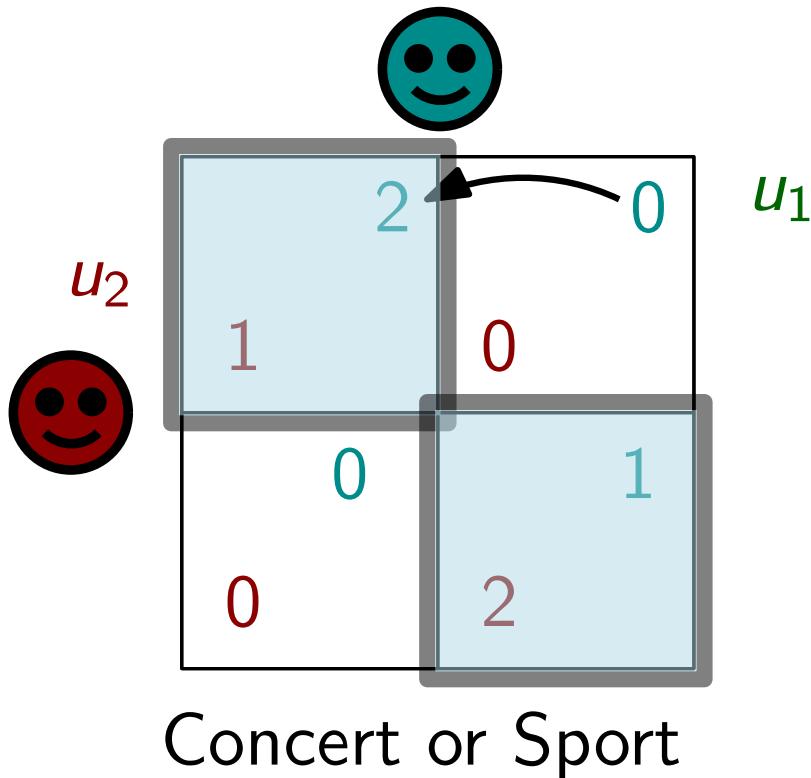
$$\underbrace{u_i(s) - u_i(s')}_{\text{player improvement}} = \text{pot}(s) - \text{pot}(s')$$



Concert or Sport

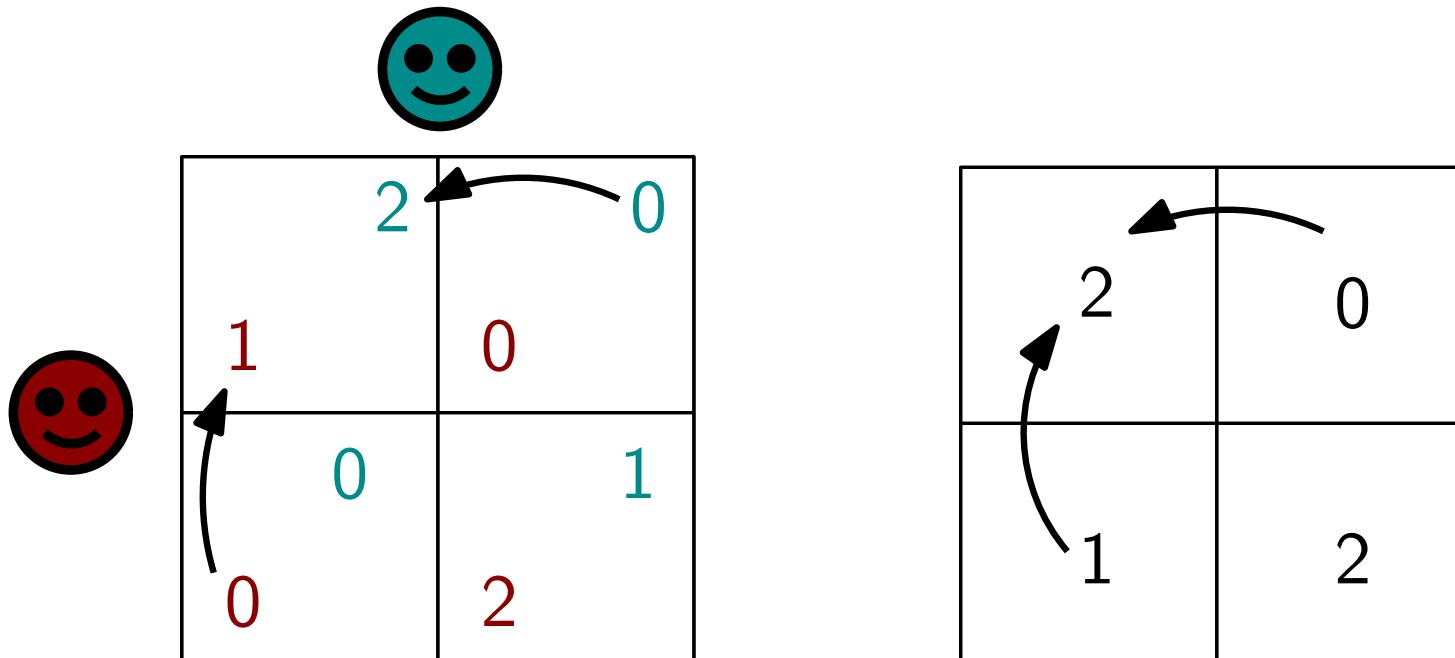
Potential Games

$$\underbrace{u_i(s) - u_i(s')}_{\text{player improvement}} = \text{pot}(s) - \text{pot}(s')$$



Potential Games

$$\underbrace{u_i(s) - u_i(s')}_{\text{player improvement}} = \text{pot}(s) - \text{pot}(s')$$

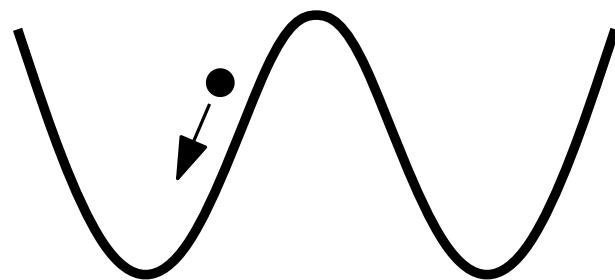


Concert or Sport

Potential Games

$$\underbrace{u_i(s) - u_i(s')}_{\text{player improvement}} = pot(s) - pot(s')$$

Better Response Converge to some Equilibrium

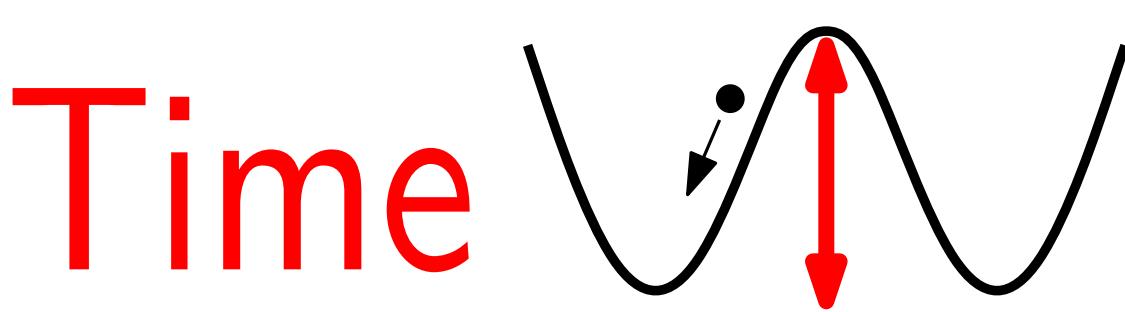


Min Potential

Potential Games

$$\underbrace{u_i(s) - u_i(s')}_{\text{player improvement}} = pot(s) - pot(s')$$

Better Response Converge to some Equilibrium



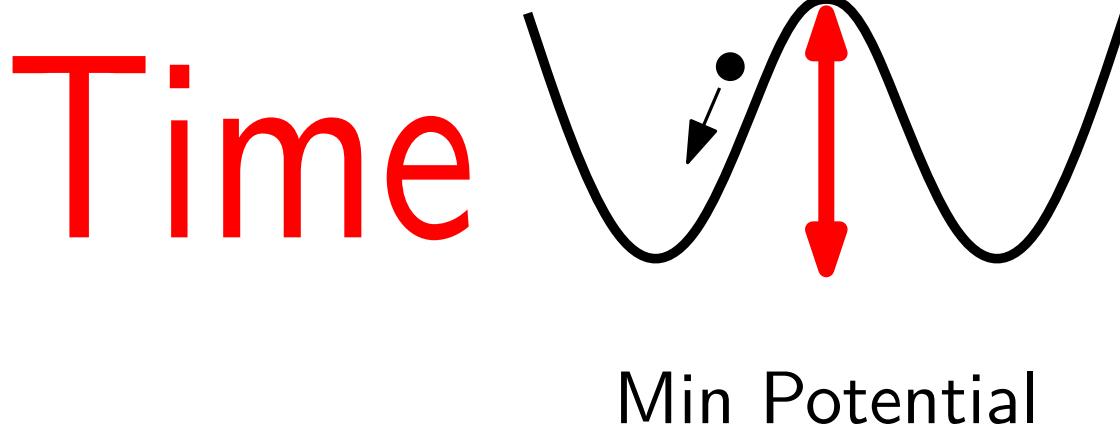
Min Potential

Potential Games

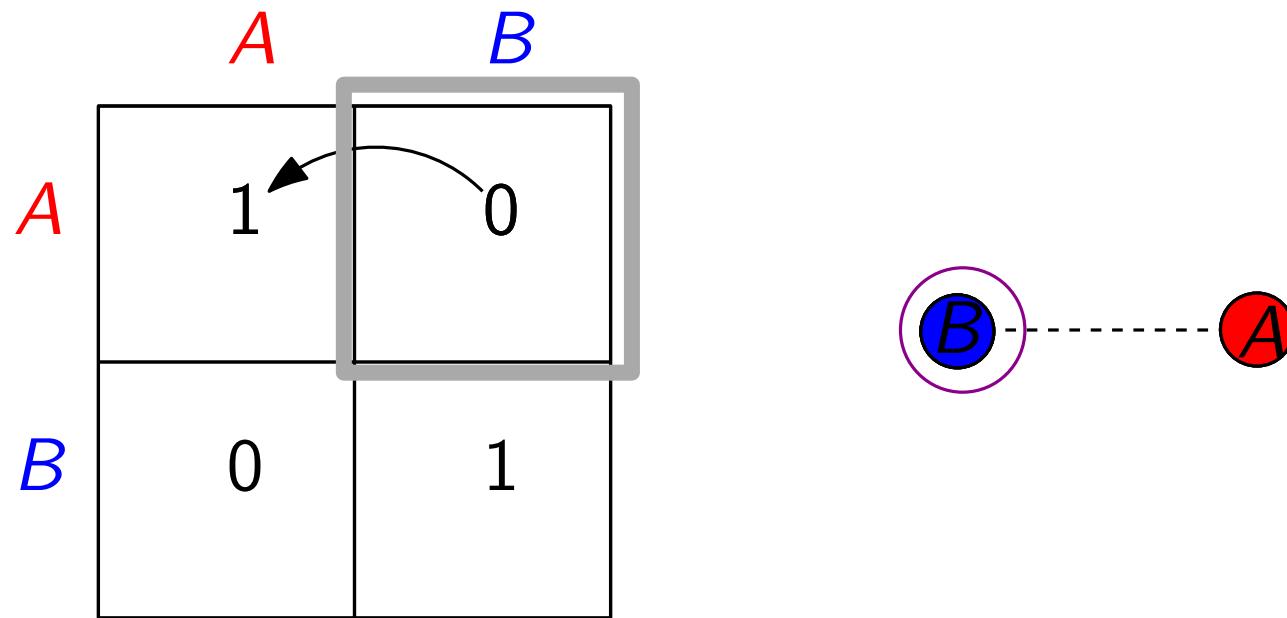
$$\underbrace{u_i(s) - u_i(s')}_{\text{player improvement}} = \text{pot}(s) - \text{pot}(s')$$

One at a time!

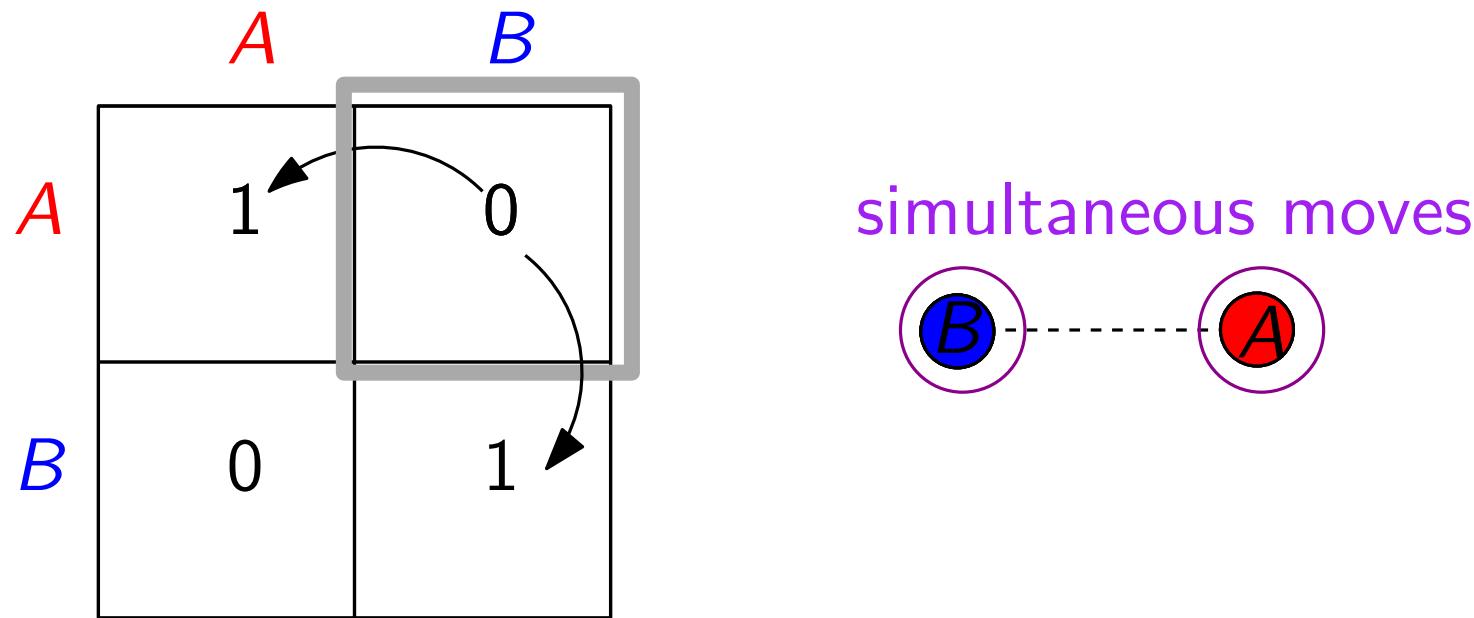
Better Response Converge to some Equilibrium



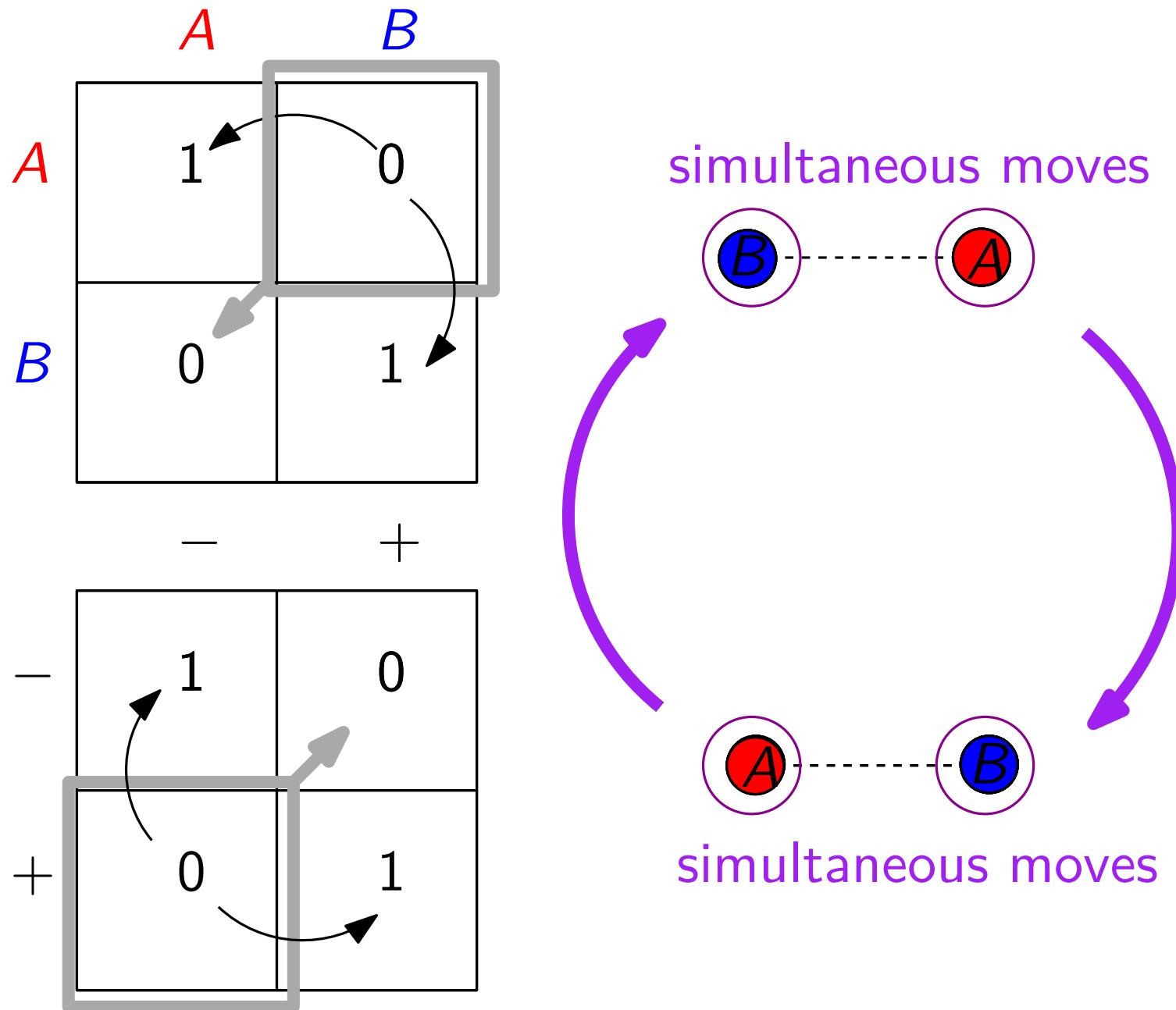
Potential Games



Potential Games



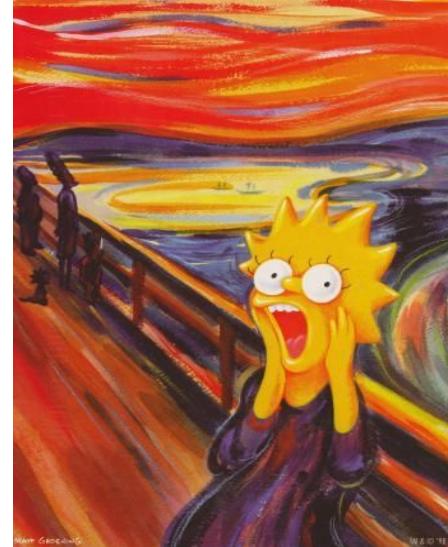
Potential Games



Potential Games

need synchronization?

