Optimal dislocation with persistent errors in subquadratic time

Barbara Geissmann, Stefano Leucci,
Chih-Hung Liu, Paolo Penna

ETH Zurich

Sorting with erroneous comparisons

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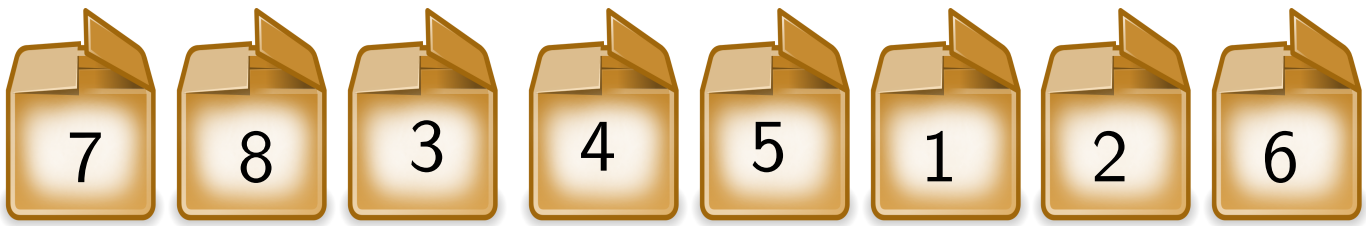
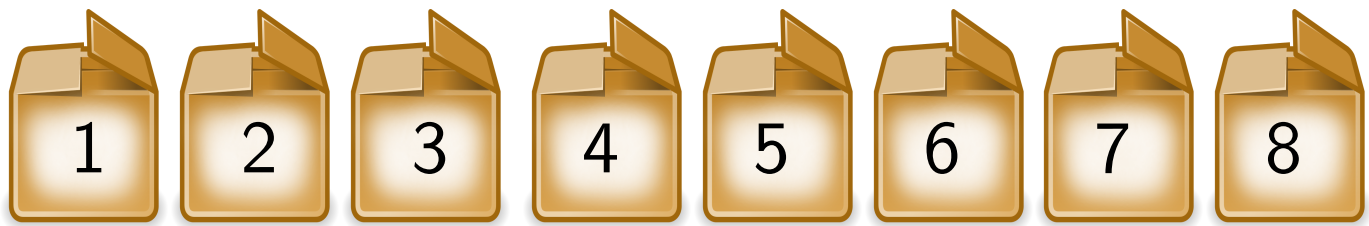
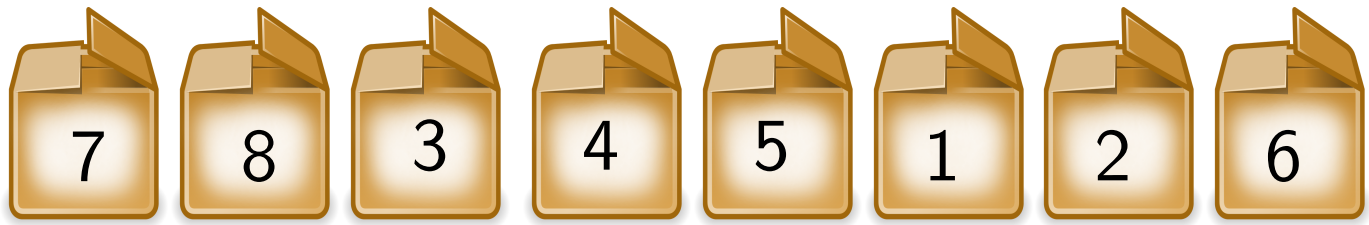


Sorting with erroneous comparisons



The Problem



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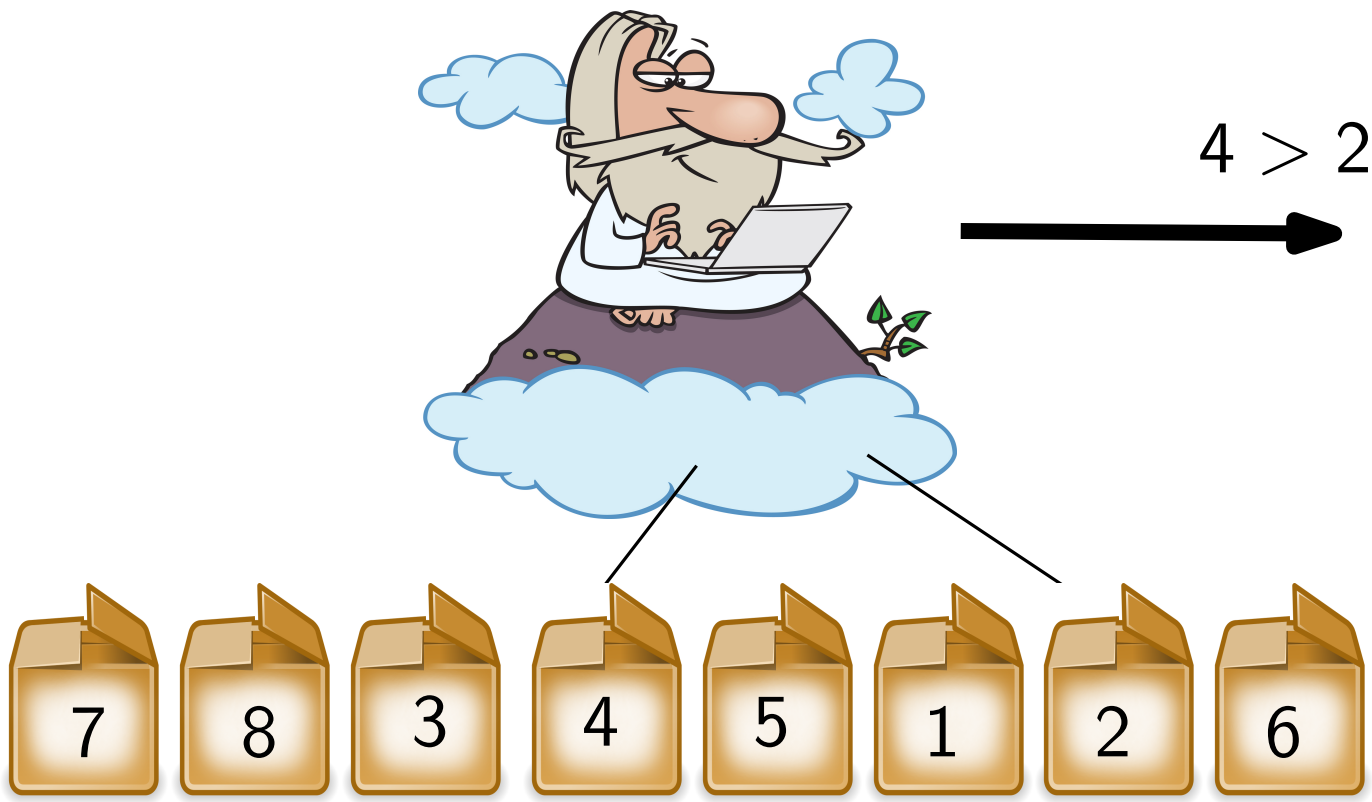
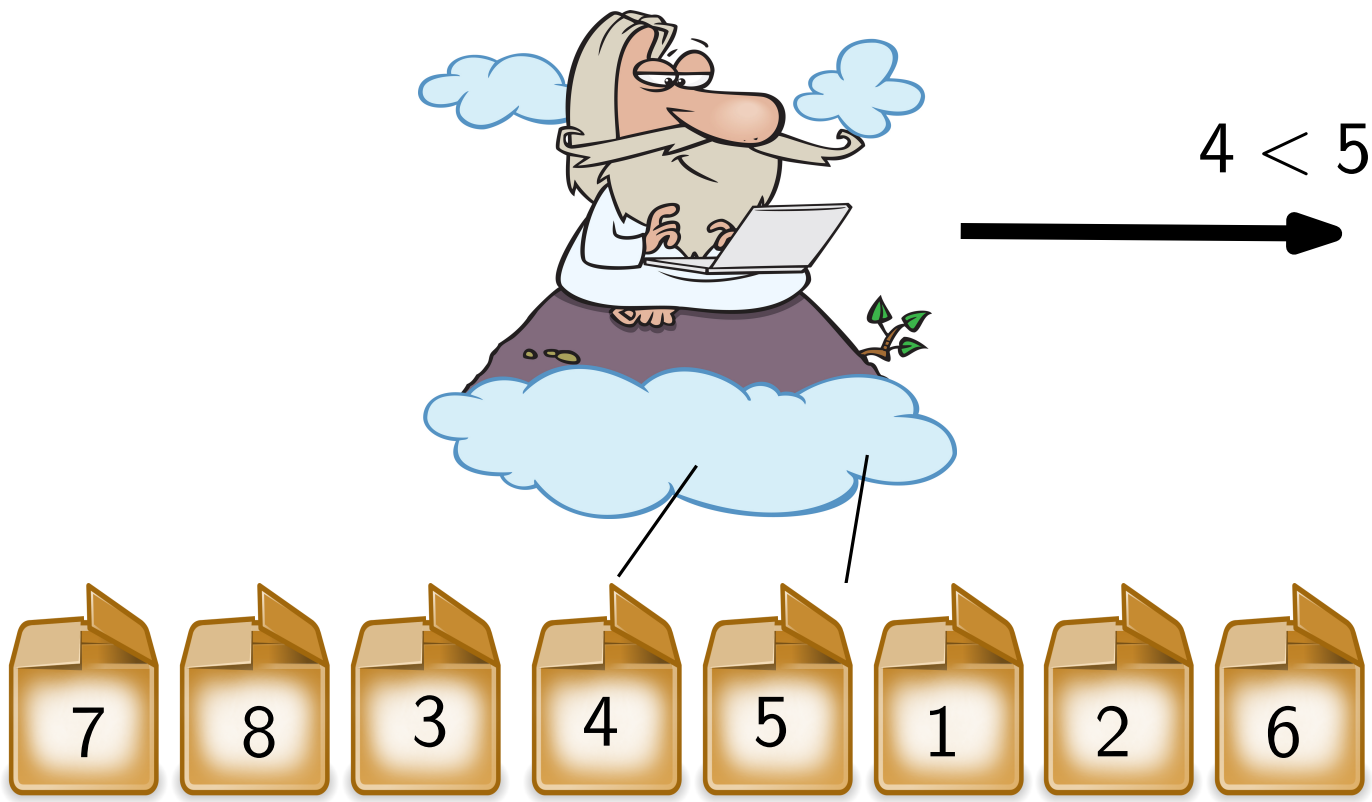
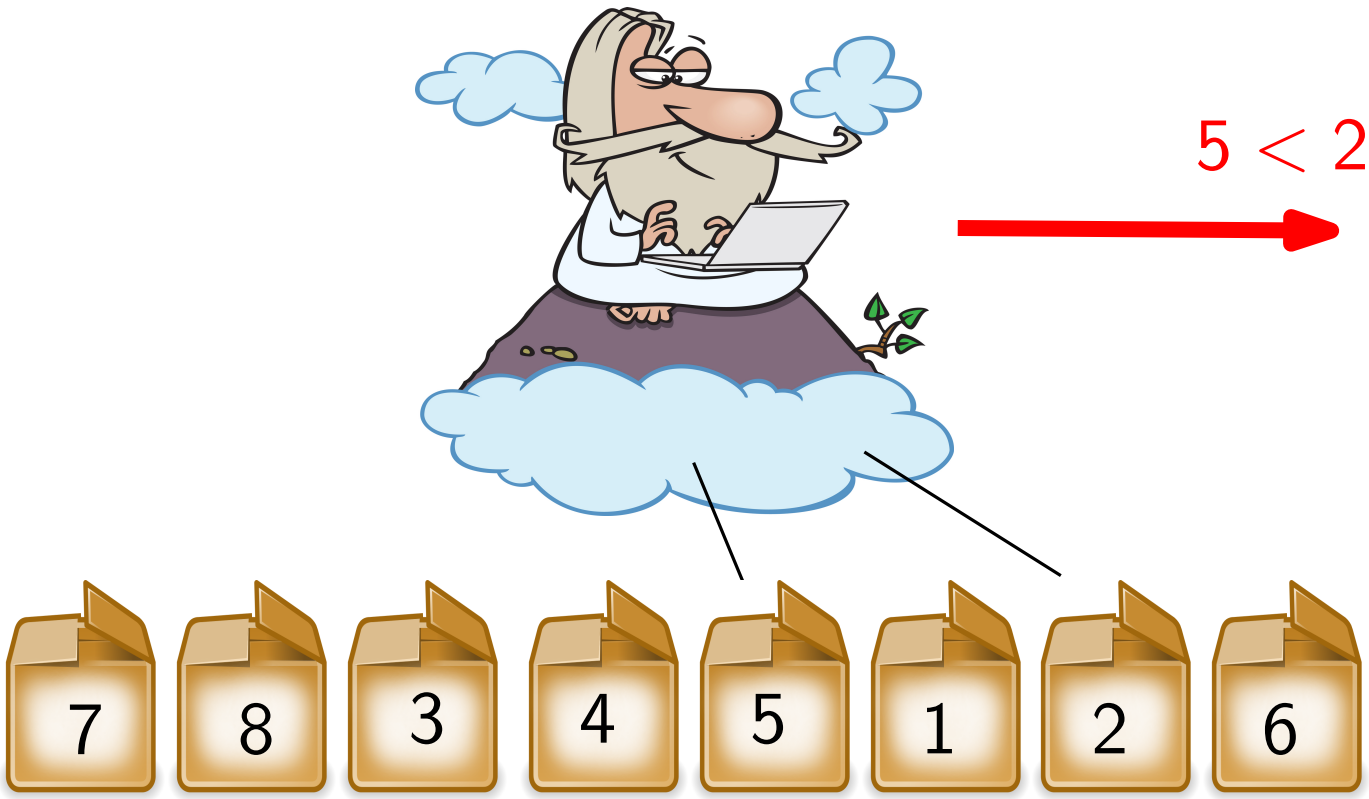


