MORE POWERFUL AND SIMPLER COST-SHARING METHODS

PAOLO PENNA and CARMINE VENTRE

UNIVERSITÀ DI SALERNO

SERVICE PROVIDER

USERS U (CUSTOMERS)

- WHO GETS SERVICED?
- HOW TO SHARE THE COST?

USERS:

v_i = USER i'S VALUATION (HOW MUCH WOULD PAY)

$$u_i = \begin{cases} v_i - P_i \\ \emptyset \quad \text{IF NOT SERVICED} \end{cases}$$

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- WHO GETS SERVICED ?
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USERS: SELFISH

v_i = USER i'S VALUATION (HOW MUCH WOULD PAY) (PRIVATE)

$$u_i = \begin{cases} v_i - P_i \\ \emptyset & \text{IF NOT SERVICED} \end{cases}$$

USER i WANTS TO MAXIMIZE HIS UTILITY

b_i = REPORTED VALUATION

MECHANISM: TAKES $b=(b_1,\dots,b_m)$

$$M=(A,P)$$

WHO GETS SERVICE
$Q(b)$

HOW MUCH EACH USER PAYS
$P_1(b)\cdots P_i(b)\cdots P_m(b)$

MECHANISM: TAKES $b = (b_1, \dots, b_m)$

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WHO GETS SERVICE
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HOW TO SERVICE $Q(b)$

$$\Downarrow$$

$$C_A(Q(b))$$

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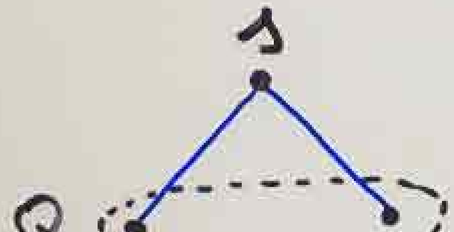
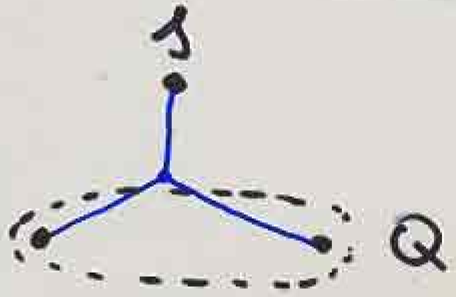


A=MST



A=OPT

GOALS

1) VOLUNTARY PARTICIPATION (VP)

$$P_i(b) \leq b_i, \quad P_i(b) = \emptyset \text{ IF } i \notin Q(b)$$

2) CONSUMER SOVEREGNTY (CS)

b_i "LARG ENOUGH" $\Rightarrow i \in Q(b)$

3) NO POSITIVE TRANSFER (NPT)

$$P_i(\cdot) \geq \emptyset$$

4) BUDGET BALANCE (BB)

$$\sum_{i \in Q(b)} P_i(b) = C_A(Q(b))$$

5) COST OPTIMALITY (CO)

$$C_A(Q(b)) = C_{OPT}(Q(b))$$

6) GROUP STRATEGYPROOF

REPORTING $b_i = v_i$ IS DOMINANT STRATEGY, ALSO FOR COALITIONS

GENERAL APPROACH

COST-SHARING METHODS:
DISTRIBUTE $C_A(Q)$ AMONG USERS IN Q

$\forall Q \subseteq U \quad \xi(Q,i) \geq \emptyset$

1) $\xi(Q,i) = 0, \; i \notin Q$

2) $\sum_{i \in Q} \xi(Q,i) = C_A(Q)$

$\xi(\cdot)$ IS "NICE" $\Longrightarrow \exists \; M(\xi)$ WHICH IS
GROUP STRATEGYPROOF
NPT, VP, CS, BB

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$\xi(Q',i) \geq \xi(Q,i)$

CROSS-MONOTONIC

* [MOULIN-SHENKER'97]

RELATED WORK

HOW TO BUILD A MECHANISM:

$\xi(\cdot)$ CROSS-MONOTONIC $\Rightarrow M(\xi)$ [MS '97]

MORE POWERFUL TECHNIQUES ?