Better Response Converge to some Equilibrium

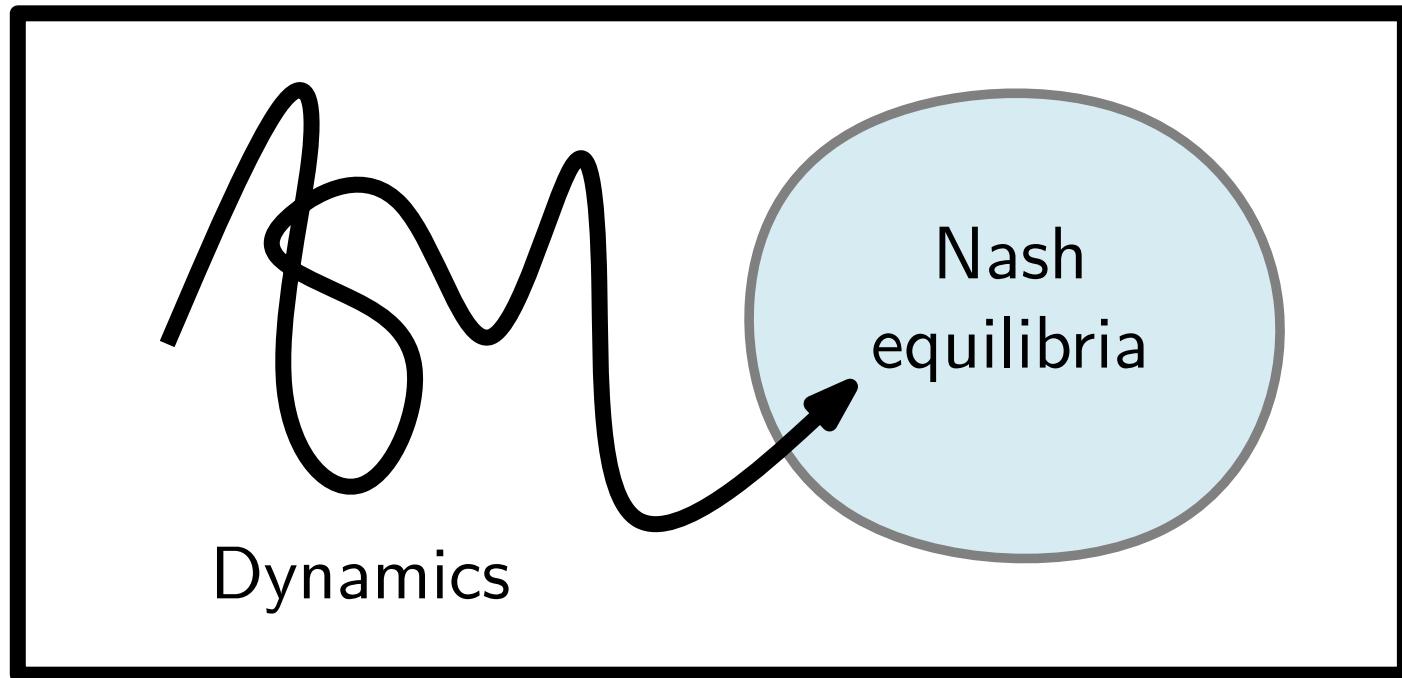


need synchronization?

Better Response Converge to some Equilibrium

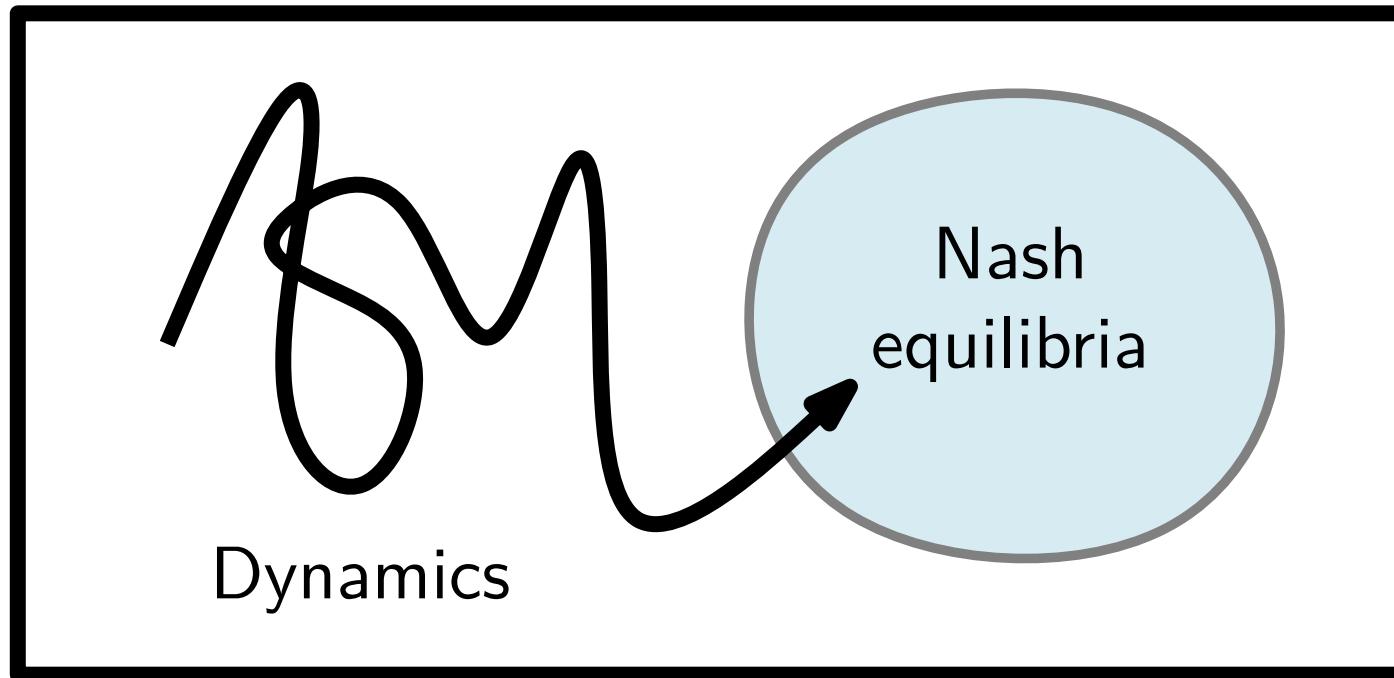
need synchronization?

Better Response Converge to some Equilibrium



need synchronization?

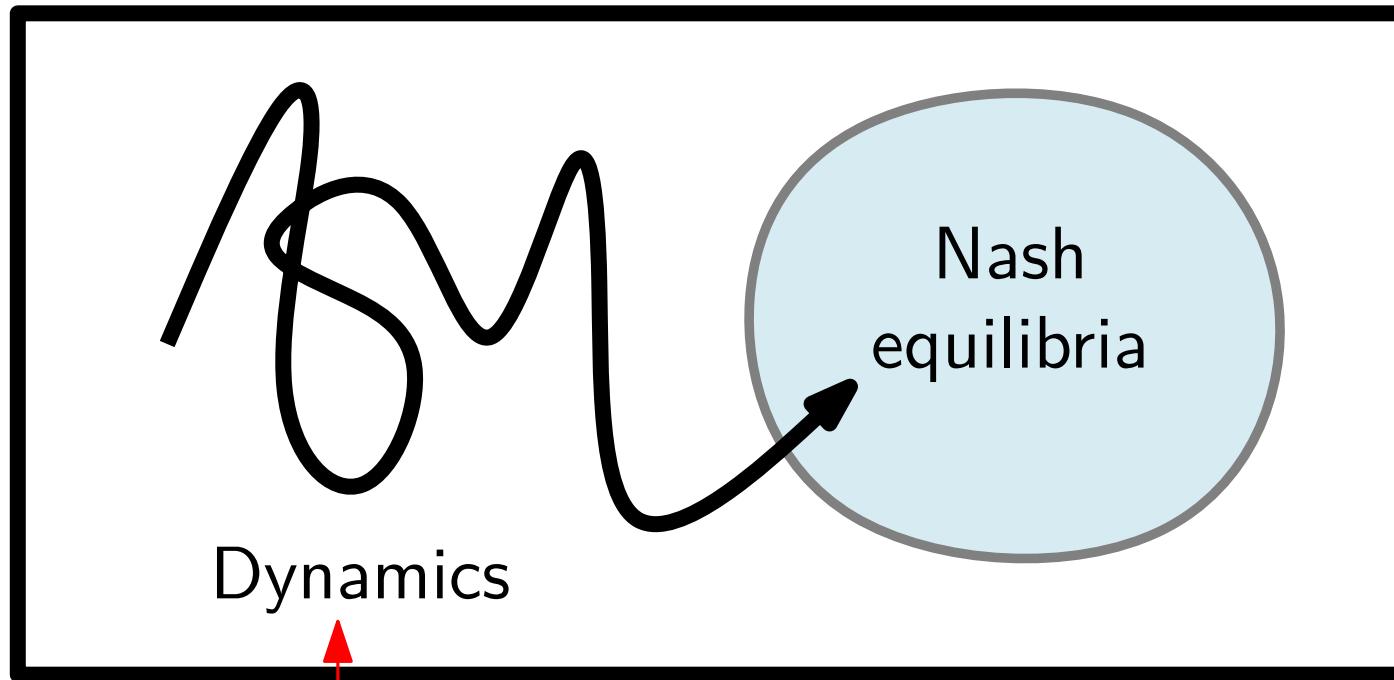
Better Response Converge to some Equilibrium



Players find “easily” an equilibrium by themselves.

need synchronization?

Better Response Converge to some Equilibrium

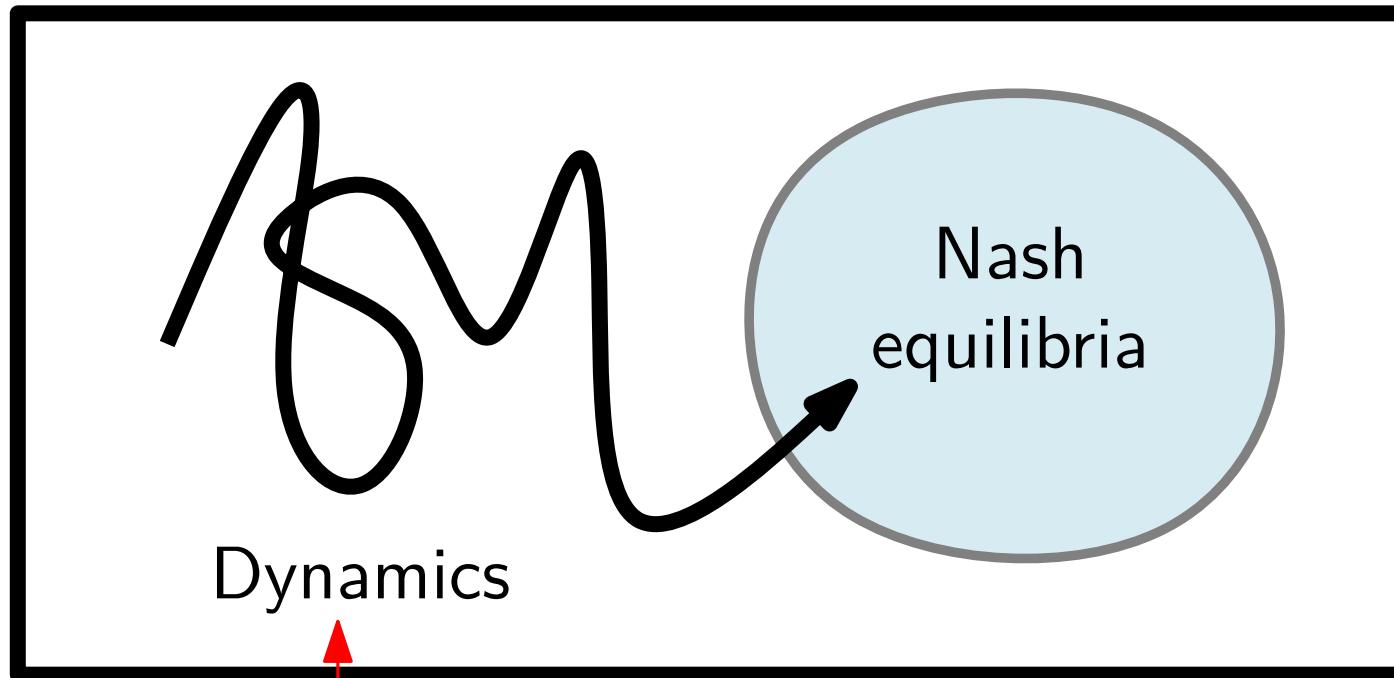


Players find “easily” an equilibrium by themselves.

Natural

need synchronization?

Better Response Converge to some Equilibrium

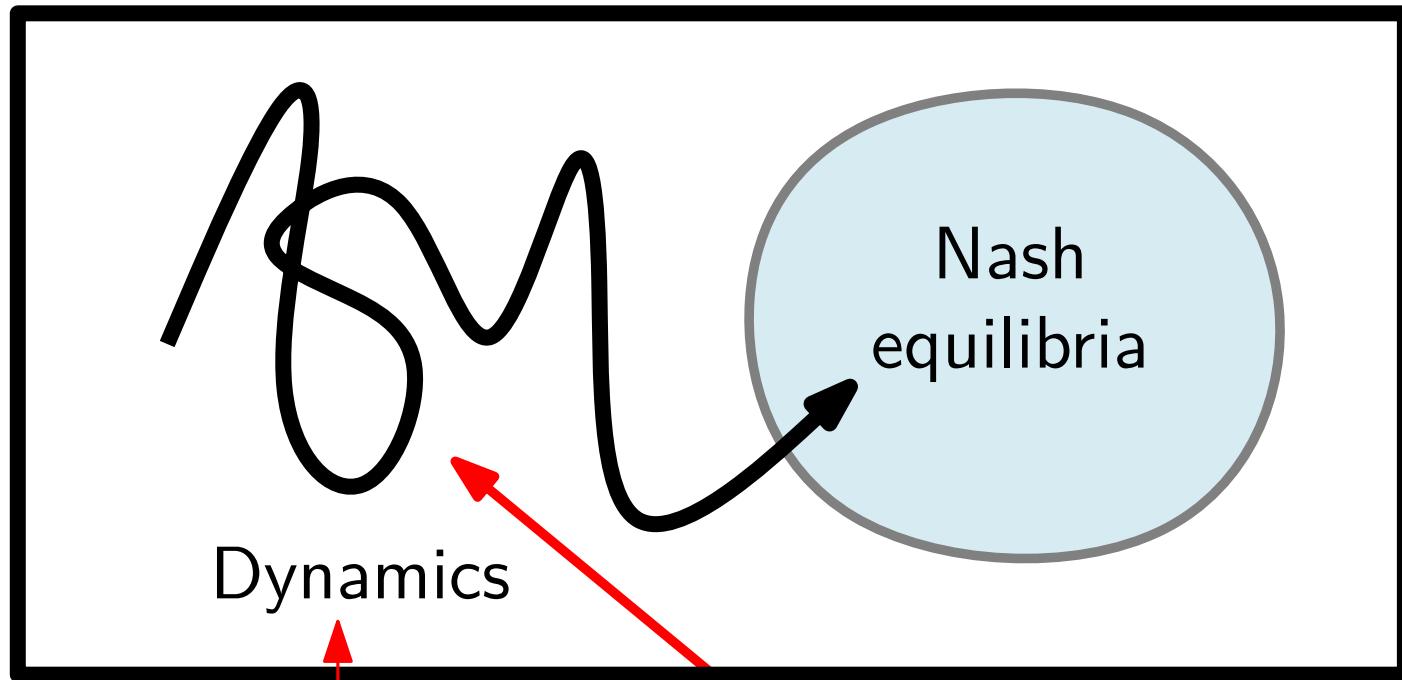


Players find “easily” an equilibrium by themselves.

Natural
NO SYNCHRONIZATION

need synchronization?

Better Response Converge to some Equilibrium

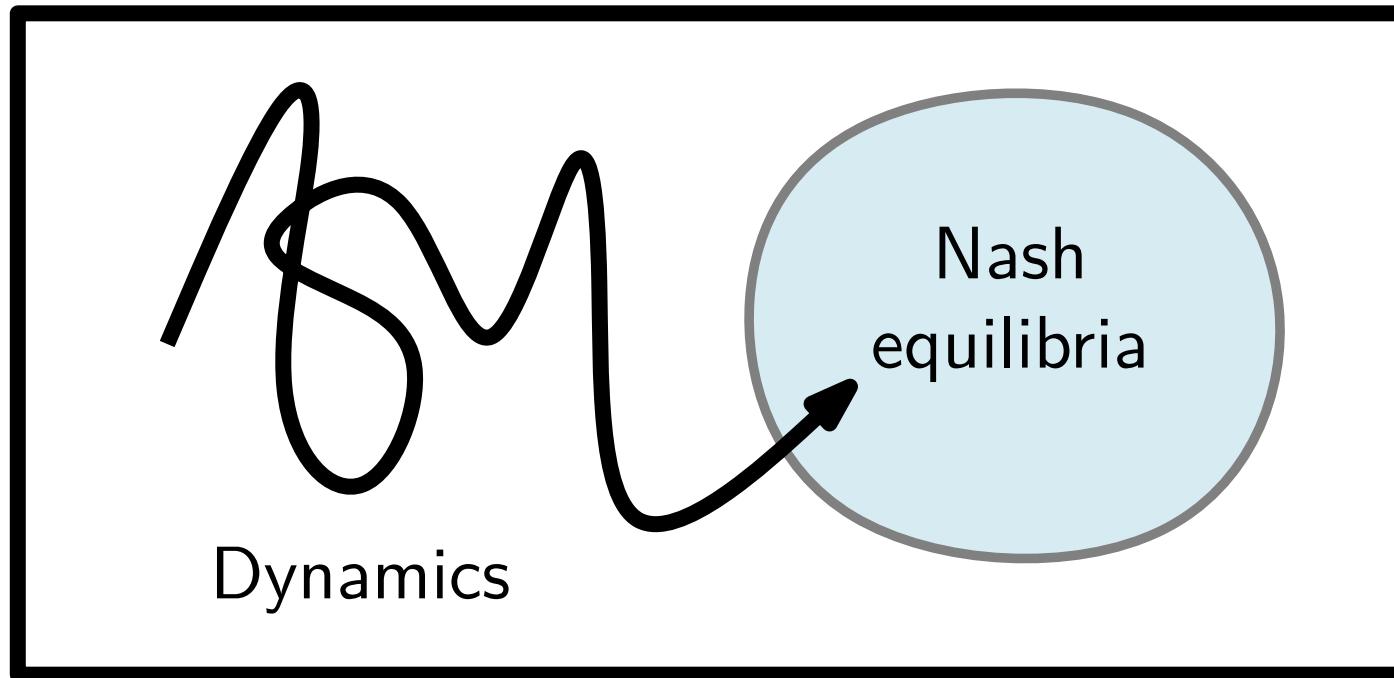


Players find “easily” an equilibrium by themselves.