The Problem



The Problem

The Problem



- error probability p constant

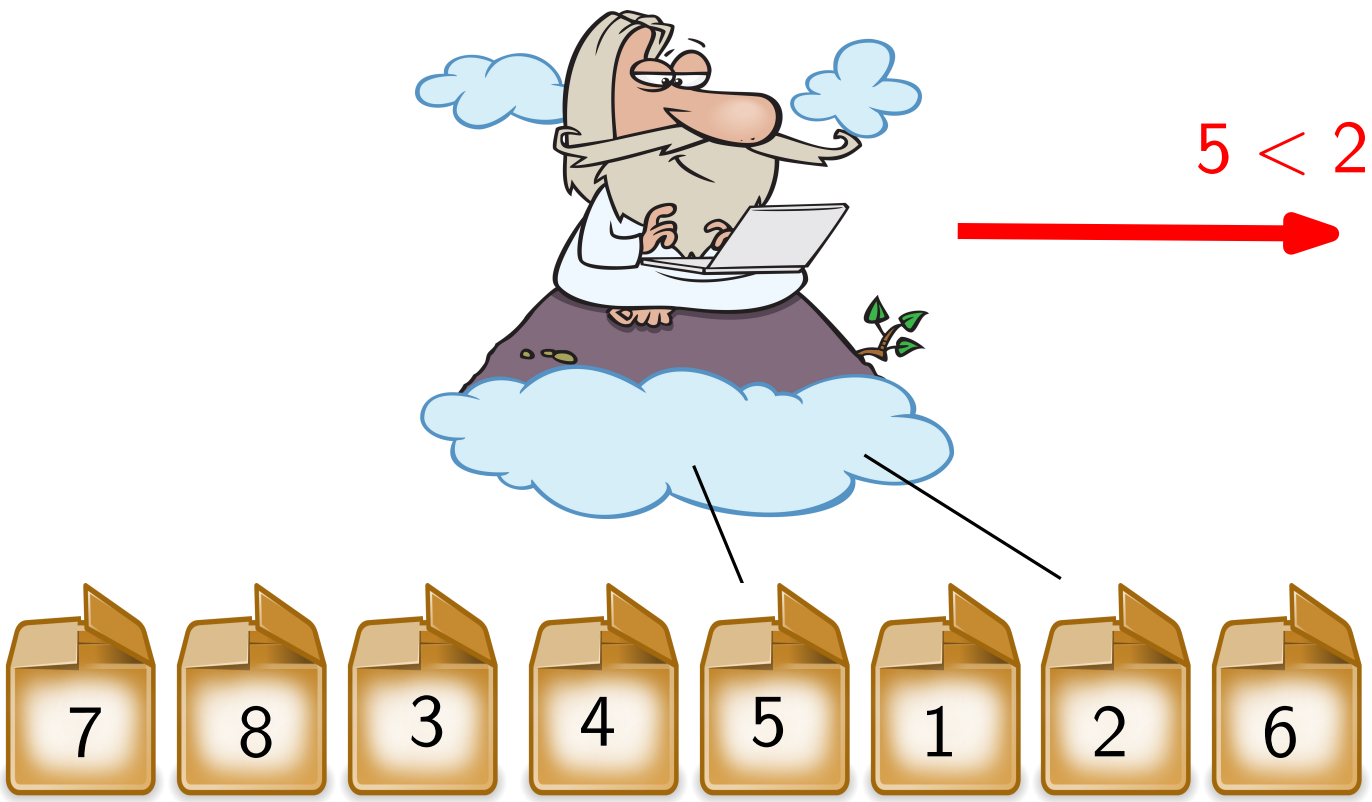
The Problem

- error probability p constant
- independent for each pair

The Problem

- error probability p constant
- independent for each pair
- persistent errors

The Problem



- error probability p constant
- independent for each pair
- persistent errors

5%

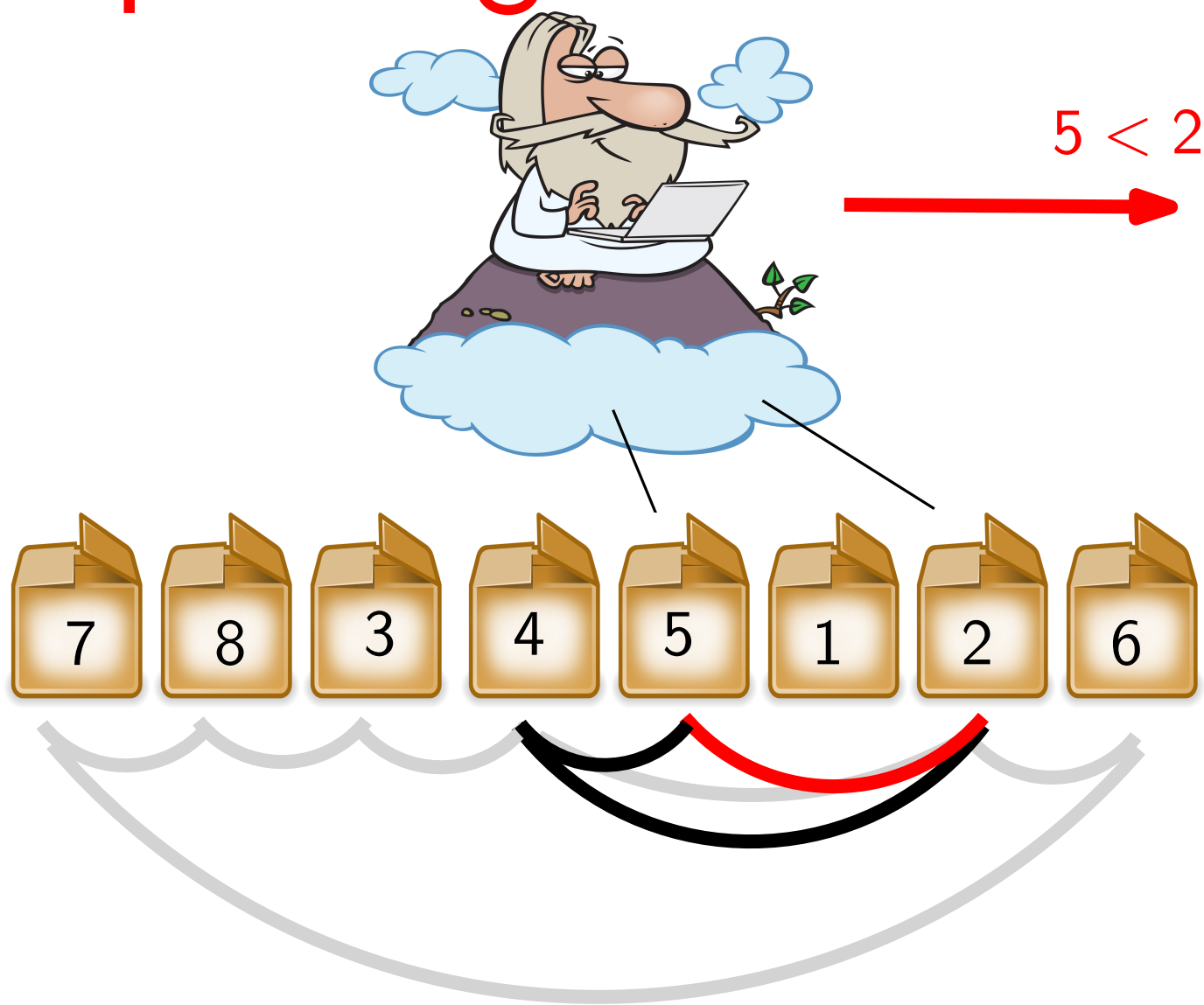
The Problem

Repeating does not help

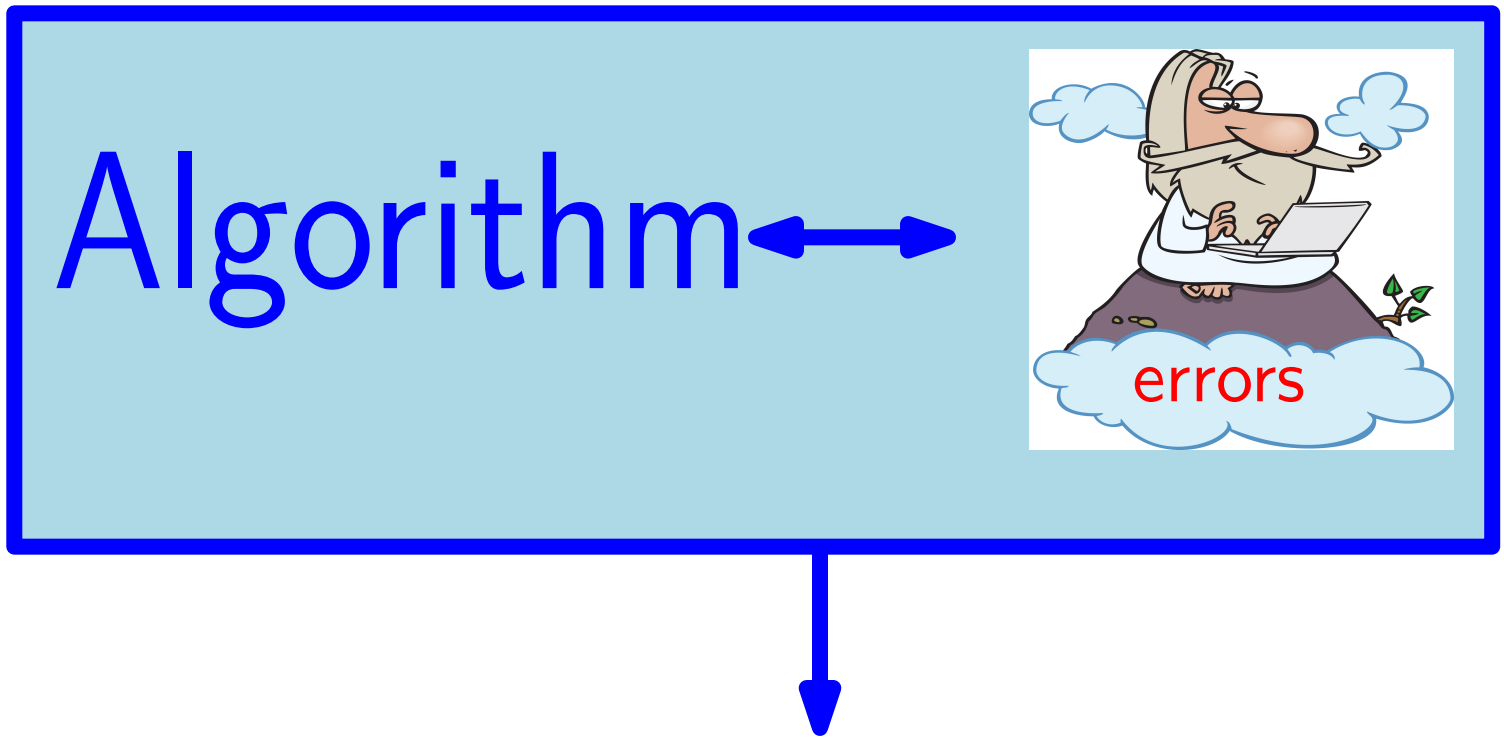
- error probability p constant
- independent for each pair
- persistent errors

The Problem

Repeating does not help



Can you sort?



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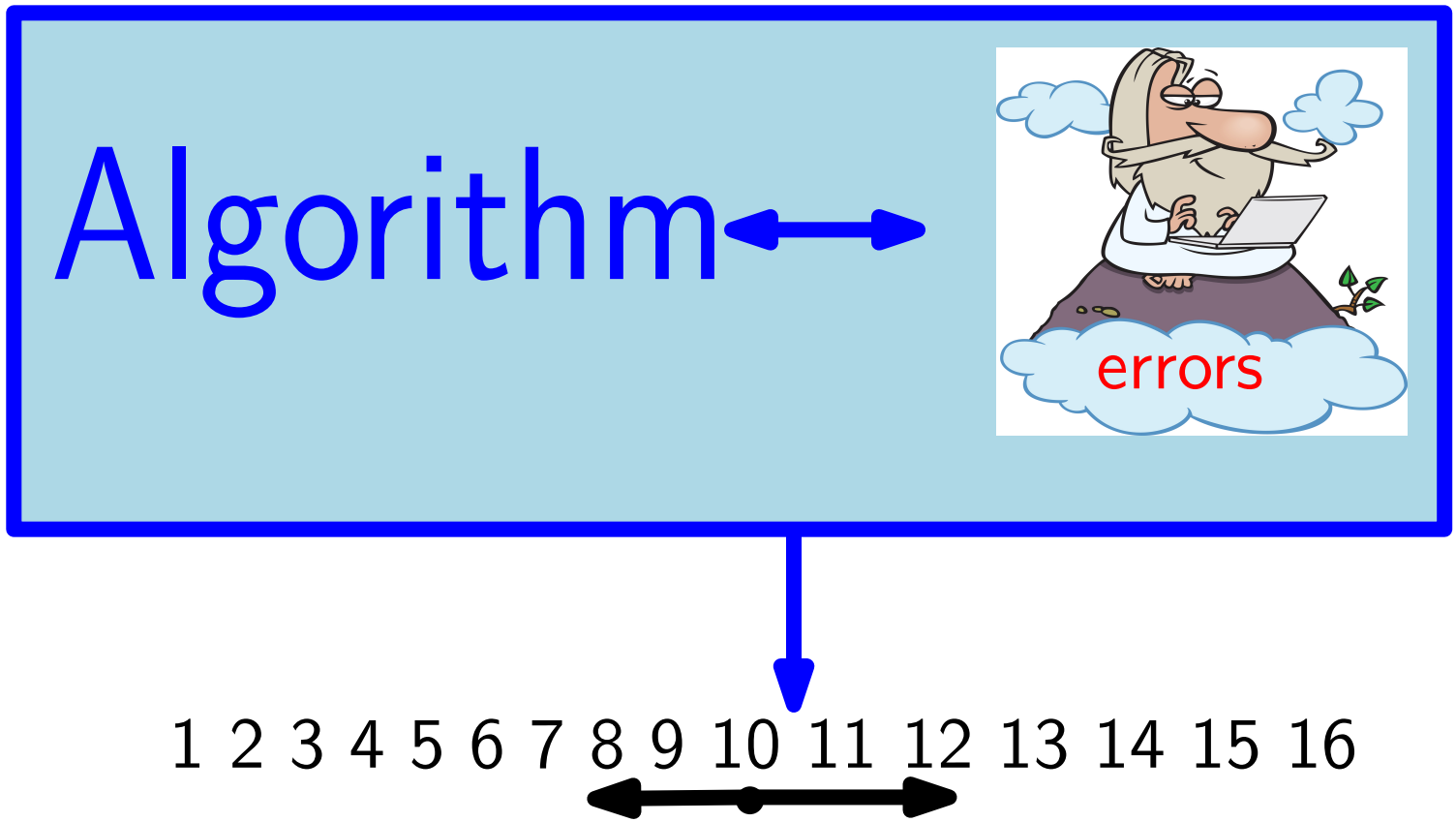

Can you sort?

Algorithm

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

1 2 3 4 5 6 7 8 9 11 12 10 13 14 15 16

Approx Sorted

Can you sort?

Algorithm

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

1 2 3 4 5 6 7 8 9 11 12 10 13 14 15 16

Approx Sorted

Can you sort?

Algorithm ⟷

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

1 2 3 4 5 6 7 8 9 11 12 10 13 14 15 16

Dislocation

What can be done?

Prior Results

Prior Results

MAX	**Dislocation**	TOTAL
$O(\log n)$		$O(n)$

Braverman & Mossel (SODA'08)

Prior Results

Time:	**Dislocation** MAX	TOTAL
$O(n^{3+C})$	$O(\log n)$	$O(n)$

Braverman & Mossel (SODA'08)

Prior Results

Time:	Dislocation: MAX	Dislocation: TOTAL
$O(n^{3+C})$	$O(\log n)$	$O(n)$

$$\frac{110525}{(1/2-p)^4}$$

Braverman & Mossel (SODA'08)

Prior Results

Time:	Dislocation: MAX	Dislocation: TOTAL
$O(n^{3+C})$	$O(\log n)$	$O(n)$
$O(n^2)$	$O(\log n)$	

Braverman & Mossel (SODA'08)
Klein, Penninger, Sohler, Woodruff (ESA'11)

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Geissmann, Leucci, Liu, Penna (ISAAC'17)

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$O(n^2)$	$O(\log n)$	$O(n)$
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Prior Results

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$O(n^2)$	$O(\log n)$	
$O(n^2)$	$O(\log n)$	$O(n)$
	$\Omega(\log n)$	$\Omega(n)$

Subquadratic time?

Our Contribution

YES

Our Contribution

YES

Time:	**Dislocation**	
	MAX	TOTAL
$O(n^{3/2})$	$O(\log n)$	$O(n)$

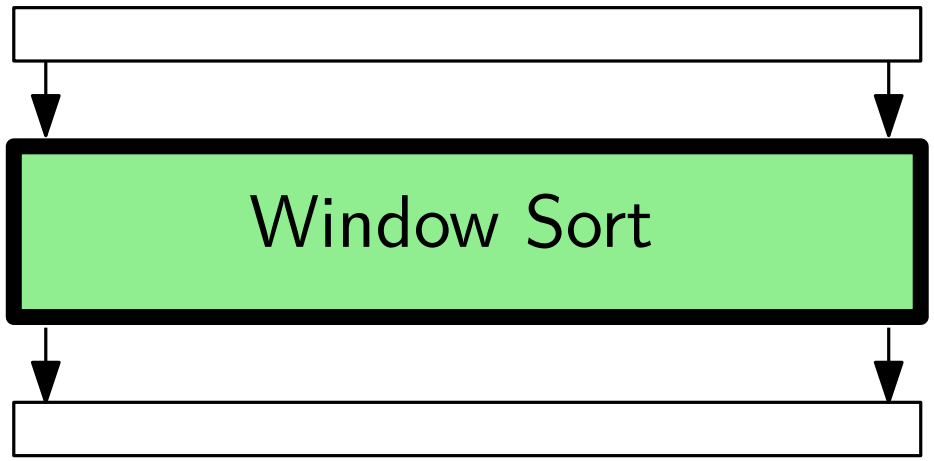
Our Contribution

YES

Time:	**Dislocation** MAX	**Dislocation** TOTAL
$O(n^{3/2})$	$O(\log n)$	$O(n)$

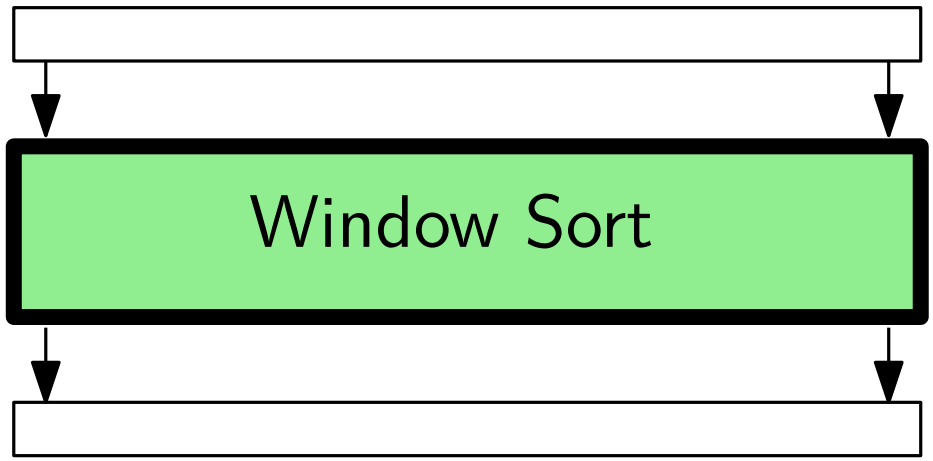
randomized algorithm ⟶ "derandomized" algorithm

$O(n^2)$-Time Algorithm



Geissmann, Leucci, Liu, Penna (ISAAC'17)

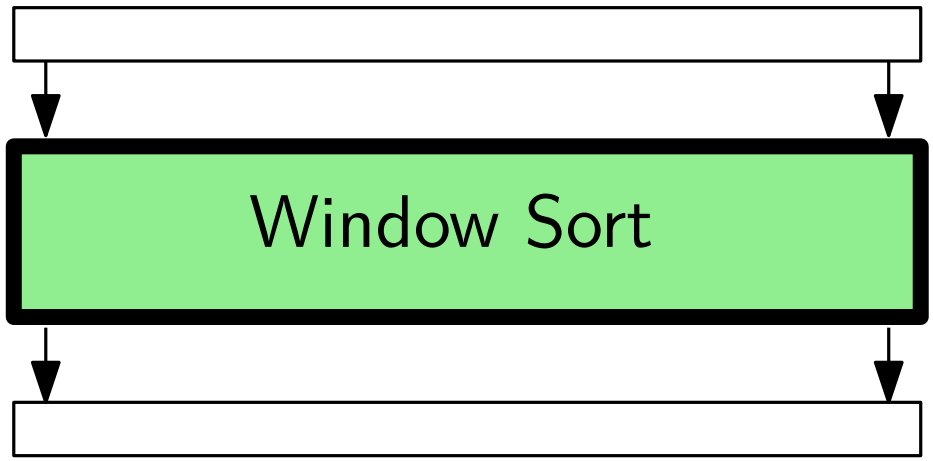
$O(n^2)$-Time Algorithm



errors well-spread $\Rightarrow$ success

Geissmann, Leucci, Liu, Penna (ISAAC'17)

$O(n^2)$-Time Algorithm

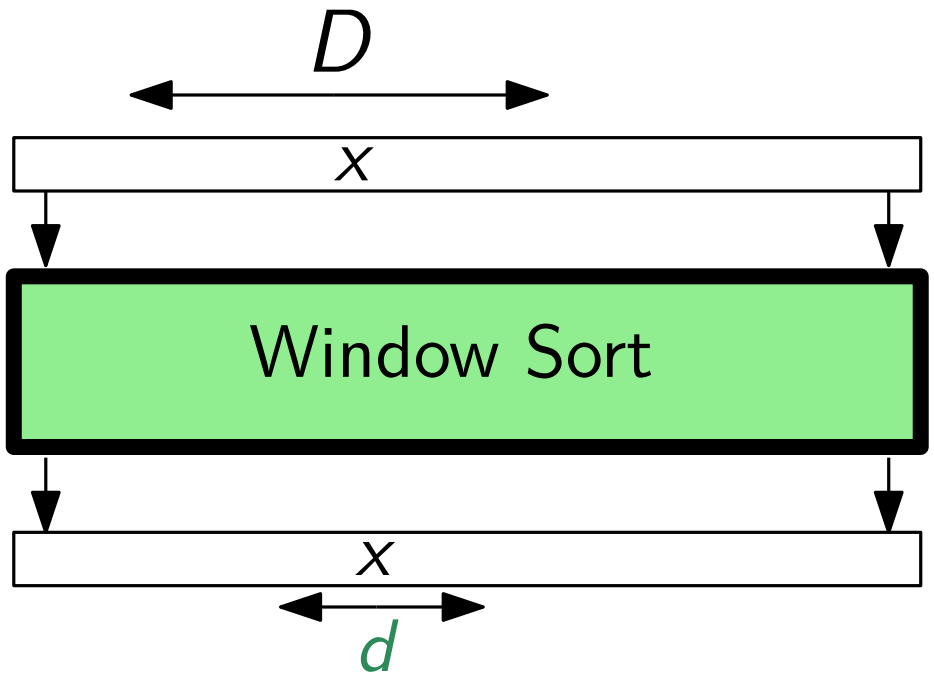


errors well-spread $\Rightarrow$ success

initial dislocation $D \Rightarrow$ time $O(Dn)$

Geissmann, Leucci, Liu, Penna (ISAAC'17)

$O(n^2)$-Time Algorithm



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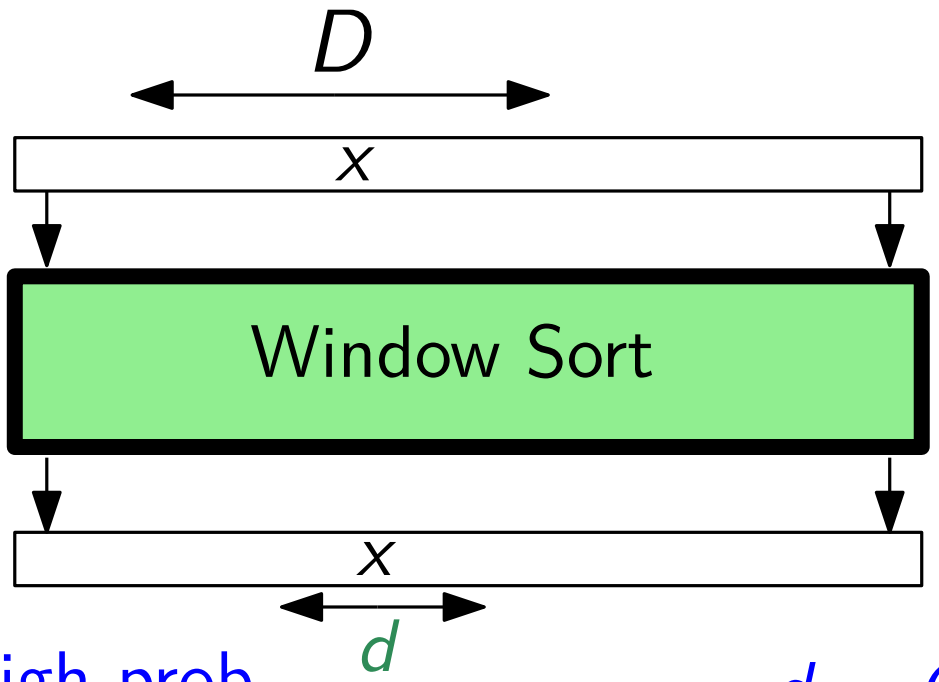


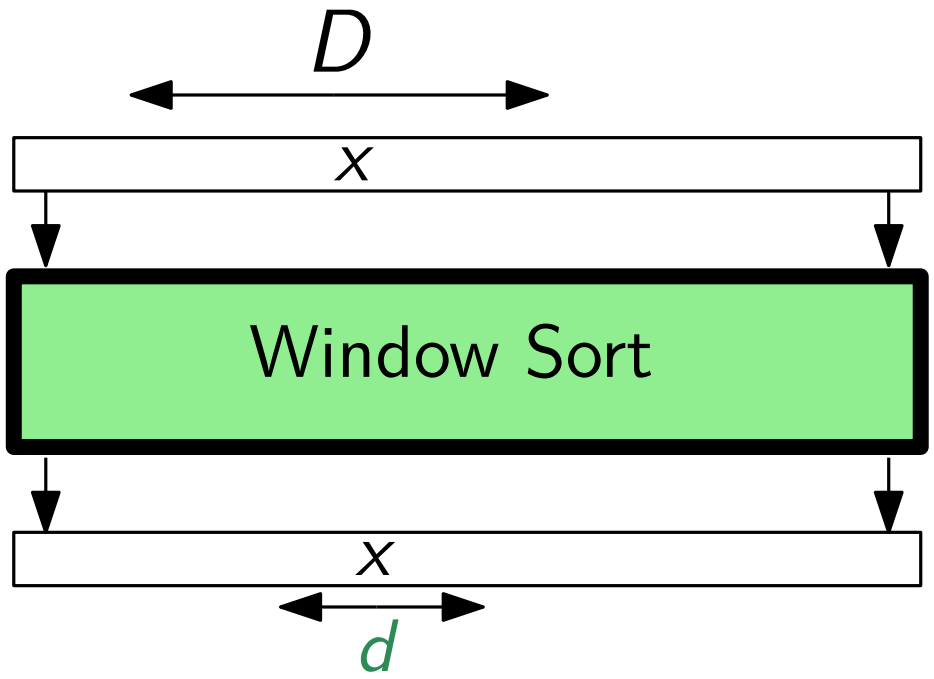
with high prob $d = O(\log n)$

errors well-spread $\Rightarrow$ success

initial dislocation $D \Rightarrow$ time $O(Dn)$

Geissmann, Leucci, Liu, Penna (ISAAC'17)