$C_A(\cdot)$ SUBMODULAR NON-DECREASING $\Rightarrow$ EVERY MECH. M IS "EQUIVALENT" TO SOME $M(\xi)$, $\xi(\cdot)$ CROSS-MONOTONIC

[MS '97]

HOW TO PROVE LOWER BOUNDS:

IF $\xi(\cdot)$ IS (WEAKLY) CROSS-MONOTONIC THEN CORE(C_A) IN <u>NOT EMPTY</u>

[BONDAREVA '63] [SHAPLEY '67]

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HOW TO PROVE LOWER BOUNDS:

IF $\xi(\cdot)$ IS (WEAKLY) CROSS-MONOTONIC

THEN CORE(C_A) IN <u>NOT EMPTY</u>

$\Downarrow$

IF CORE(C_A) $= \emptyset \Rightarrow$ NO M.S. RESULT

RELATED WORK

STEINER TREE GAME:

GIVEN: $G=(U\cup\{s\}, E, c)$

COMPLETE WEIGHTED UNDIRECTED

GOAL: CONNECT $Q(b)$ TO s
(MIN COST STEINER TREE)

OBSTACLES:

- $C_A(Q) = C_{OPT}(Q)$ NP-HARD!

- $C_{OPT}(\cdot)$ EMPTY CORE [MEGIDDO'78]

WE NEED APPROXIMATION

α-APPROXIMATE BB

$$C_A(Q(b)) \leq \sum_{i\in Q(b)} P_i(b) \leq \alpha \cdot C_{OPT}(Q(b))$$

2-APX BB [JAIN-VAZIRANI'01]

MAIN RESULTS

HOW TO BUILD A MECHANISM:

$\xi(\cdot)$ SELF CROSS-MONOTONIC $\Longrightarrow$ $\underset{\Uparrow}{M}(\xi)$

SAME MECHANISM
SAME PROPERTIES

MAIN RESULTS

HOW TO BUILD A MECHANISM:

$$\underbrace{\xi(\cdot) \text{ SELF CROSS-MONOTONIC}}_{\Uparrow \atop \text{RELAXATION}} \Longrightarrow \underset{\Uparrow}{M}(\xi)$$

SAME MECHANISM
SAME PROPERTIES

- MUCH SIMPER TO OBTAIN (SUFFICIENT CONDITION)

MAIN RESULTS

HOW TO BUILD A MECHANISM:

$\underbrace{\xi(\cdot) \text{ SELF CROSS-MONOTONIC}}_{\Uparrow \text{ RELAXATION}} \Longrightarrow \underbrace{M}_{\Uparrow}(\xi)$

SAME MECHANISM
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IS THIS MORE POWERFUL?

STEINER TREE GAME MECHANISM:

POLYNOMIAL-TIME, CO,

BB, VP, CS, NPT

MAIN RESULTS

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RELAXATION

SAME MECHANISM
SAME PROPERTIES

- MUCH SIMPER TO OBTAIN (SUFFICIENT CONDITION)

IS THIS MORE POWERFUL?

STEINER TREE GAME MECHANISM:

POLYNOMIAL-TIME, CO,
BB, VP, CS, NPT

NOT POSSIBLE WITH CROSS-MONOTONIC $\xi(\cdot)$, EVEN IN EXP TIME!
(EMPTY CORE)

MECHANISM $M(\xi)$

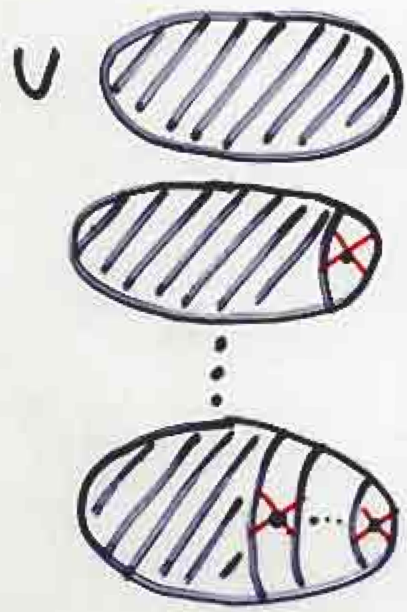
1) INITIALIZE $Q \leftarrow U$

2) WHILE $\exists\, i \in Q$ s.t.

