$$\boxed{\begin{array}{l}(s_1, s_2) \\ s_1 < s_2\end{array} \text{ IS PNE} \Rightarrow s_2 = s_1 + 1}$$

(A PLAYER USES THE MIN ENERGY TO BEAT THE OTHER)

PROOF: IF $s_2 \geq s_1 + 2$

THEN $(s_1, s_1 + 1)$ IS

A BETTER RESPONSE FOR 2

$$\boxed{\begin{array}{l}(s_1, s_2) \\ s_1 < s_2\end{array} \text{ IS PNE} \Rightarrow s_1 = 0}$$

(IF I LOOSE, IT'S BETTER TO JUST NOT TRANSMIT)

PROOF: IF $s_1 \geq 1$

THEN $(0, s_2)$ IS

BETTER RESPONSE FOR 1

THUS THE ONLY POSSIBLE PNE ARE:

$(0, 1)$, $(1, 0)$ AND PERHAPS (e, e)

PART 1

$\boxed{(e,e) \quad \text{NOT PNE} \quad e>0}$

(BOTH LOOSE AND SPEND ENERGY ⇒ BETTER USE "0")

ONLY THESE CAN BE PNE:

$$(0,1)\ ,\ (1,0)\ ,\ (0,0)$$

IF $k>2$ NONE IS A PNE:

REPLACE "0" BY "2" IS A BETTER RESPONSE

$$\underset{\substack{\uparrow \\ \text{COST} \\ k}}{(0}, 1) \longrightarrow \underset{\substack{\uparrow \\ \text{COST} \\ 2}}{(2}, 1)$$

IF $k=2$ THE ONLY PNE ARE

$$(0,1)\ ,\ (1,0)$$

IF $k=1$ ALL THREE ARE PNE

$\boxed{\text{PART 1 CONTD}}$

$\boxed{k=2}$

THE ABOVE PROOF SAYS THAT THERE ARE ONLY 2 PNE:

$$(0,1) \quad \text{AND} \quad (1,0)$$

WHOSE SOCIAL COST IS

$$1 + K$$

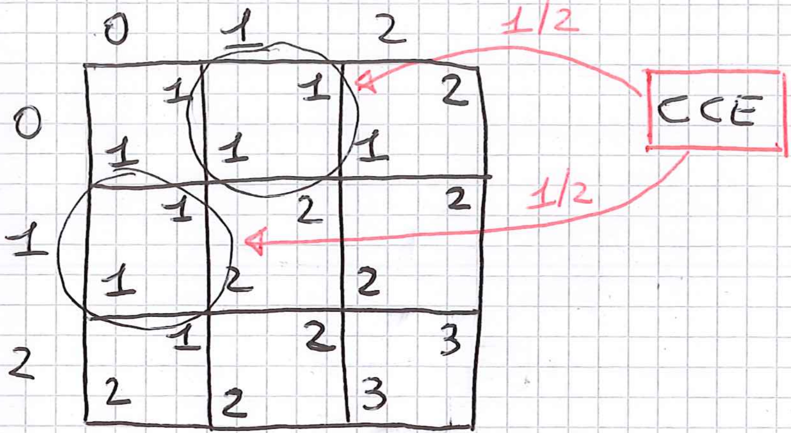
THIS IS OPTIMAL AS ONE OF THE TWO PLAYERS MUST NOT SUCCEED IN TRANSMITTING (COST k) AND THE OTHER MUST PAY AT LEAST THE MIN POWER (COST 1)

$\boxed{k=1}$ THE PNE $(0,0)$ HAS ALSO OPTIMAL COST: $k+k = 1+1 = 2$

$\boxed{\text{PART 2}}$

IN BOTH CASES, ALL PNE ARE OPT:

$$PoA_{PNE} = 1$$



NOTE: THIS IS A SOLUTION IF YOU WANT A CCE WHICH IS NOT A MNE OR PNE (PROOF BELOW)

SIMPLER ANSWER:

ANY OF THE THREE PNE

(I OVERLOOKED THIS POSSIBILITY, BUT A STUDENT FIND IT)

IN CCE BOTH PLAYERS HAVE EXPECTED COST

$$\frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 1 = \underline{1}$$

PART 3

DEVIATING TO SOME s_i^* NOT IMPROVING:

$$\left.\begin{aligned}
0 &\Rightarrow \frac{1}{2}\cdot 0\,\overset{1}{\boxed{1}} + \frac{1}{2}\cdot 0\,\overset{0}{\boxed{1}} = 1 \\
1 &\Rightarrow \frac{1}{2}\cdot 1\,\overset{1}{\boxed{2}} + \frac{1}{2}\cdot 1\,\overset{0}{\boxed{1}} > 1 \\
2 &\Rightarrow \frac{1}{2}\cdot 2\,\overset{1}{\boxed{2}} + \frac{1}{2}\cdot 2\,\overset{0}{\boxed{2}} > 1
\end{aligned}\right\}\ \text{EXPECTED COST OF PLAYER 1 IF TAKING } s_i^*$$

$$\boxed{\text{CCE IS INDEED A COARSE CORRELATED EQUILIBRIUM}}$$

PART 3 CONTD