$O(n^2)$-Time Algorithm

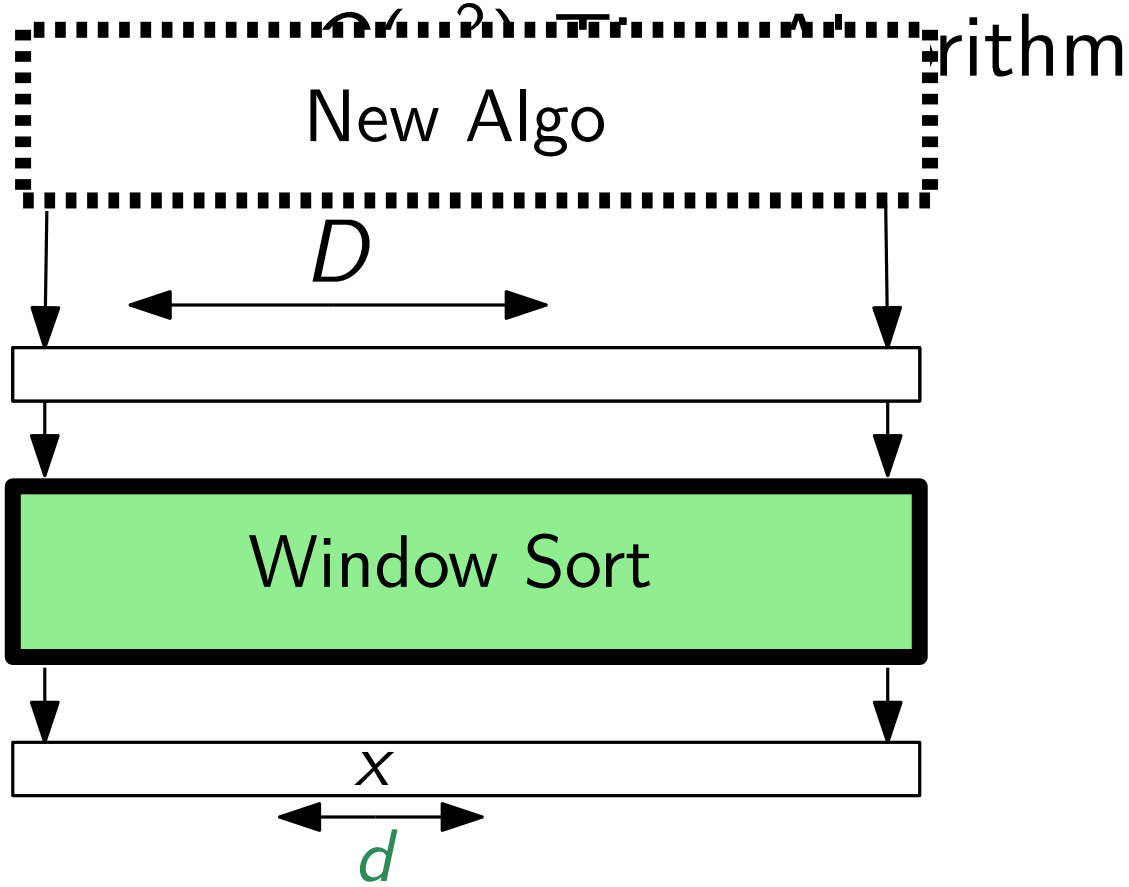


n

errors well-spread $\Rightarrow$ success

initial dislocation $D \Rightarrow$ time $O(Dn)$

Geissmann, Leucci, Liu, Penna (ISAAC'17)



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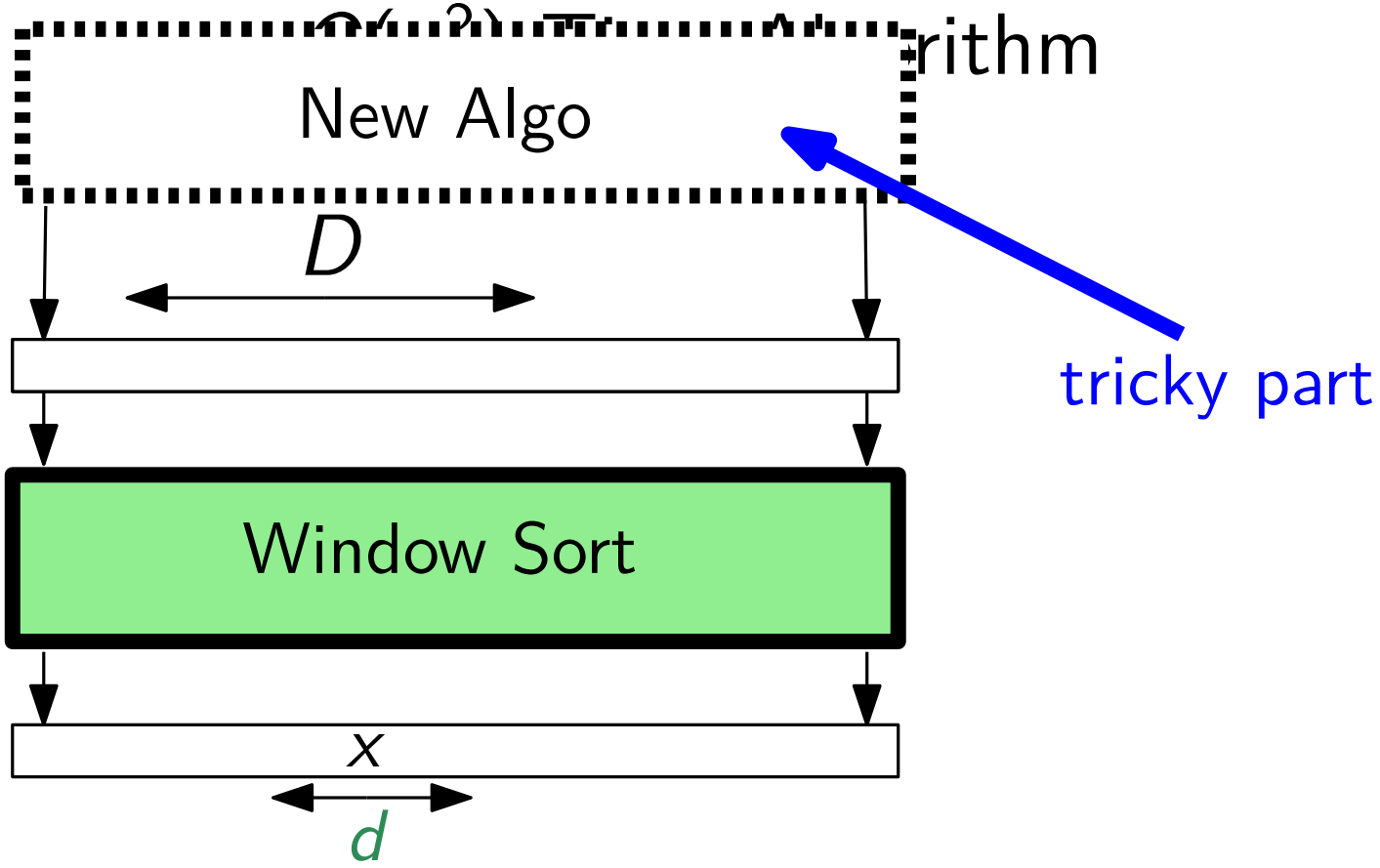

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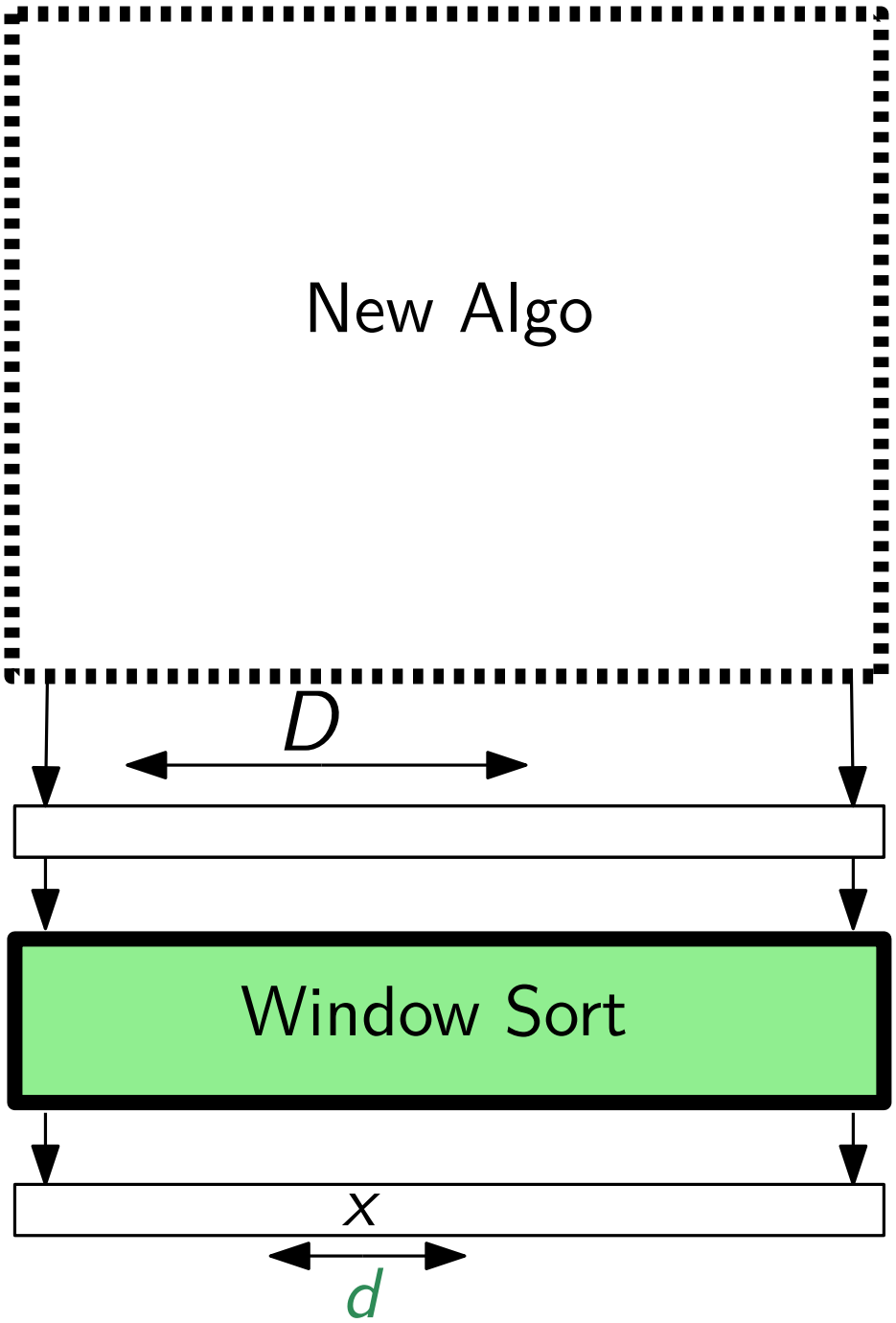
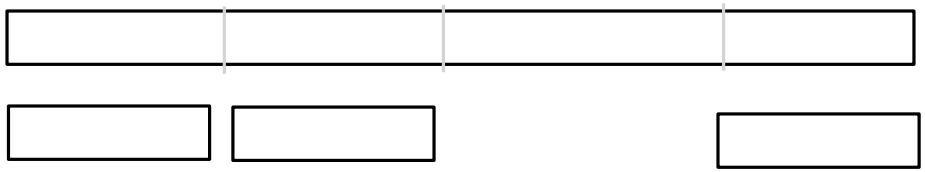
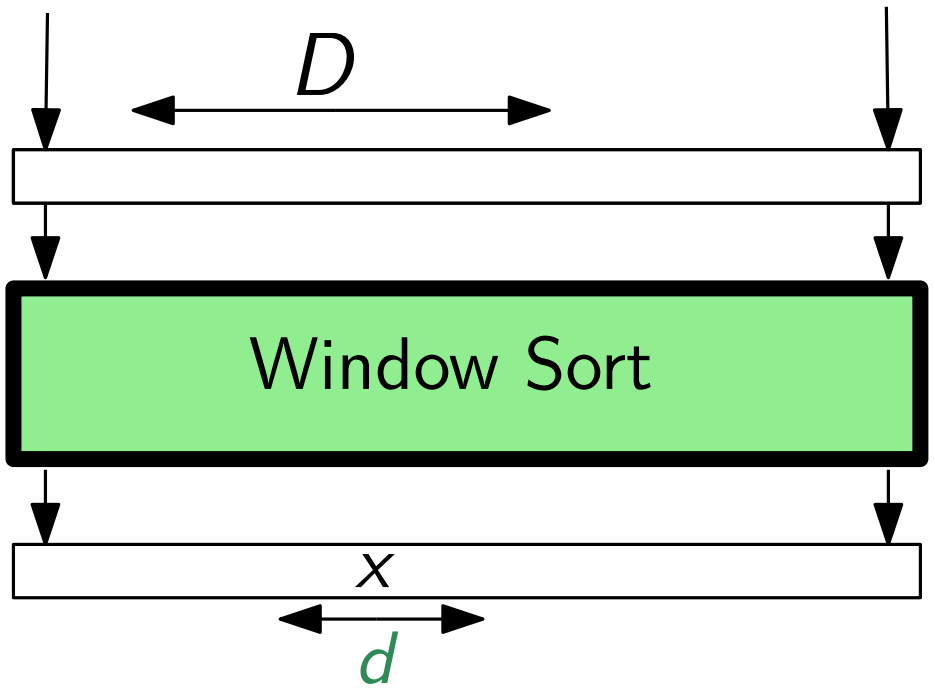
Simple Faster Algo



Simple Faster Algo





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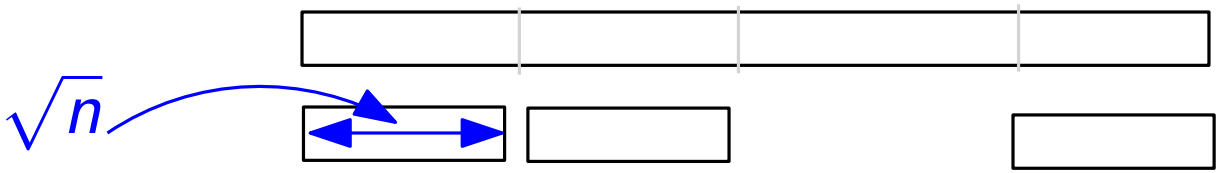

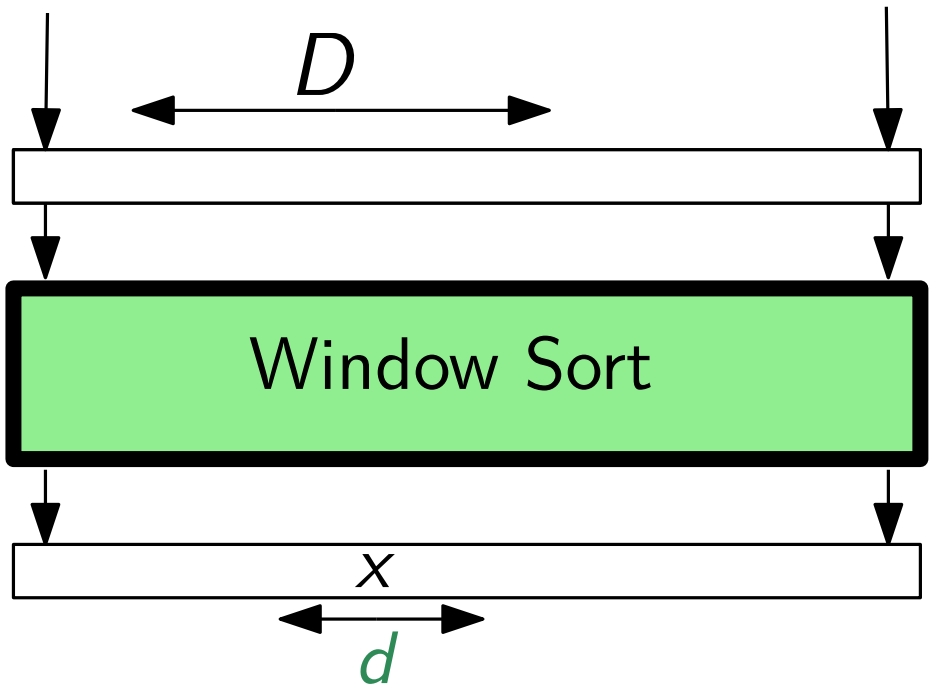

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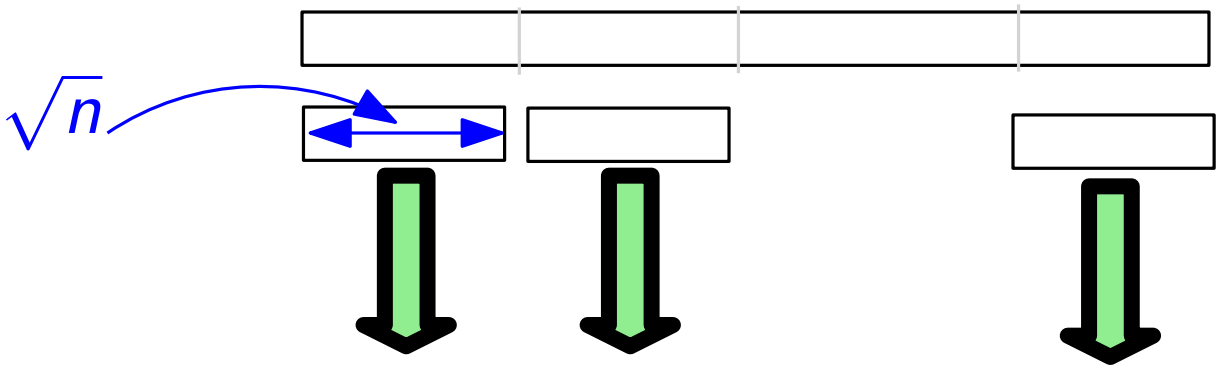

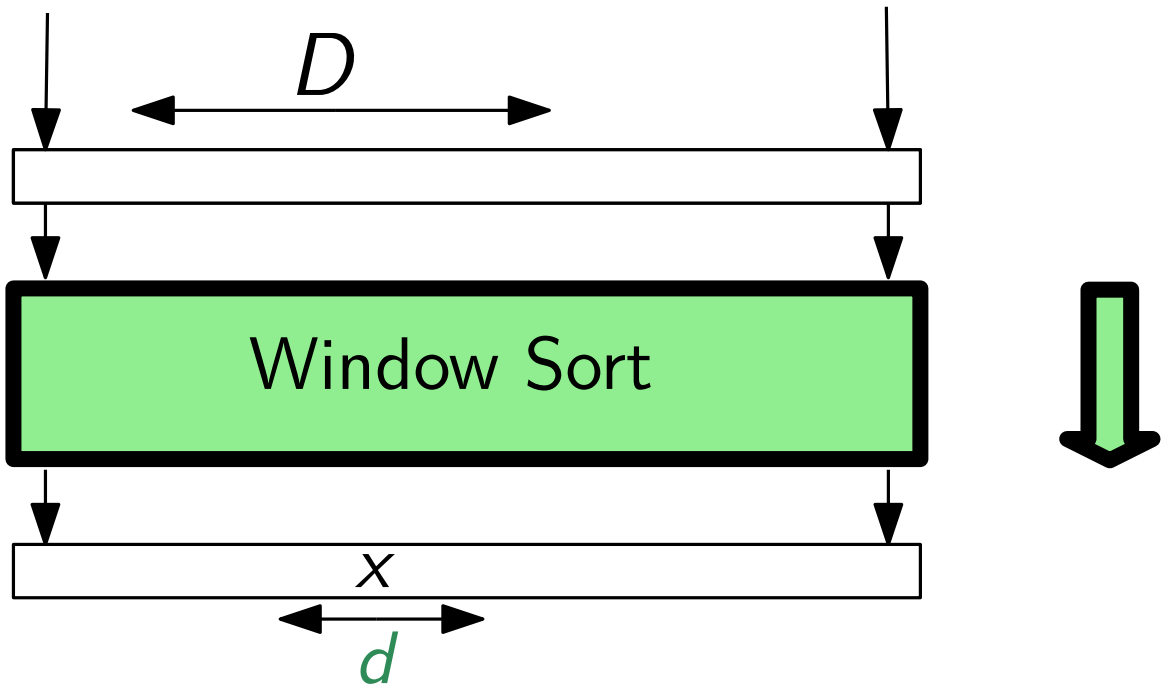

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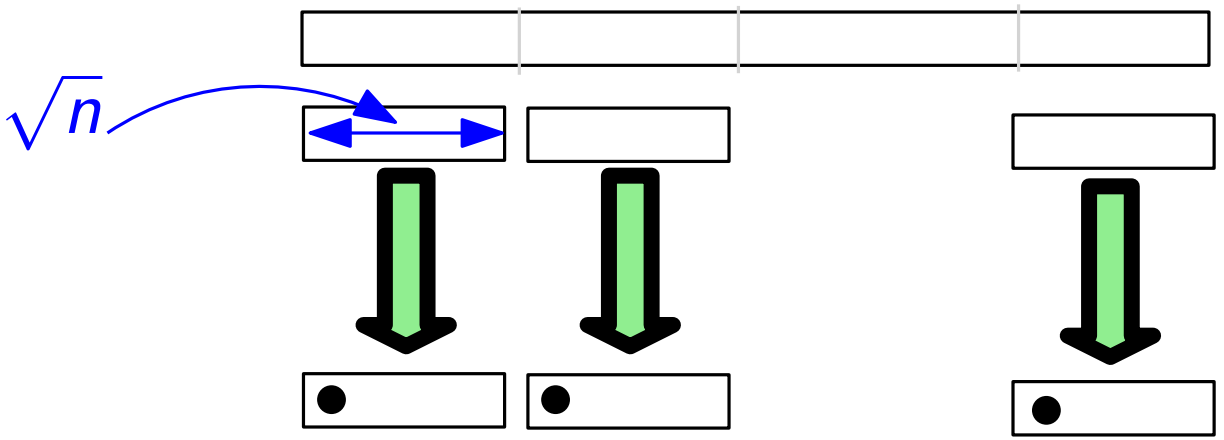