$$\xi(Q,i) > b_i$$

DROP i : $Q \leftarrow Q \setminus \{i\}$

3) RETURN Q, $P_i = \xi(Q,i)$



$Q' \subset Q$

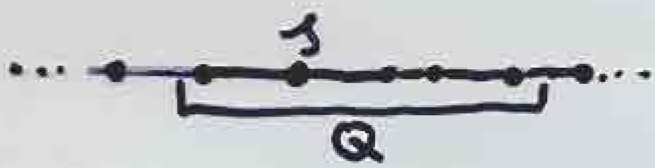
$\forall i \in Q',\ \xi(Q',i) \geq \xi(Q,i)$

CROSS-MONOTONIC

PRICES GO UP!

NAIVE SOLUTIONS

SOL1:



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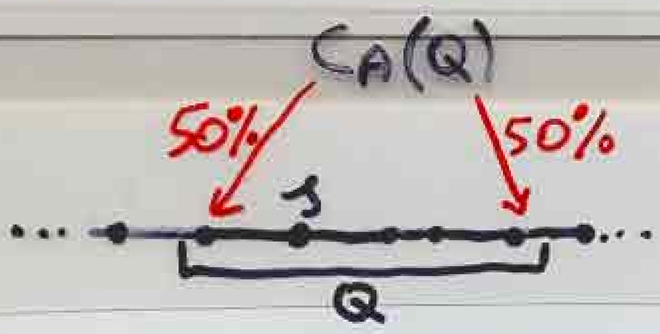

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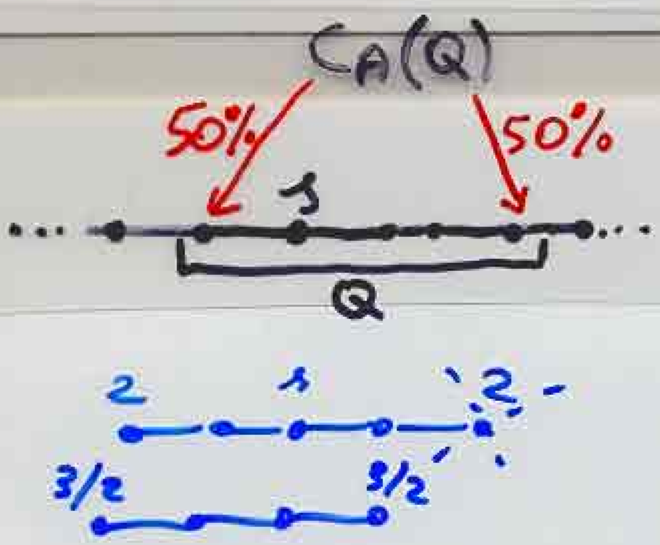

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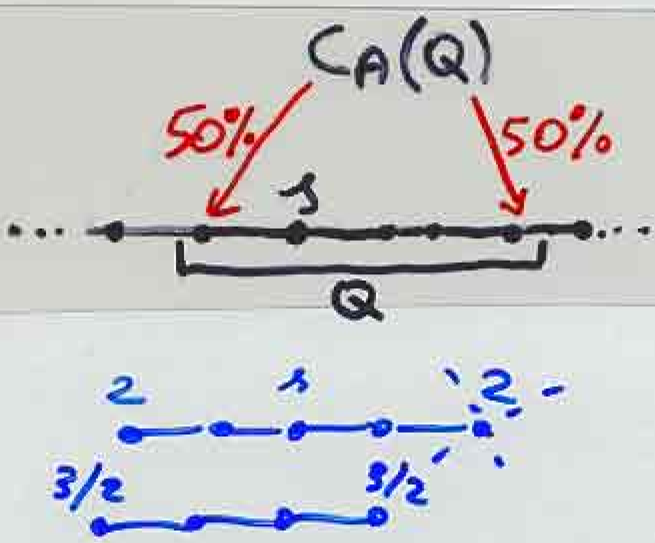
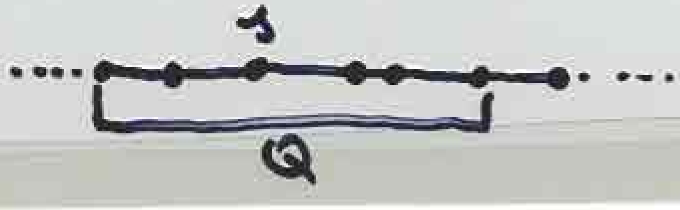