Natural Time
NO SYNCHRONIZATION

need synchronization?

Better Response Converge to some Equilibrium

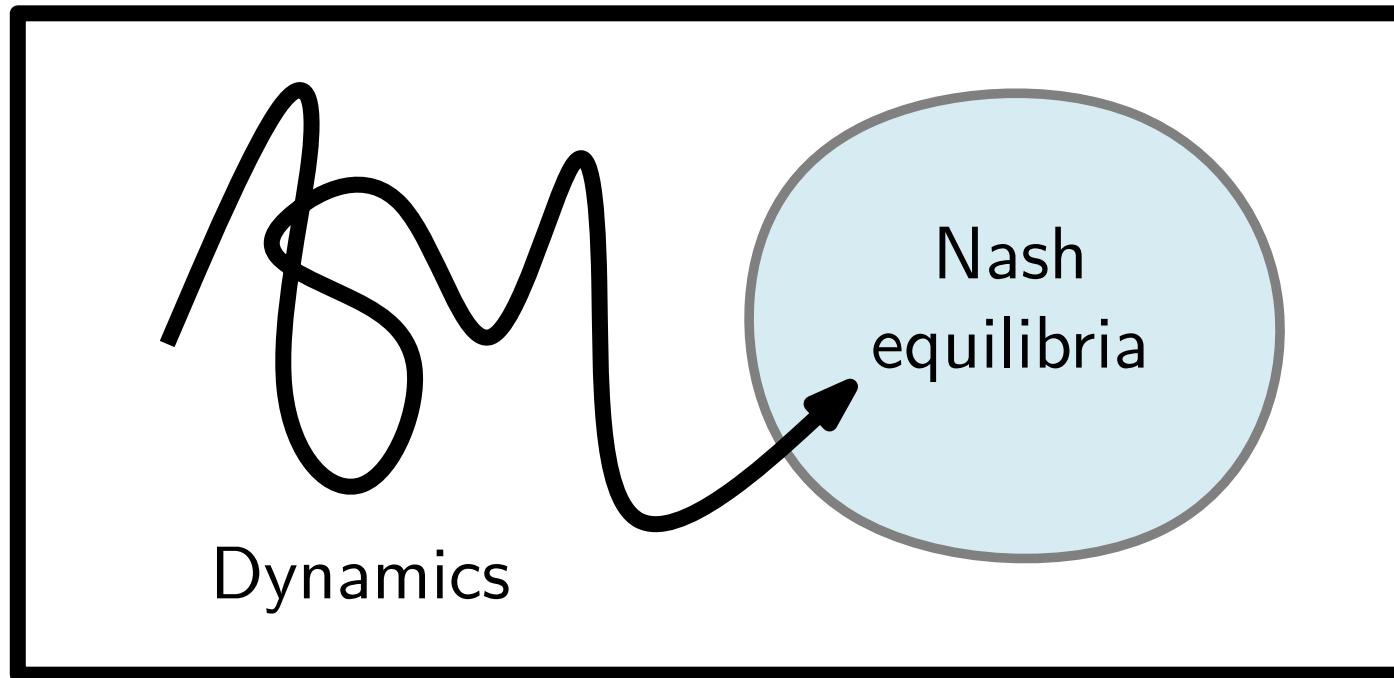


Players find “easily” an equilibrium by themselves.

Relax “synchronization”
(Coucheney - Durand - Gaujal - Touati, 2014)

need synchronization?

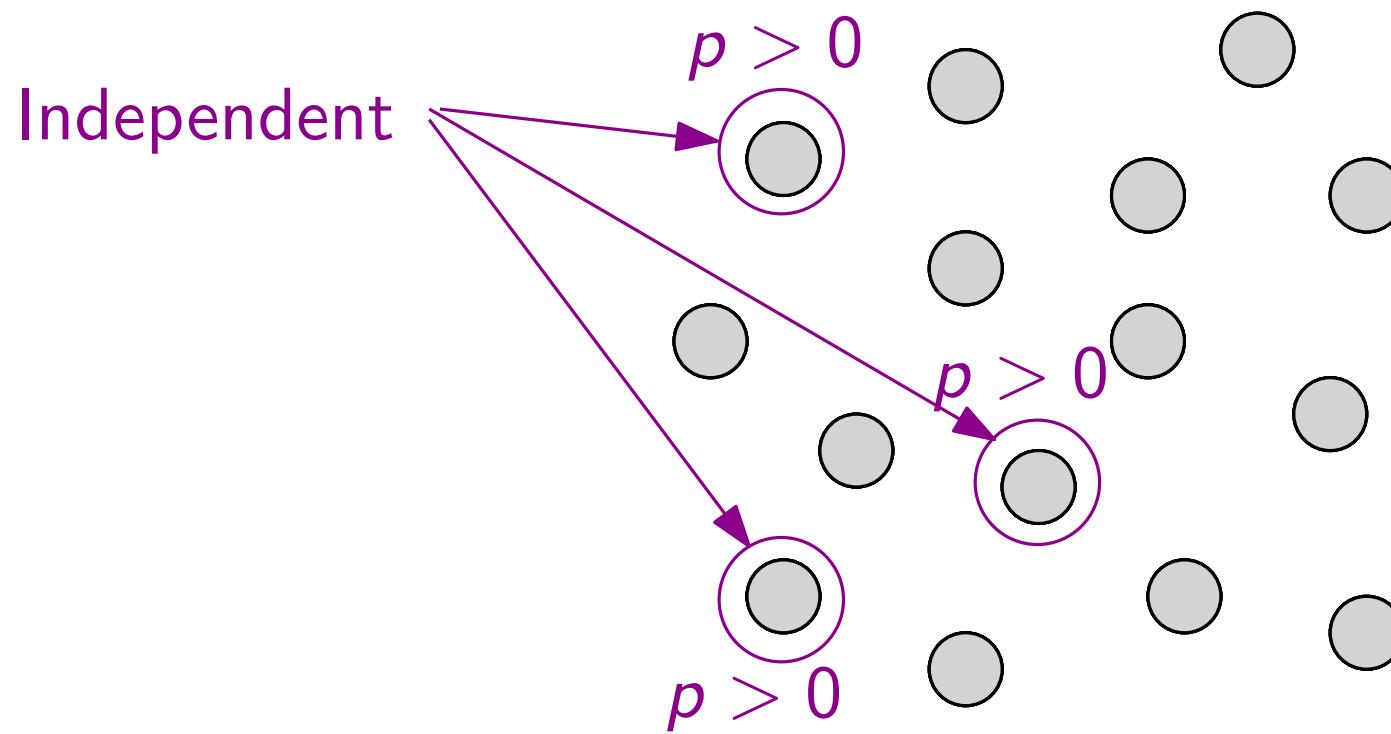
Better Response Converge to some Equilibrium



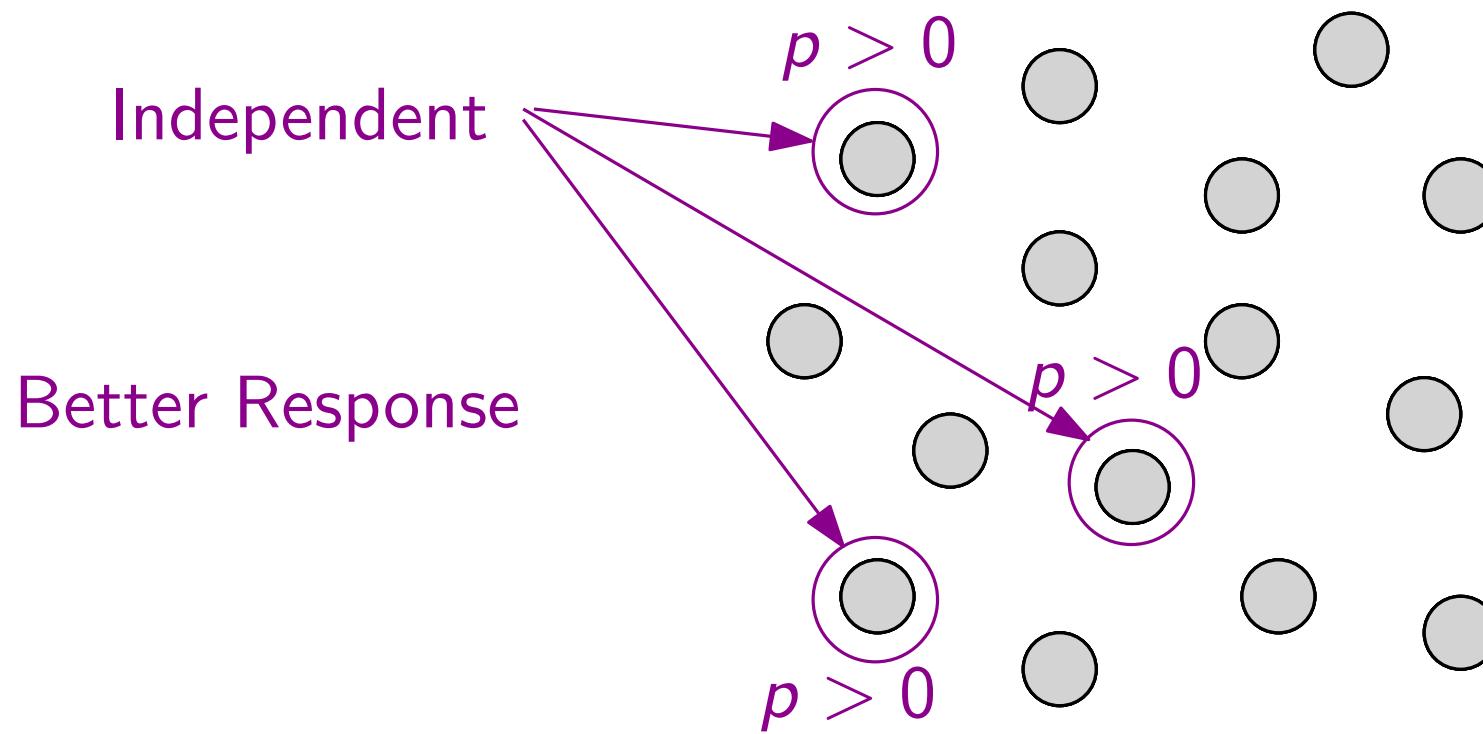
Players find “easily” an equilibrium by themselves.

Limit “simultaneous moves”
(Fotakis - Kaporis -Spirakis, 2014)
(Fanelli - Moscardelli - Skopalik, 2012)