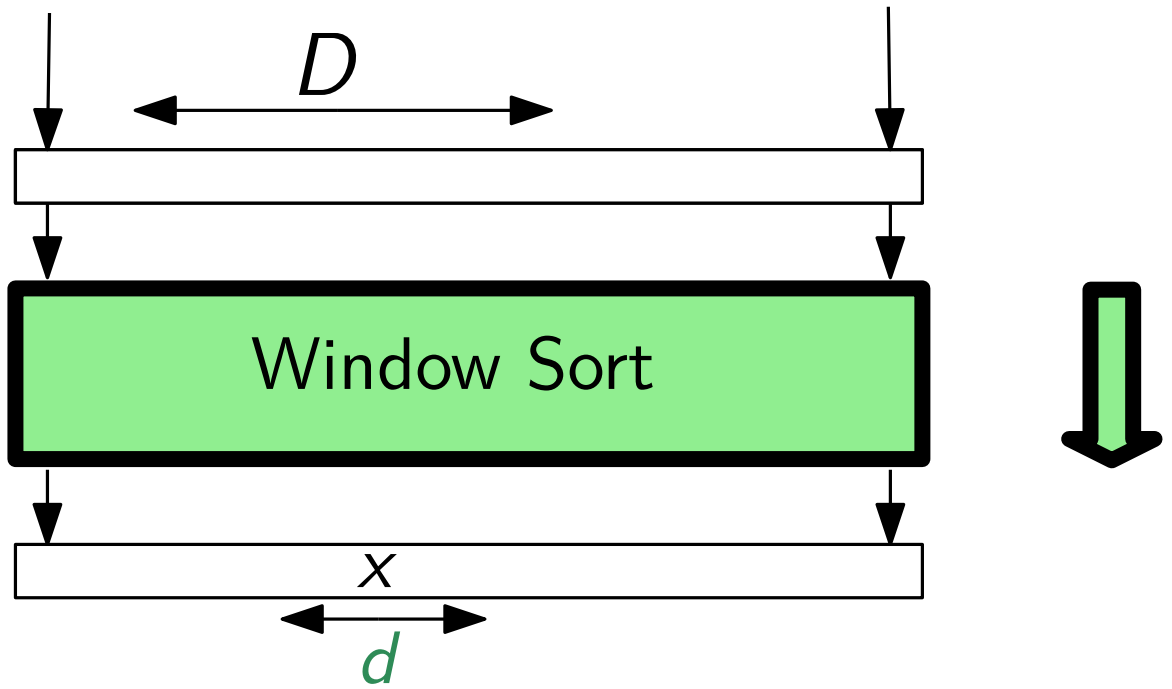

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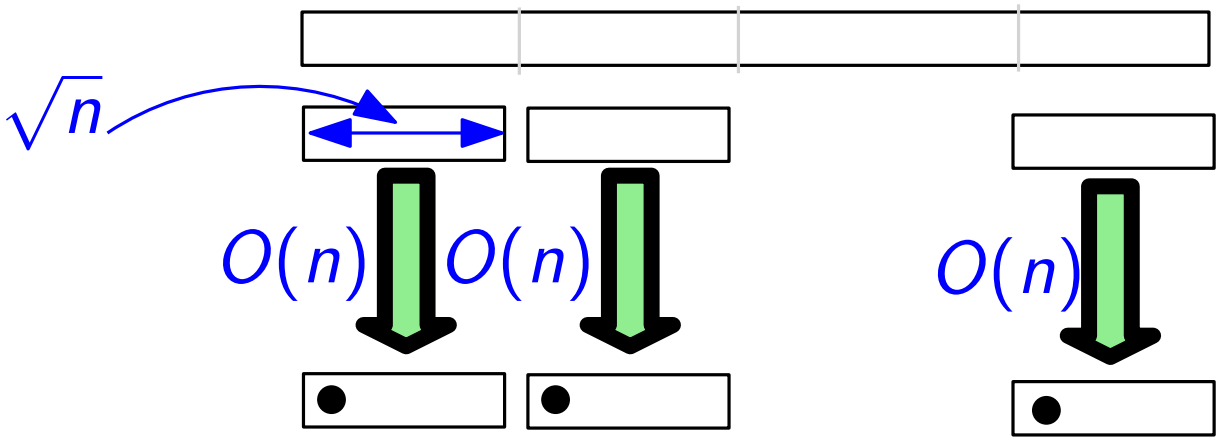

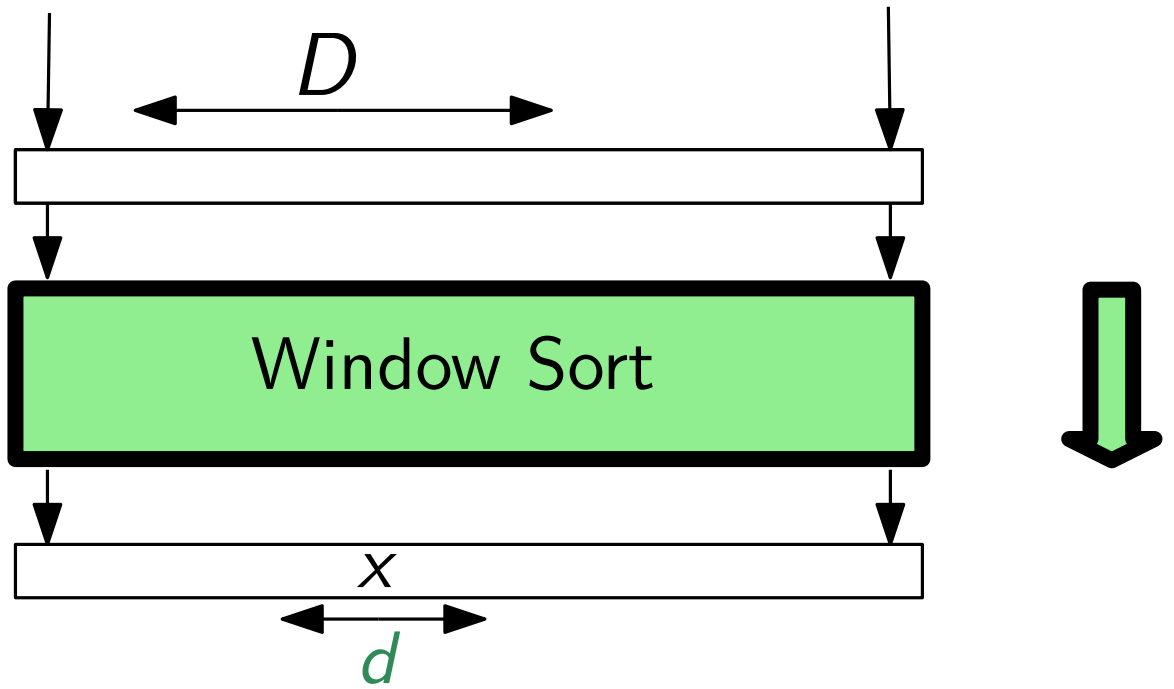

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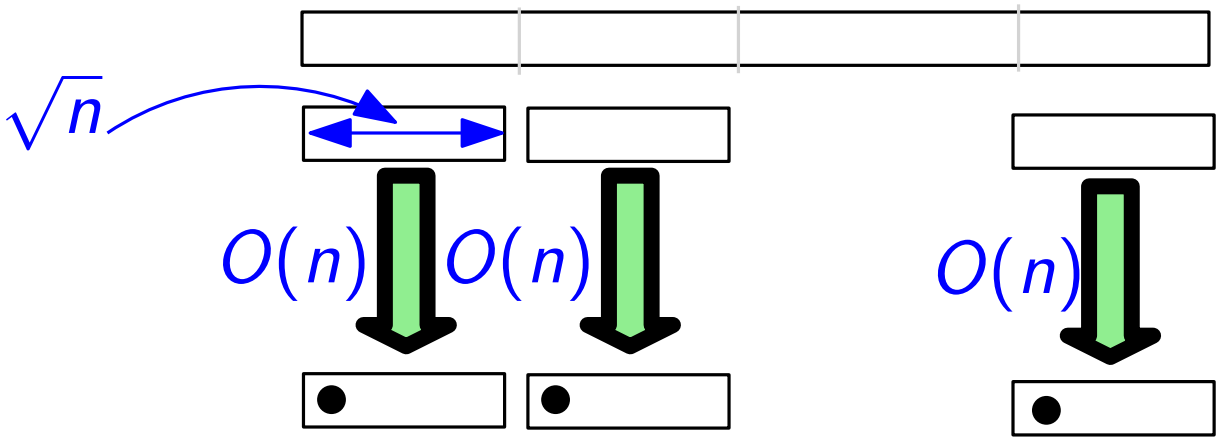
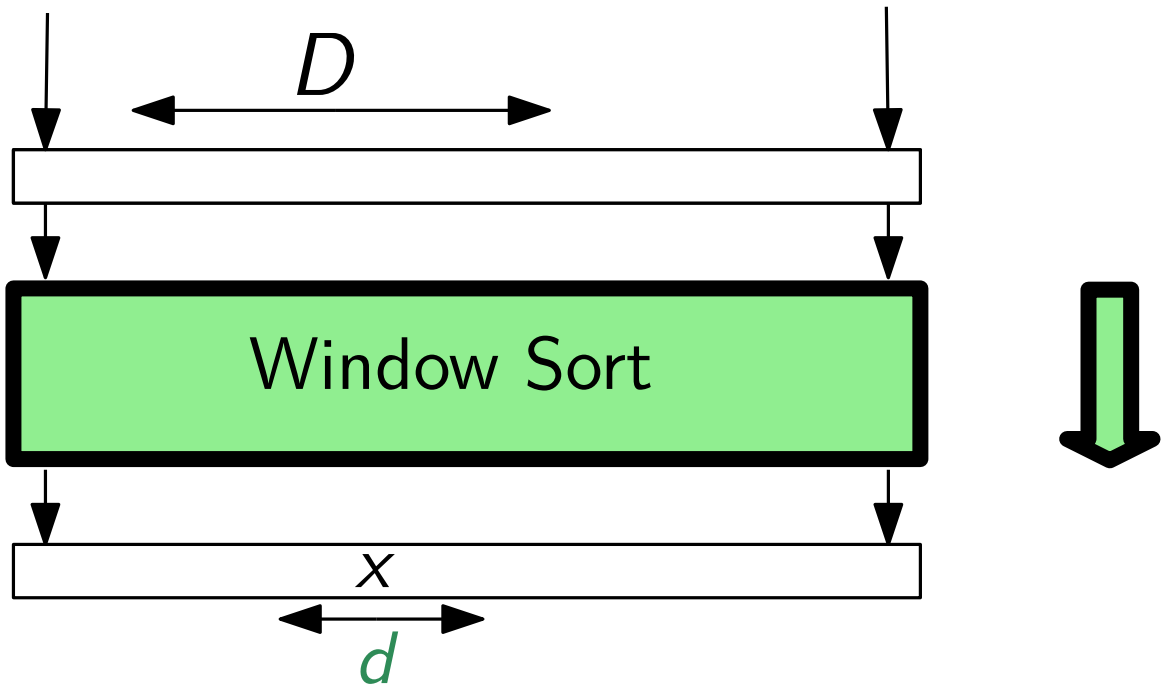


$O(n^{3/2})$



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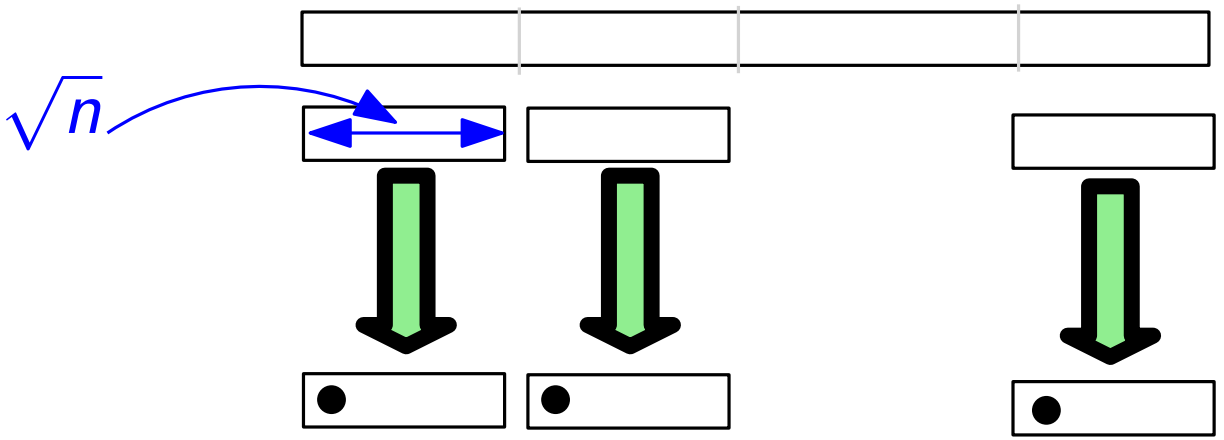
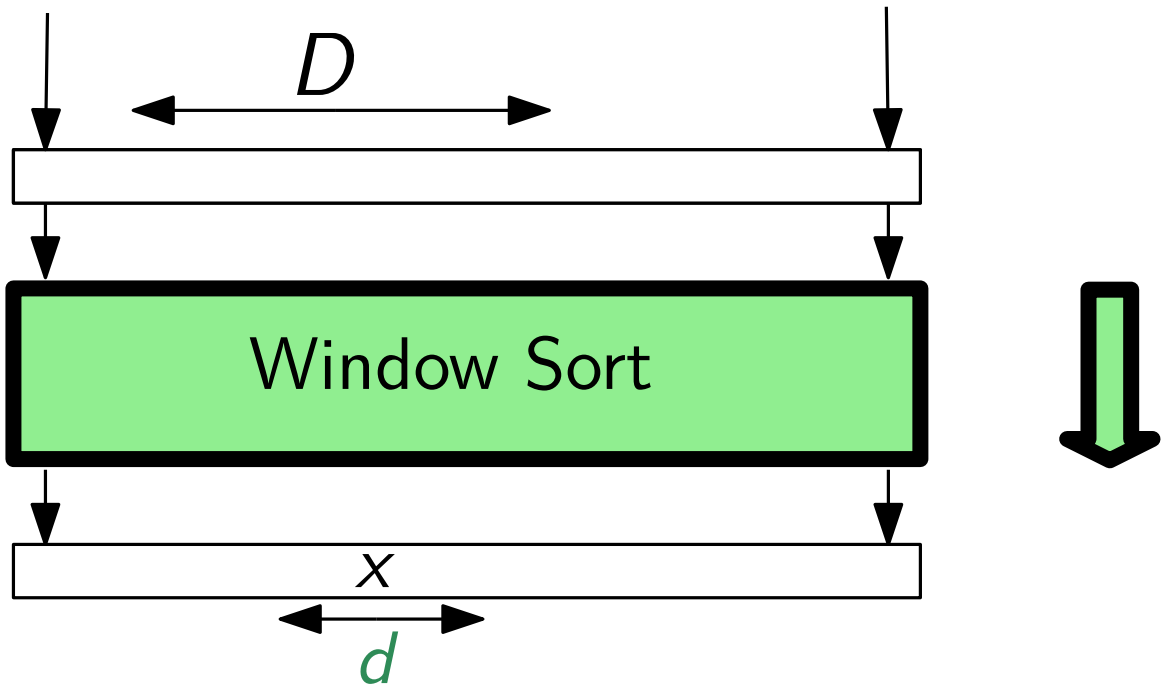


$O(n^{3/2})$

NOT ENOUGH!



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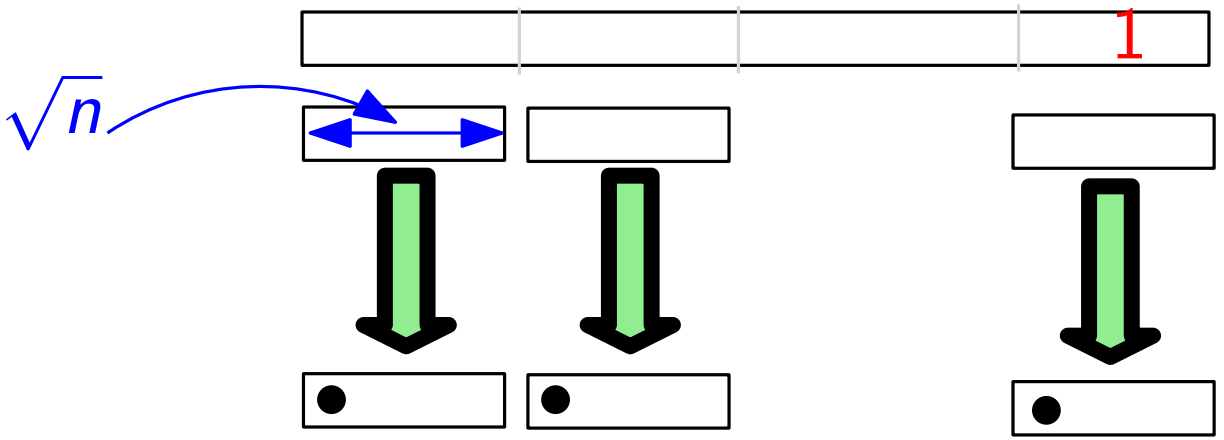

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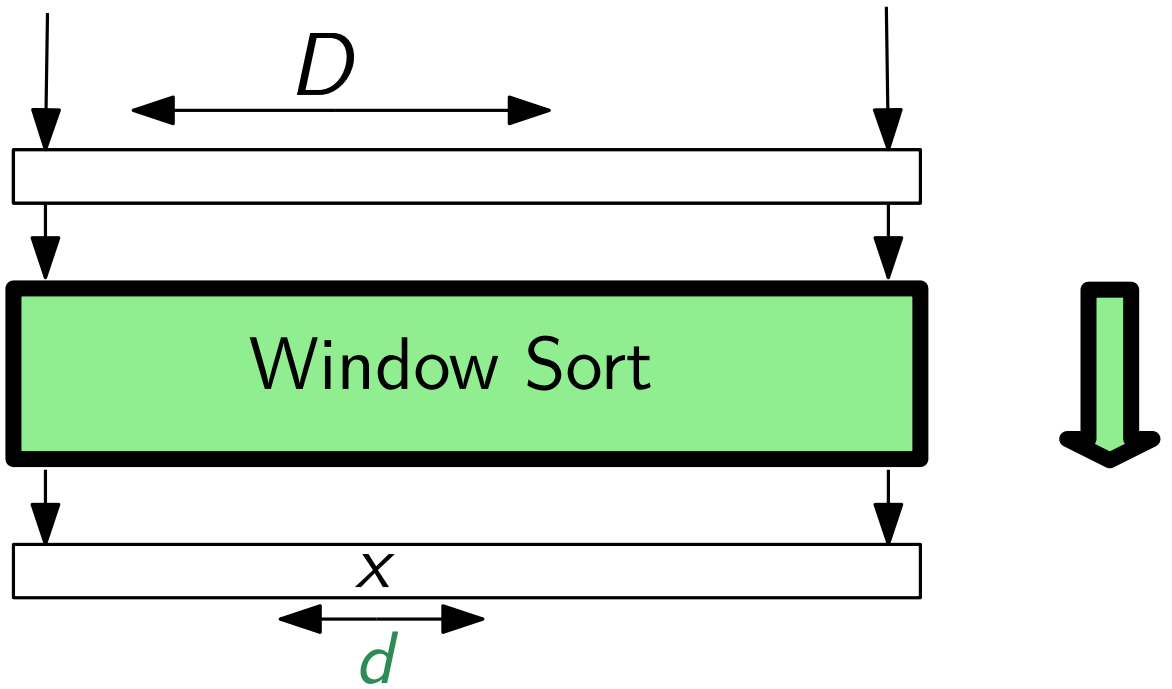