SOL2:



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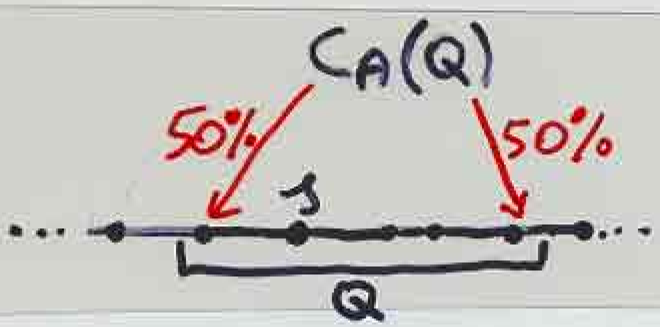

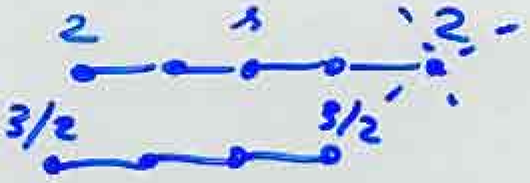

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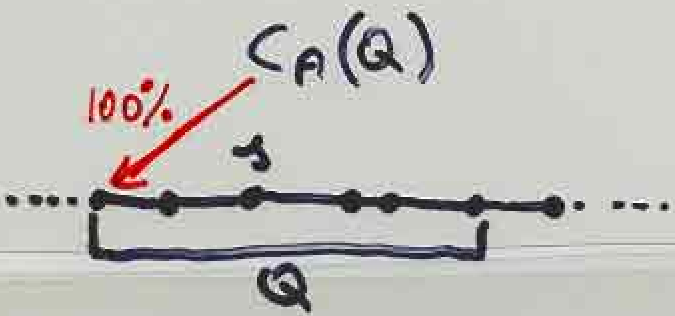

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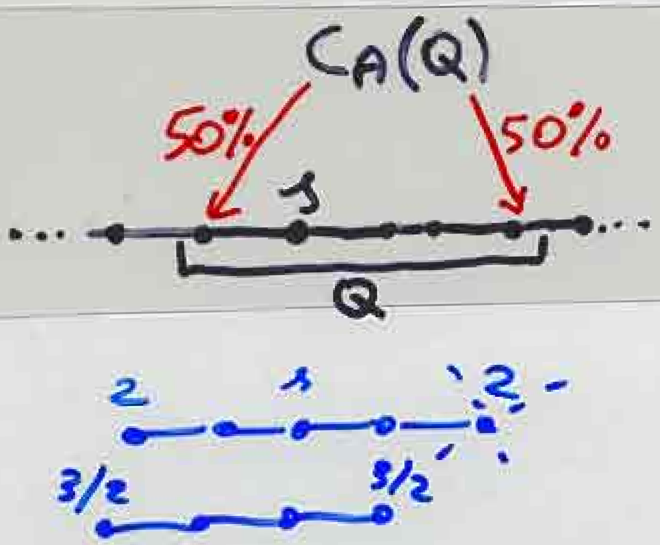

