Tighter Bounds on the Inefficiency Ratio of Stable Equilibria in Load Balancing Games

Akaki Mamageishvili   Paolo Penna
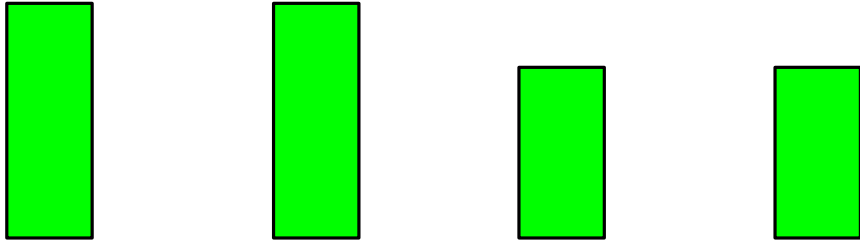
ETH Zurich

Outline
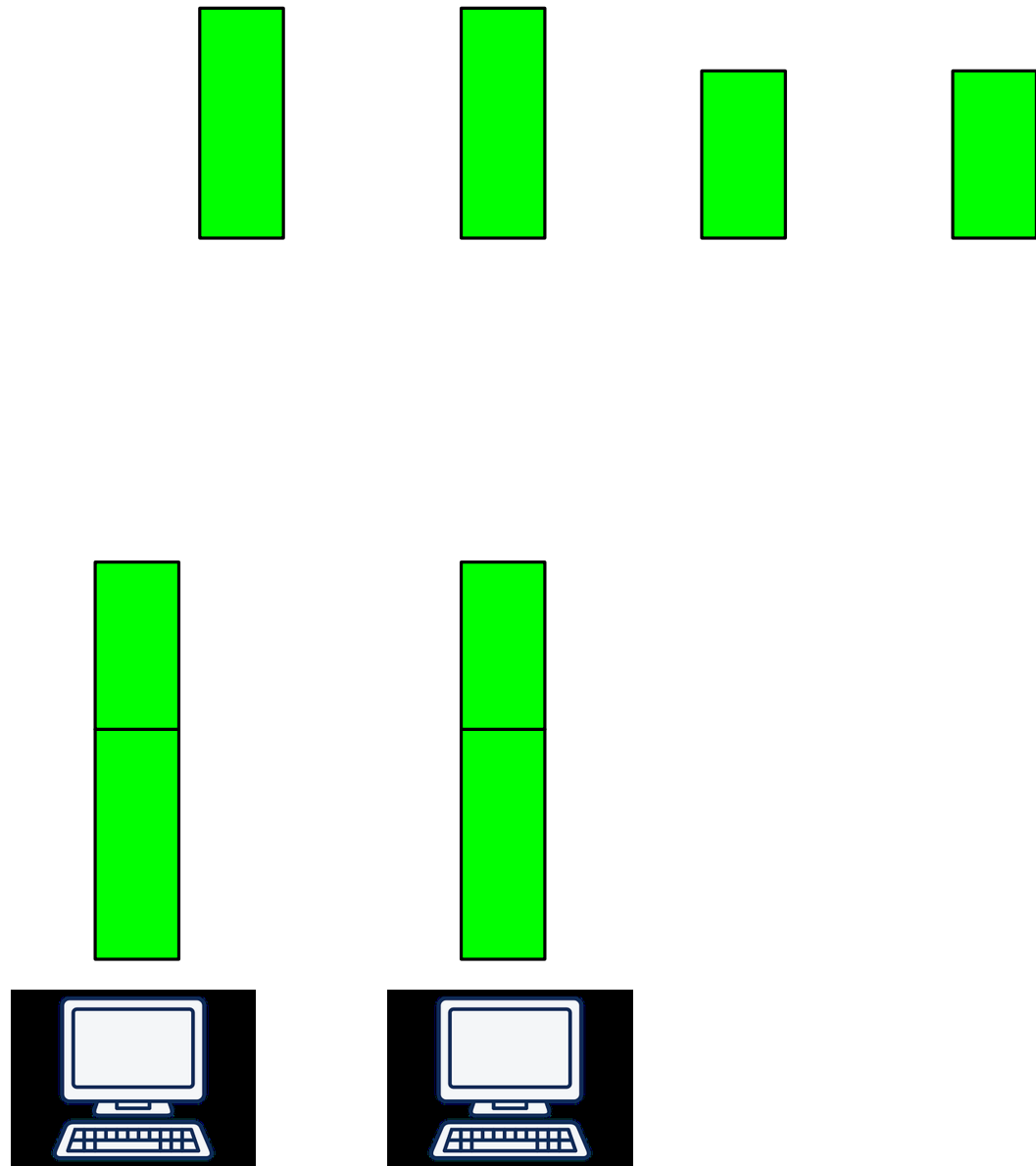
Load Balancing Games

Inefficiency Ratio of Stable Equilibria
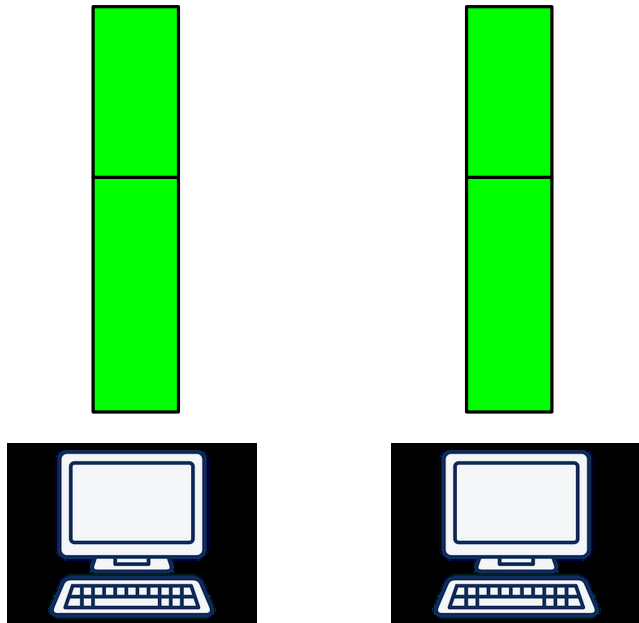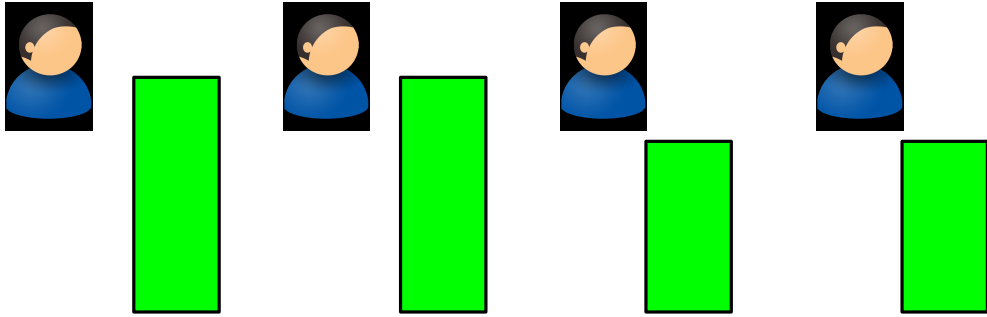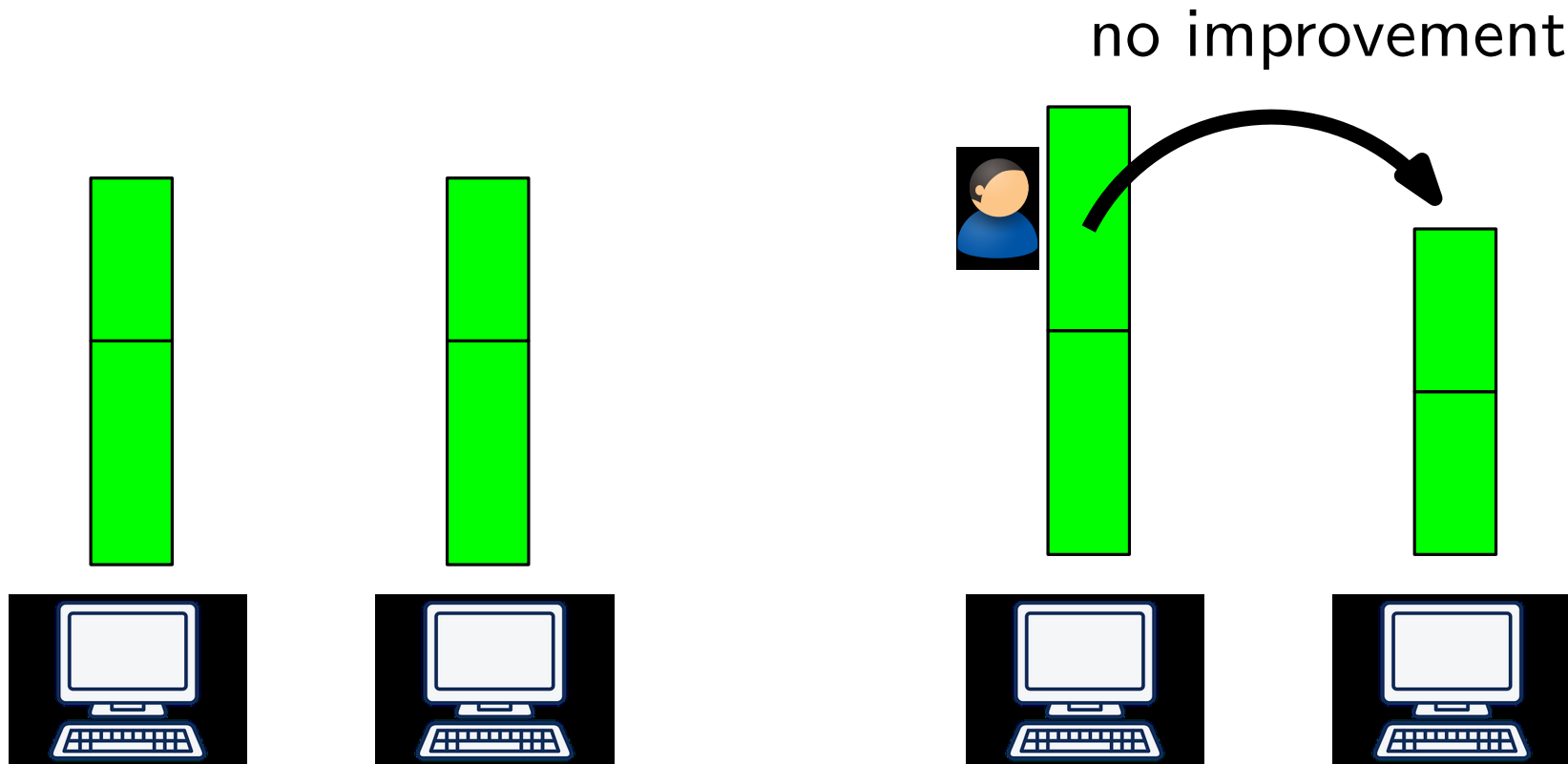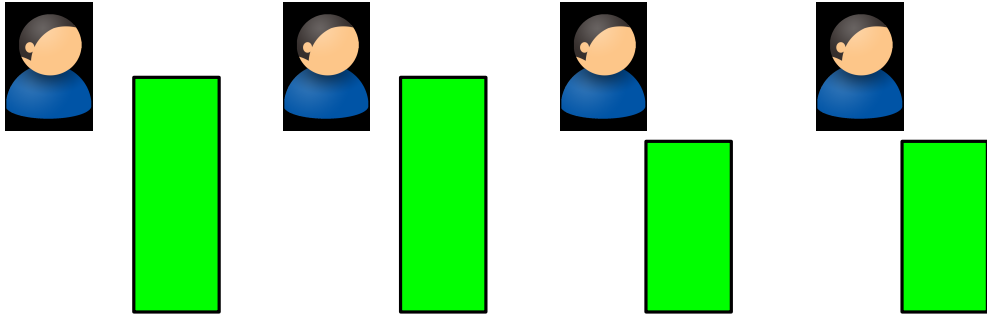
Tighter Bounds for IRSE   (Our Contribution)
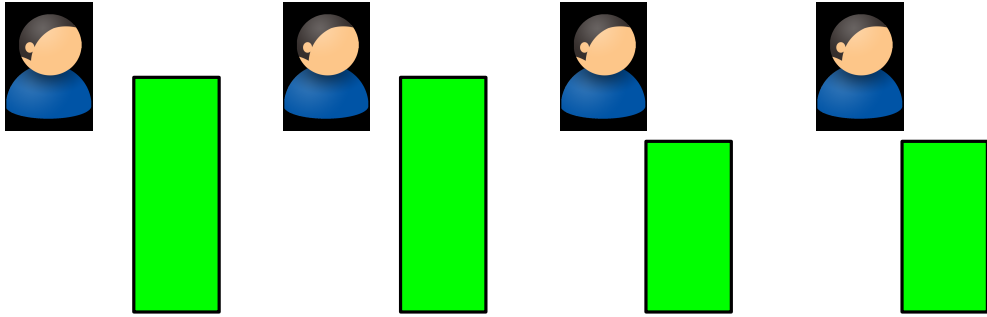
Load Balancing (Games)
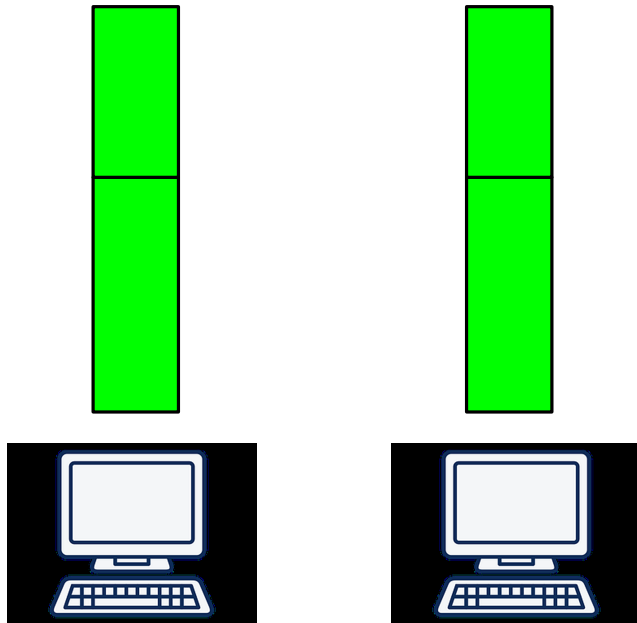
Load Balancing (Games)