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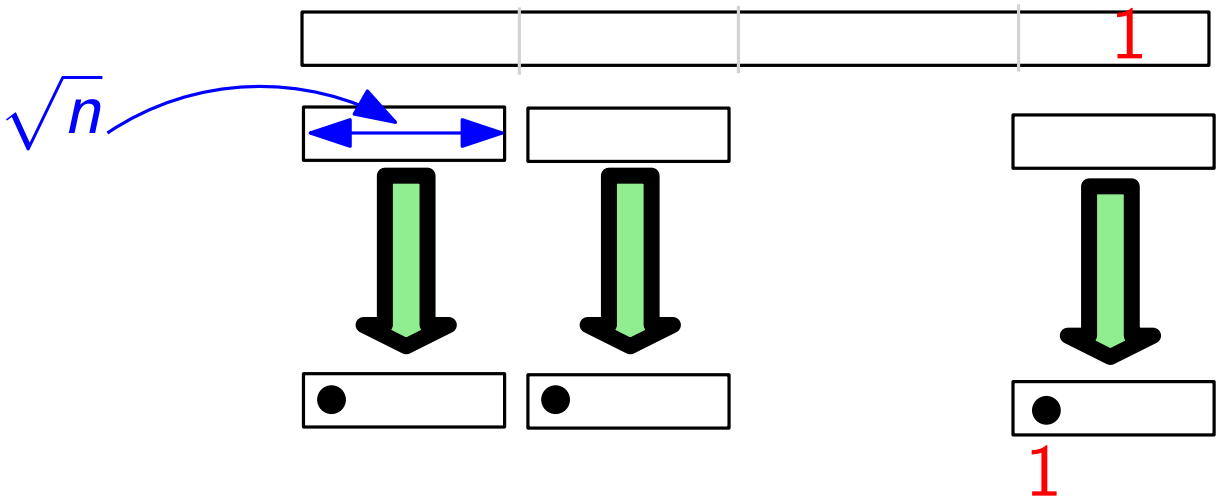

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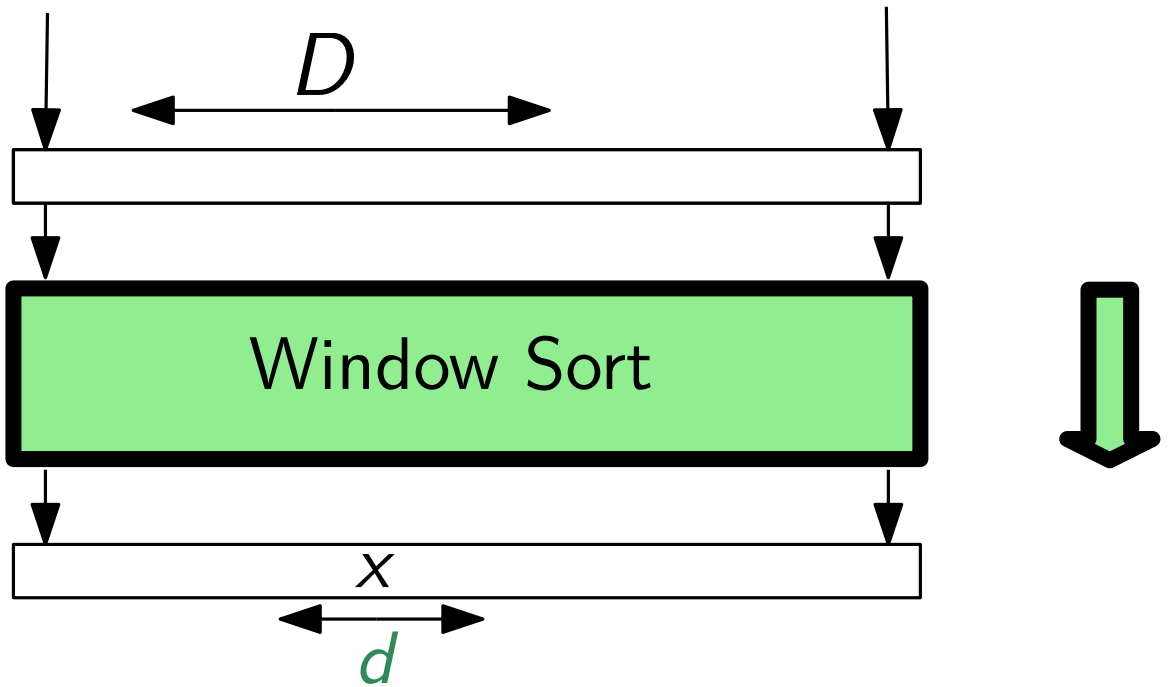

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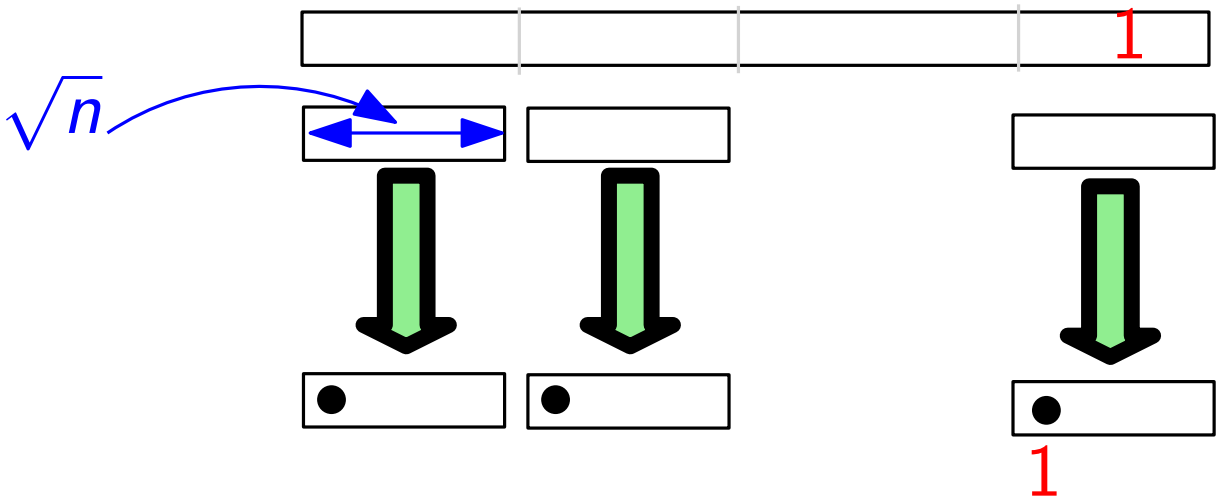

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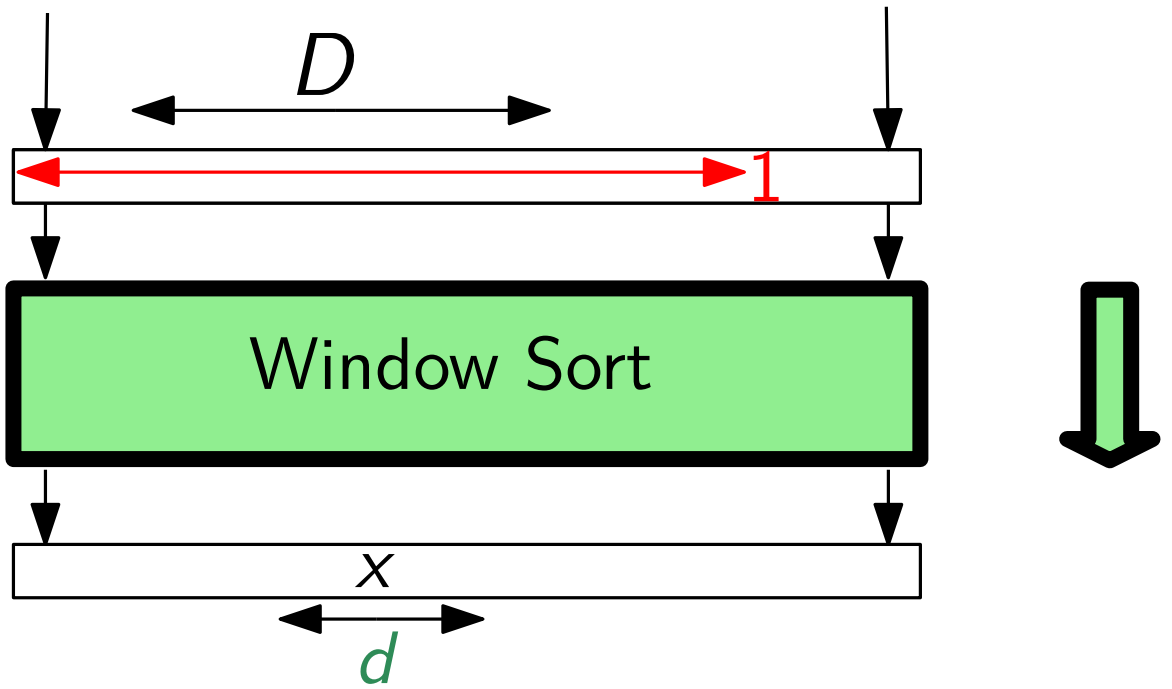

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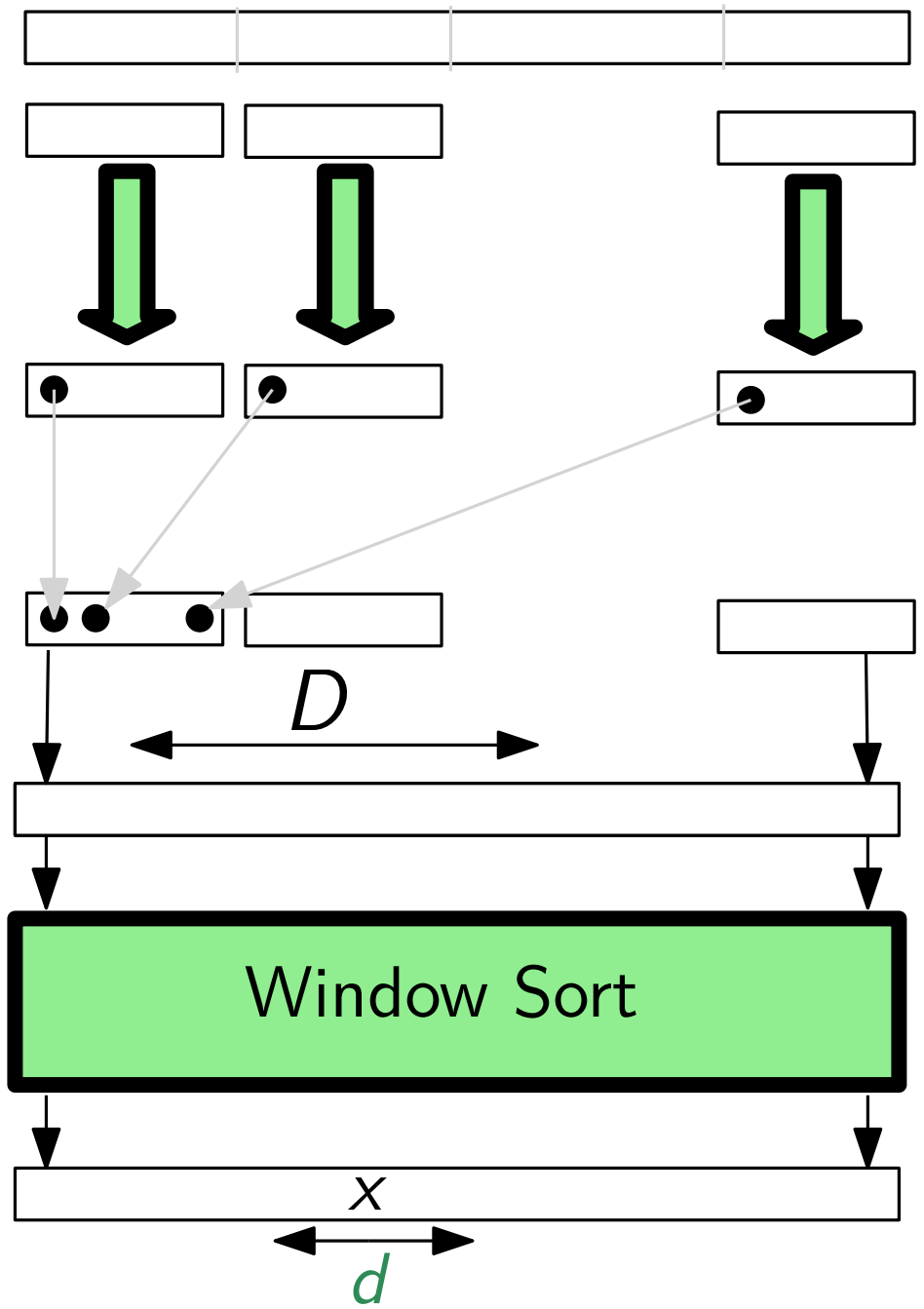

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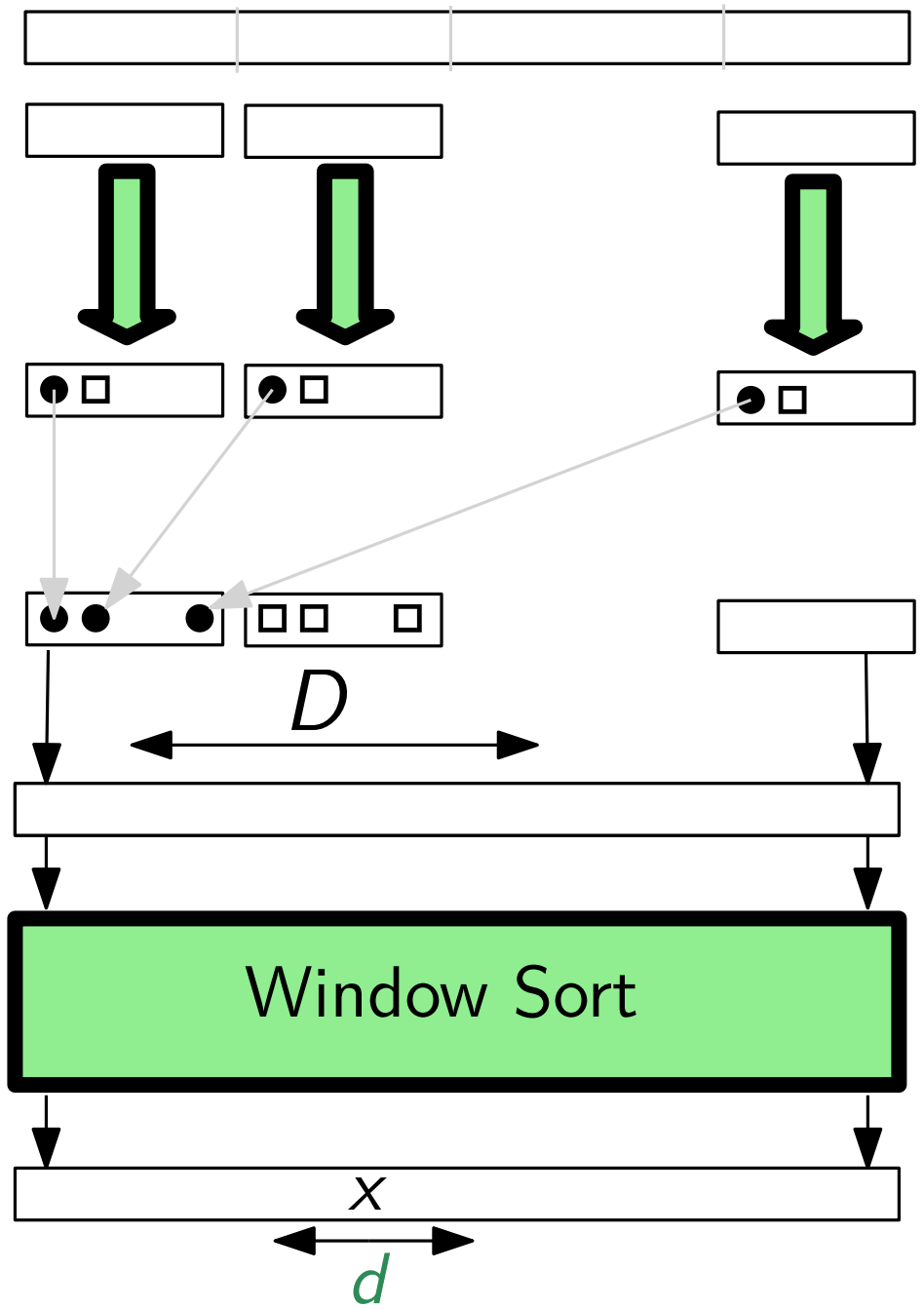
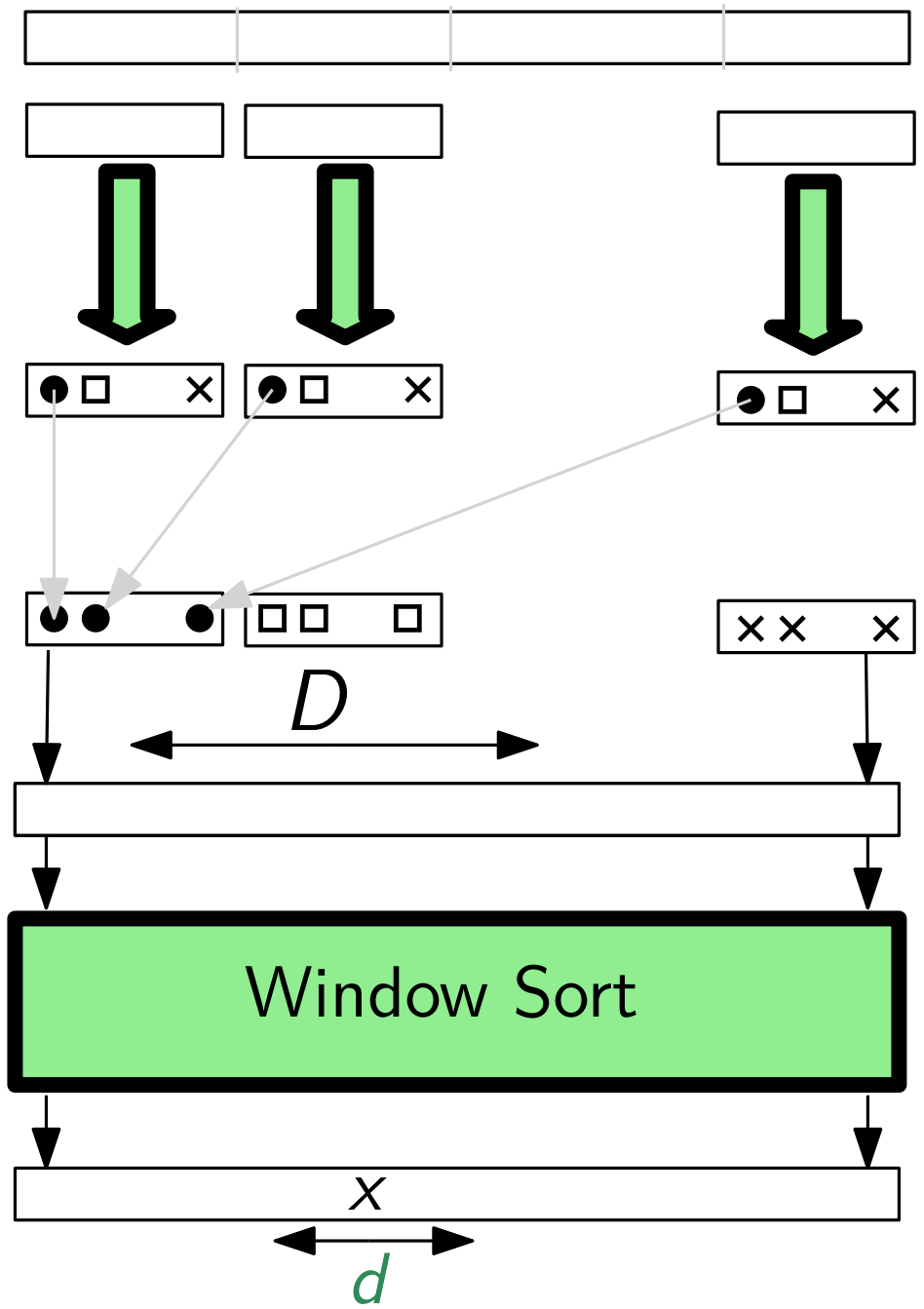