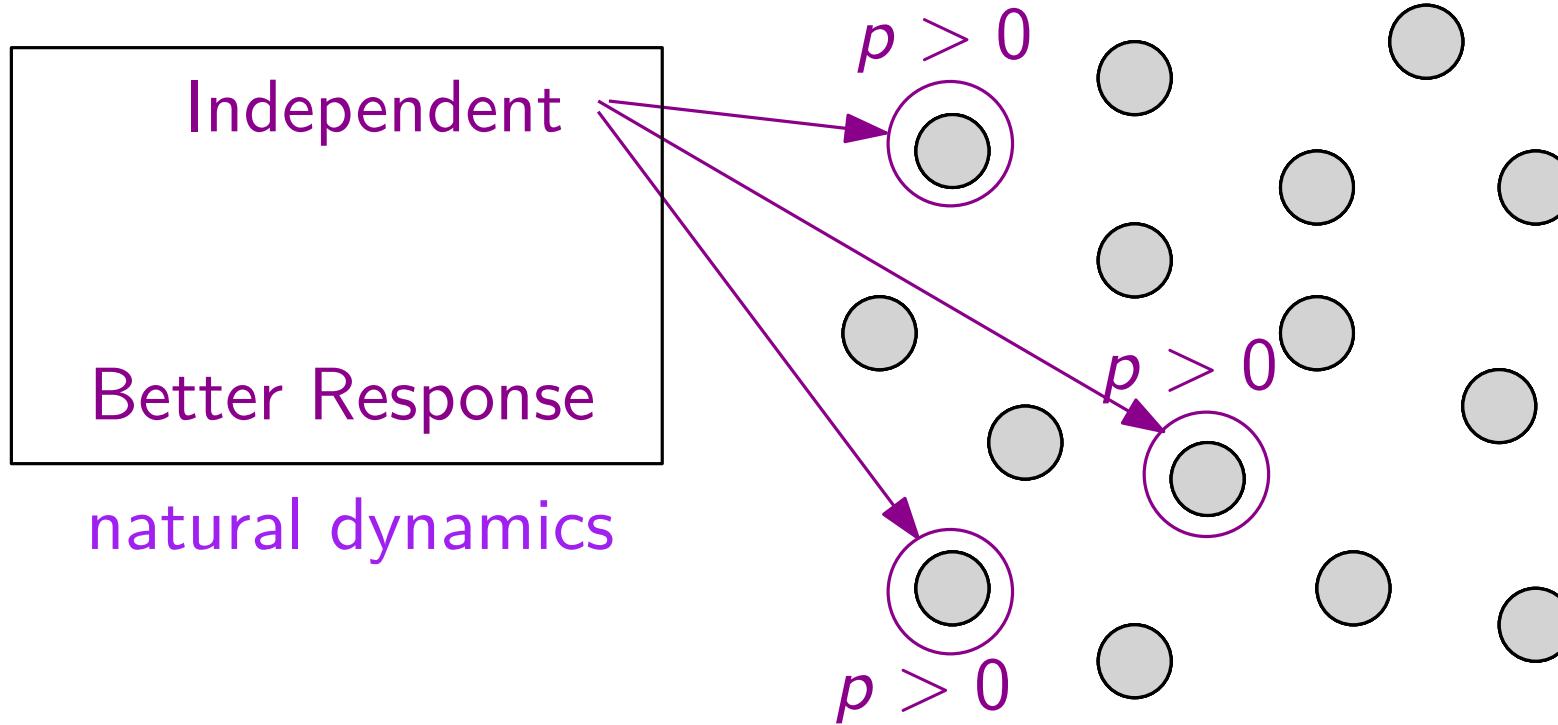
Our Model



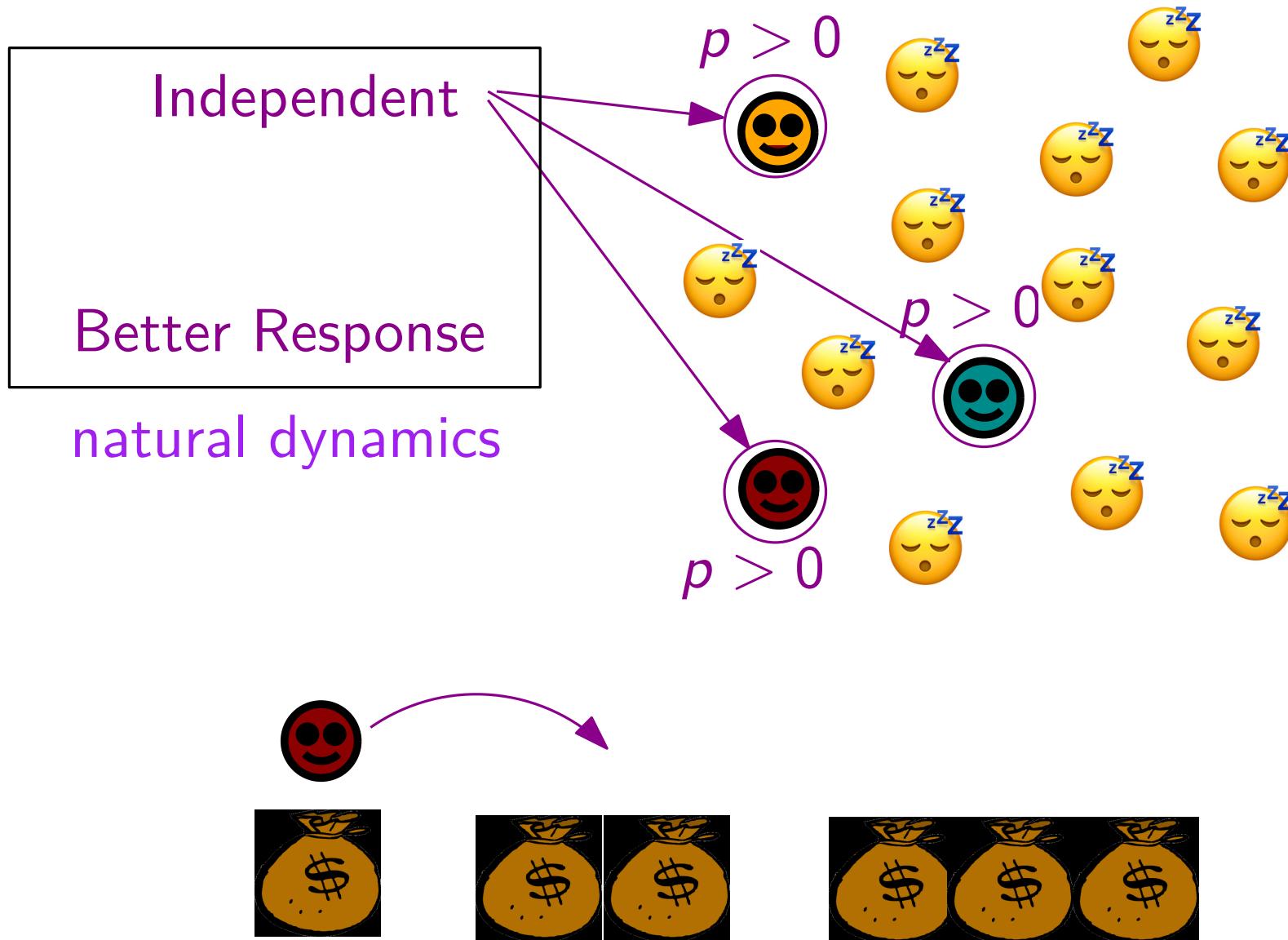
Our Model



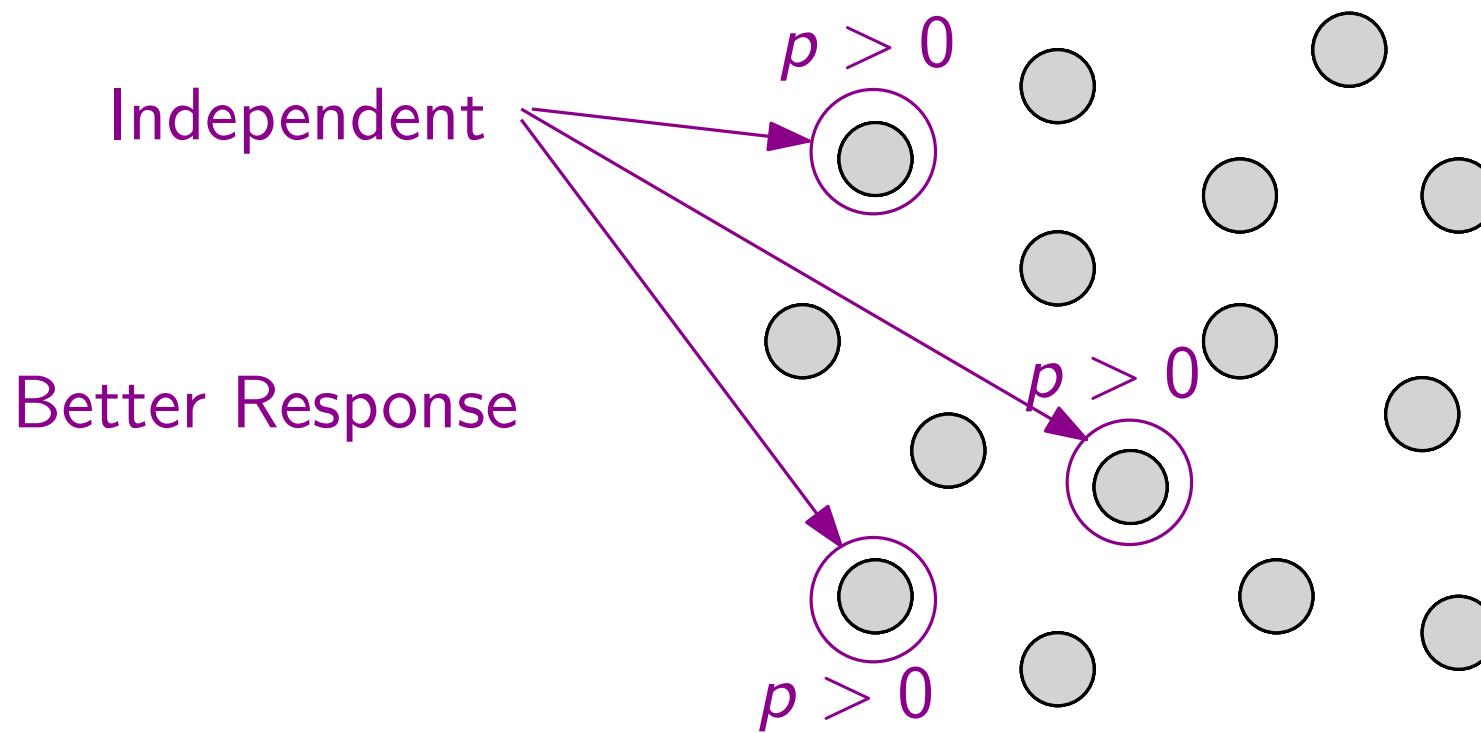
Our Model



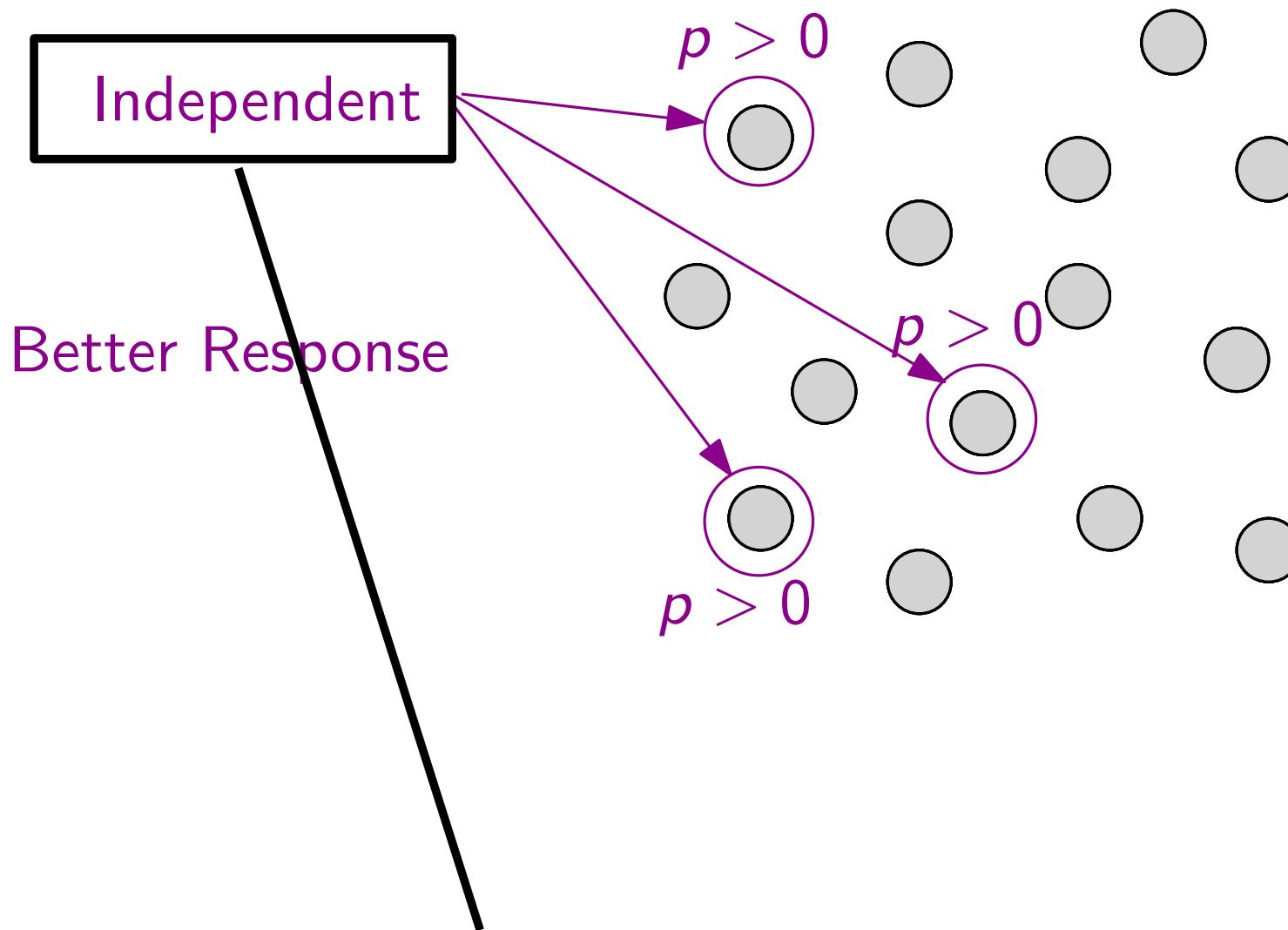
Our Model



Our Model

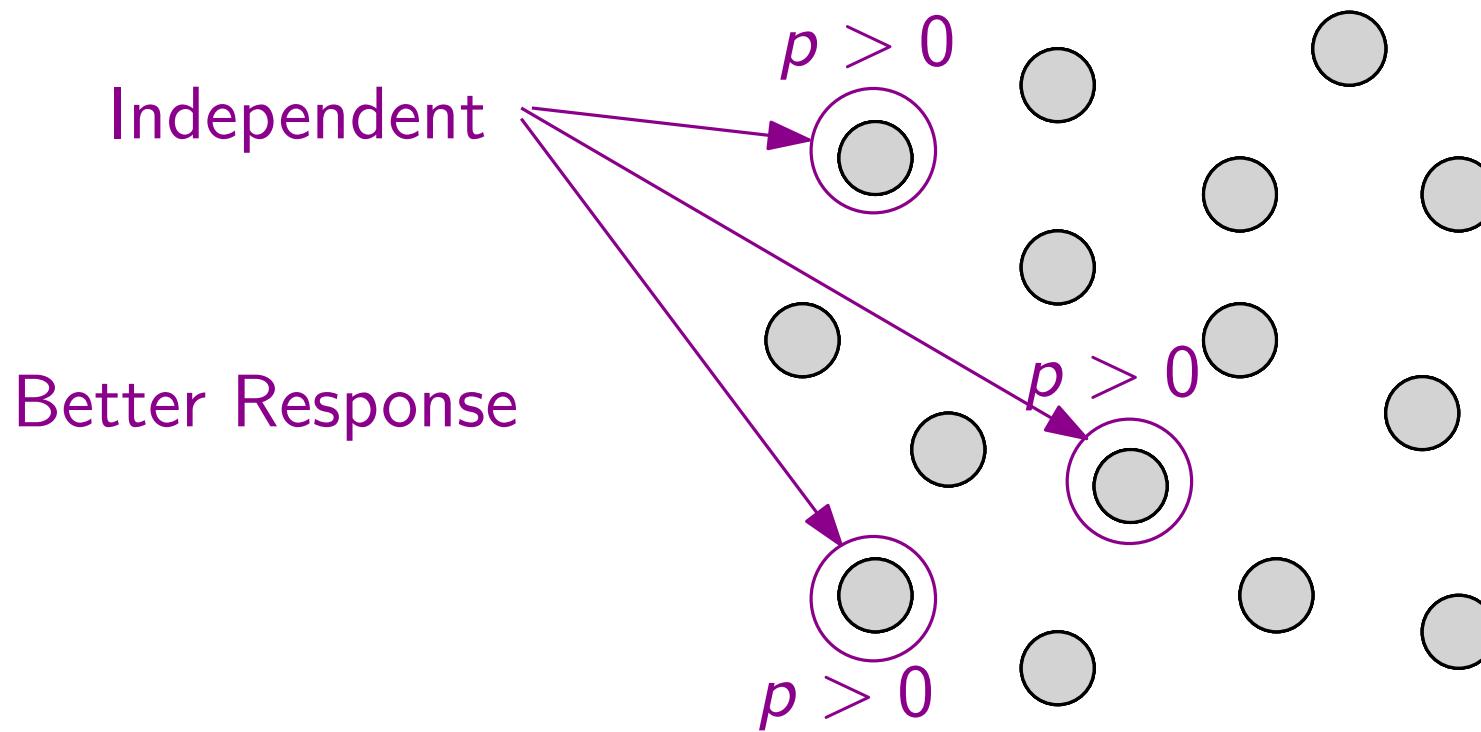


Our Model

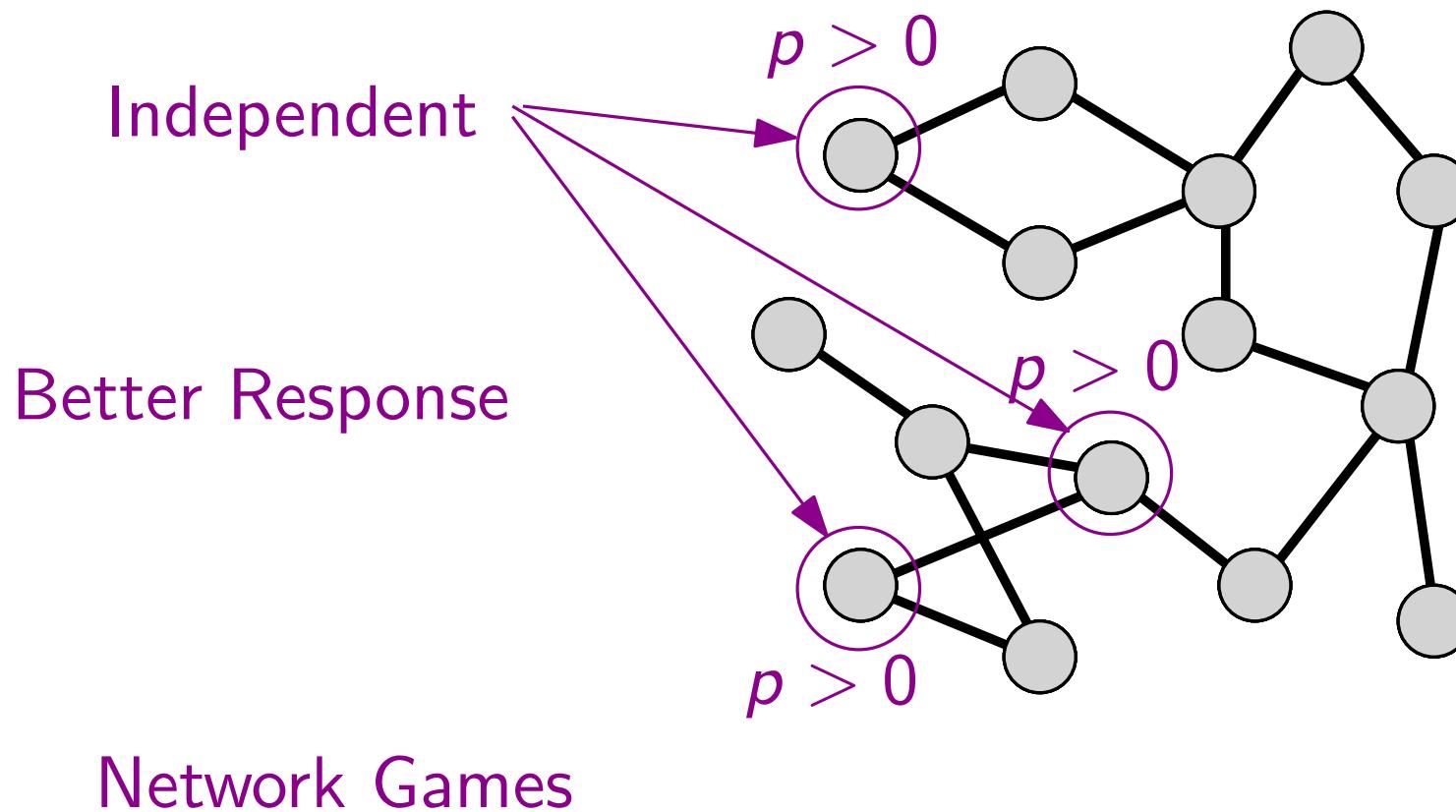


(Alós-Ferrer and Netzer, 2010)