Load Balancing (Games)

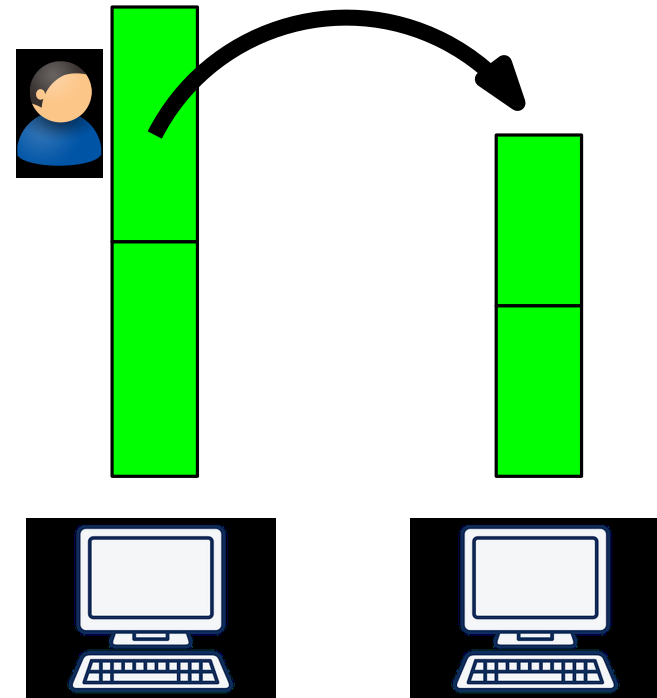# Load Balancing (Games)
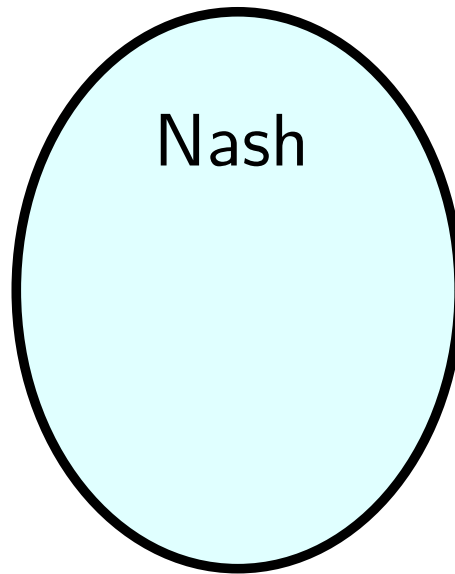

no improvement

Load Balancing (Games)

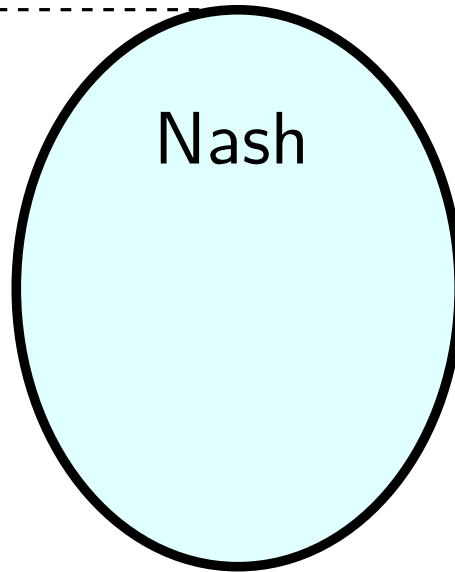OPT

Nash Equilibrium

Quality of Equilibria

Price of Anarchy: Let the players choose some equilibrium by themselves. How bad this can be?

Nash

Quality of Equilibria

Price of Anarchy: Let the players choose some equilibrium by themselves. How bad this can be?
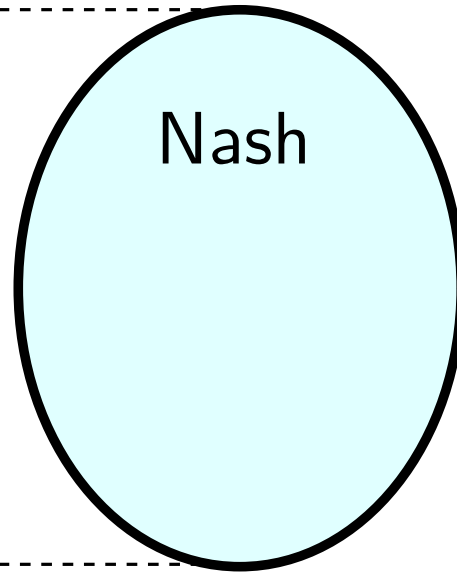
$$PoA = \frac{worst\ Nash}{Opt}$$

Nash

Quality of Equilibria

Price of Anarchy: Let the players choose some equilibrium by themselves. How bad this can be?

$$PoA = \frac{worst\ Nash}{Opt}$$

Nash

$$PoS = \frac{best\ Nash}{Opt}$$

Load Balancing (Games)

Load Balancing (Games)

$$PoA = \frac{2x}{1+x} \to \frac{4}{3}$$
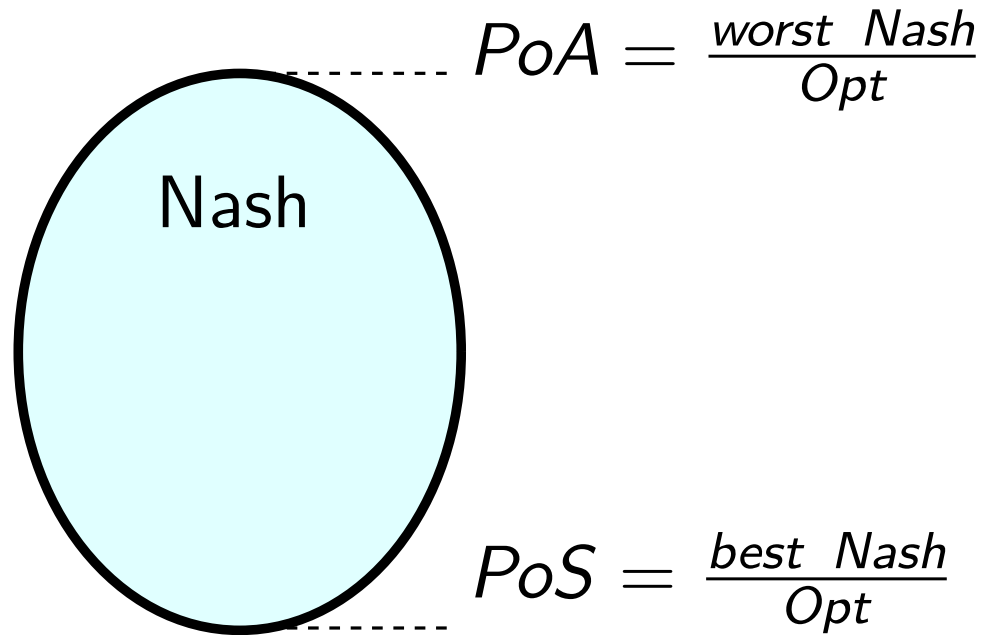
Load Balancing (Games)

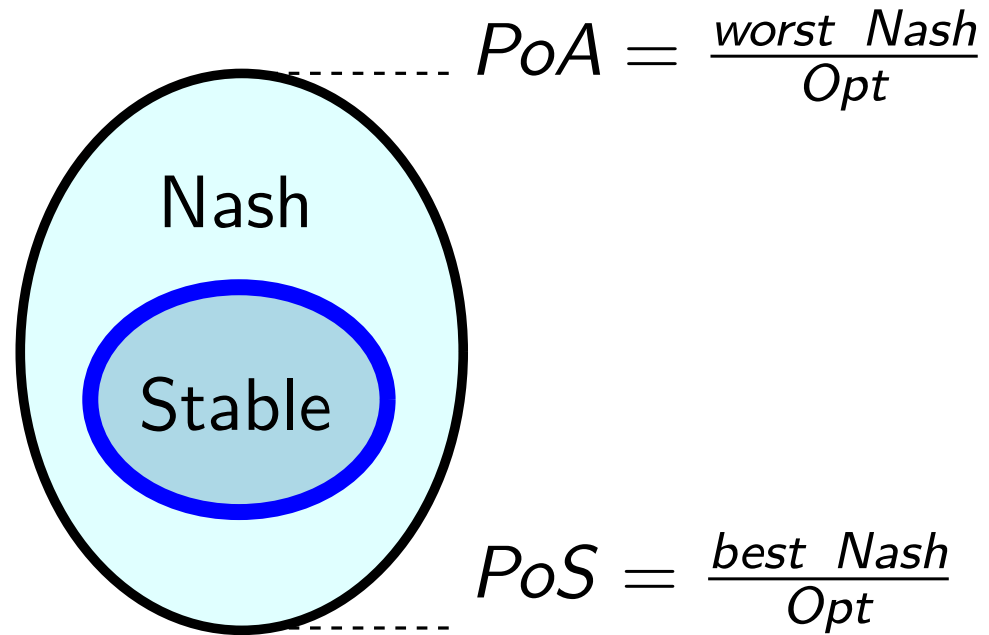$$PoA = \frac{2x}{1+x} \rightarrow \frac{4}{3}$$

In general: $PoA = 2\left(1 - \frac{1}{m+1}\right)$

$$PoS = 1$$

Inefficiency Ratio of Stable Equilibria

$PoA = \frac{worst\ Nash}{Opt}$

Nash

$PoS = \frac{best\ Nash}{Opt}$

Inefficiency Ratio of Stable Equilibria
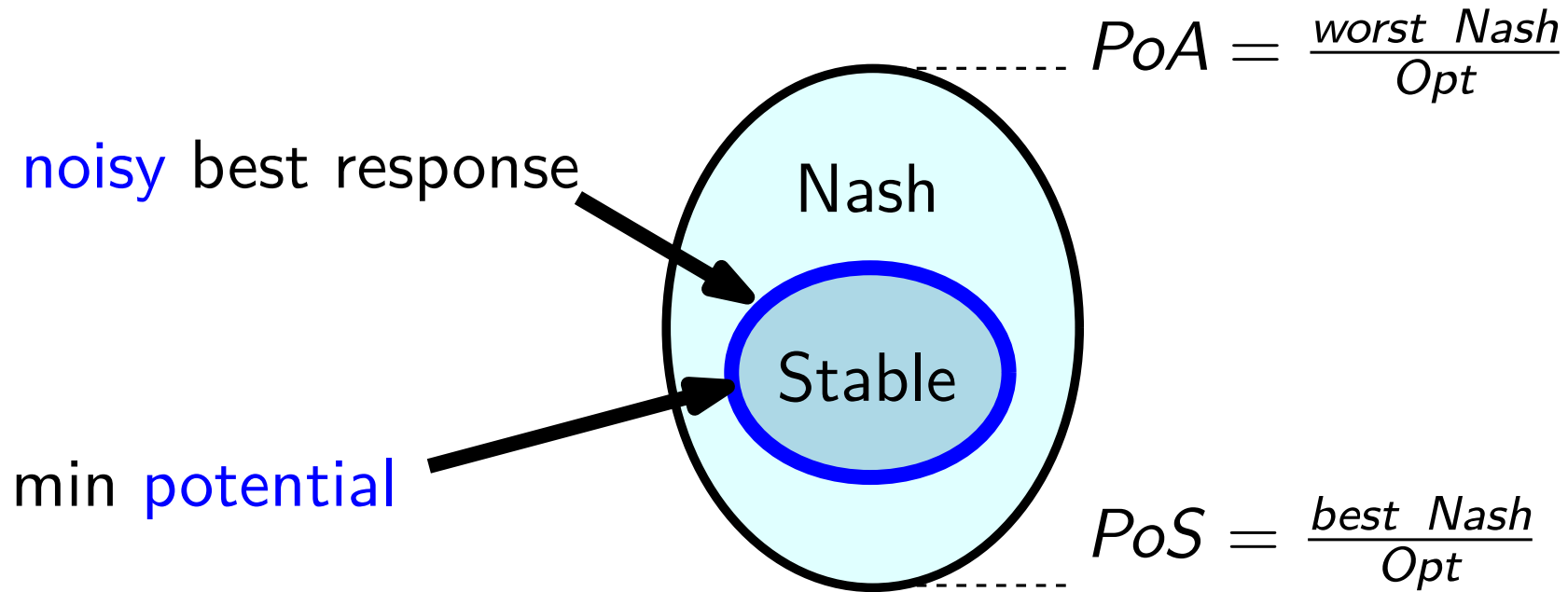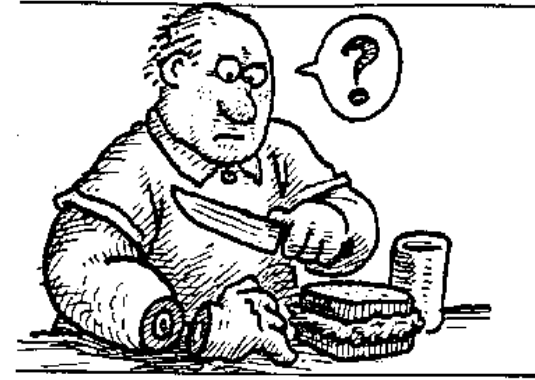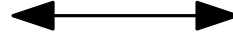