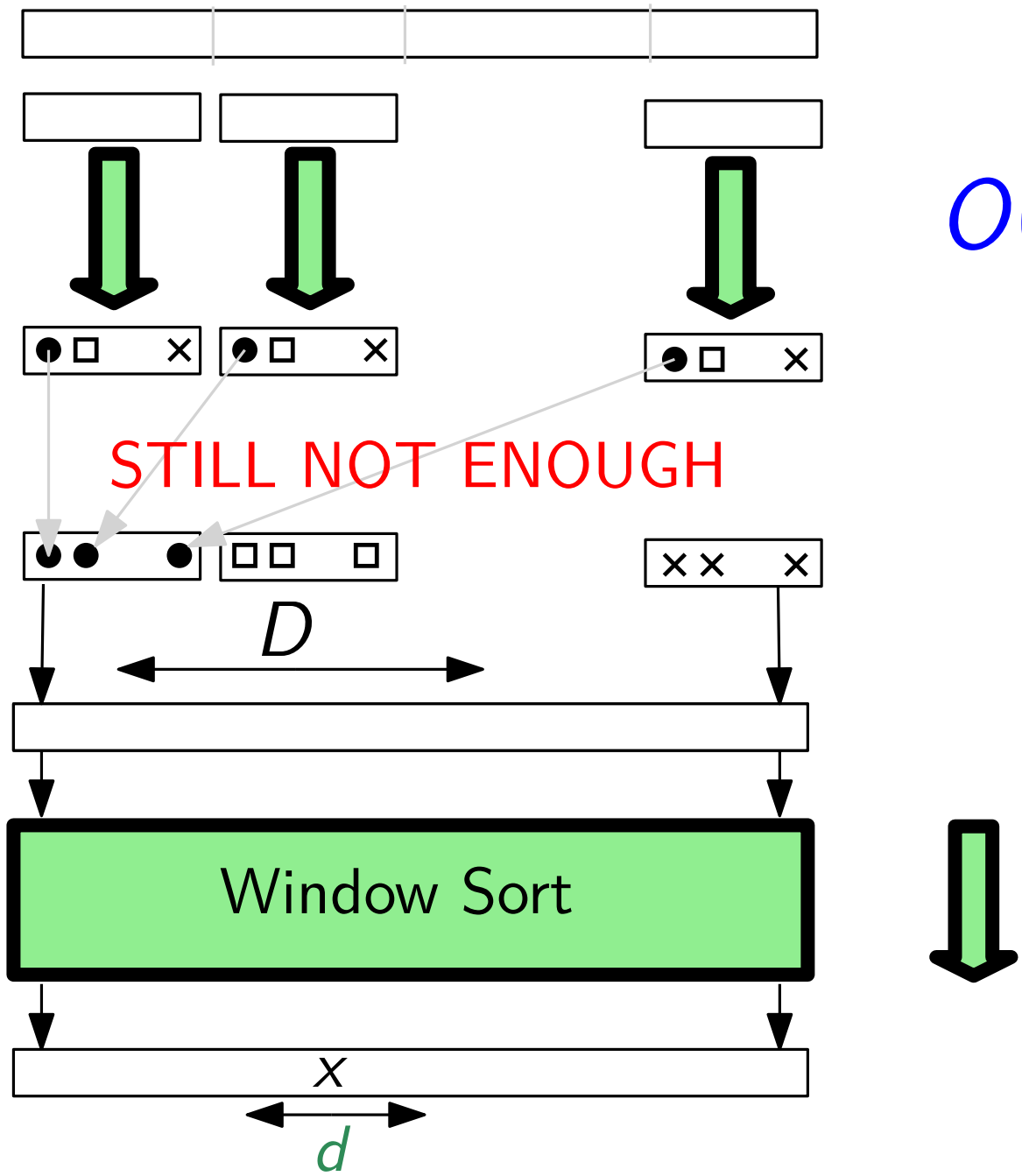
$O(n^{3/2})$

Simple Faster Algo



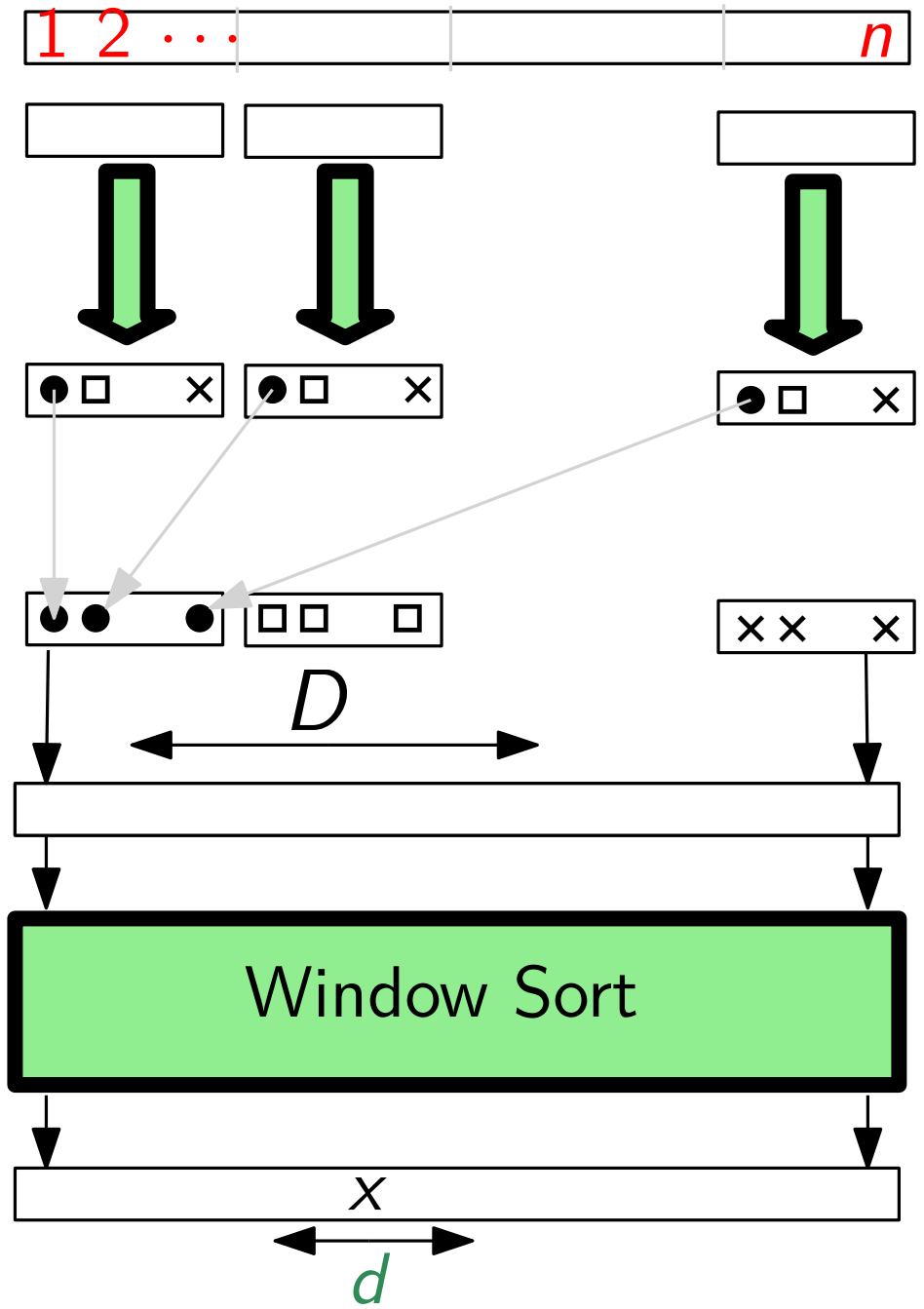
$O(n^{3/2})$

Simple Faster Algo



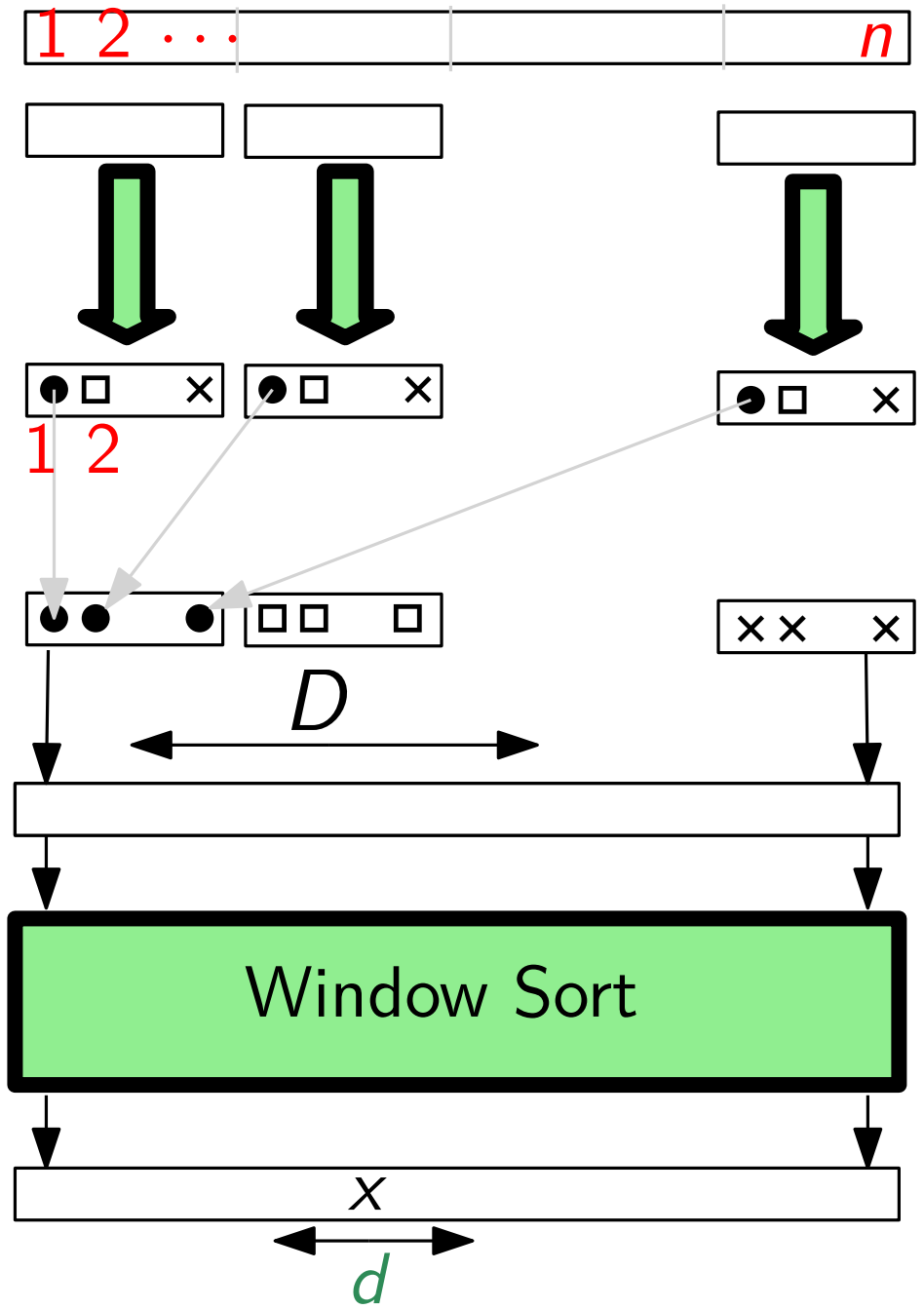
$O(n^{3/2})$

Simple Faster Algo



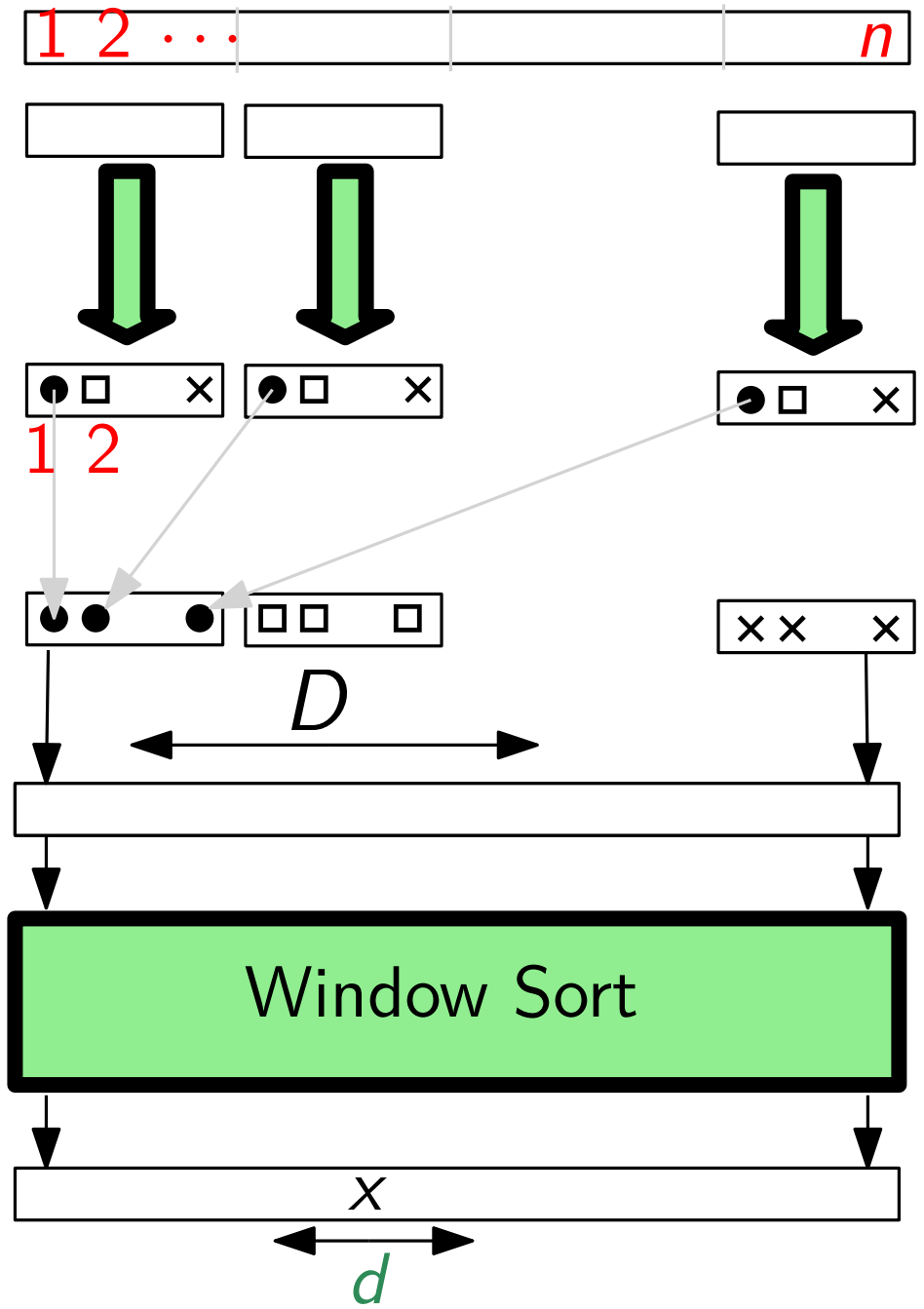
$O(n^{3/2})$

Simple Faster Algo



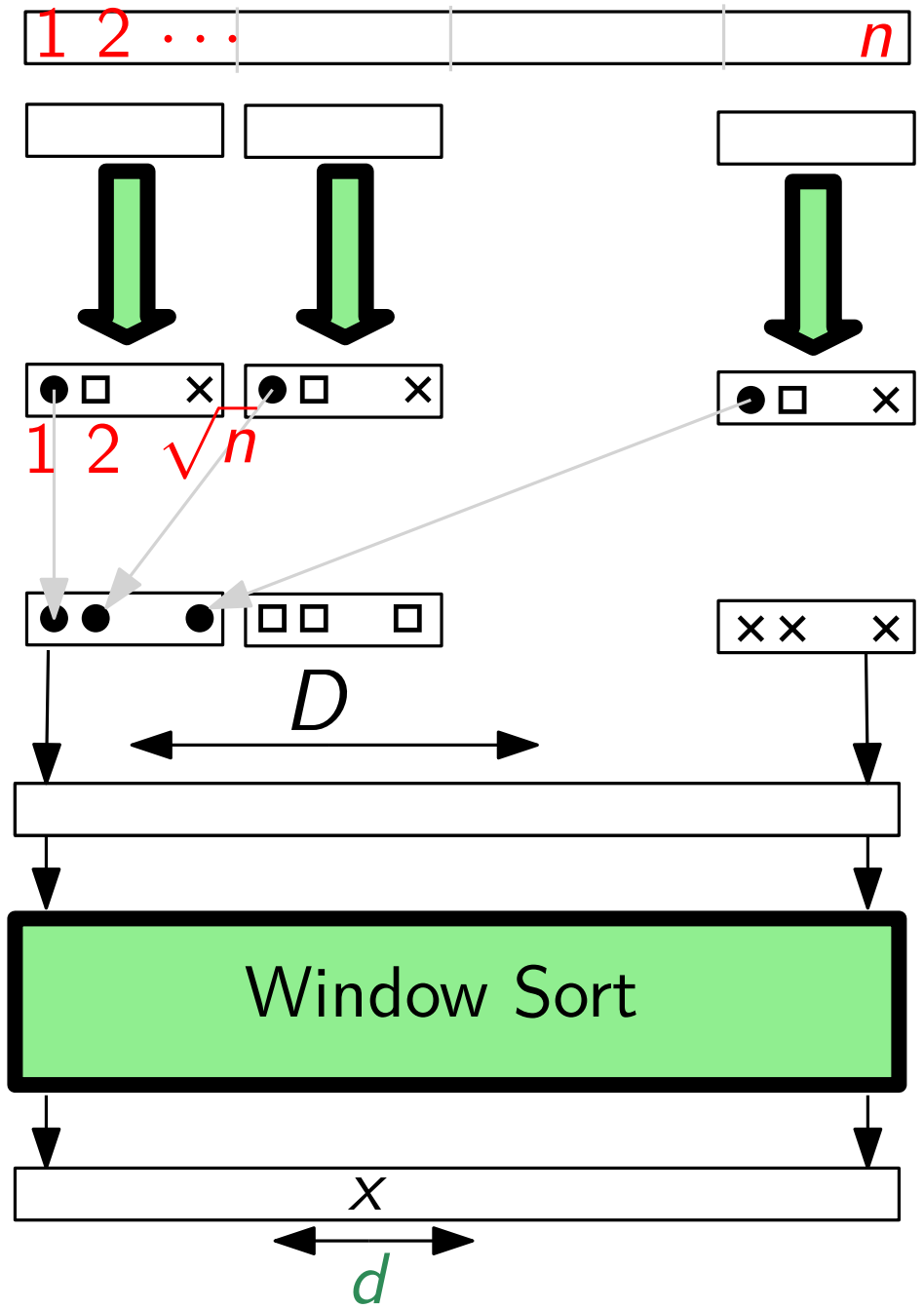
$O(n^{3/2})$

Simple Faster Algo



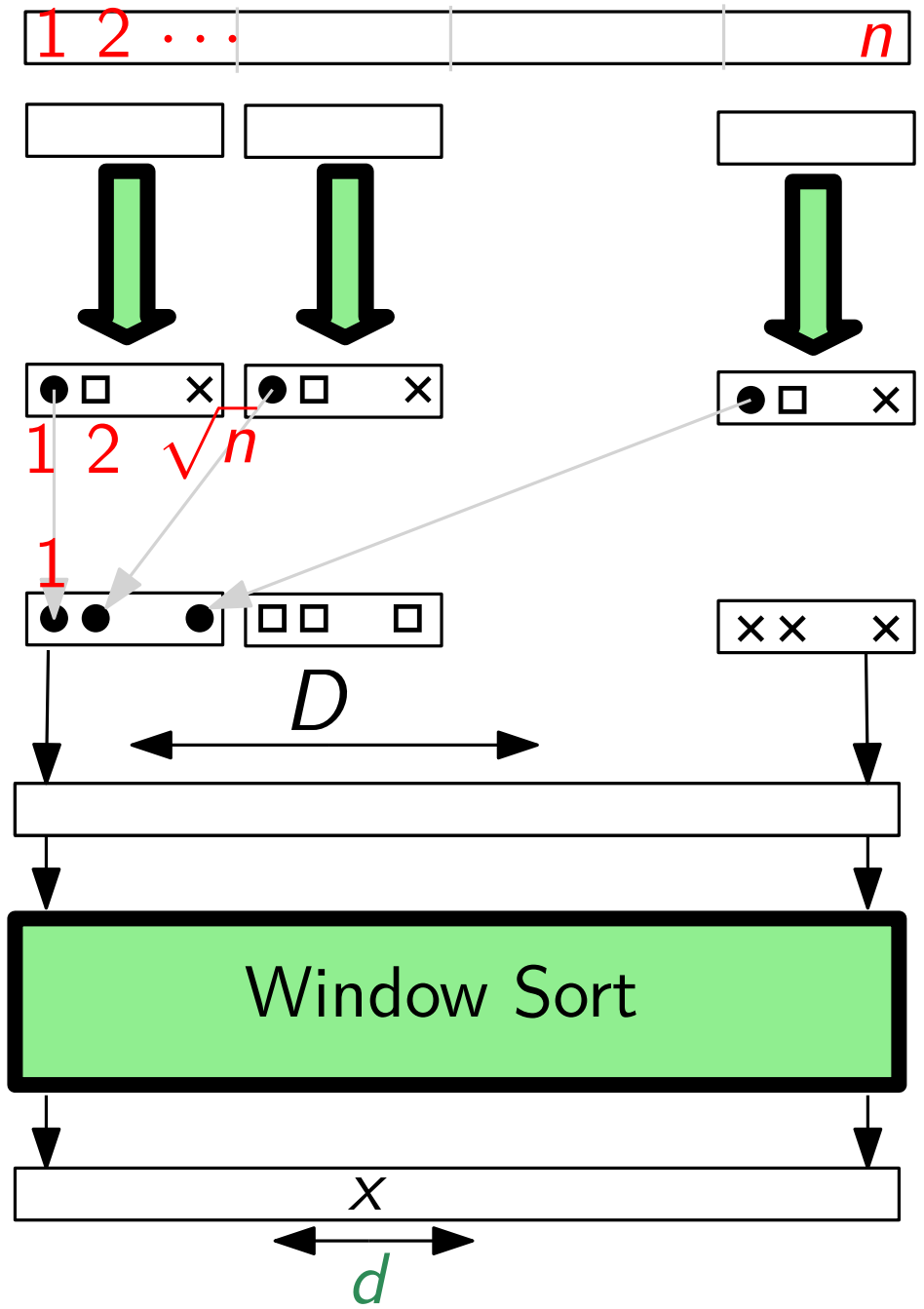
$O(n^{3/2})$

Simple Faster Algo



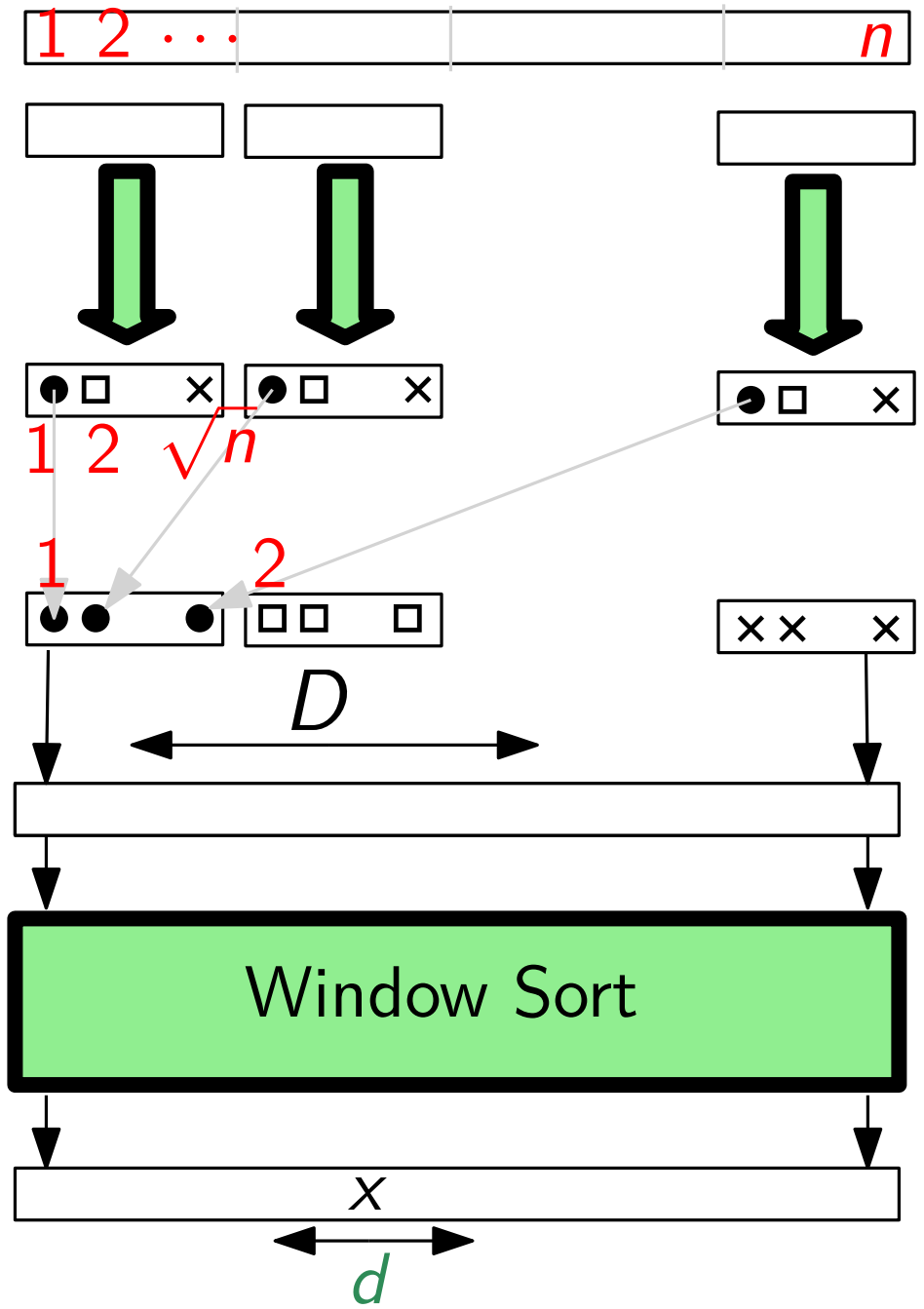
$O(n^{3/2})$

Simple Faster Algo



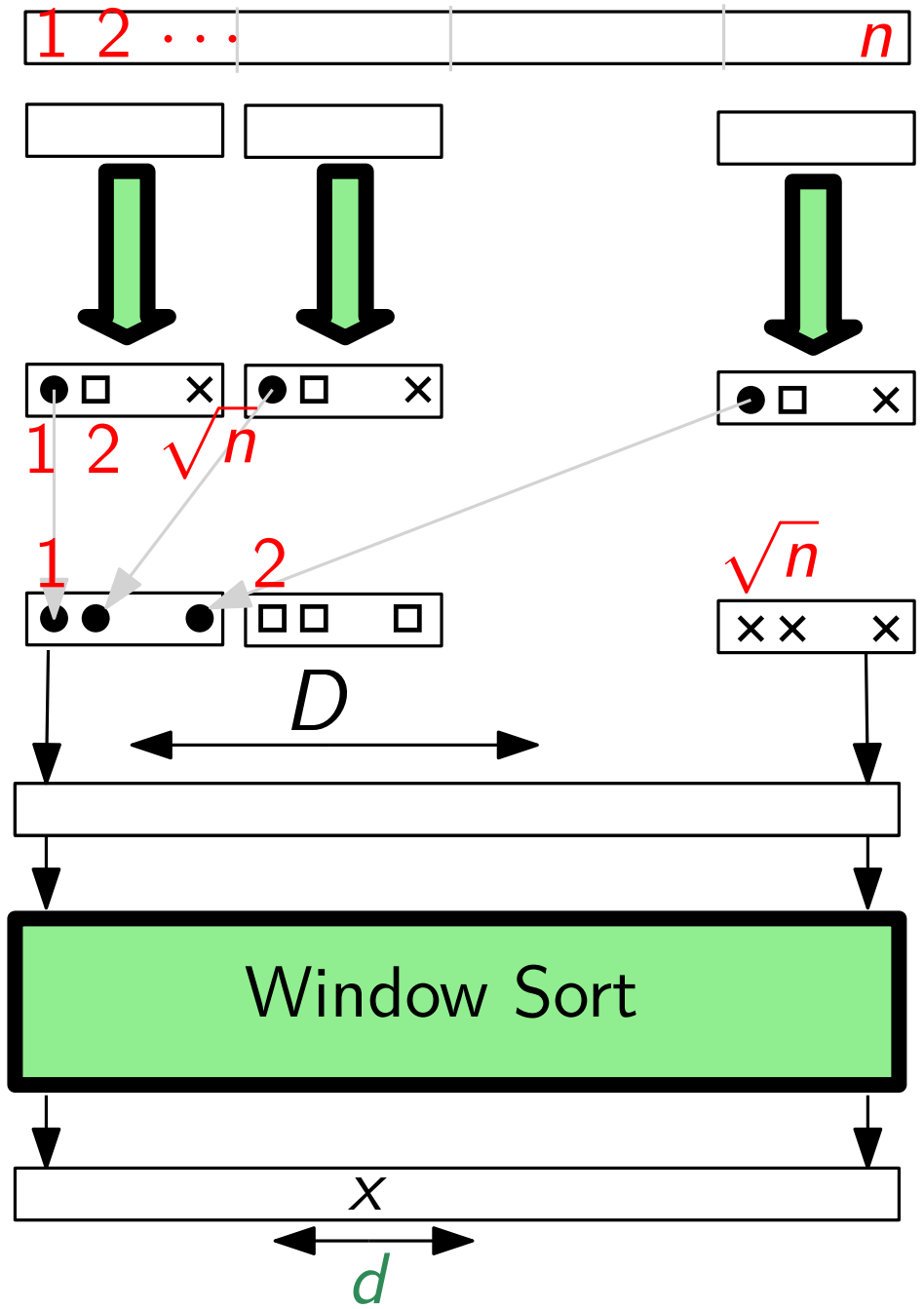
$O(n^{3/2})$

Simple Faster Algo



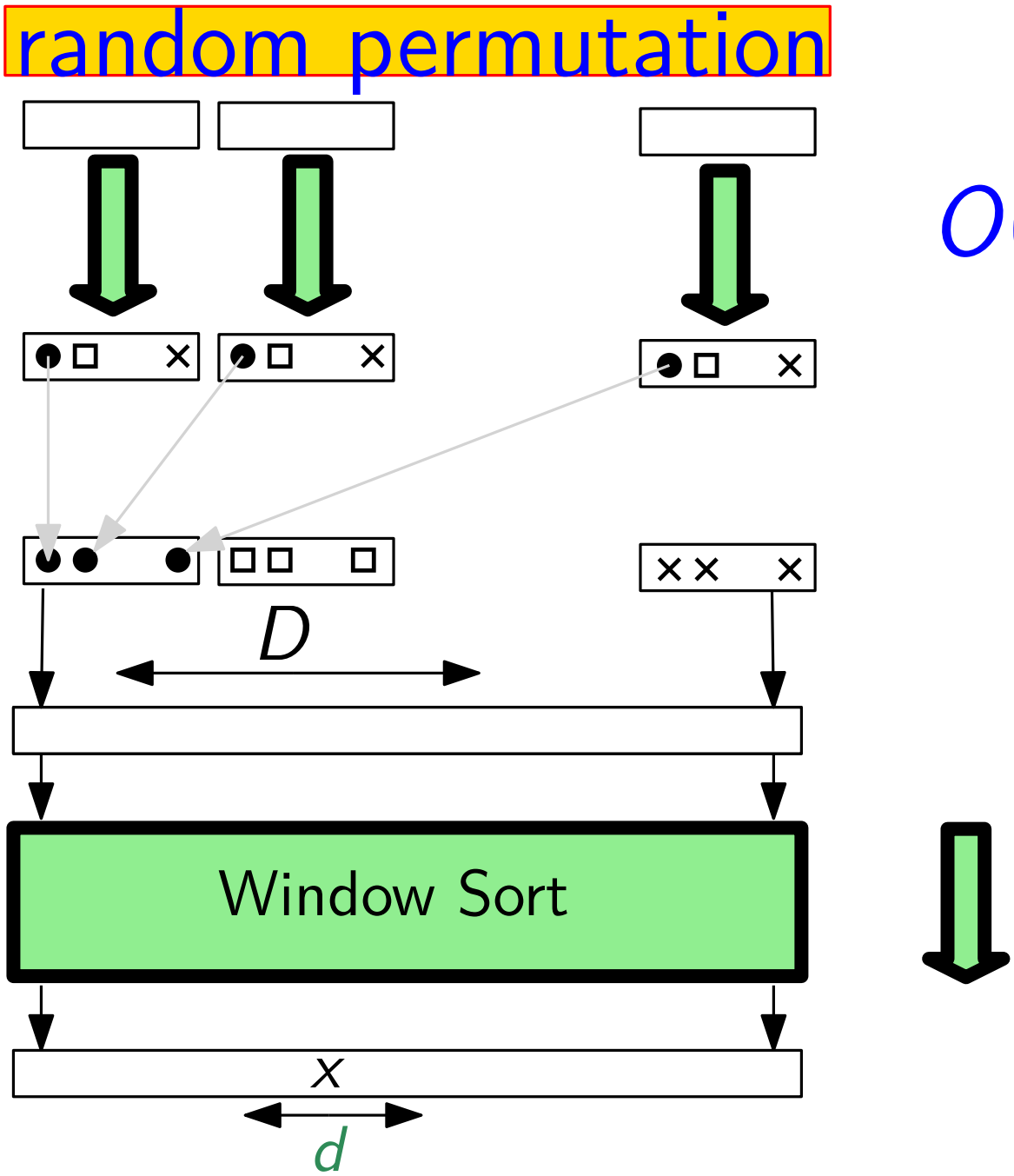