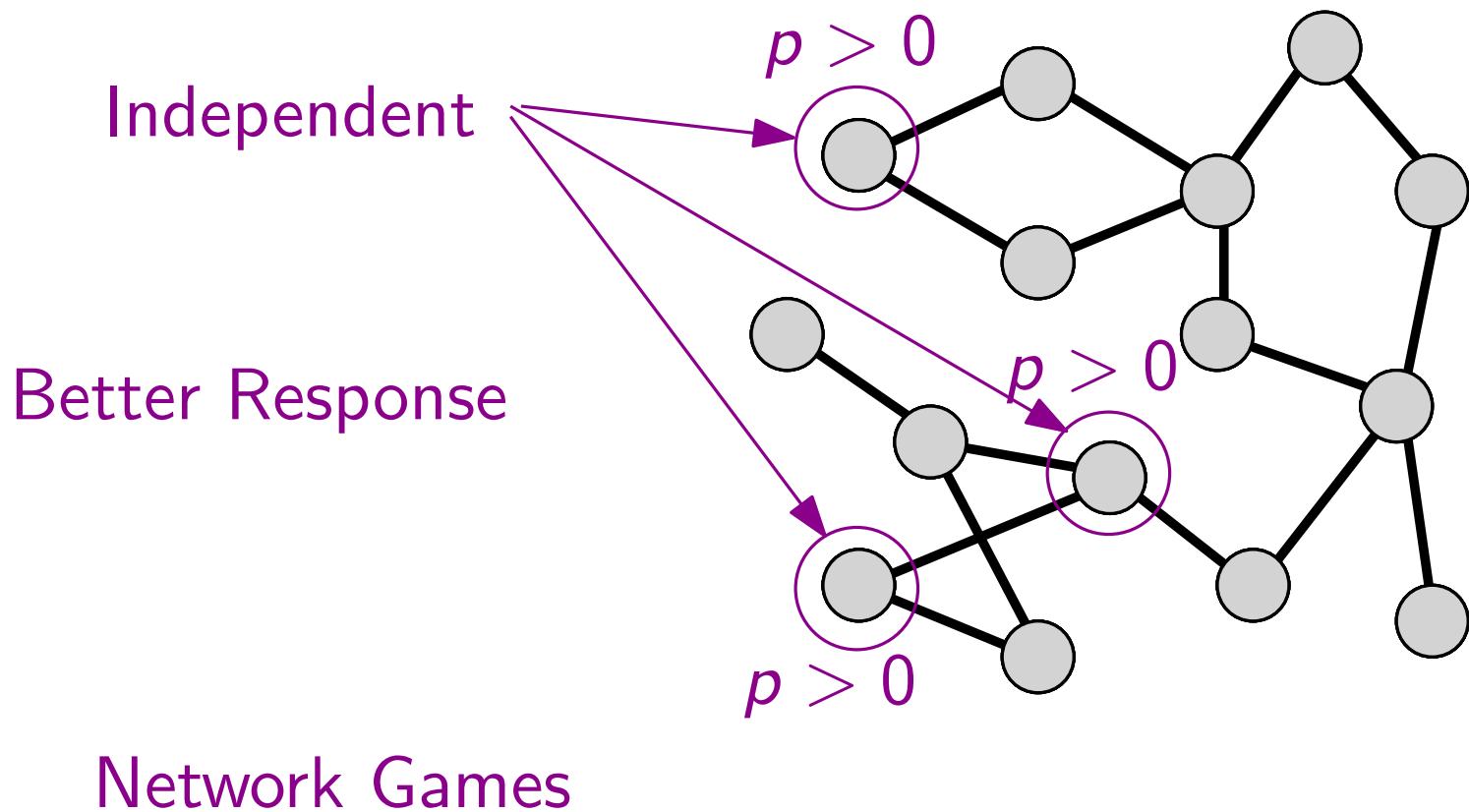
Our Model



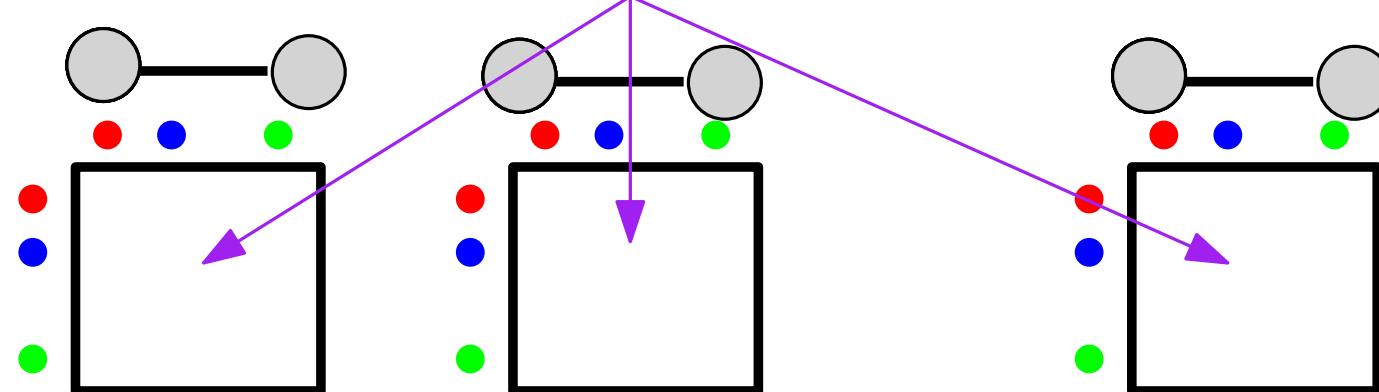
Our Model



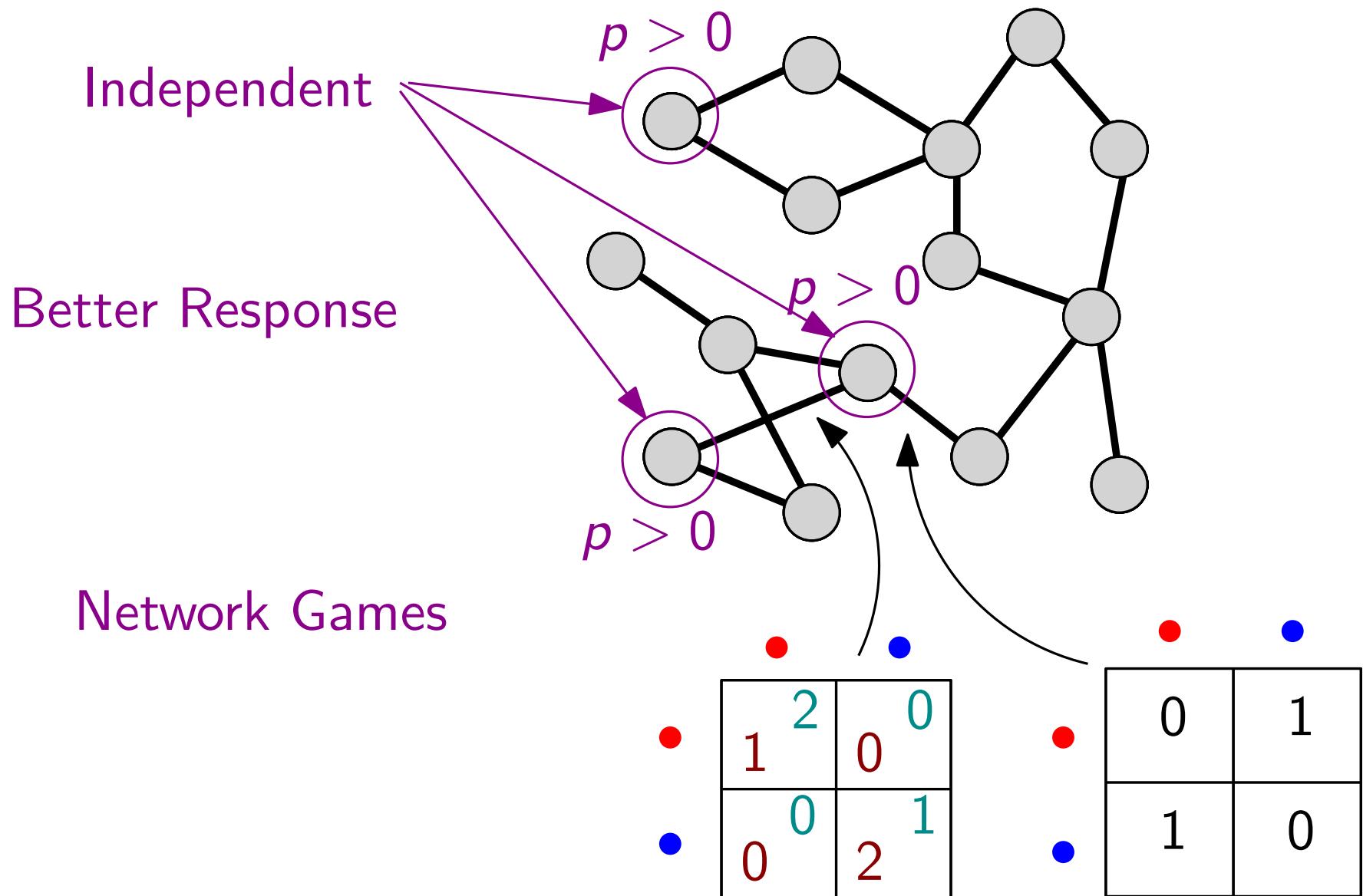
Our Model



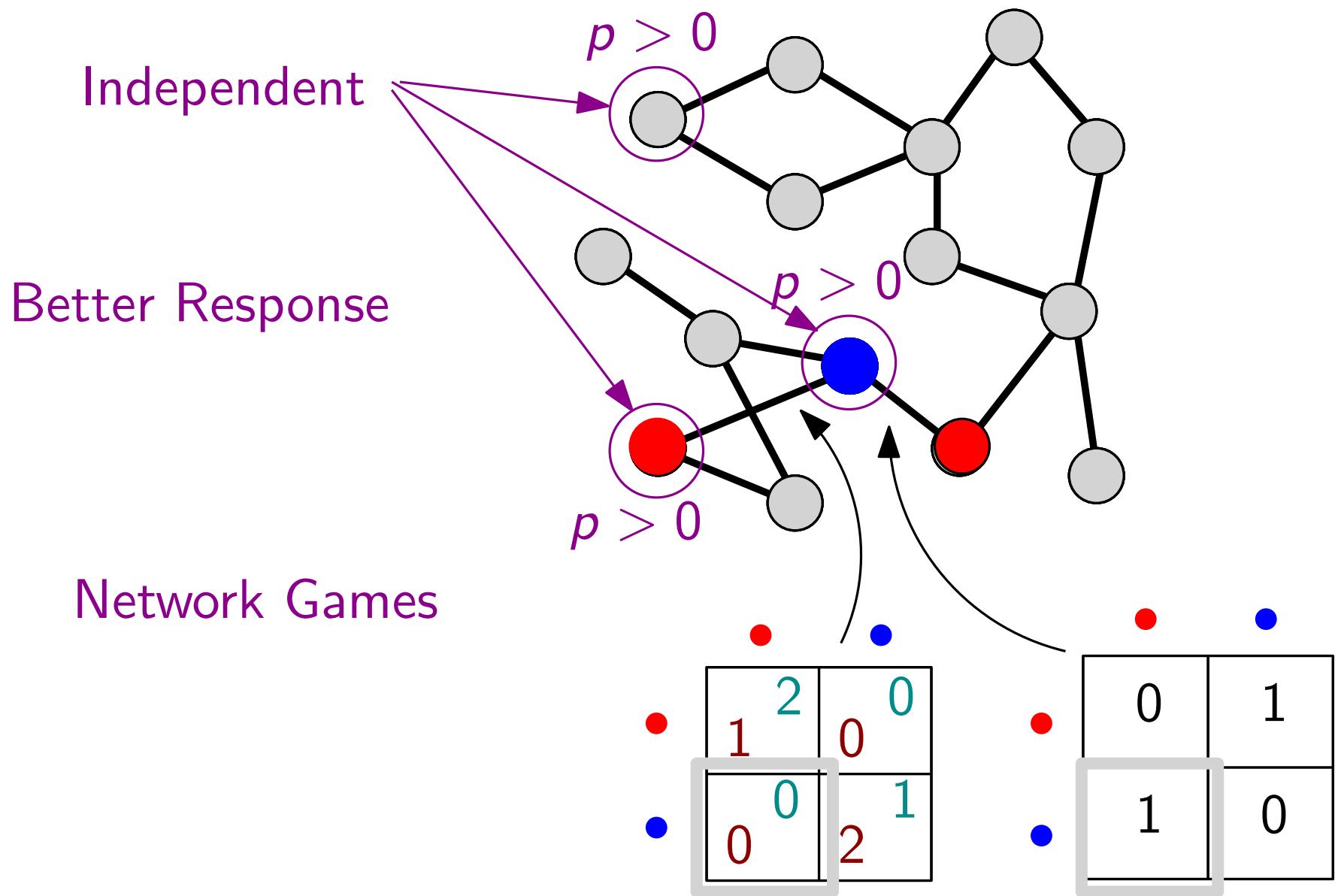
potential games



Our Model

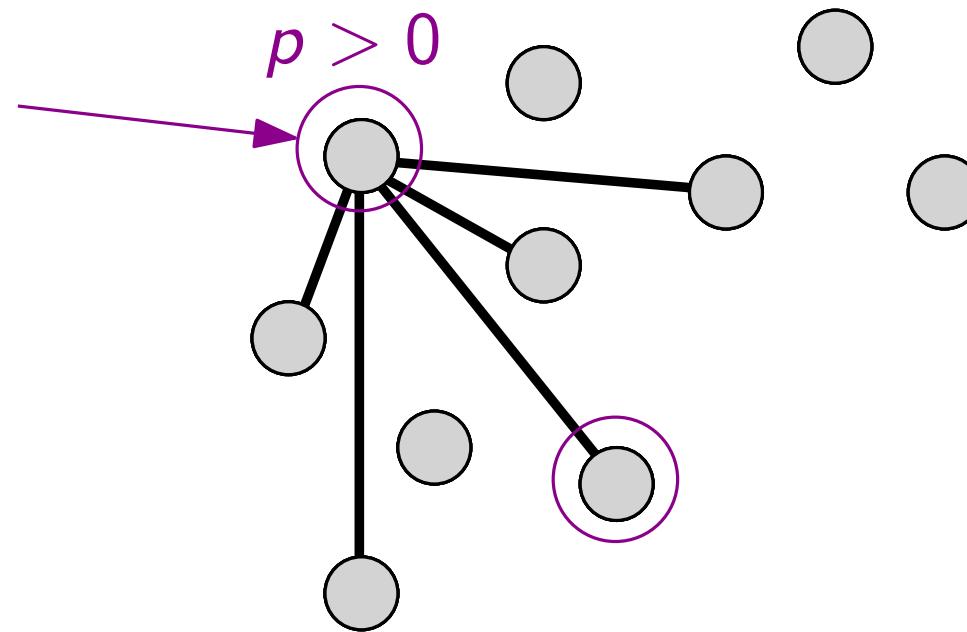


Our Model



Results

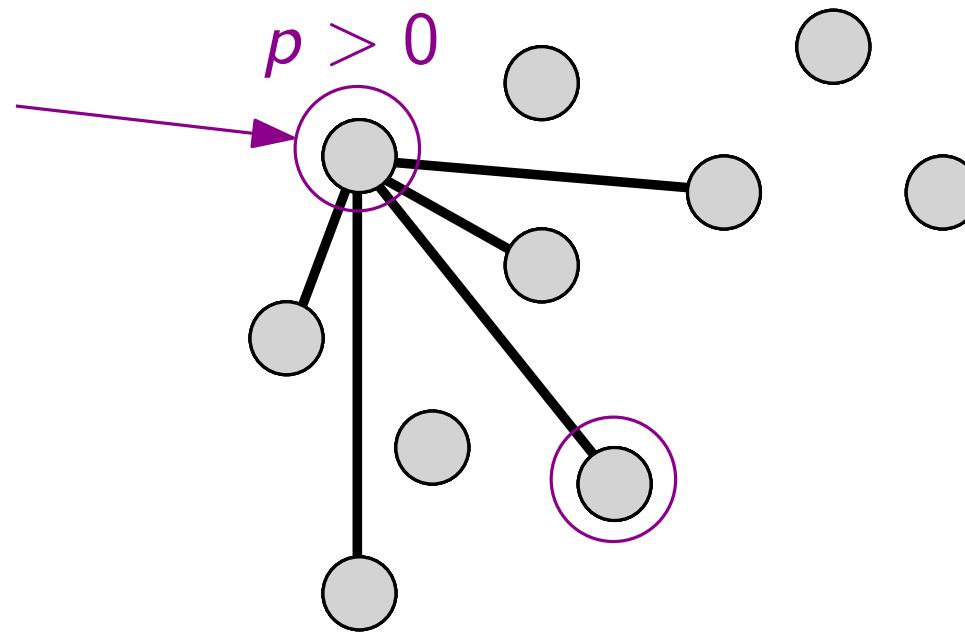
Independent



Results

Independent

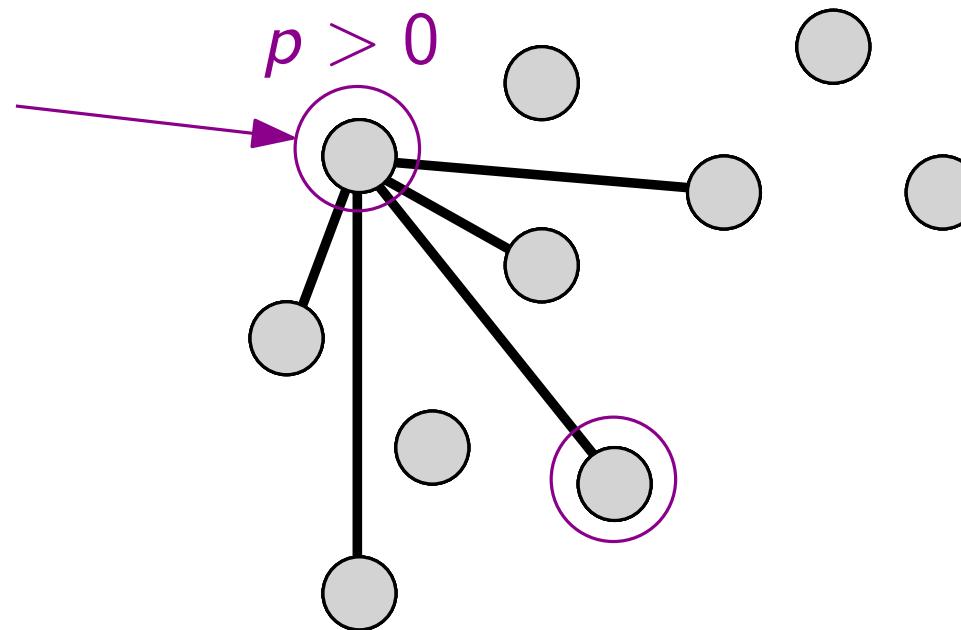
Lazy
(p small)



Results

Independent

Lazy
(p small)