$PoA = \frac{worst\ Nash}{Opt}$

$PoS = \frac{best\ Nash}{Opt}$

Inefficiency Ratio of Stable Equilibria

noisy best response

Nash

Stable

min potential

$PoA = \frac{worst\ Nash}{Opt}$

$PoS = \frac{best\ Nash}{Opt}$

Bounded Rationality

Bounded Rationality

Noisy best response

€10 $prob \propto e^{10/noise}$
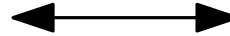
€1 $prob \propto e^{1/noise}$
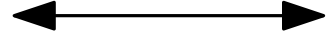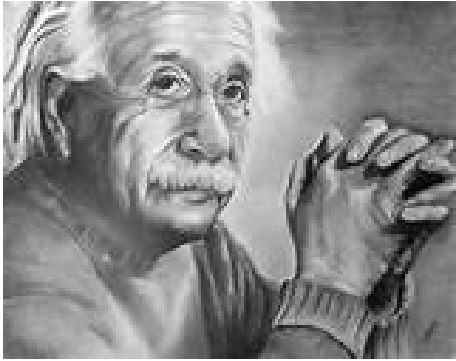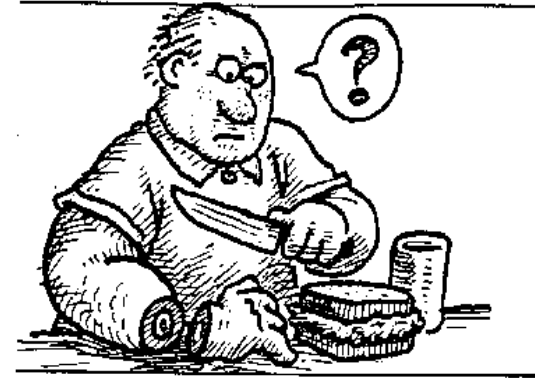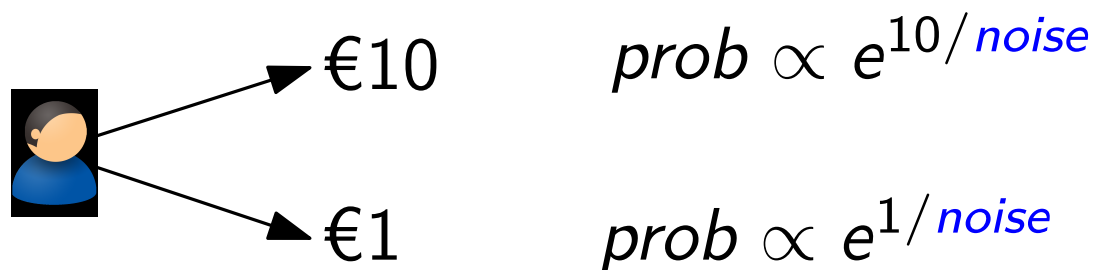
Strategies with higher payoff chosen with higher probability

Inefficiency Ratio of Stable Equilibria
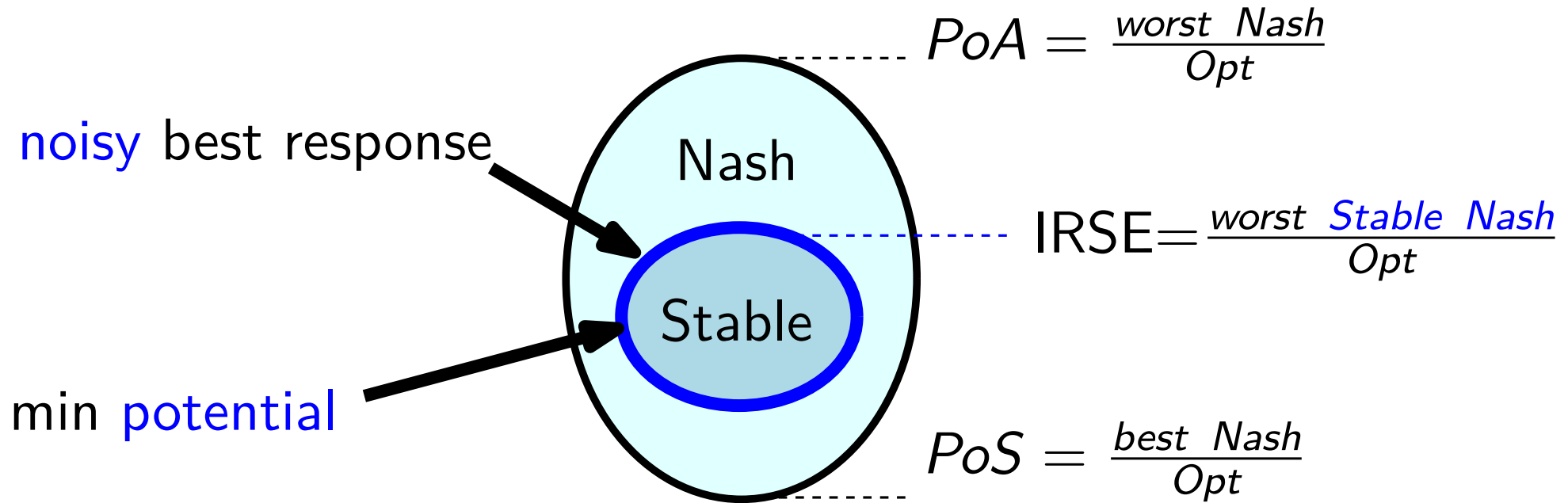
$PoA = \frac{worst\ Nash}{Opt}$

noisy best response

Nash

Stable

min potential

$PoS = \frac{best\ Nash}{Opt}$

Inefficiency Ratio of Stable Equilibria

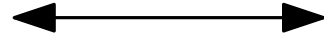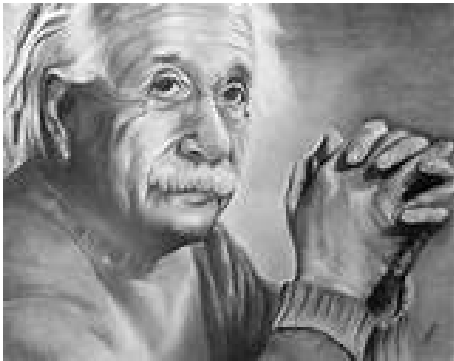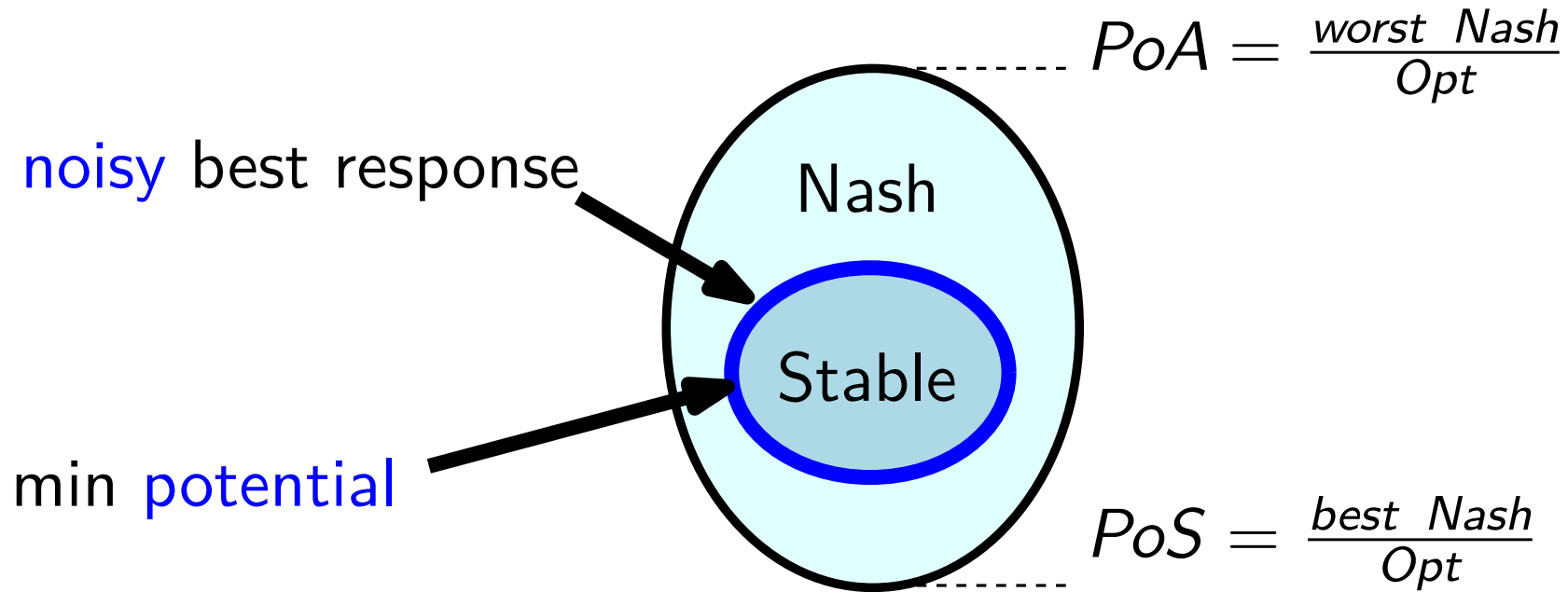$$PoA = \frac{worst\ Nash}{Opt}$$

noisy best response

$$IRSE = \frac{worst\ Stable\ Nash}{Opt}$$

min potential

$$PoS = \frac{best\ Nash}{Opt}$$

Nash

Stable

(Asadpour, Saberi, 2009)

Inefficiency Ratio of Stable Equilibria

$$PoA = \frac{worst\ Nash}{Opt}$$

noisy best response

Nash

$$IRSE = \frac{worst\ Stable\ Nash}{Opt}$$

Stable

min potential

$$PoS = \frac{best\ Nash}{Opt}$$

L_2-norm

L_∞-norm

(Asadpour, Saberi, 2009)

