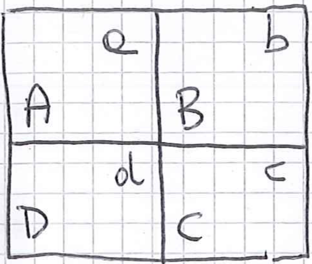
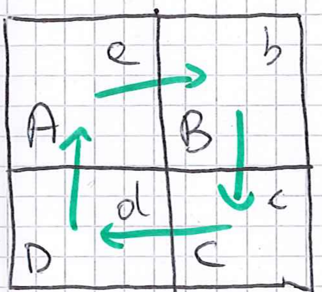
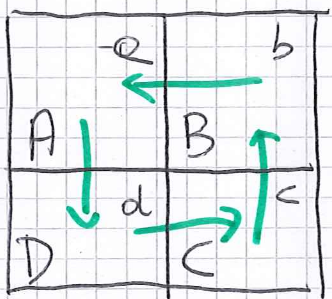
PROBLEM 2



IF ONE OF THESE STATES IS A PNE THEN WE ARE DONE (MNE INCLUDE PNE)

OTHERWISE ONE OF THESE TWO CYCLES OF (STRICT) BEST-RESPONSE MUST EXIST:





(PROOF AT THE END)

CYCLE $\Rightarrow$ MNE

ROW \ COL	q	$1-q$
p	A, a	B, b
$1-p$	D, d	C, c

CONDITIONS (FOR MNE):

$$\underbrace{qA + (1-q)B = qD + (1-q)C}_{\text{ROW IS INDIFFERENT BETWEEN THE TWO STRATEGIES}} \quad (1)$$

$$\underbrace{pa + (1-p)d = pb + (1-p)c}_{\text{COL IS INDIFFERENT BETWEEN THE TWO STRATEGIES}} \quad (2)$$

(1) HAS A SOLUTION:

$$(1) \iff (1-q)(C-B) = q(A-D)$$

$$\iff (1-q)\frac{C-B}{A-D} = q \qquad \left(\underset{\text{FROM CYCLE}}{A \neq D}\right)$$

$$\iff \frac{C-B}{A-D} = \frac{q}{1-q} \qquad \text{FOR } q \neq 1$$

THIS IS > 0 BECAUSE

$$\boxed{\begin{matrix} C > B \\ A > D \end{matrix}} \quad \text{OR} \quad \boxed{\begin{matrix} C < B \\ A < D \end{matrix}}$$

CLOCKWISE CYCLE

COUNTERCLOCKWISE CYCLE

AND THE FUNCTION $\frac{q}{1-q}$ CAN TAKE ANY VALUE FROM 0 TO ∞ FOR $q \in [0, 1)$

(2) HAS A SOLUTION
(SAME ARGUMENTS FOR (1)) :

$$(2) \Longleftrightarrow (1-p)(c-d) = p(e-b)$$

$$\Longleftrightarrow (1-p)\frac{c-d}{e-b} = p \qquad \begin{pmatrix} e \neq b \\ \text{IN CYCLES} \end{pmatrix}$$

$$\Longleftrightarrow \frac{c-d}{e-b} = \frac{p}{1-p} \qquad \text{FOR } p \neq 1$$

THIS IS > 0 BECAUSE

$$\boxed{\begin{matrix} d > c \\ b > e \end{matrix}} \quad \text{OR} \quad \boxed{\begin{matrix} d < c \\ b < e \end{matrix}}$$

CLOCKW. CYCLE — COUNTERCL. CYCLE

NO PNE $\Rightarrow$ CYCLE
(CLOCKWISE OR COUNTER CLOCKWISE)

BY DEF OF BEST-RESPONSE, ANY SEQUENSE OF BR MUST ALTERNATE

COL, ROW, COL, ROW, ... (I)

OR

ROW, COL, ROW, COL, ... (II)

BECAUSE THERE ARE ONLY TWO PLAYERS

START FROM STATE $\boxed{A}$

(I) GIVES A CLOCKWISE CYCLE

(II) " " COUNTERCLOCKWISE CYCLE