Lazy
 $p \sim 1/\Delta$

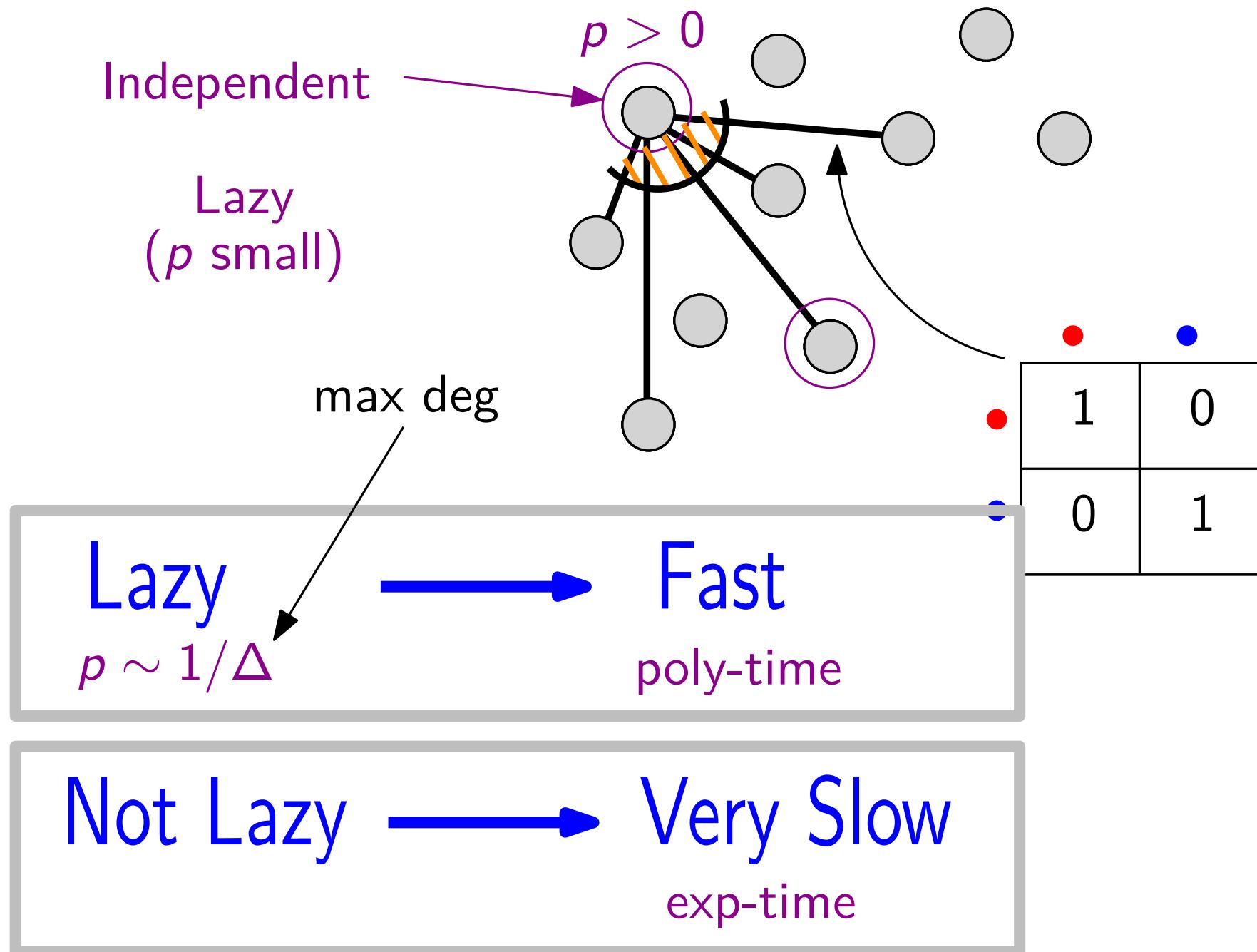
Fast
poly-time

Not Lazy

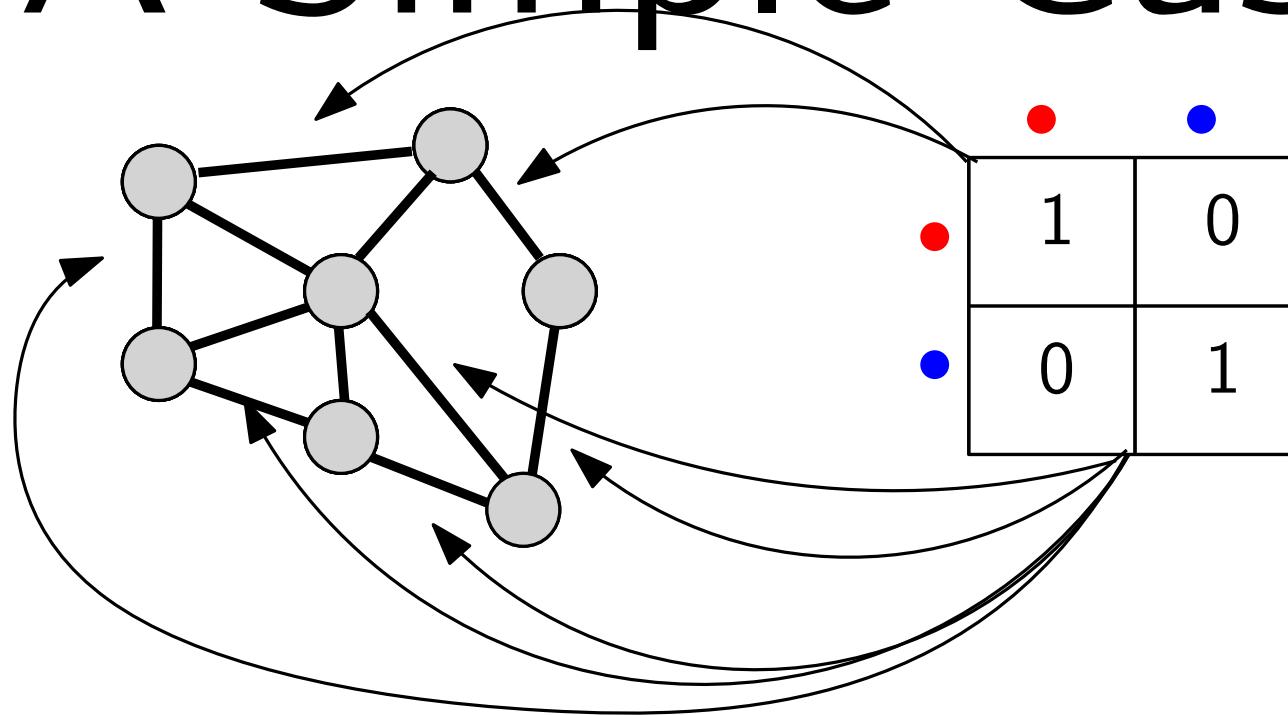


Very Slow
exp-time

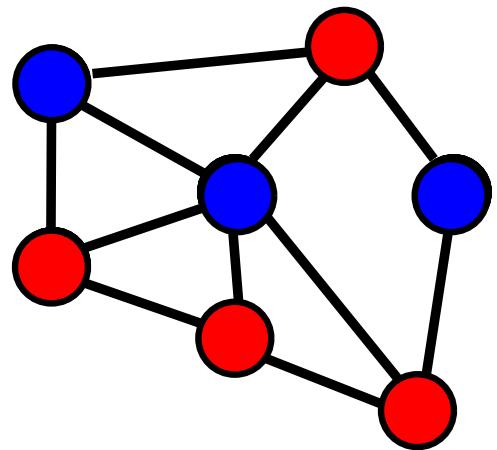
Results



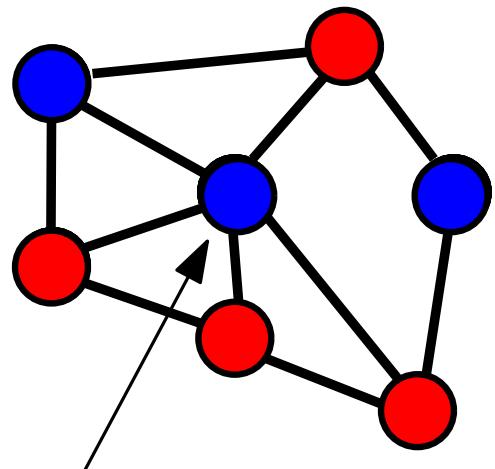
A Simple Case



A Simple Case

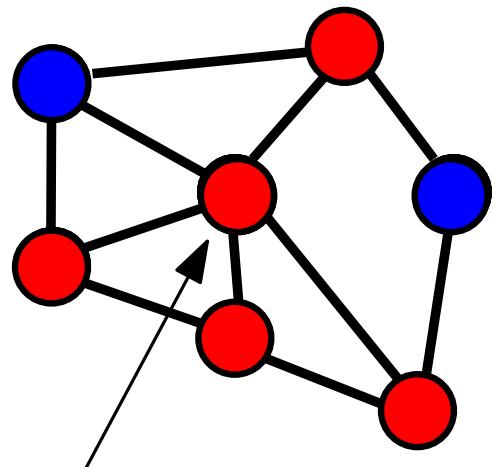


A Simple Case



Best Response = Majority

A Simple Case



Best Response = Majority

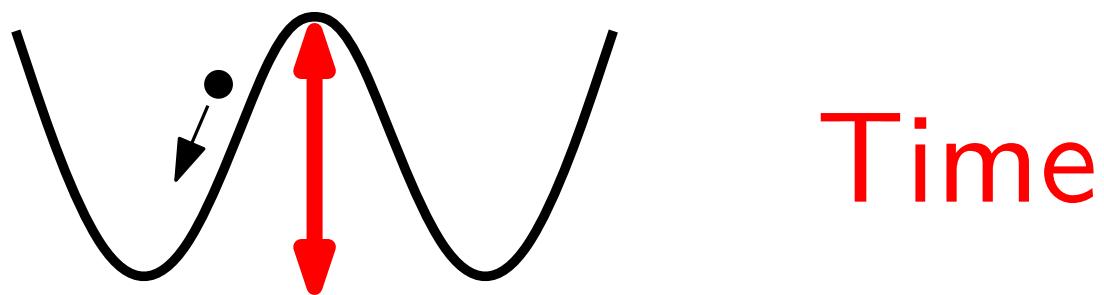
Upper Bound

Lazy
 $p \sim 1/\Delta$

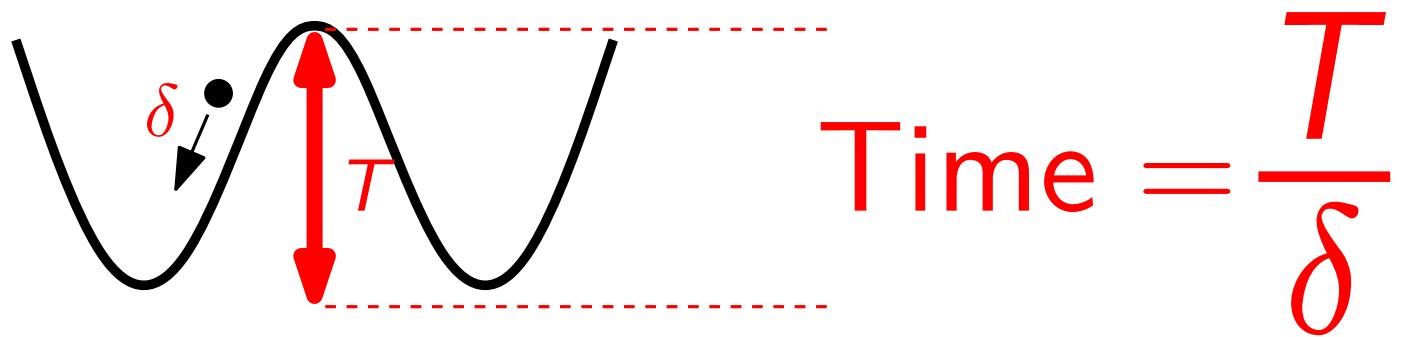


Fast
 $O(\Delta m)$

Upper Bound

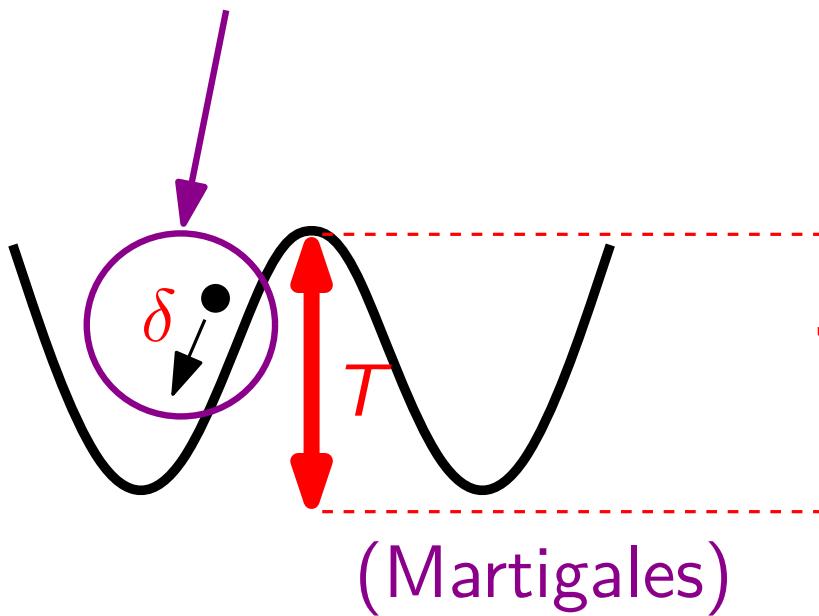


Upper Bound



Upper Bound

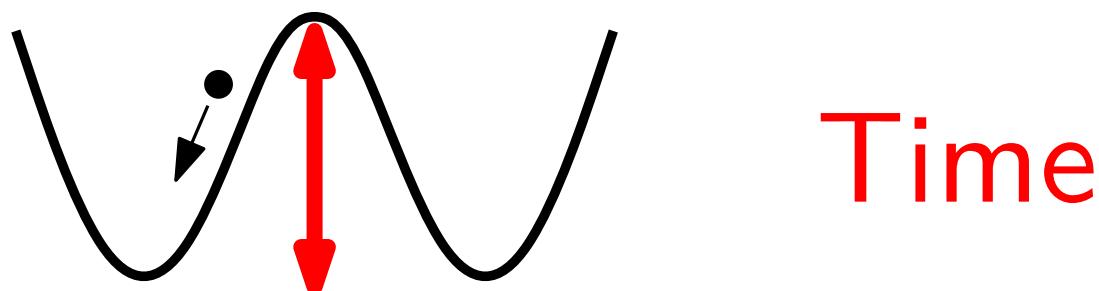
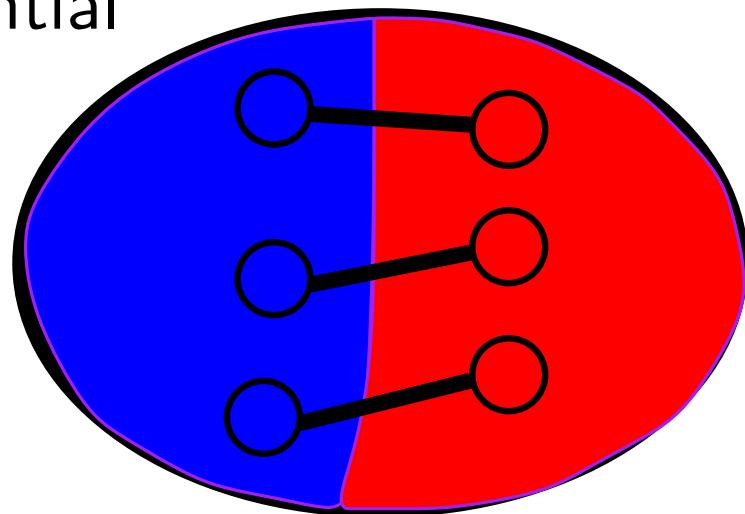
in expectation



$$\text{Time} = \frac{T}{\delta}$$

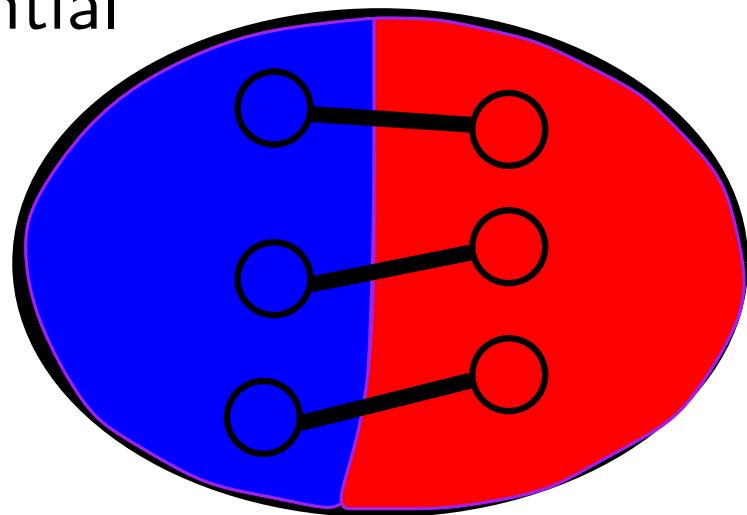
Upper Bound

Potential

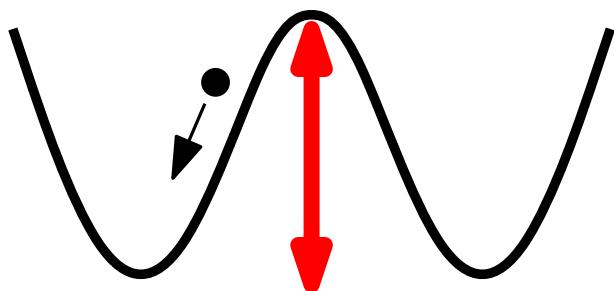
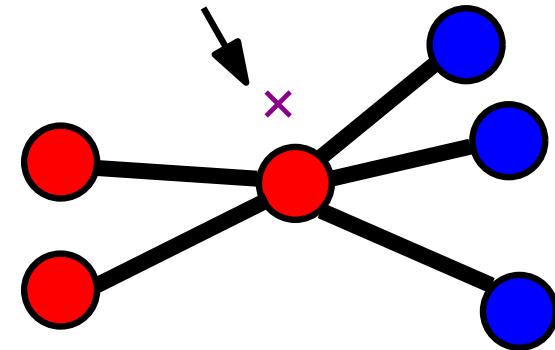


Upper Bound

Potential



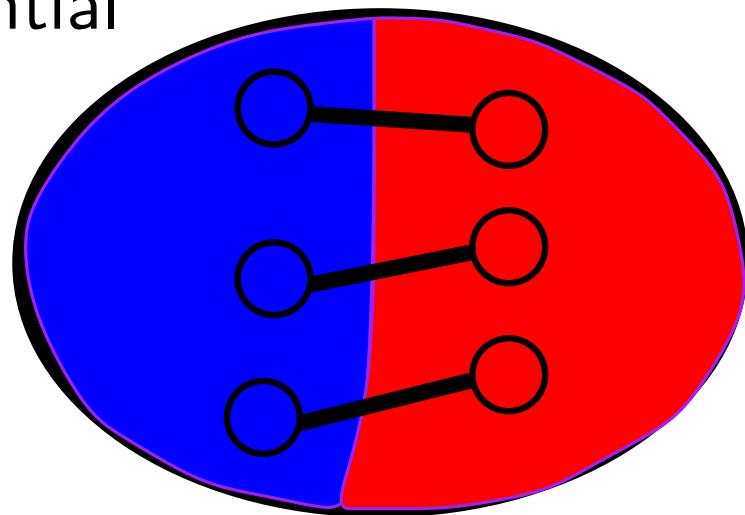
Unstable



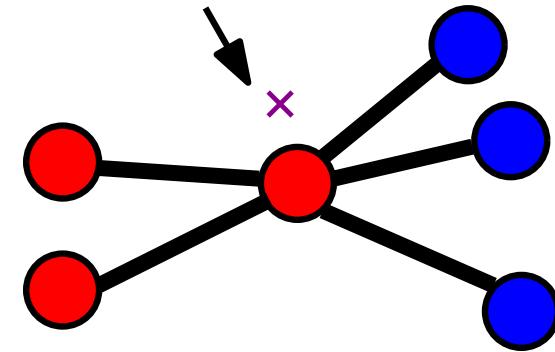
Time

Upper Bound

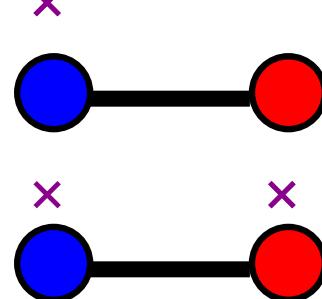
Potential



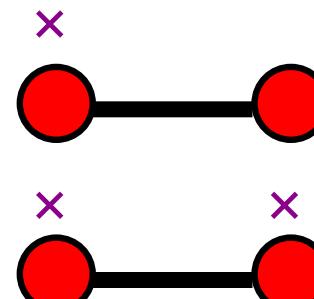
Unstable



Good/Gain
↓

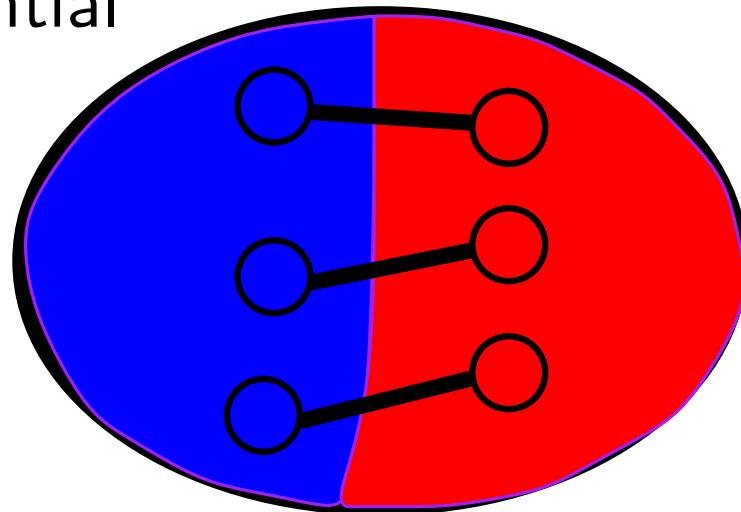


Bad/Loose
↑

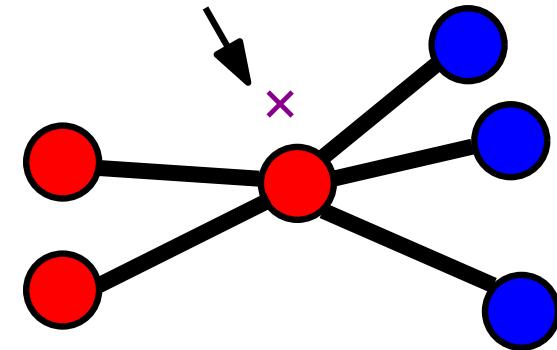


Upper Bound

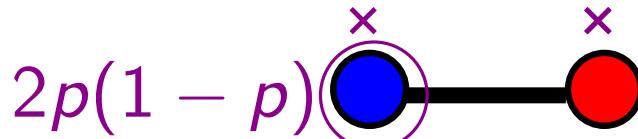
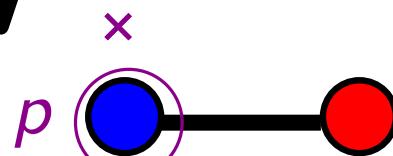
Potential



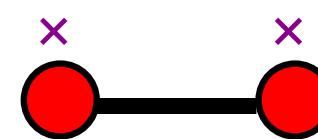
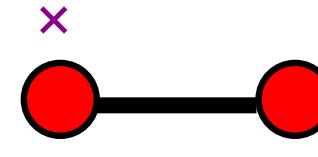
Unstable



Good/Gain
↓

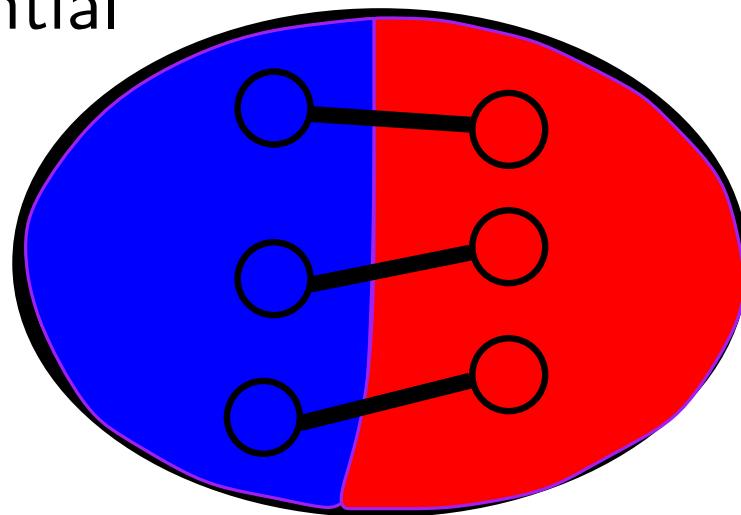


Bad/Loose
↑

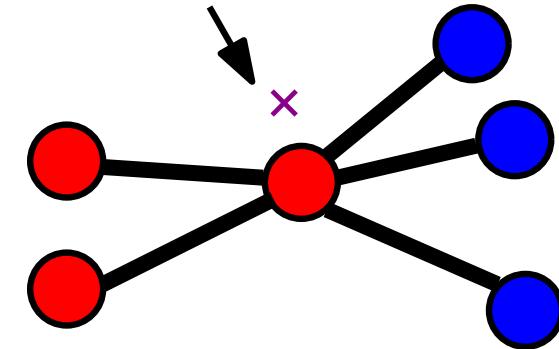


Upper Bound

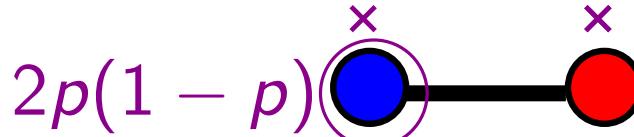
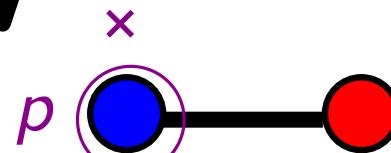
Potential