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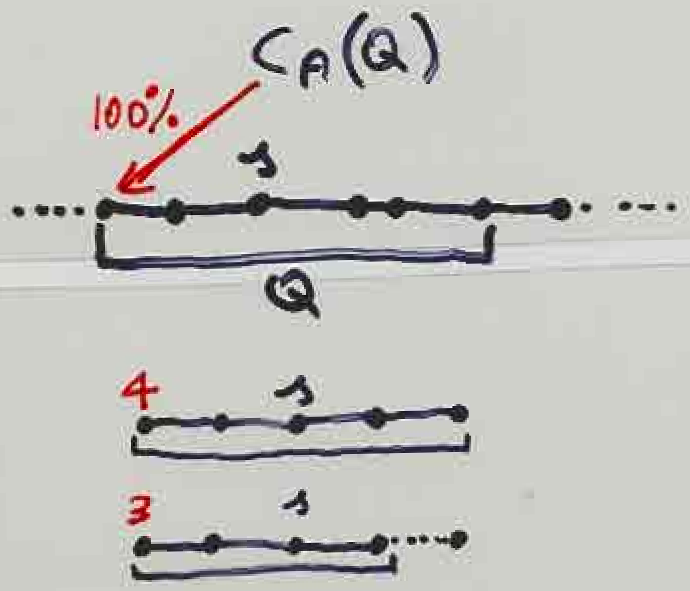

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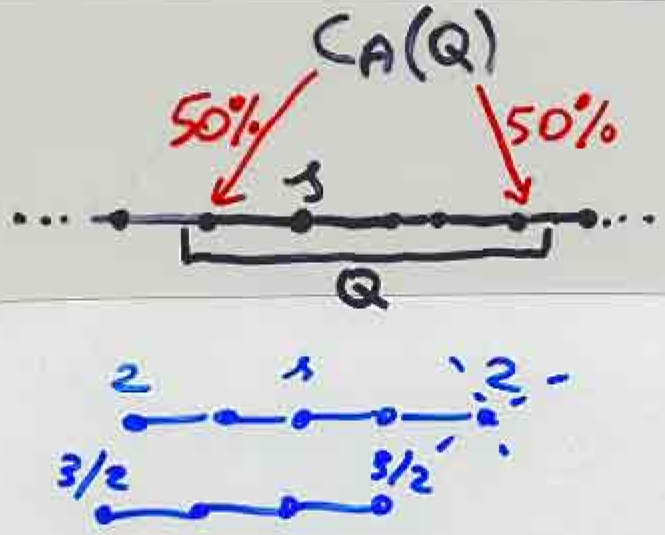

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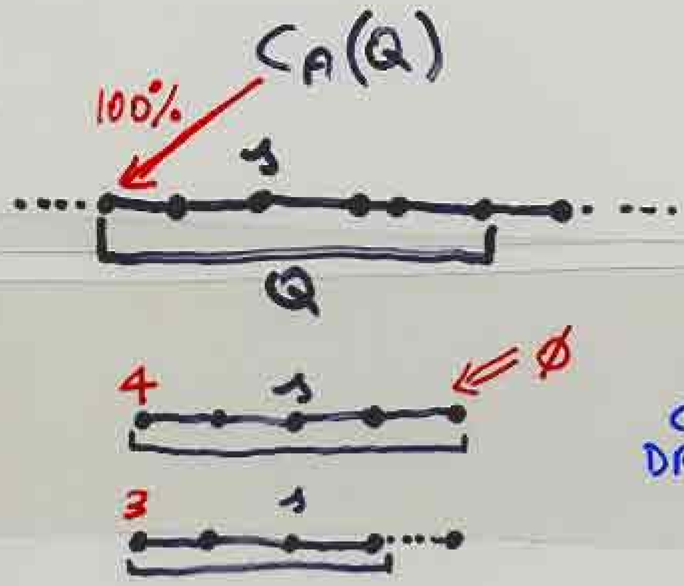

$M(\varepsilon)$
CANNOT
DROP THIS

NAIVE SOLUTIONS

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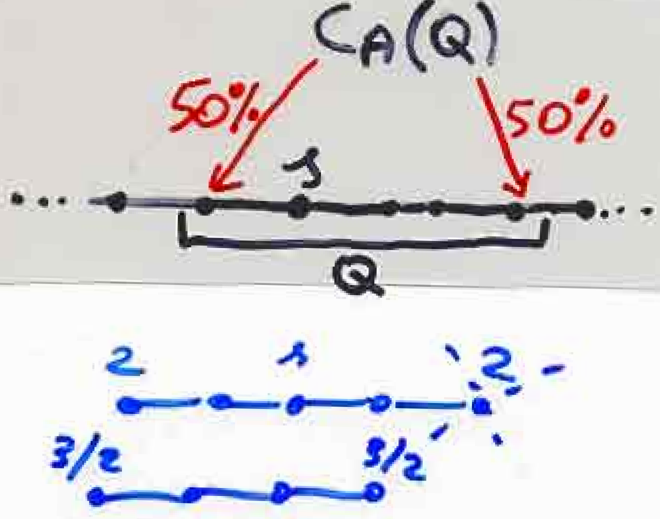