Inefficiency Ratio of Stable Equilibria

$PoA = \frac{worst\ Nash}{Opt}$

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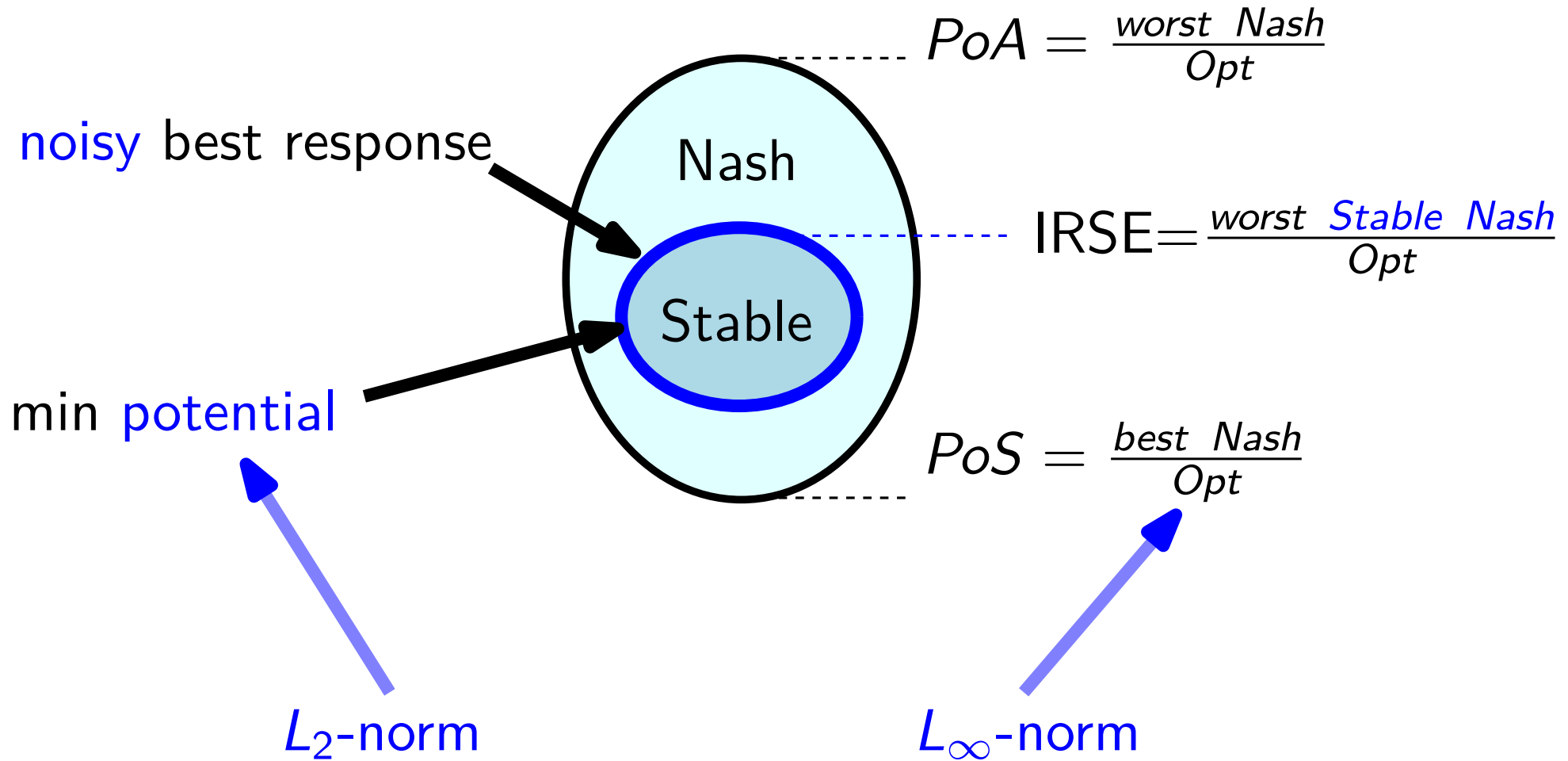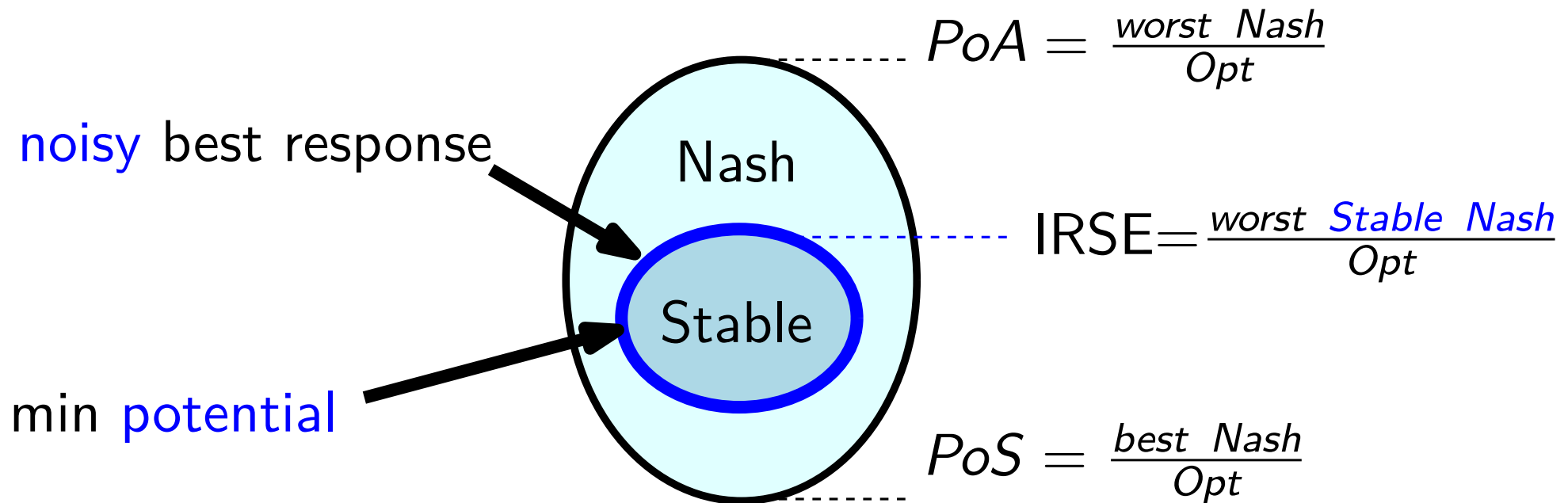
Minimize L_2-norm $\Rightarrow$ also good for L_∞-norm (makespan)?

(Alon, Azar, Woeginger, Yadid, 1997)   (Asadpour, Saberi, 2009)

Our Contribution

$$7/6 \leq \text{IRSE} \leq 4/3$$

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mininimize L_2-**norm** automatically
4/3-**APX** for L_∞-**norm**

Our Contribution

$$7/6 \leq \text{IRSE} \leq 4/3$$

mininimize L_2-**norm** sometimes
at least 7/6-**APX** of L_∞-**norm**

Our Contribution

$$7/6 \leq \text{IRSE} \leq 4/3$$

Previous bunds:

$$19/18 \leq \text{IRSE} \leq 3/2$$

Our Contribution

$$7/6 \le \text{IRSE} \le 4/3$$

Previous bunds:

$$19/18 \le \text{IRSE} \le 3/2$$

Asadpour - Saberi, WINE 2009

Alon - Azar - Woeginger - Yadid, SODA 1997

Our Contribution

$$7/6 \leq \text{IRSE} \leq 4/3$$

mininimize L_2-**norm** sometimes
at least 7/6-**APX** of L_∞-**norm**

Lower Bound (IRSE ≥ 7/6)

Lower Bound (IRSE ≥ 7/6)

$$\frac{2}{\frac{9}{2}}$$

$$\frac{3}{2}$$

$$\frac{2}{3}$$
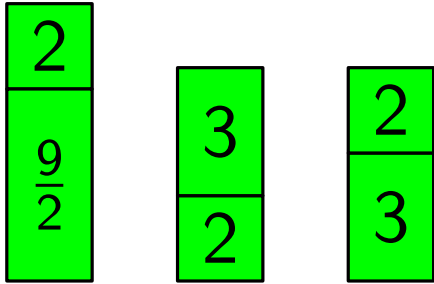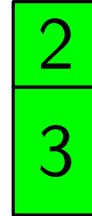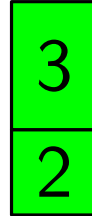
min potential
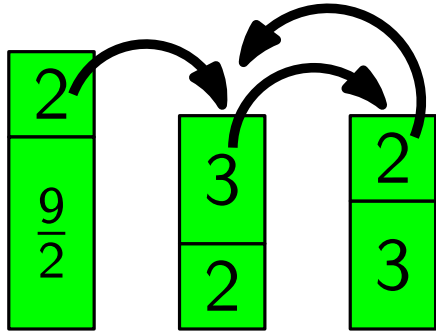
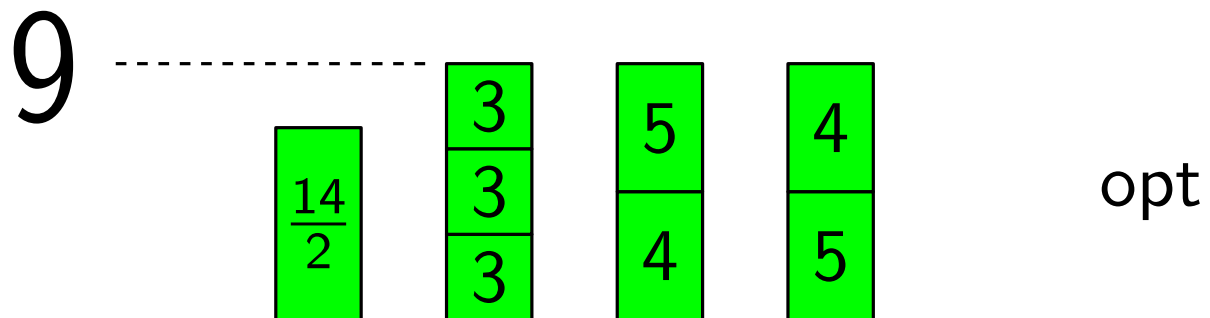# Lower Bound (IRSE ≥ 7/6)

min potential
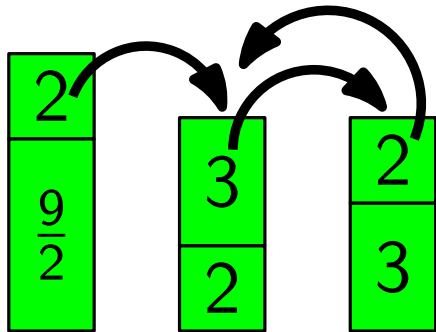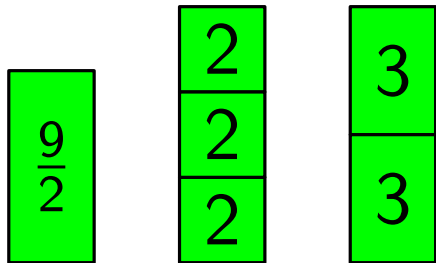
Lower Bound (IRSE $\geq 7/6$)

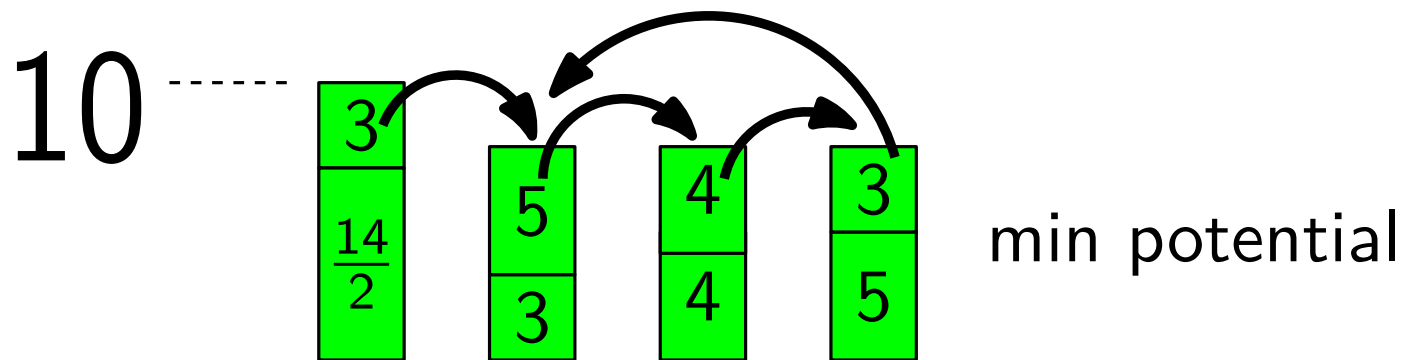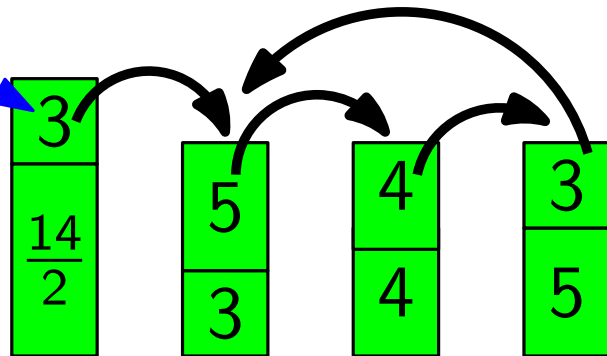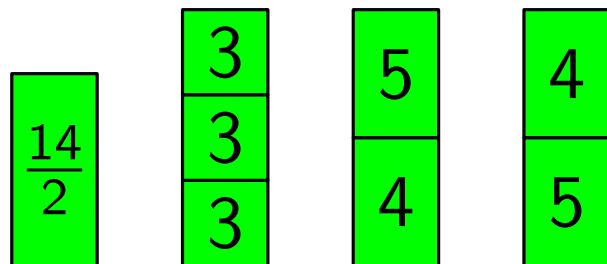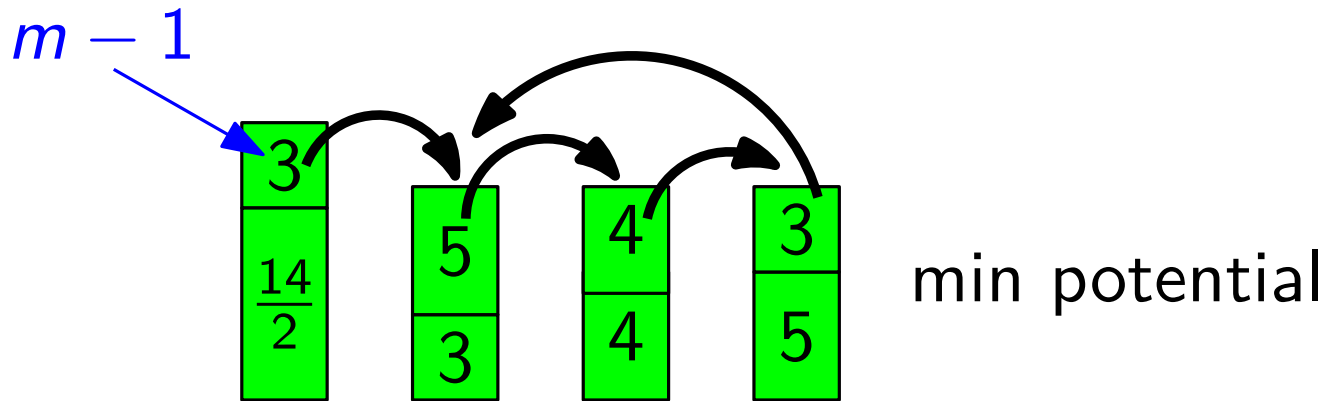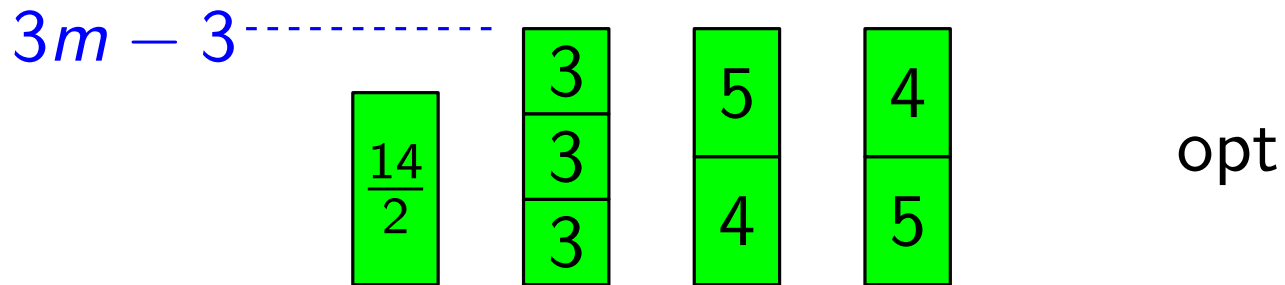
min potential