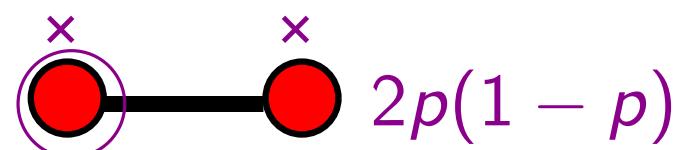
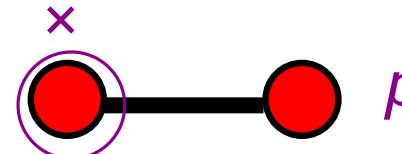
Unstable



Good/Gain
↓

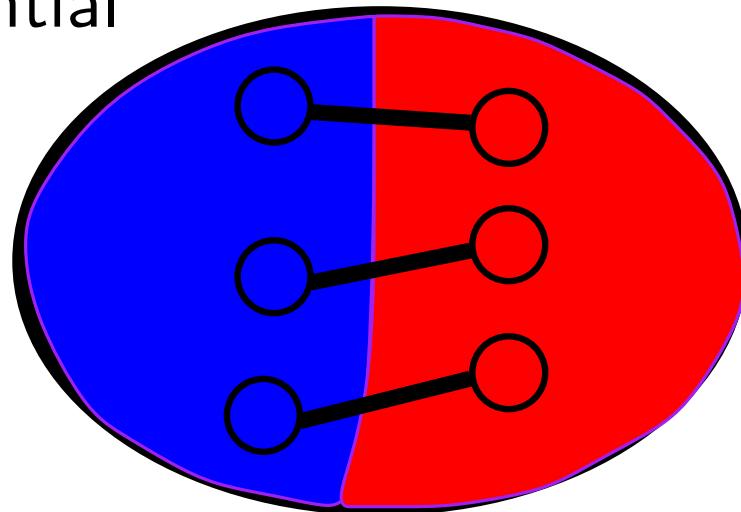


Bad/Loose
↑

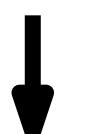
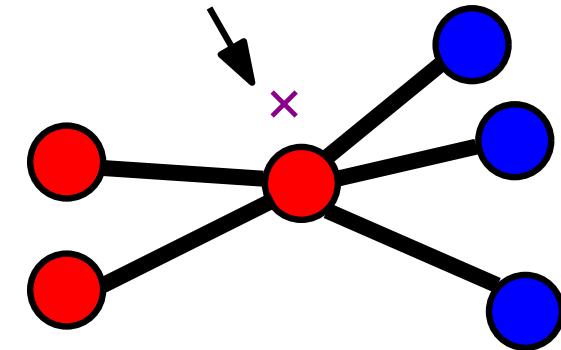


Upper Bound

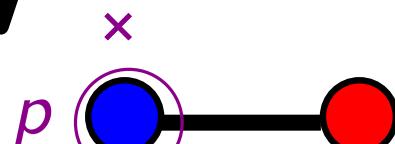
Potential



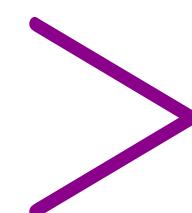
Unstable



Good/Gain



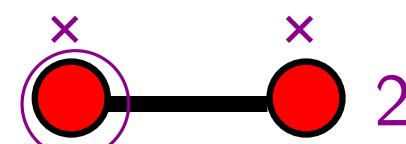
$$2p(1 - p)$$



Bad/Loose

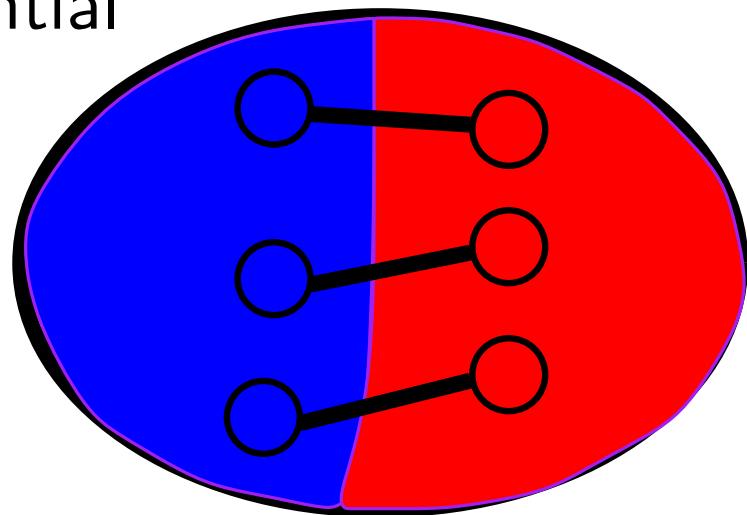


$$2p(1 - p)$$

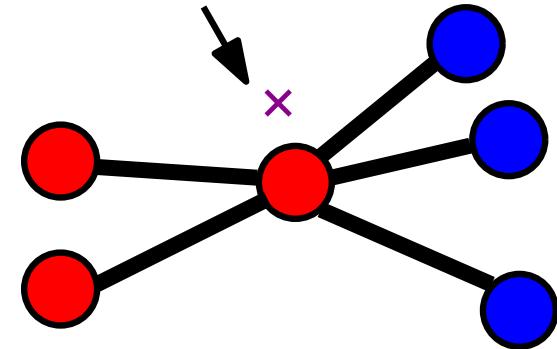


Upper Bound

Potential



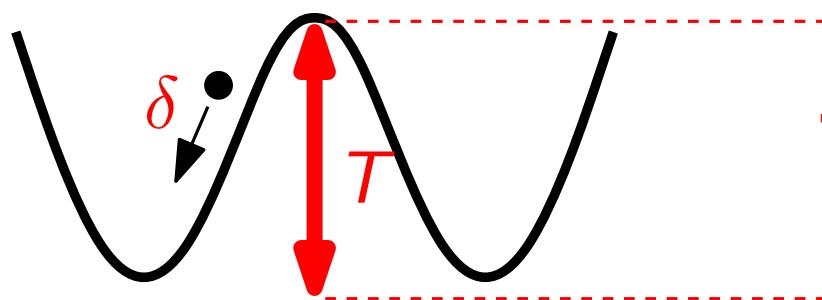
Unstable



$$p = \frac{1-\epsilon}{\Delta}$$

$$\delta = \frac{\epsilon(1-\epsilon)}{\Delta}$$

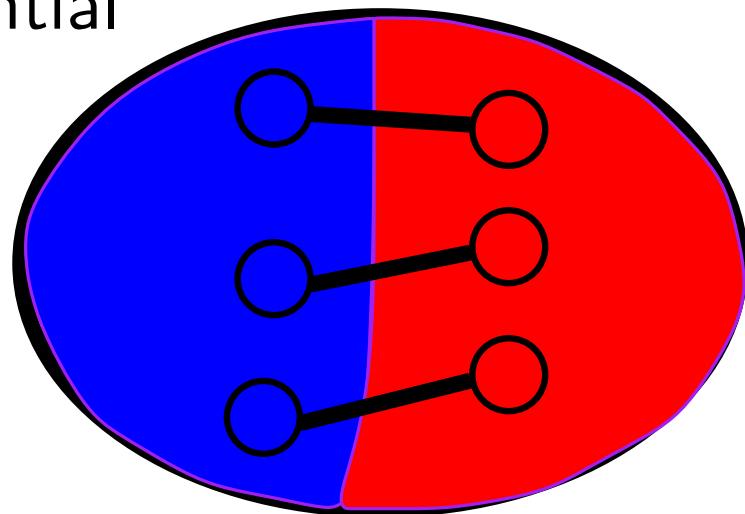
$$T/\delta = m/\delta = O(\Delta m)$$



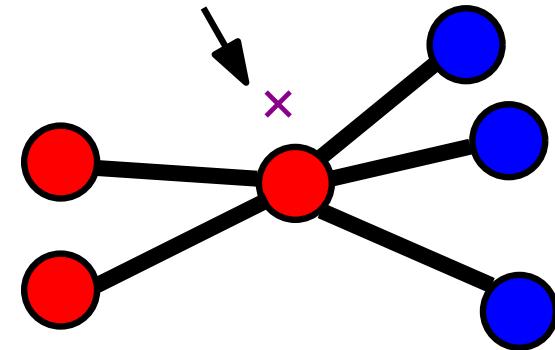
$$\text{Time} = \frac{T}{\delta}$$

Upper Bound

Potential



Unstable



Lazy

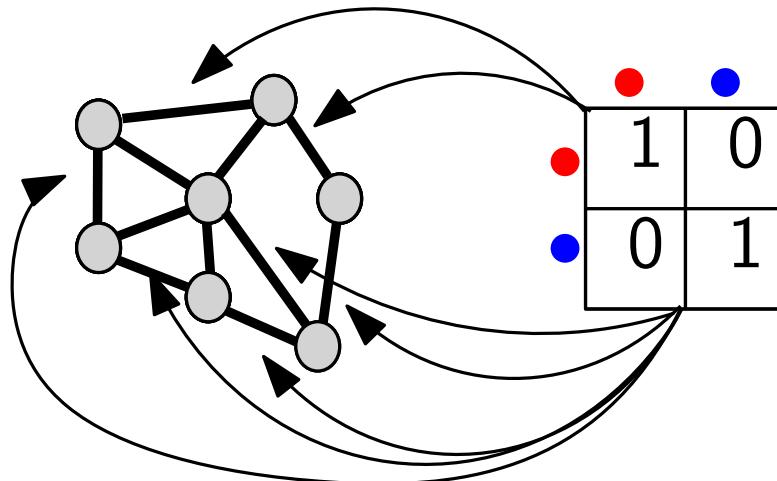
$$p \sim 1/\Delta$$



Fast

$$O(\Delta m)$$

Upper Bound



Lazy
 $p \sim 1/\Delta$

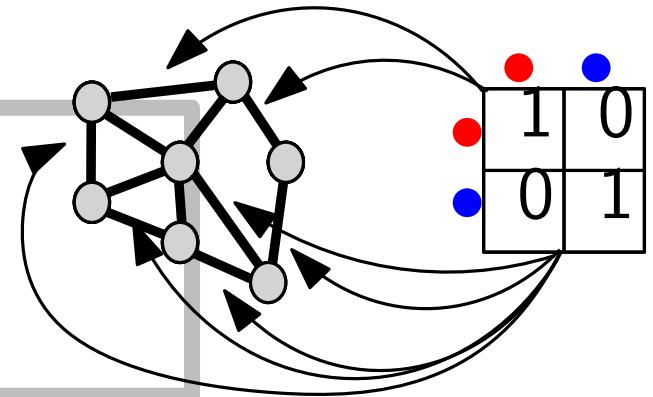


Fast
 $O(\Delta m)$

Lazy
 $p \sim 1/\Delta$



Fast
 $O(\Delta m)$



Lazy

$$p \sim 1/\Delta$$

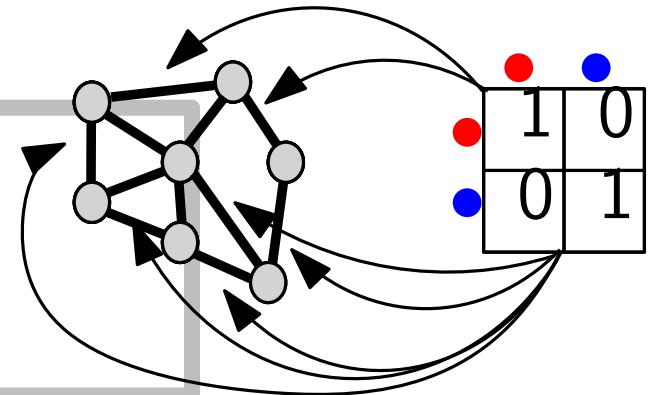


Fast

$$O(\Delta m)$$

$$p_u \sim 1/d_u$$

$$O(\Delta m)$$



Lazy

$$p \sim 1/\Delta$$

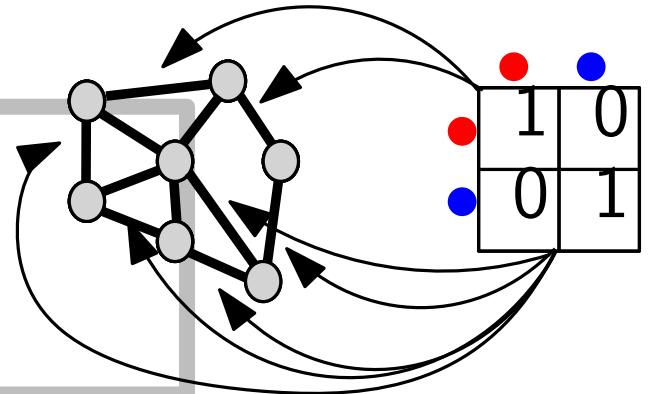


Fast

$$O(\Delta m)$$

$$\begin{aligned} p_u &\sim 1/d_u \\ p_u^t &\sim 1/d_u^t \end{aligned}$$

$$\begin{aligned} O(\Delta m) \\ O(m) \end{aligned}$$



Lazy

$$p \sim 1/\Delta$$



Fast

$$O(\Delta m)$$

$$\begin{aligned} p_u &\sim 1/d_u \\ p_u^t &\sim 1/d_u^t \end{aligned}$$

$$\begin{aligned} O(\Delta m) \\ O(m) \end{aligned}$$

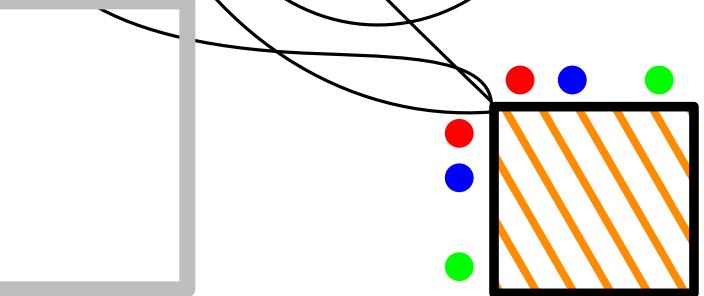
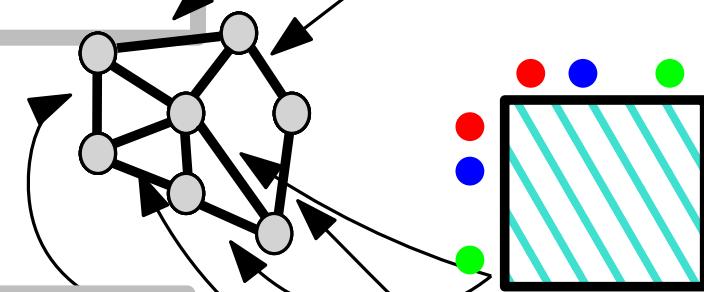
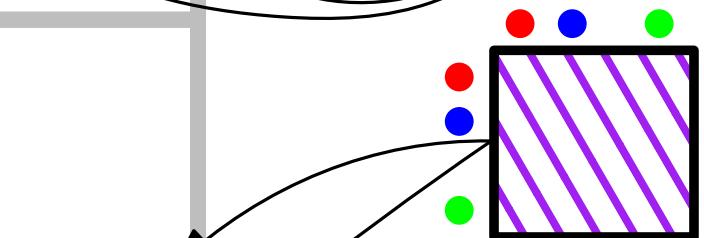
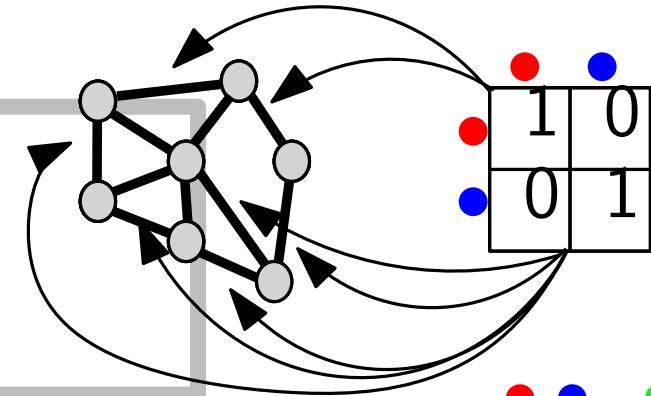
Lazy