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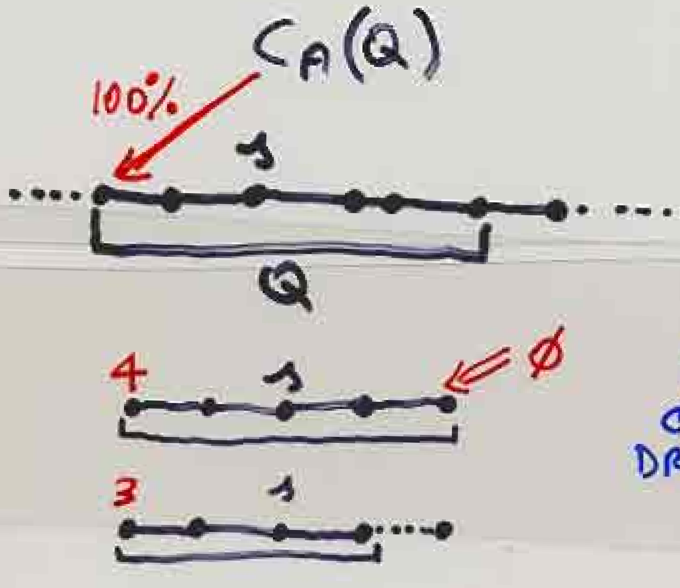


$M(\xi)$ CANNOT DROP THIS

IDEA: SOME QCU DO NOT "APPEAR"

$\Downarrow$

MONOTONE ONLY FOR "POSSIBLE" SUBSETS GENERATED BY $M(\xi)$

$$\xi(\cdot) \Longrightarrow \mathcal{P}^{\xi} \text{ POSSIBLE SUBSETS}$$

$$\mathcal{P}_0^{\xi} = U$$

$$\mathcal{P}_1^{\xi} = \{U \setminus \{i\} \mid \xi(U,i) > 0\}$$

$$\vdots$$

$$\mathcal{P}_j^{\xi} = \{Q_{j-1} \setminus \{i\} \mid \xi(Q_{j-1}, i) > 0,\ Q_{j-1} \in \mathcal{P}_{j-1}^{\xi}\}$$

$$\vdots$$

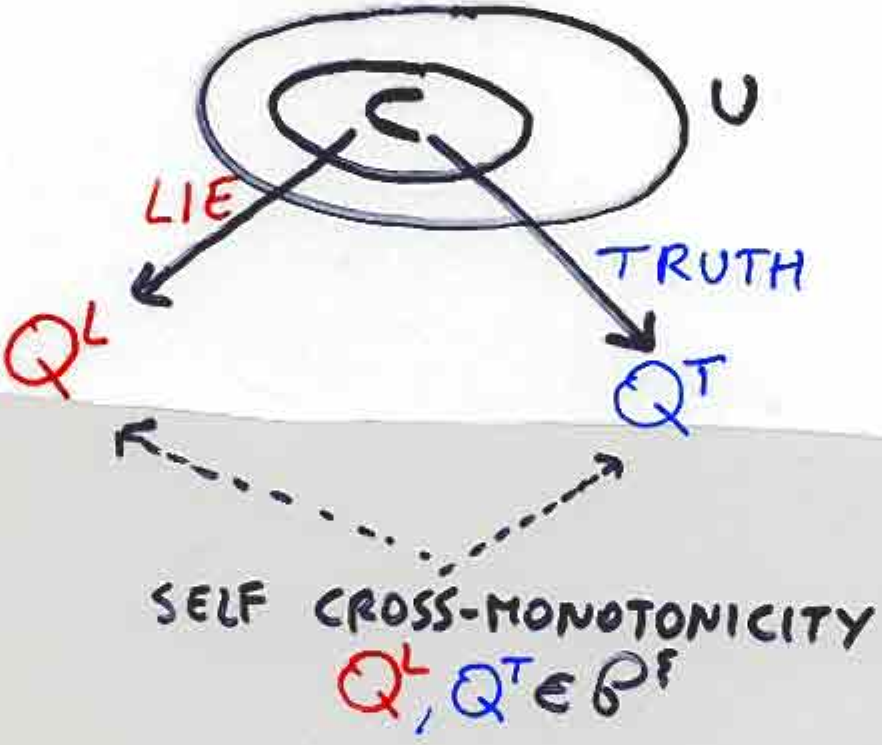
$$\mathcal{P}^{\xi} = \bigcup_{j=0}^{m} \mathcal{P}_j^{\xi}$$