opt
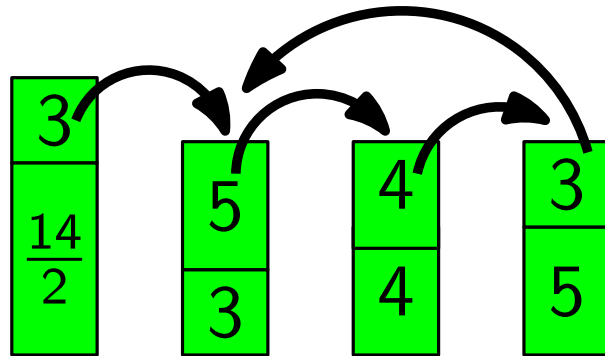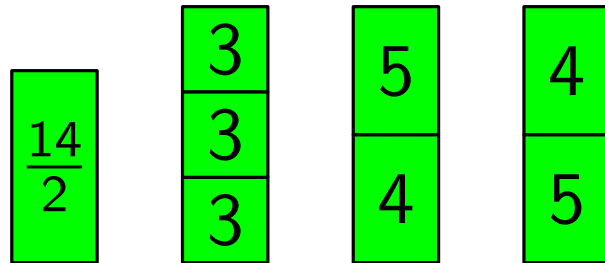
Lower Bound (IRSE $\geq 7/6$)

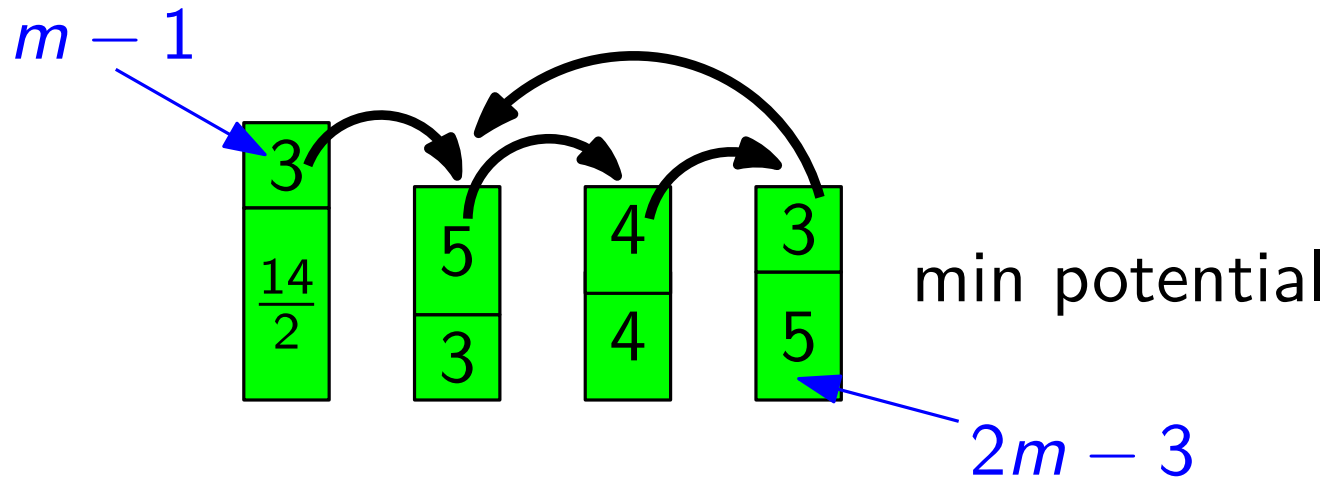# Lower Bound (IRSE $\geq$ 7/6)

Lower Bound (IRSE ≥ 7/6)
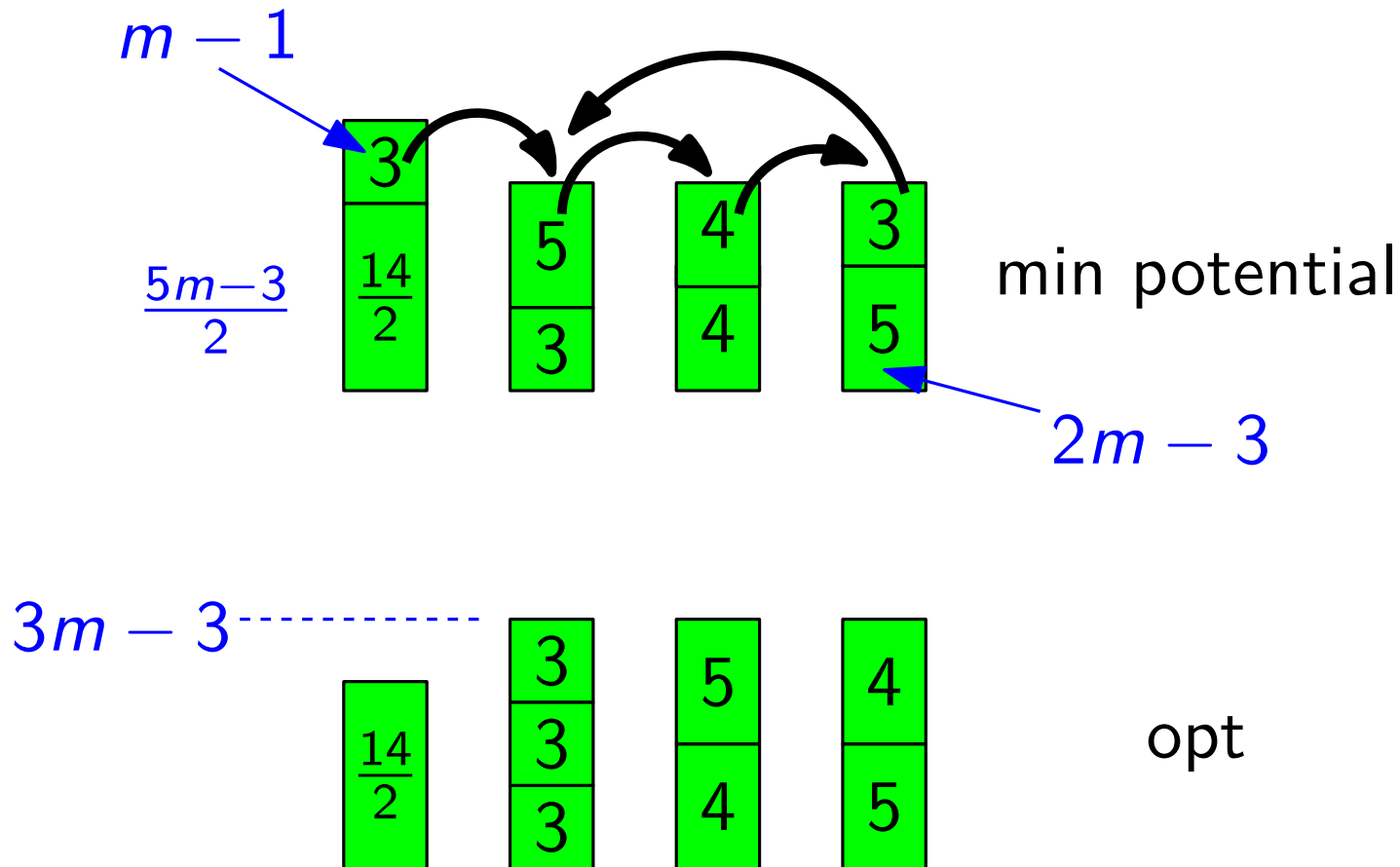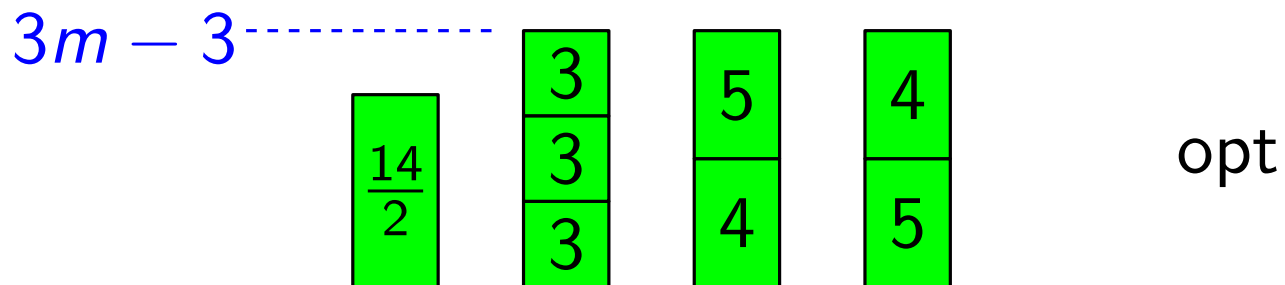
$m-1$

min potential

opt
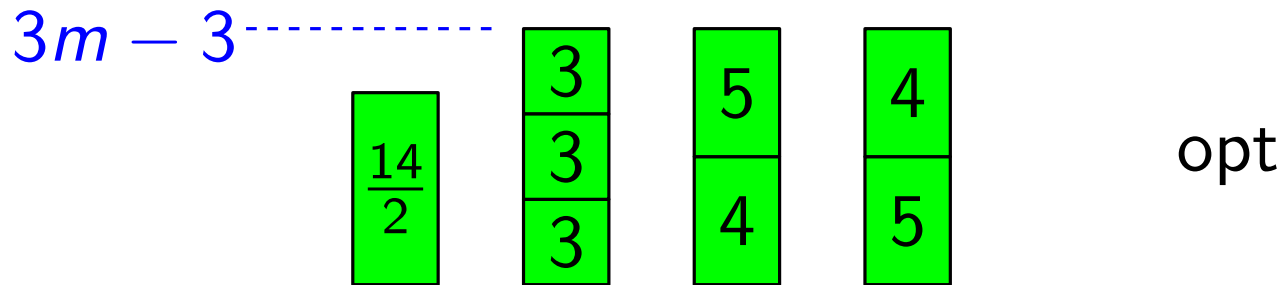
Lower Bound (IRSE $\geq 7/6$)

Lower Bound (IRSE $\geq 7/6$)

Lower Bound (IRSE ≥ 7/6)

Lower Bound (IRSE $\geq 7/6$)