$$p \sim 1/\Delta_W$$



Fast

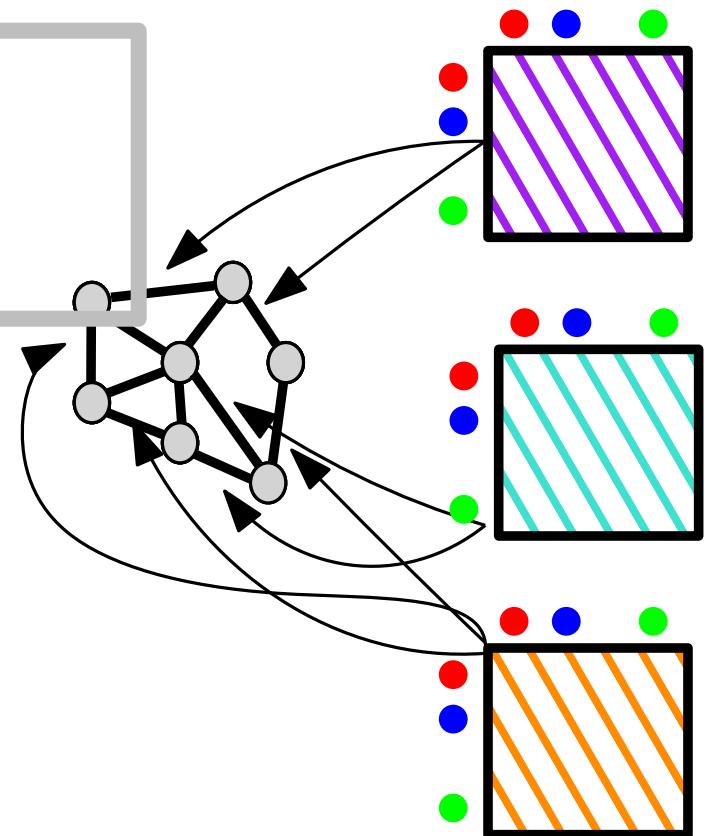
$$O(\Delta_W^2 n)$$



Lazy
 $p \sim 1/\Delta_W$



Fast
 $O(\Delta_W^2 n)$



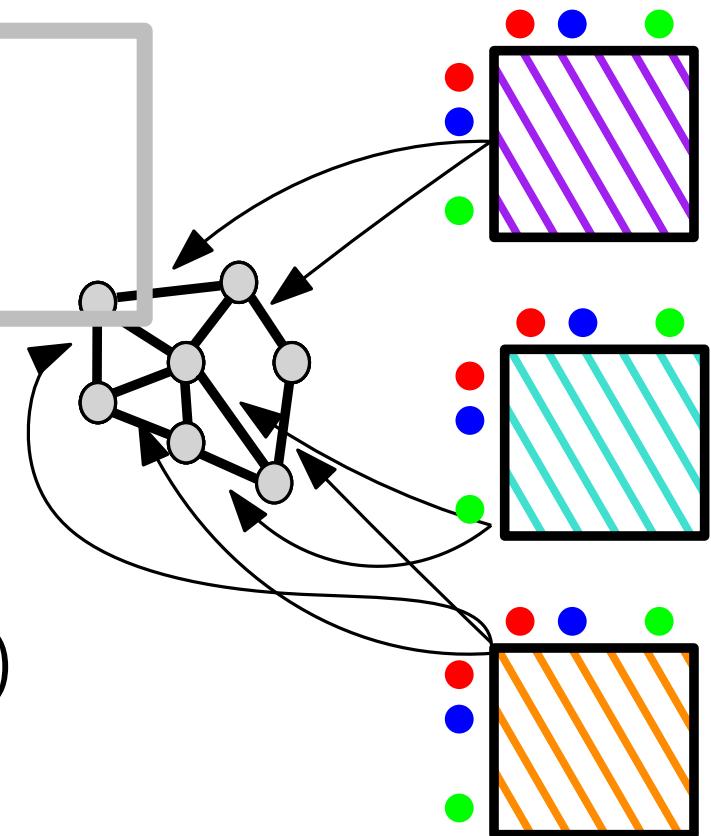
Lazy

$$p \sim 1/\Delta_W$$

Fast

$$O(\Delta_W^2 n)$$

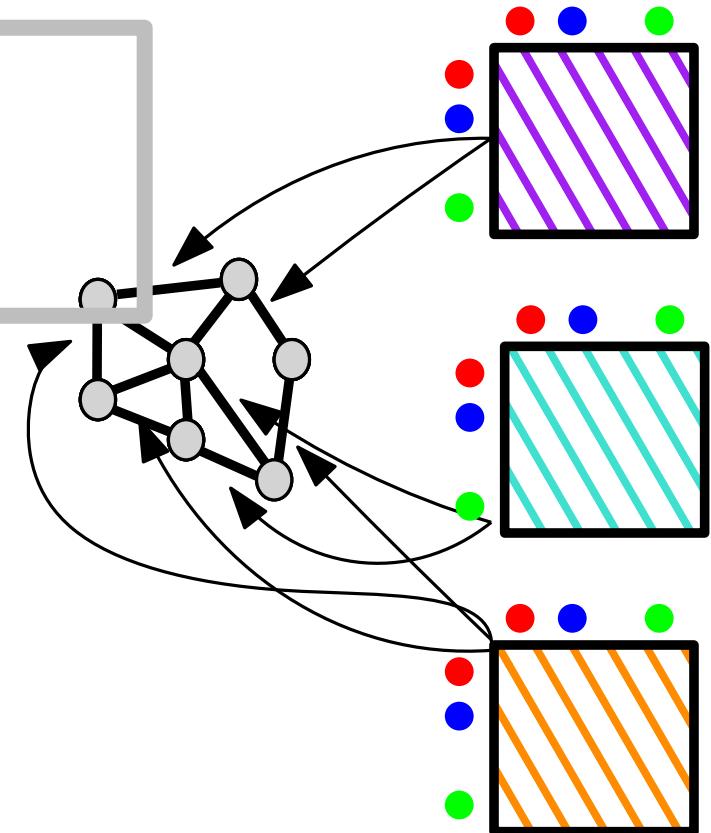
Large (not poly in general)



Lazy
 $p \sim 1/\Delta_W$



Fast
 $O(\Delta_W^2 n)$



Max-Cut games (PLS-complete)
(Schaeffer and Yannakakis, 1991)

Lazy
 $p \sim 1/\Delta_W$

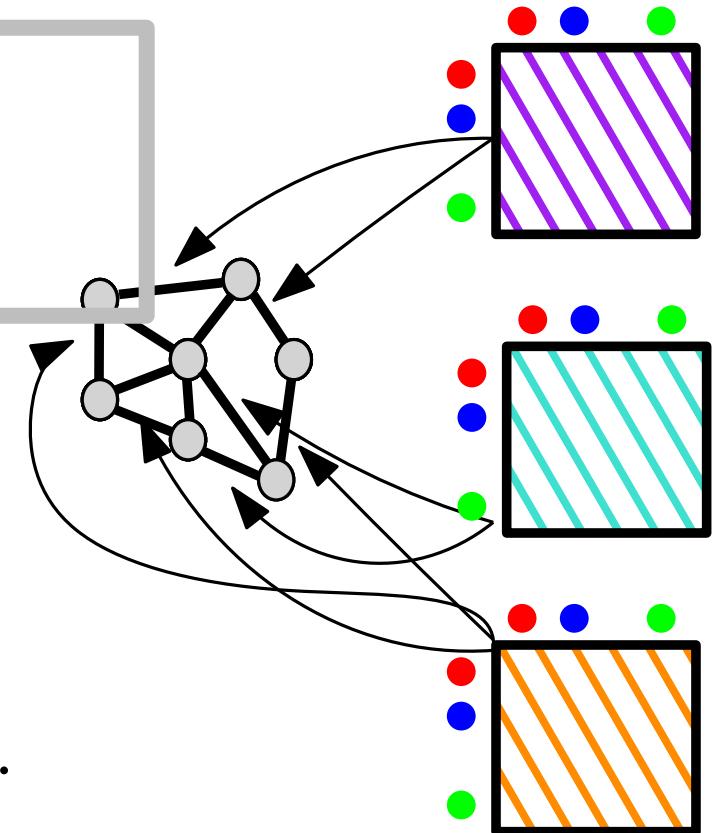


Fast
 $O(\Delta_W^2 n)$

	•	•	•
•	0	W_1	•
•	W_1	0	•
•	•	•	•

	•	•	•
•	0	W_2	•
•	W_2	0	•
•	•	•	•

...

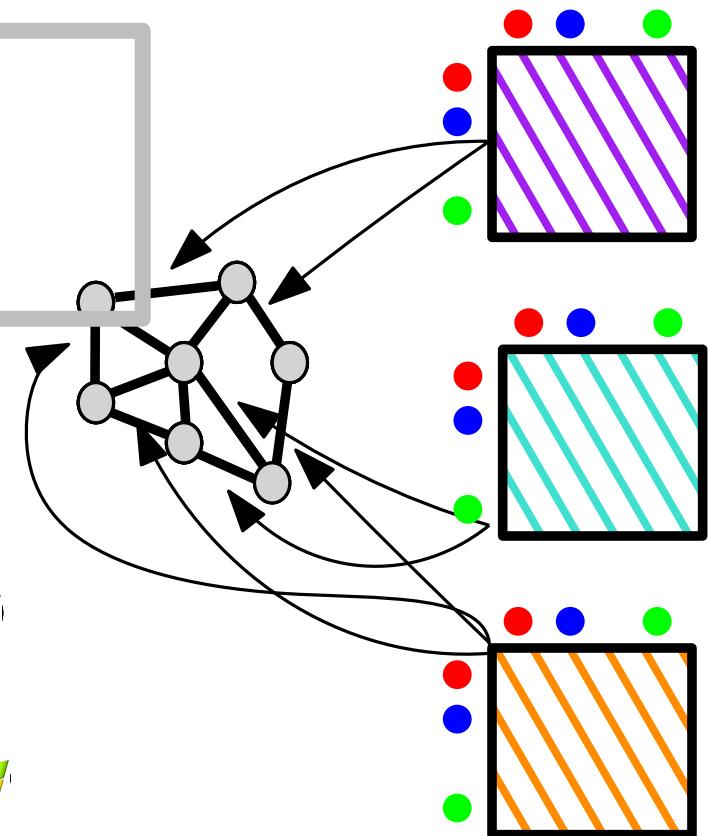
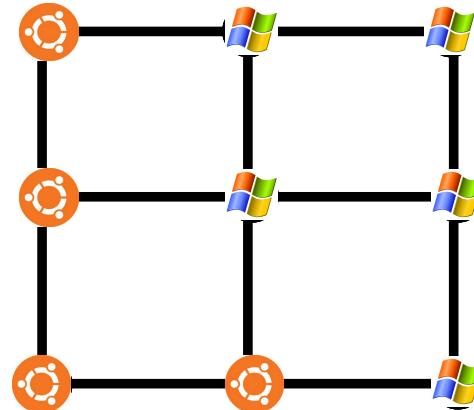
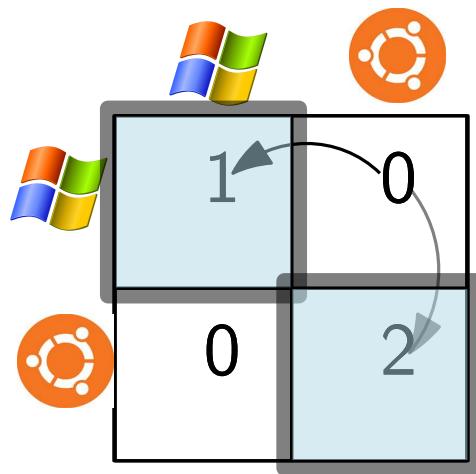


Max-Cut games (PLS-complete)
(Schaeffer and Yannakakis, 1991)

Lazy
 $p \sim 1/\Delta_W$



Fast
 $O(\Delta_W^2 n)$

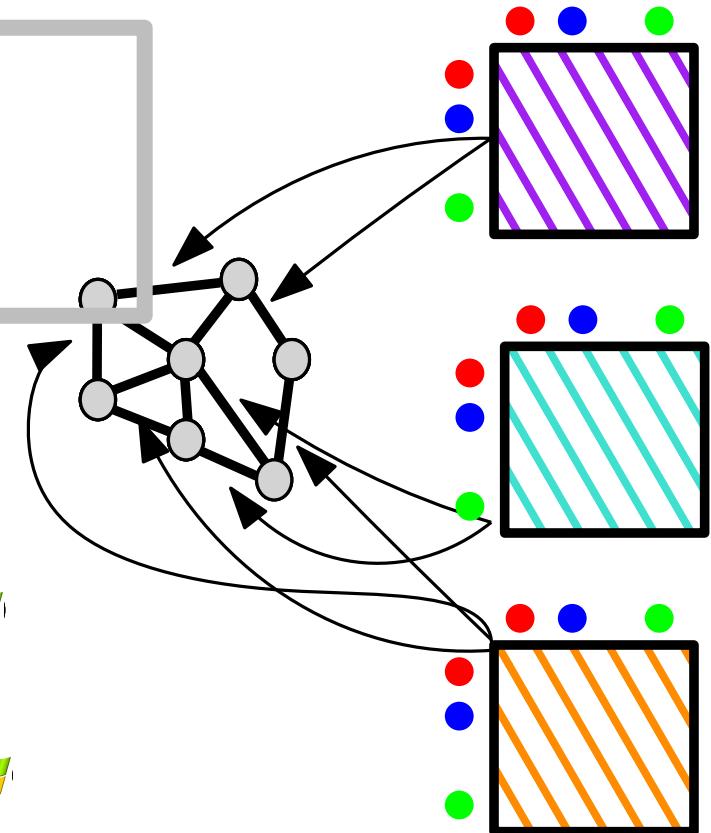
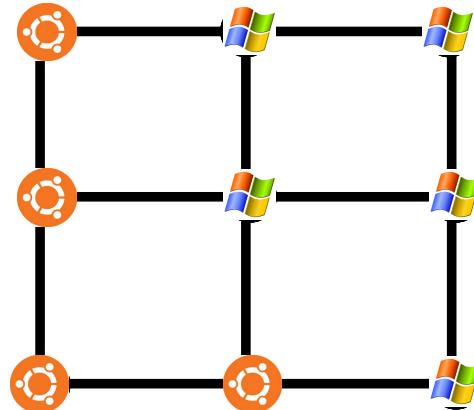
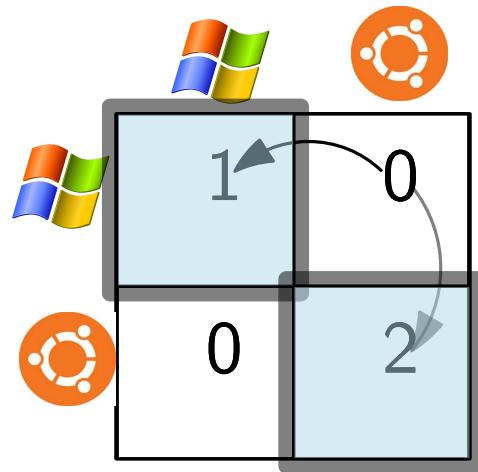


Spread of technologies poly-time
...(Montanari - Saberi, 2010)...

Lazy
 $p \sim 1/\Delta_W$



Fast
 $O(\Delta_W^2 n)$



Opinion games

poly-time

...(Ferraioli - Goldberg - Ventre, 2016)

Open Questions

$\Delta \approx n$

poly-time

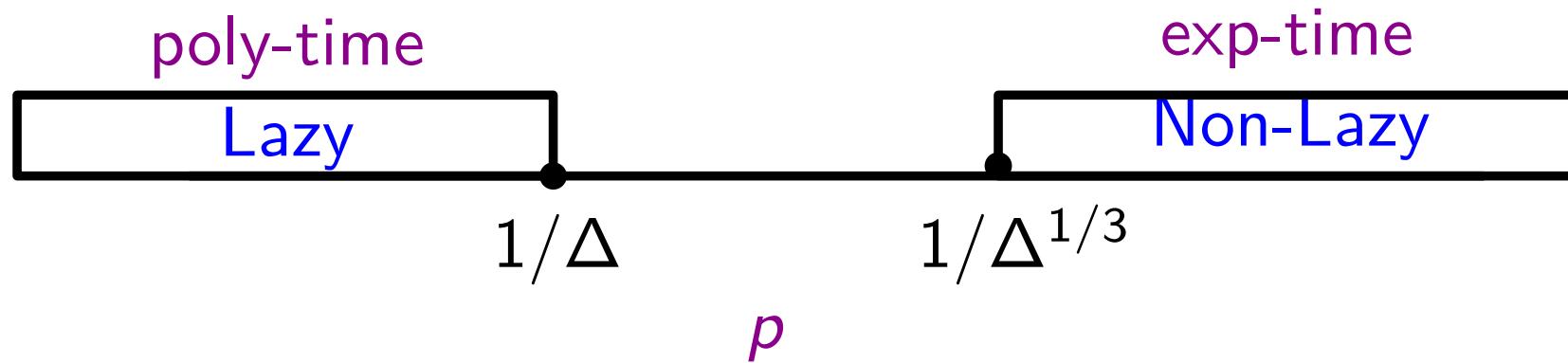
Lazy

$1/\Delta$

p

Open Questions

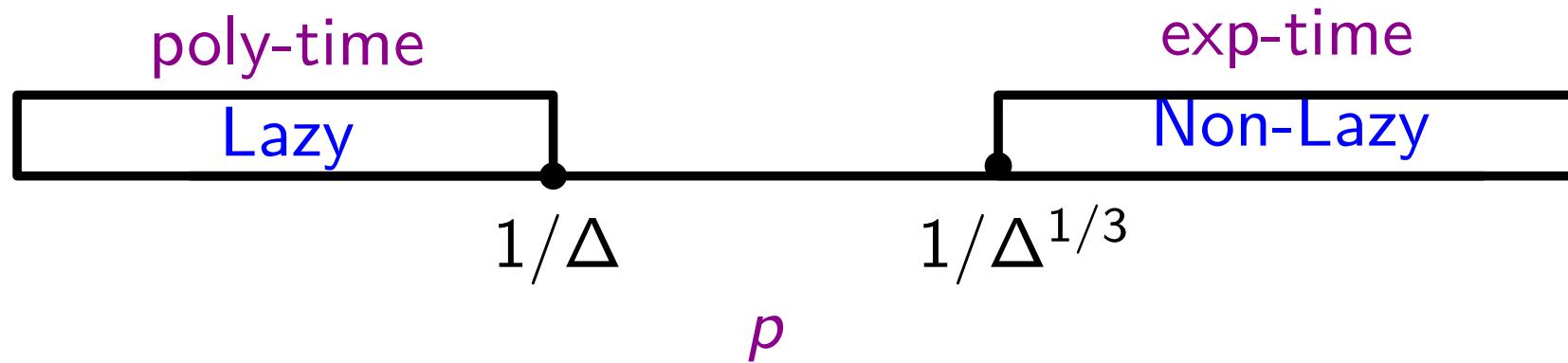
$\Delta \approx n$



$\text{poly-time} \leftrightarrow p \sim 1/\Delta$

Open Questions

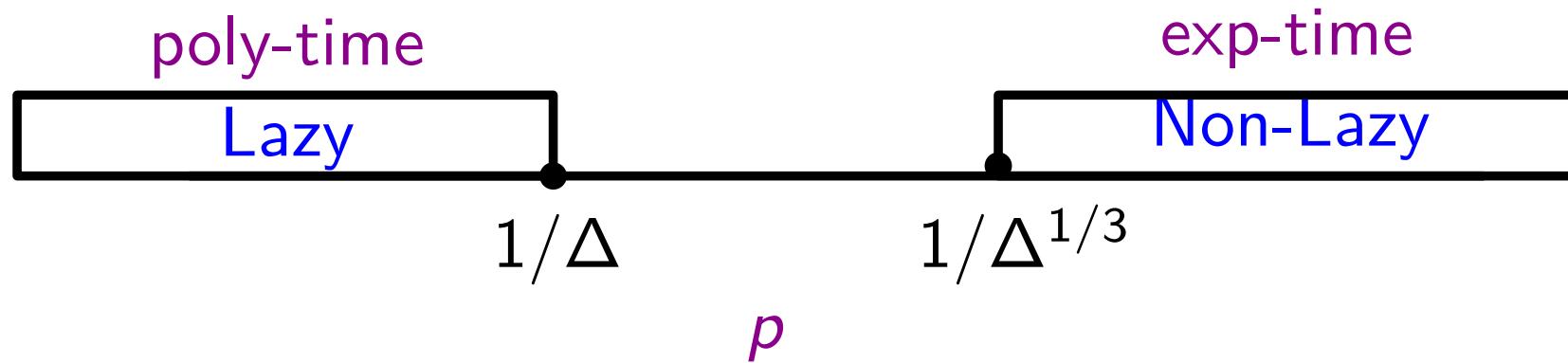
$\Delta \approx n$



$\text{poly-time} \leftrightarrow p \sim 1/\Delta$

Open Questions

$\Delta \approx n$



$$\text{poly-time} \leftrightarrow p \sim 1/\Delta$$

General Potential Games?

- Graphical Games?
(Kearns, - Littman - Singh, 2001)

Thank You!!