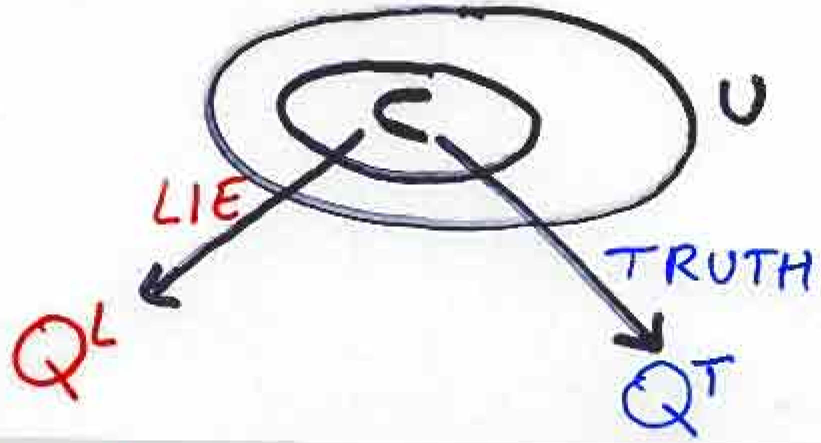
DEF: $\xi(\cdot)$ IS SELF CROSS-MONOTONIC IF

$$\forall Q', Q \in \mathcal{P}^{\xi},\ Q' \subset Q$$

$$\xi(Q', i) \geq \xi(Q, i),\ i \in Q'$$

THM: $\xi(\cdot)$ SELF CROSS-MONOTONIC COST-SHARIG METHOD FOR $C_A(\cdot) \Longrightarrow M(\xi)$ "WORKS": VP, CS, NPT, BB, GROUP STR-PROOF

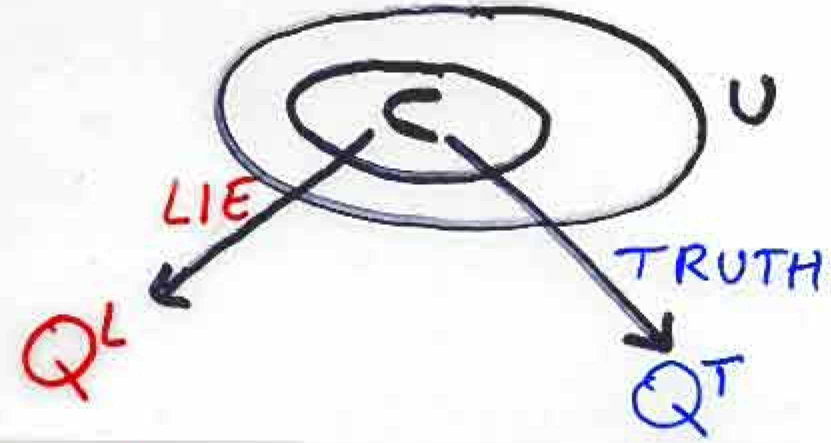

U
C
LIE
TRUTH
Q^L
Q^T
SELF CROSS-MONOTONICITY
$Q^L, Q^T \in \mathcal{P}^F$



CASE 1: $Q^L \subseteq Q^T$

$\xi(Q^L, i) \geq \xi(Q^T, i)$

"NOBODY IMPROVES"