$\frac{7m-4}{2}$

$\frac{5m-3}{2}$

min potential

$2m-3$

$3m-3$

opt
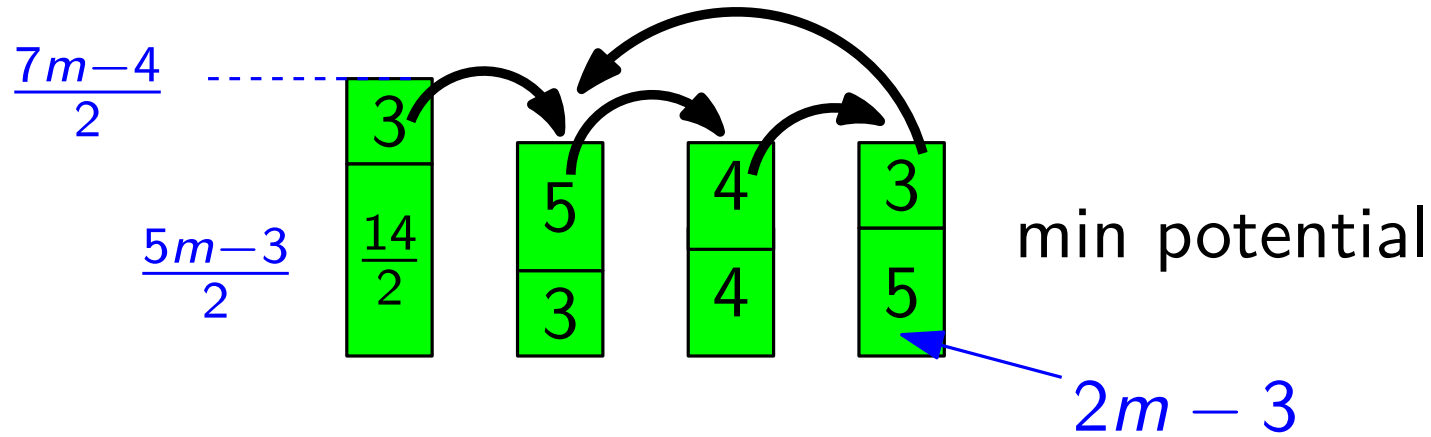
Our Contribution

$$7/6 \leq \text{IRSE} \leq 4/3$$

mininimize L_2-**norm** automatically
4/3-**APX** for L_∞-**norm**

Upper Bound (IRSE $\leq 4/3$)
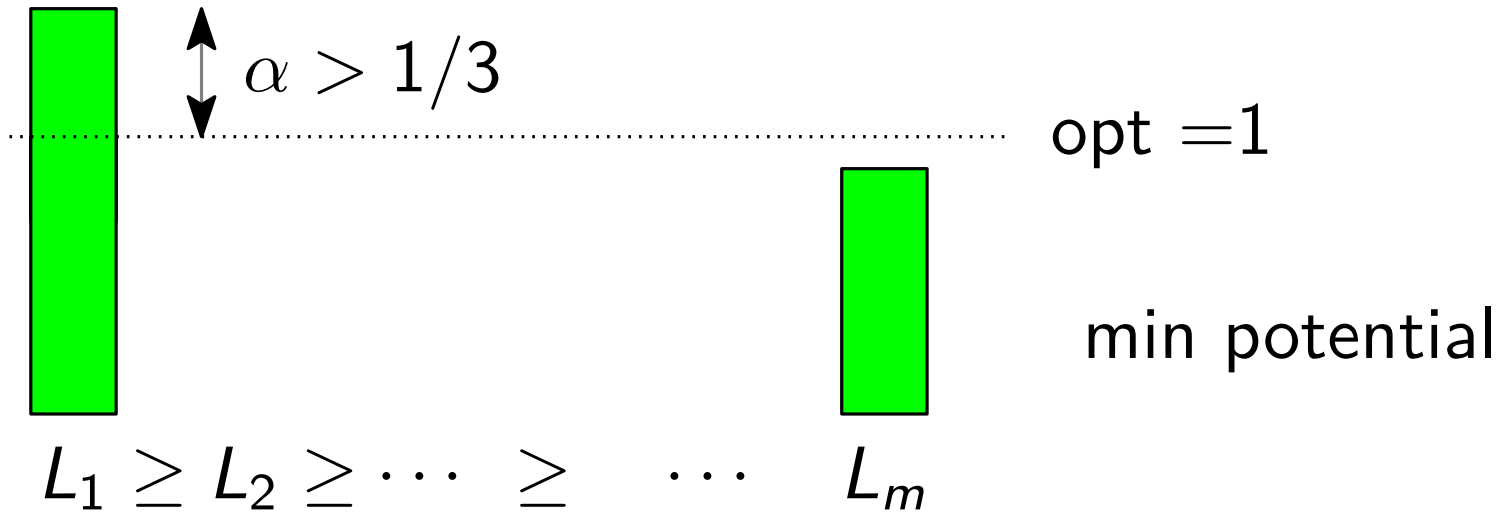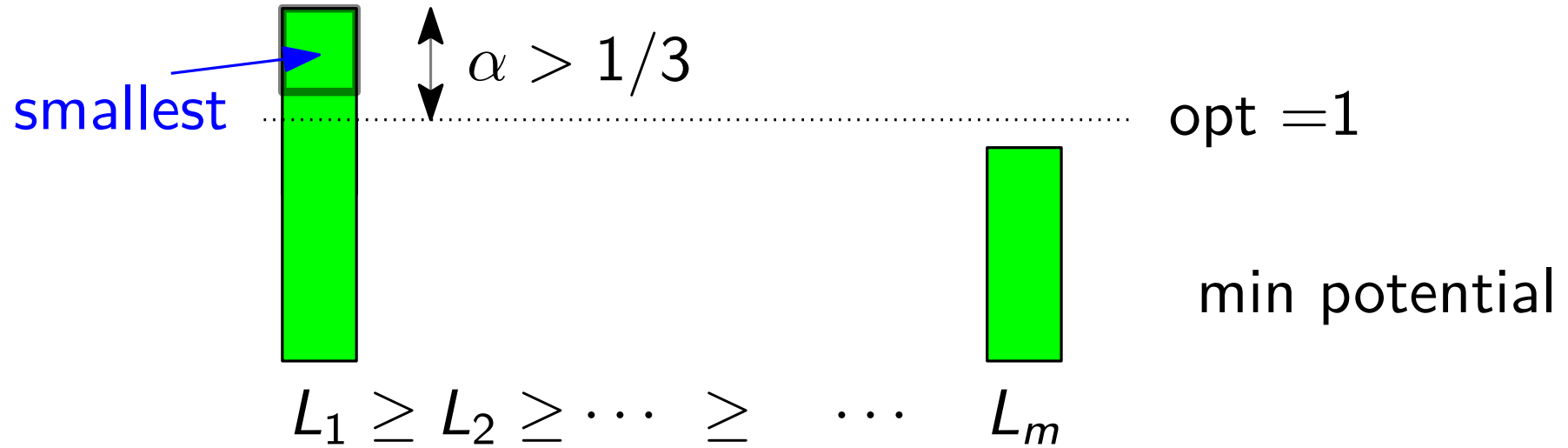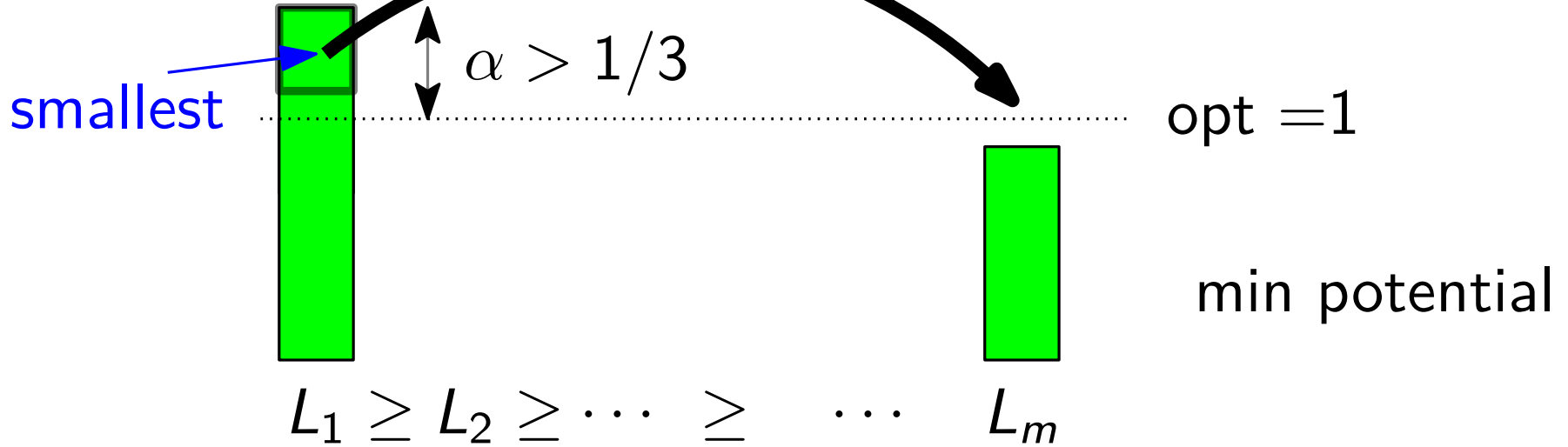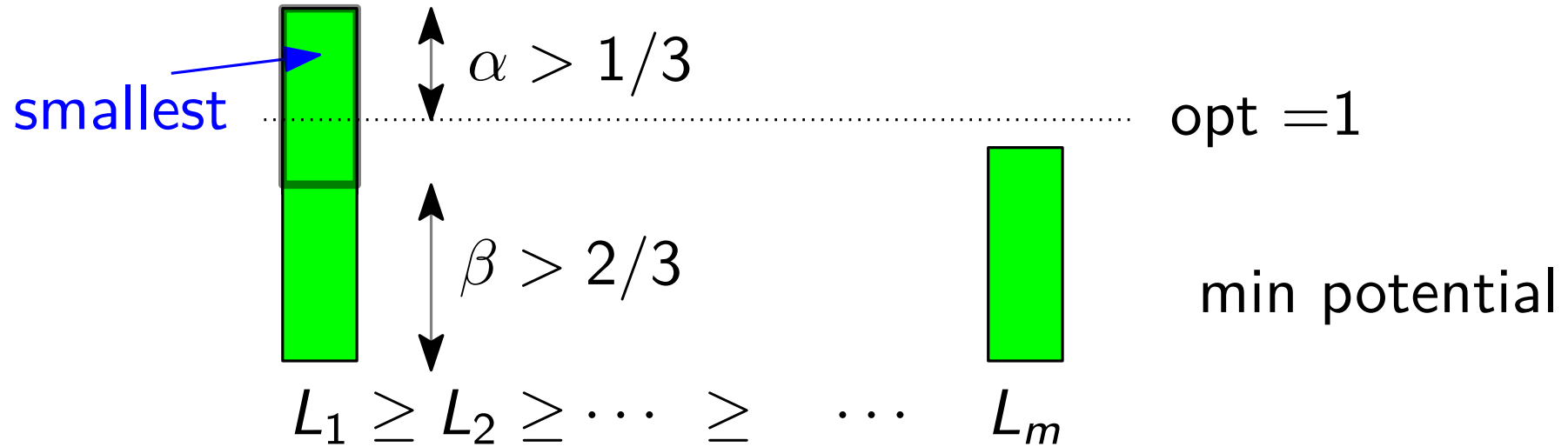
$\alpha > 1/3$

opt $= 1$

min potential

$L_1 \geq L_2 \geq \cdots \geq \quad \cdots \quad L_m$

Upper Bound (IRSE ≤ 4/3)

smallest

$\alpha > 1/3$

opt $=1$

min potential

$L_1 \geq L_2 \geq \cdots \geq \quad \cdots \quad L_m$

Upper Bound (IRSE ≤ 4/3)

smallest

$\alpha > 1/3$

opt $= 1$

min potential

$L_1 \geq L_2 \geq \cdots \geq \quad \cdots \quad L_m$

Upper Bound (IRSE ≤ 4/3)

$$\text{smallest}$$

$$\alpha > 1/3$$

$$\text{opt} = 1$$

$$\beta > 2/3$$

$$\text{min potential}$$

$$L_1 \geq L_2 \geq \cdots \geq \quad \cdots \quad L_m$$
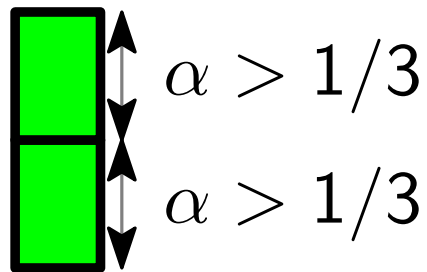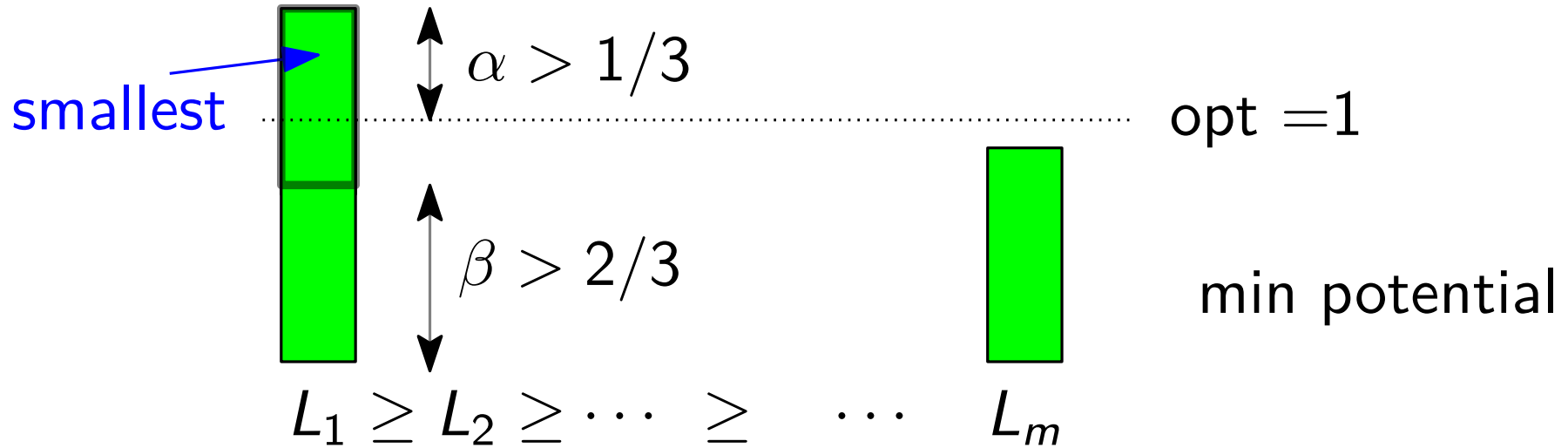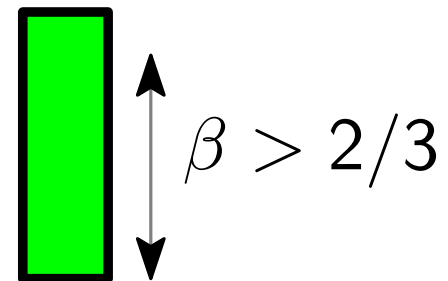
Upper Bound (IRSE $\leq 4/3$)