$O(n^{3/2})$

Simple Faster Algo



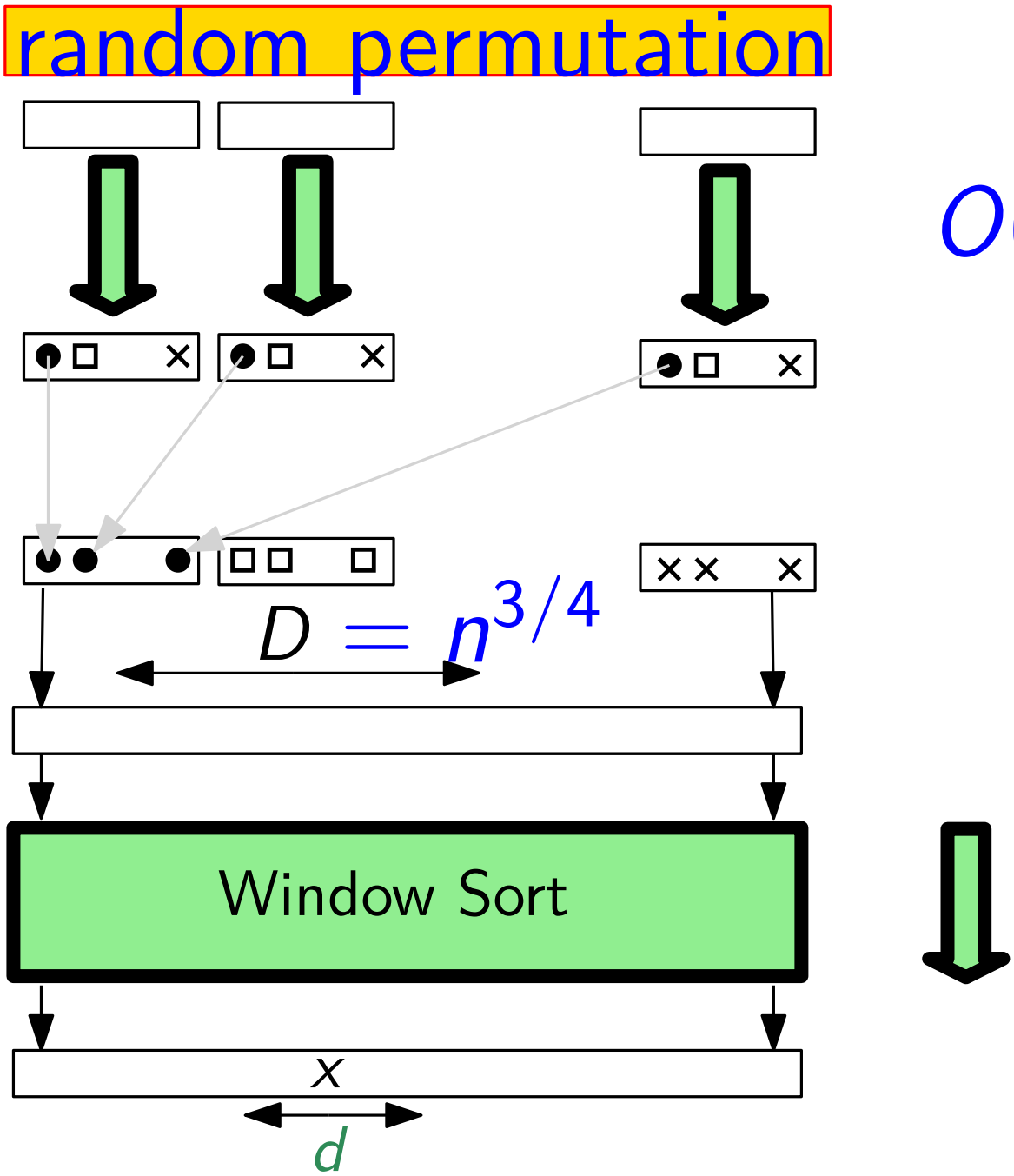
$O(n^{3/2})$

Simple Faster Algo



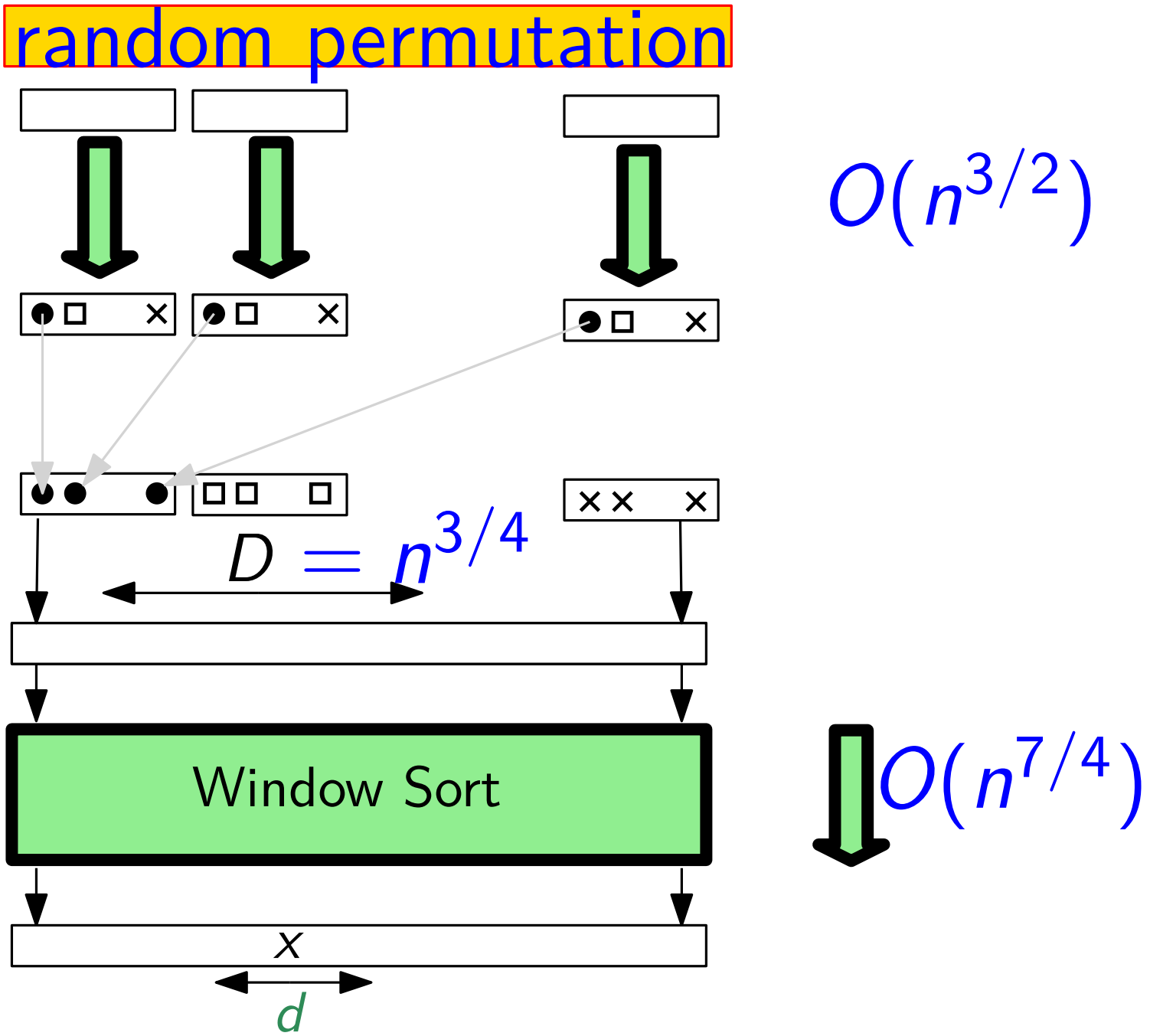
$O(n^{3/2})$

Simple Faster Algo

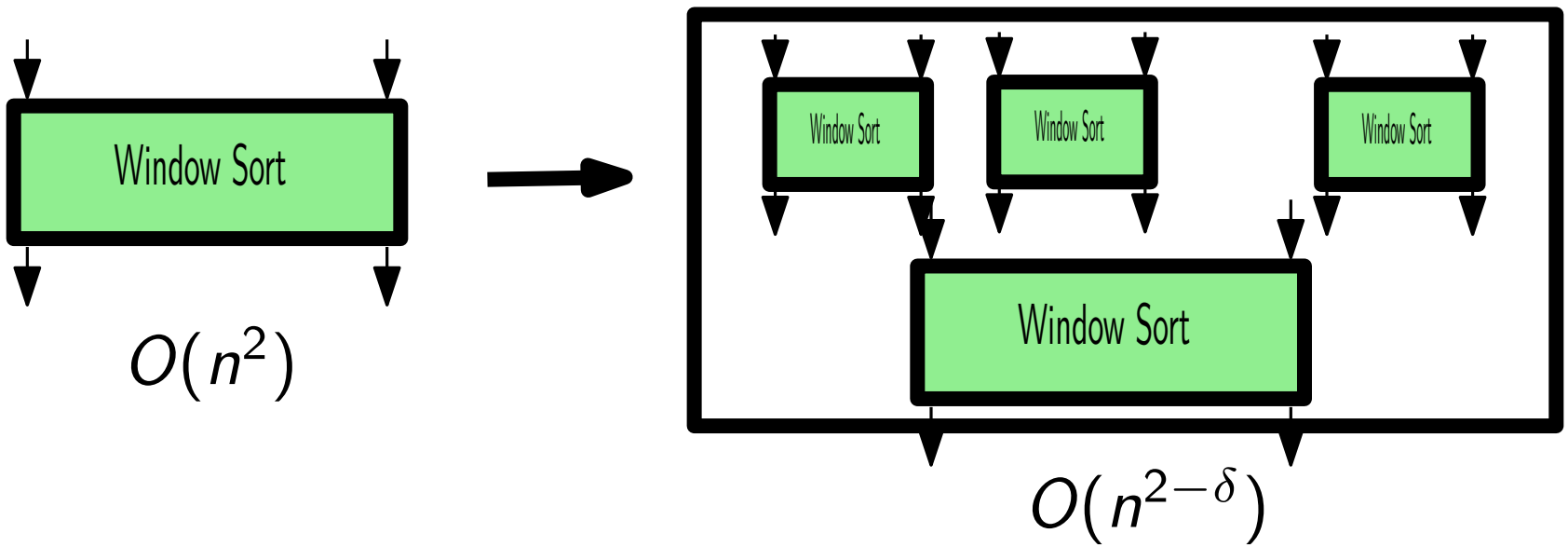


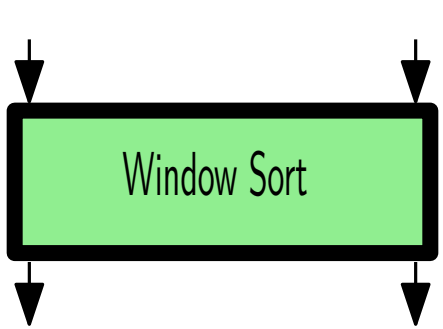
$O(n^{3/2})$

Simple Faster Algo

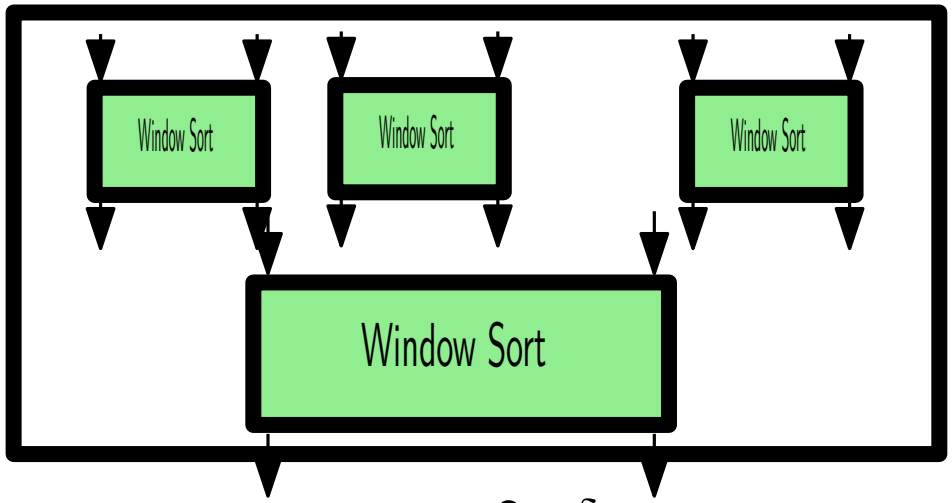


That was the **simple** version...

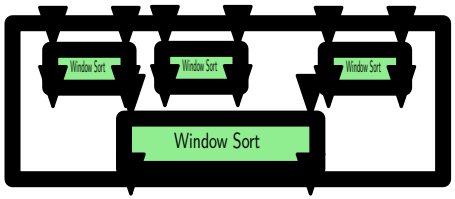

Window Sort
$O(n^2)$
Window Sort
Window Sort
Window Sort
Window Sort
$O(n^{2-\delta})$



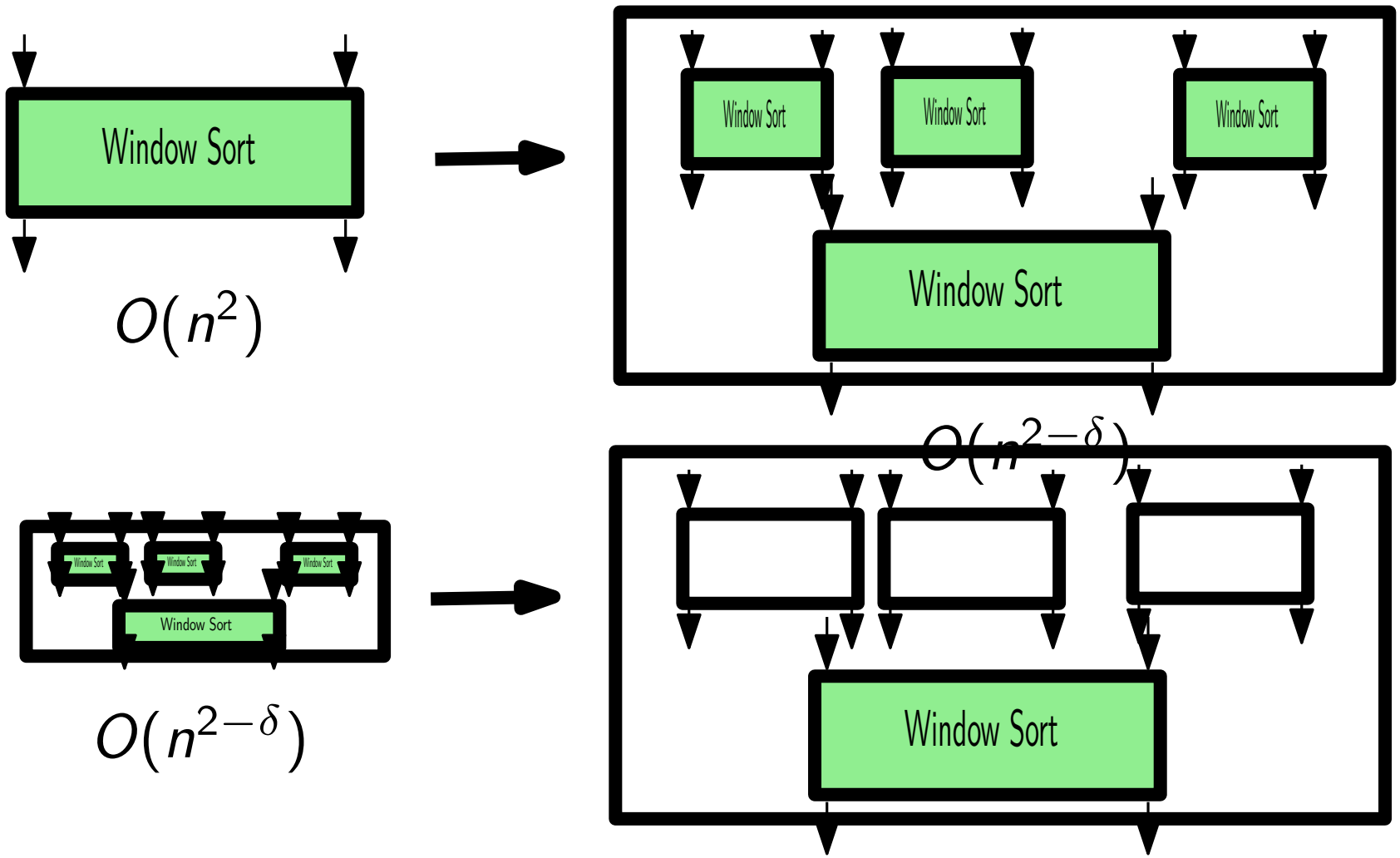
$O(n^2)$

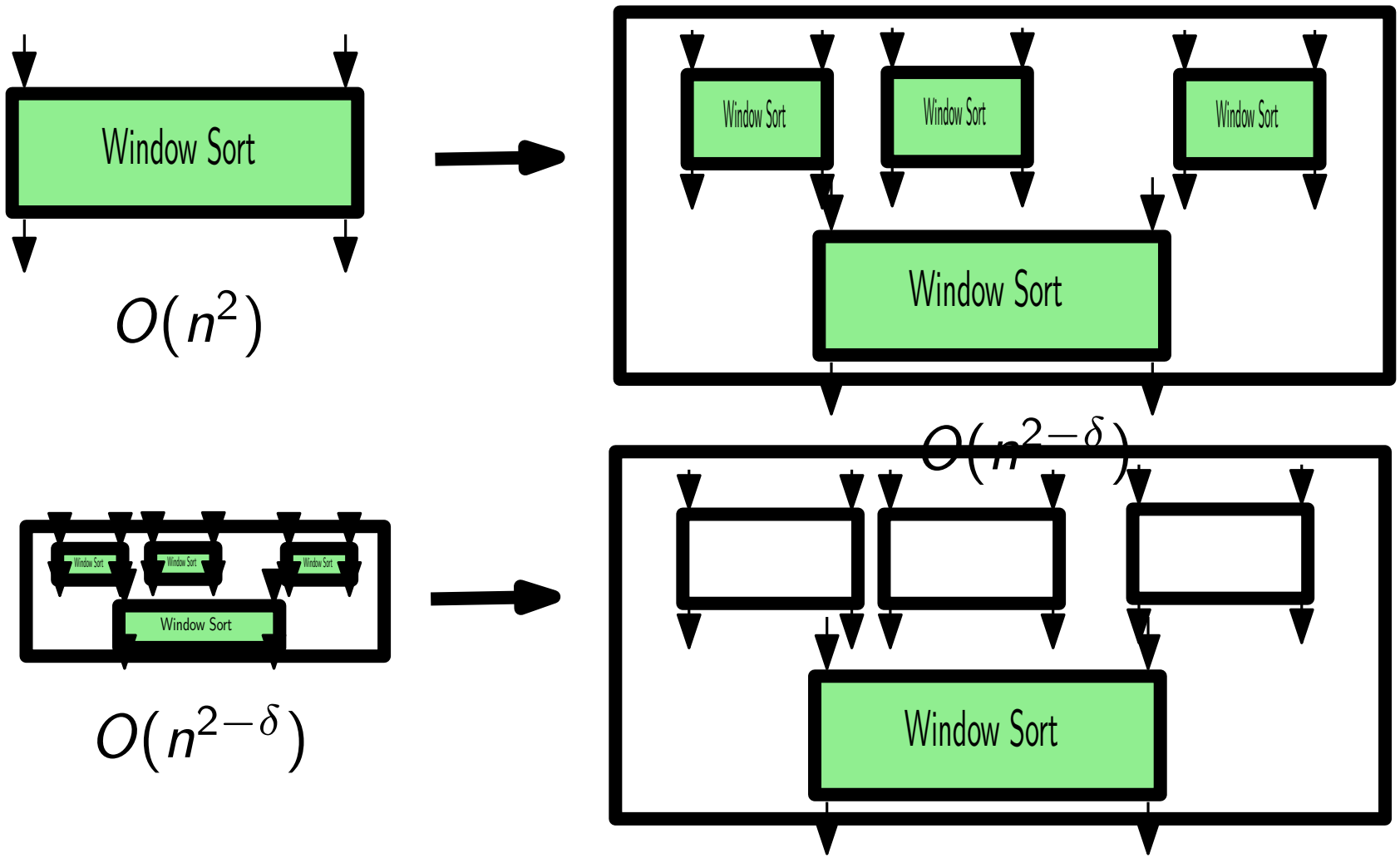


$O(n^{2-\delta})$