CASE 1: $Q^L \subseteq Q^T$

$$\xi(Q^L, i) \geq \xi(Q^T, i)$$

"NOBODY IMPROVES"

CASE 2: $Q^L \not\subseteq Q^T$

$$\Downarrow$$

SOME i IS NOT DROPPED

$$\Downarrow$$

$$b_i > v_i$$

$$\Downarrow$$

$$\xi(Q^F, i) \geq b_i > v_i$$

"REASONABLE" ALGORITHM:

CAN DROP USERS 1-BY-1

DEF: A IS <u>REASONABLE</u> IF

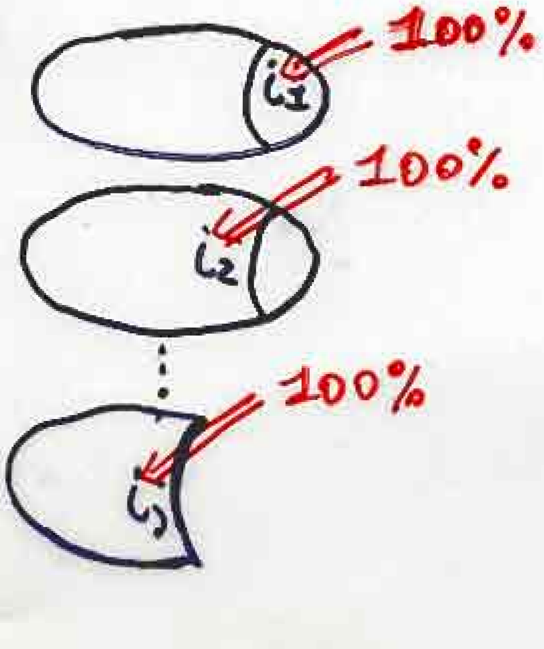
$\exists\ i_1, \ldots, i_m$ s.t.

A CAN COMPUTE A FEAS SOLUTION FOR

$$Q_j = U \setminus \{i_1, \ldots, i_j\}$$

THM: A REASONABLE $\Rightarrow$ $\xi(\cdot)$ SELF CROSS-MONOTONIC FOR $C_A(\cdot)$

"PROOF":



A REASONABLE, OPT

$\Downarrow$

$\xi(\cdot)$ SELF CROSS-MONOTONIC FOR $C_A(\cdot)$

$\Downarrow$

$M(\xi)$ IS GROUP STRATEGYPROOF

BB, VP, CS, NPT, CO

A REASONABLE, OPT

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$\xi(\cdot)$ SELF CROSS-MONOTONIC FOR $C_A(\cdot)$

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NEXT:

STEINER TREE GAME

$\exists$ A POLYTIME, REASONABLE

$$C_A(Q(b)) = C_{OPT}(Q(b))$$

ONLY ON $\mathcal{P}^{\xi}$

PRIM'S MST ALGORITHM

ADDED NODES:

$1, e_1, e_2, \dots, e_n$

HOW DO I DROP USERS?

PRIM'S MST ALGORITHM

ADDED NODES:

$1, e_1, e_2, \ldots, e_n$

$\longleftarrow$

$i_n, \ldots, i_2, i_1$

HOW DO I DROP USERS?

PRIM'S MST ALGORITHM

ADDED NODES:

$$1, e_1, e_2, \dots, e_m$$

$$\longleftarrow$$

$$i_m, \dots, i_2, i_1$$

HOW DO I DROP USERS?

$$\underbrace{1, e_1, \dots, e_j}_{Q_j}$$

$MST(Q_j)$ IS AN OPTIMAL STEINER TREE FOR Q_j

$$\Downarrow$$

$$C_{MST}(Q_j) = C_{OPT}(Q_j)$$

FUTURE PLANS

OTHER PROBLEMS:

STEINER FOREST
[KÖNEMANN, LEONARDI, SCÄRF '04]

CONNECTED FACILITY LOCATION
[PÁL-TARDOS '03]

SINGLE-SOURCE RENT-OR-BUY
[GUPTA-KUMAR-ROUGHGARDEN '03]
⋮

DISTRIBUTED MECHANISMS?

FAIRNESS

- EGALITARIAN
- CORE