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Upper Bound (IRSE $\leq 4/3$)

smallest

$\alpha > 1/3$

opt $= 1$

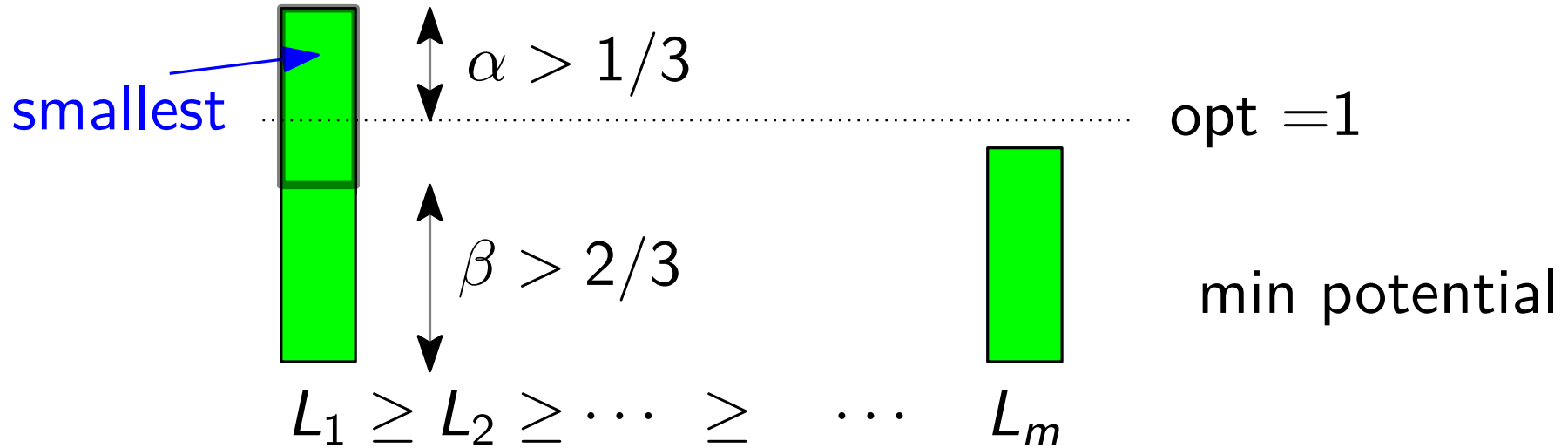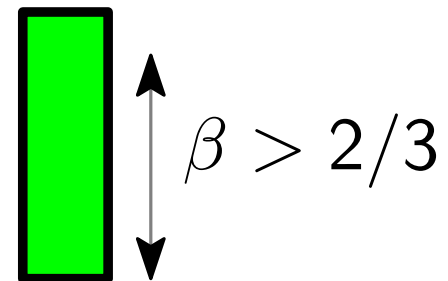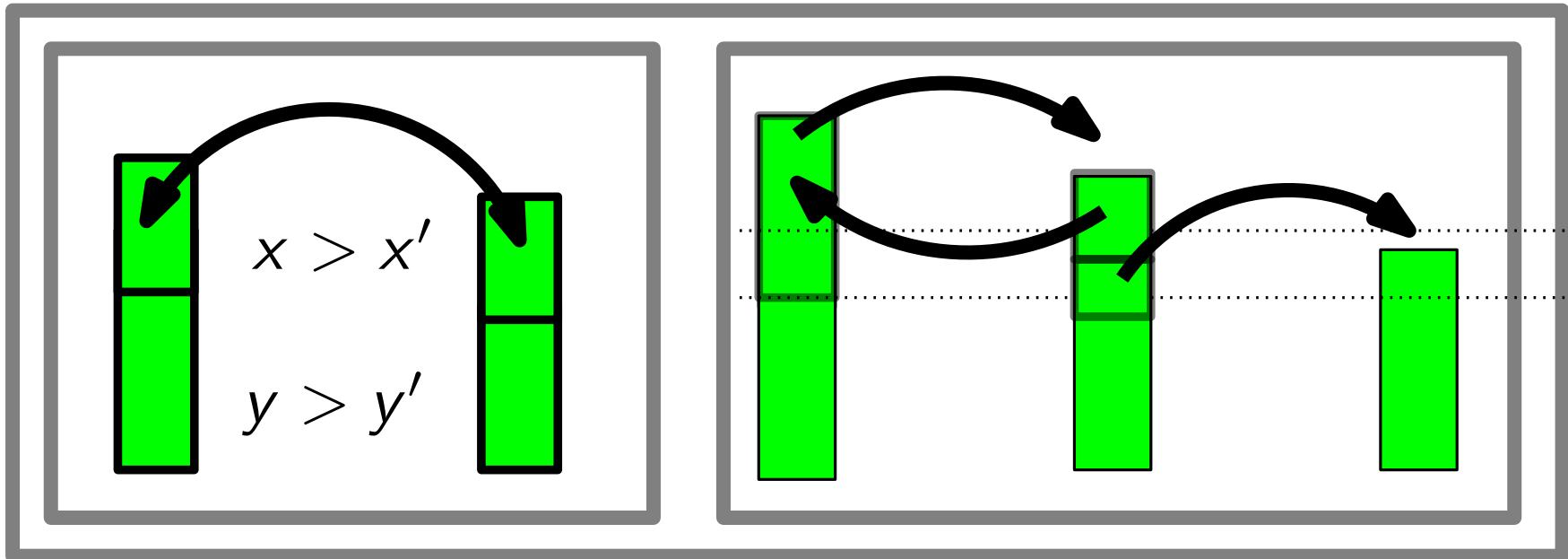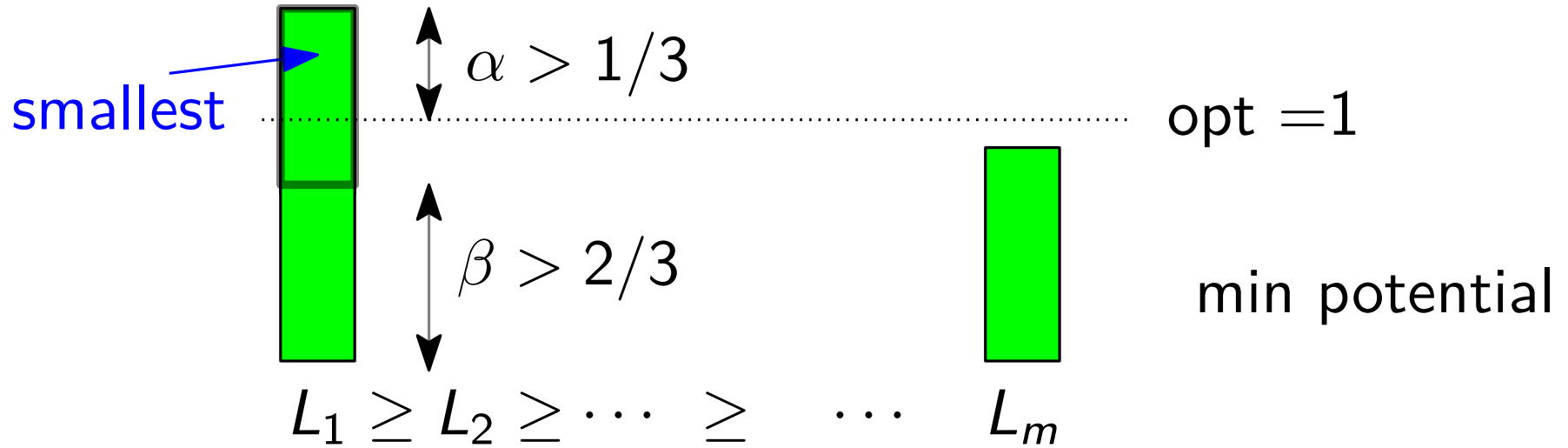$\beta > 2/3$

min potential

$L_1 \geq L_2 \geq \cdots \geq \quad \cdots \quad L_m$

$x > x'$

$y > y'$

Upper Bound (IRSE ≤ 4/3)

smallest

$\alpha > 1/3$

opt $= 1$

$\beta > 2/3$

min potential

$L_1 \geq L_2 \geq \cdots \geq \cdots \quad L_m$

$x > x'$

$y > y'$

Conclusions

Nash

Stable

$PoA \approx 2$

$PoS = 1$
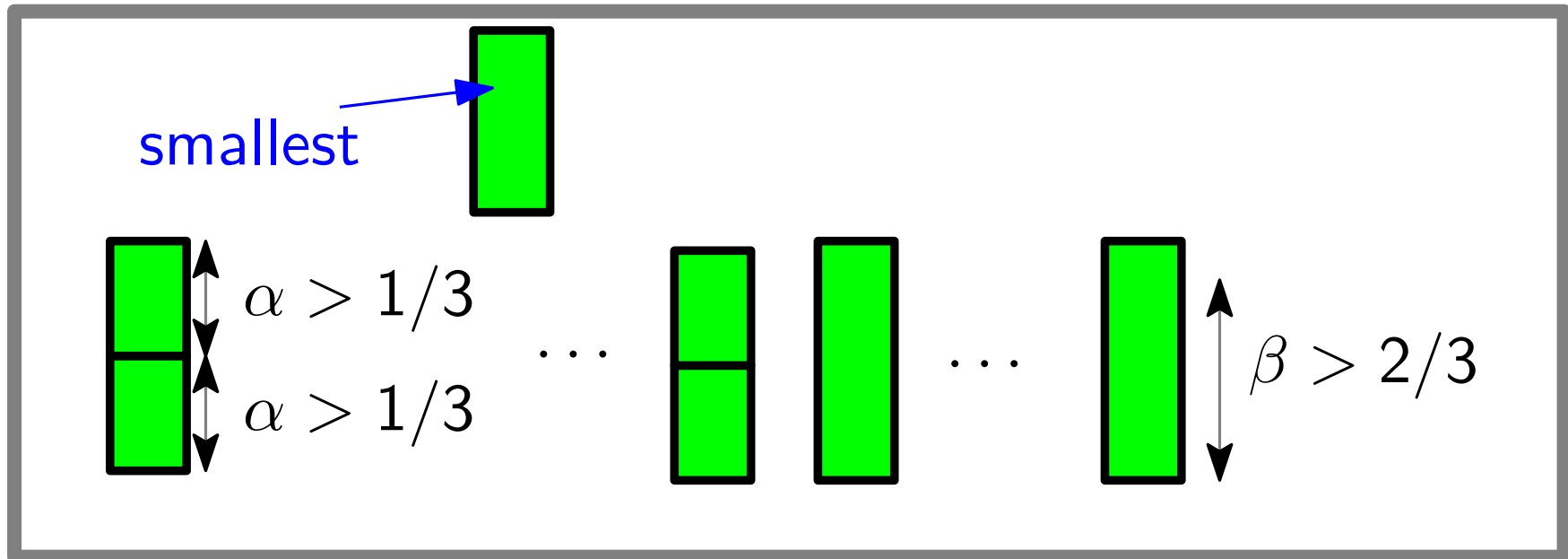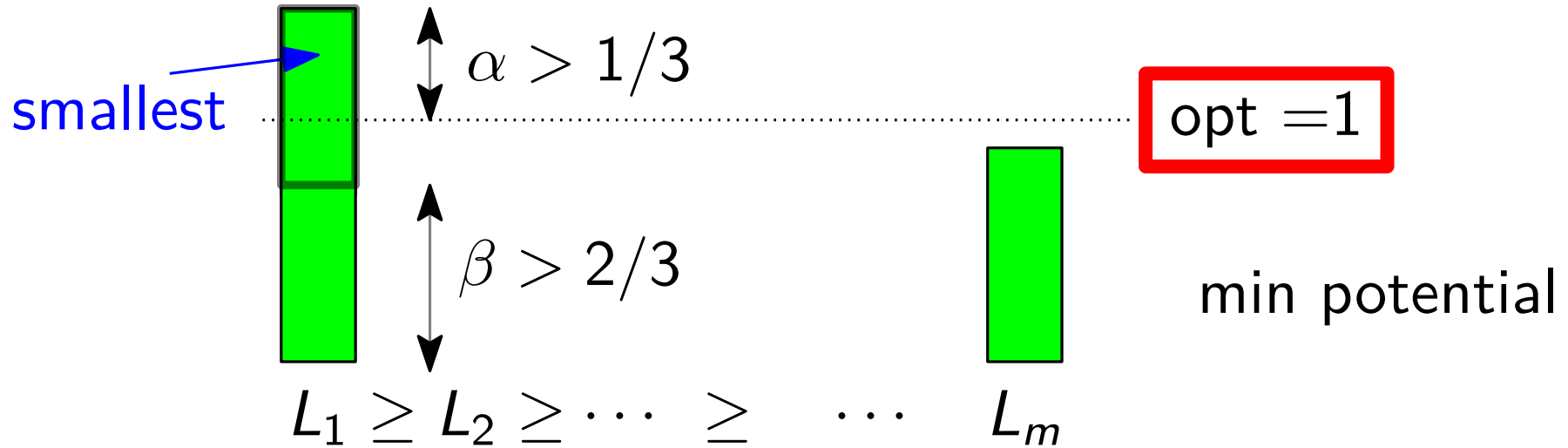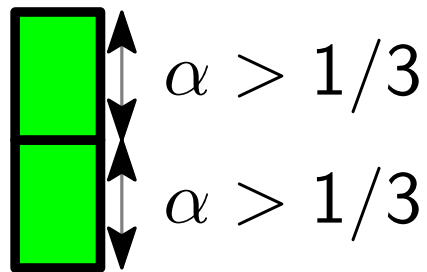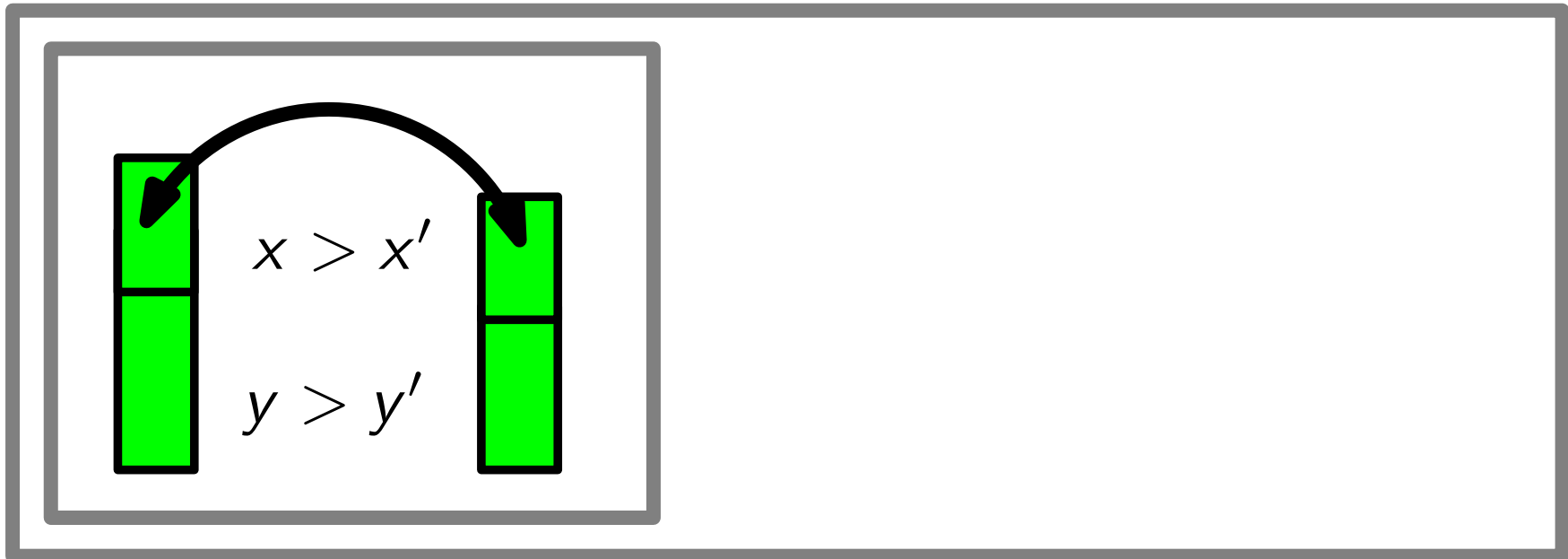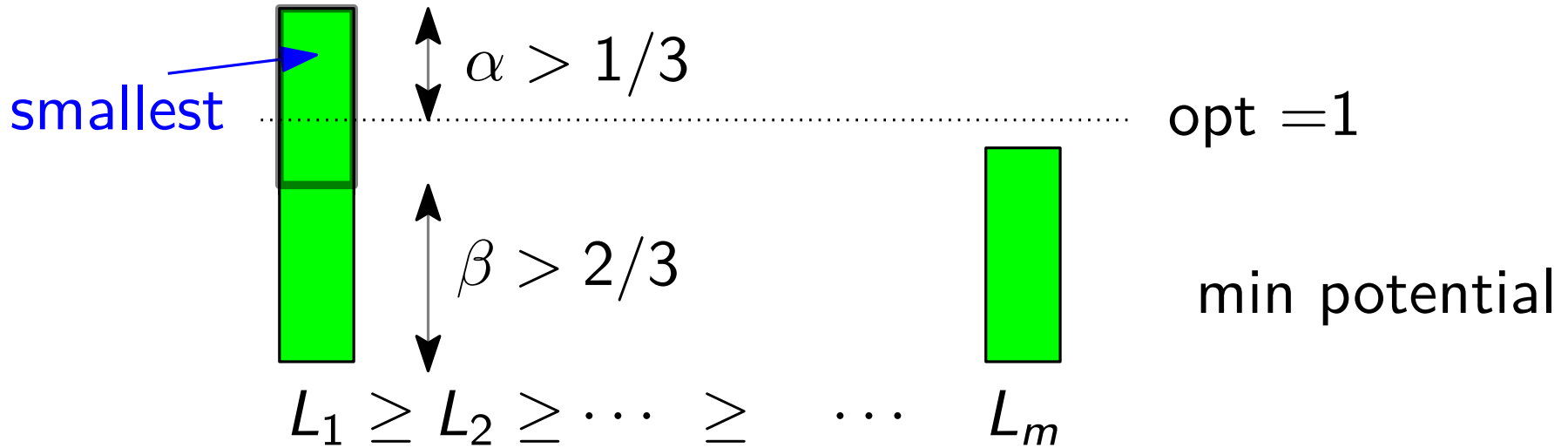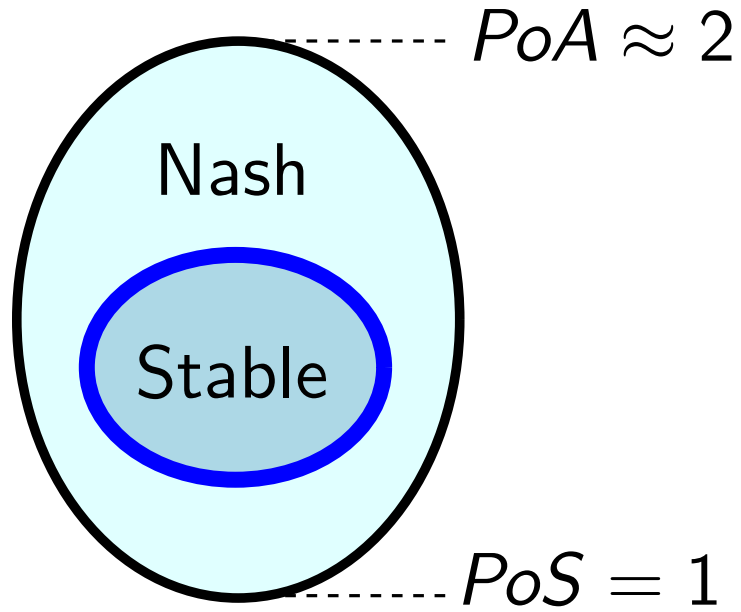
Conclusions

Conclusions

$PoA \approx 2$

Nash

Stable

IRSE

$4/3$

$7/6$

$> ?$

$PoS = 1$
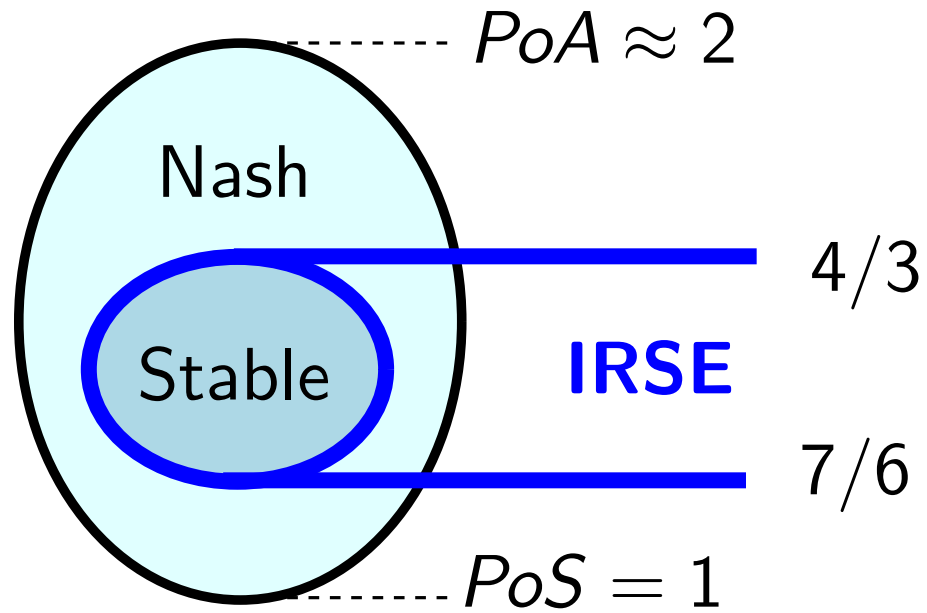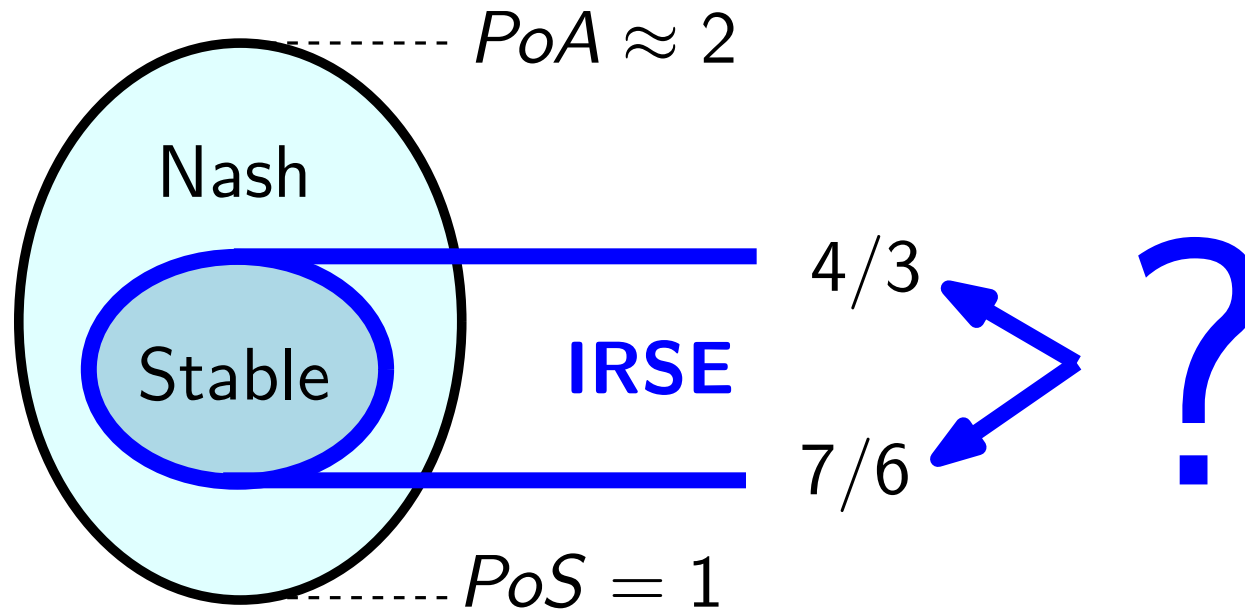
Minimize L_2-norm $\Rightarrow$ also good for L_∞-norm (makespan)?

Conclusions

$PoA \approx 2$

Nash

Stable

IRSE

$4/3$

$7/6$

$>$?

$PoS = 1$
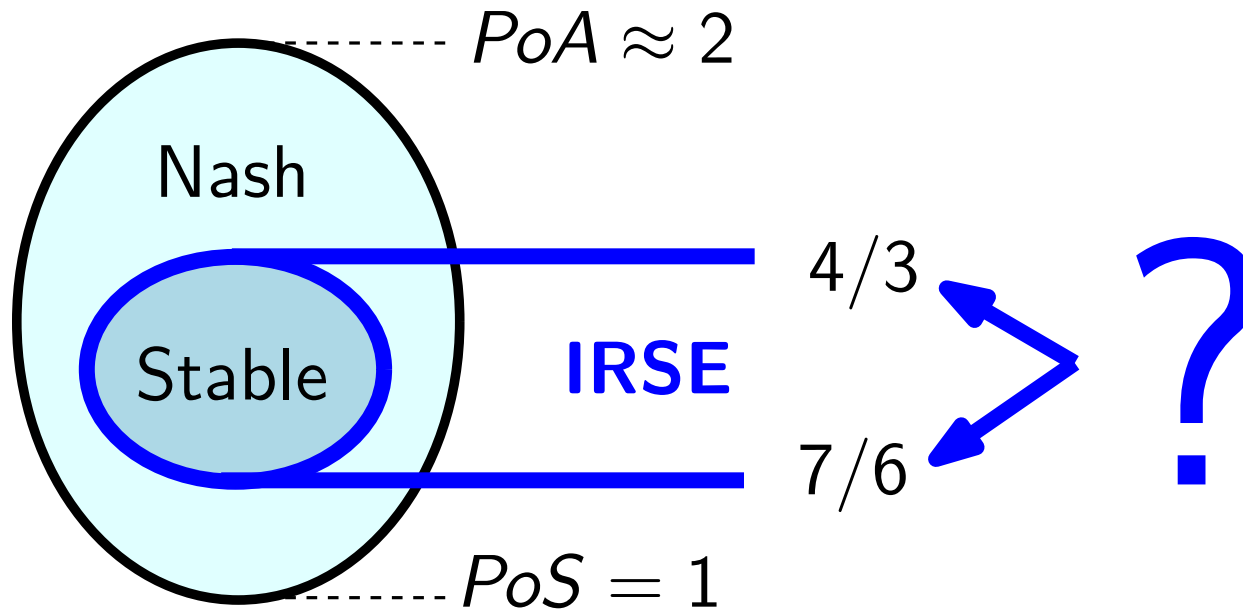
Global properties?

Thank You!!