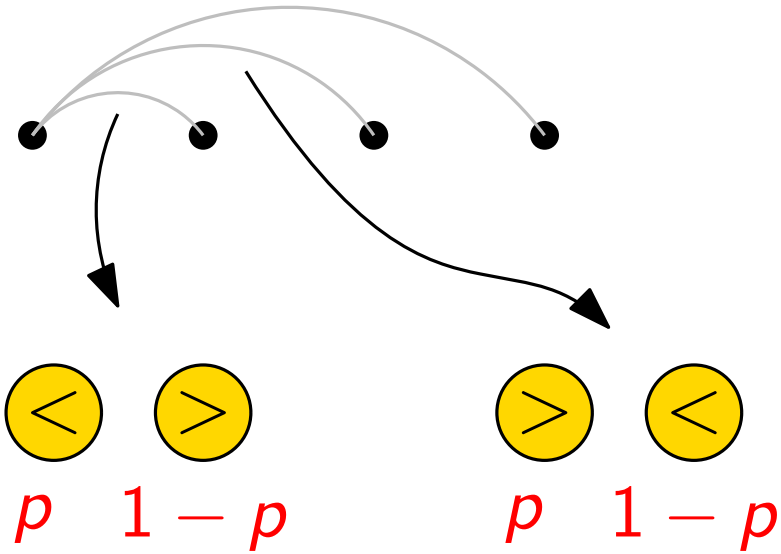
$O(n^{2-\delta})$

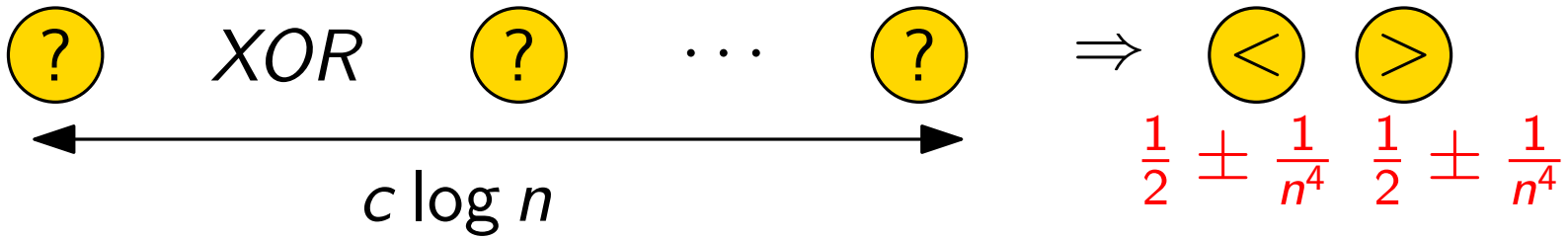

Window Sort
$O(n^2)$
Window Sort
Window Sort
Window Sort
Window Sort
$O(n^{2-\delta})$
Window Sort
Window Sort
Window Sort
Window Sort
$O(n^{2-\delta})$
Window Sort


Window Sort
$O(n^2)$
Window Sort
Window Sort
Window Sort
Window Sort
$O(n^{2-\delta})$
Window Sort
Window Sort
Window Sort
Window Sort
$O(n^{2-\delta})$
Window Sort
Window Sort
Window Sort
Window Sort
$\cdots O(n^{3/2})$

Part II: Derandomization

Comparisons $\Rightarrow$ Randomness

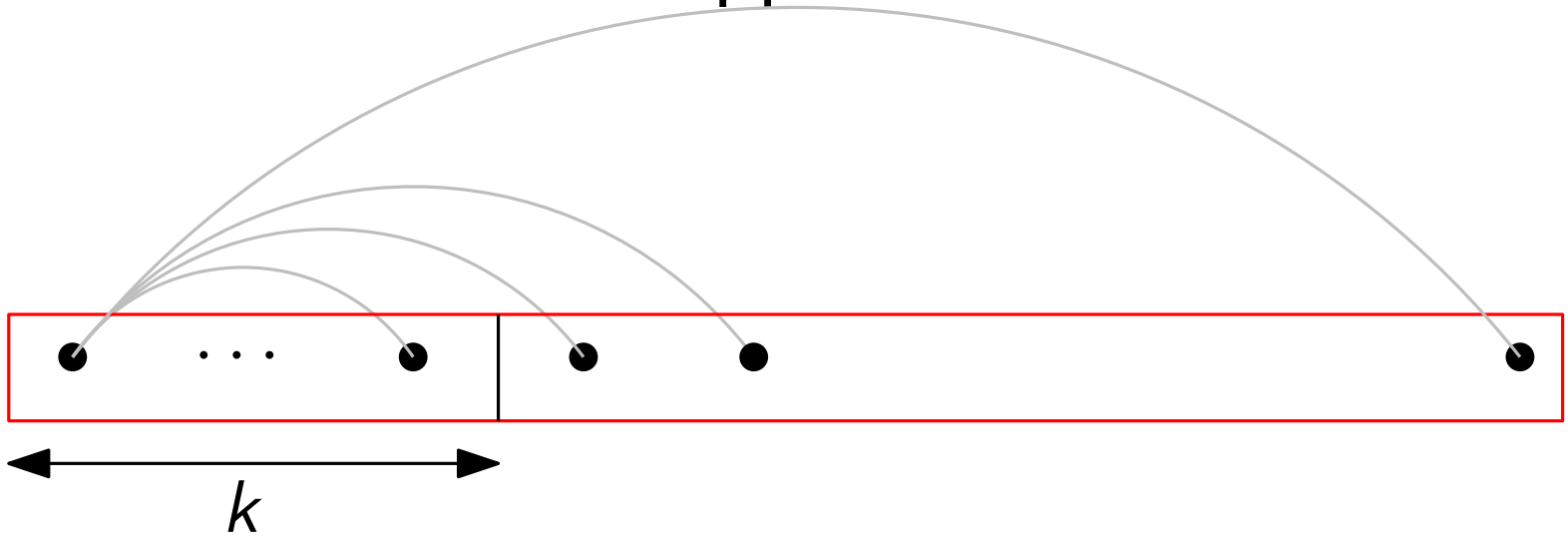


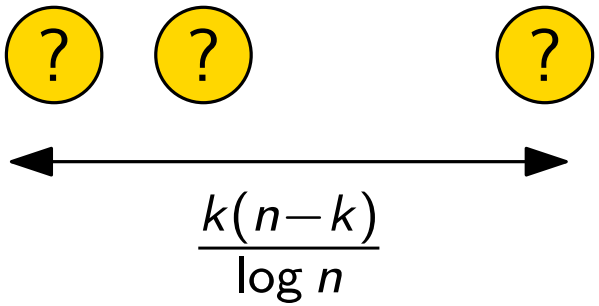


Comparisons

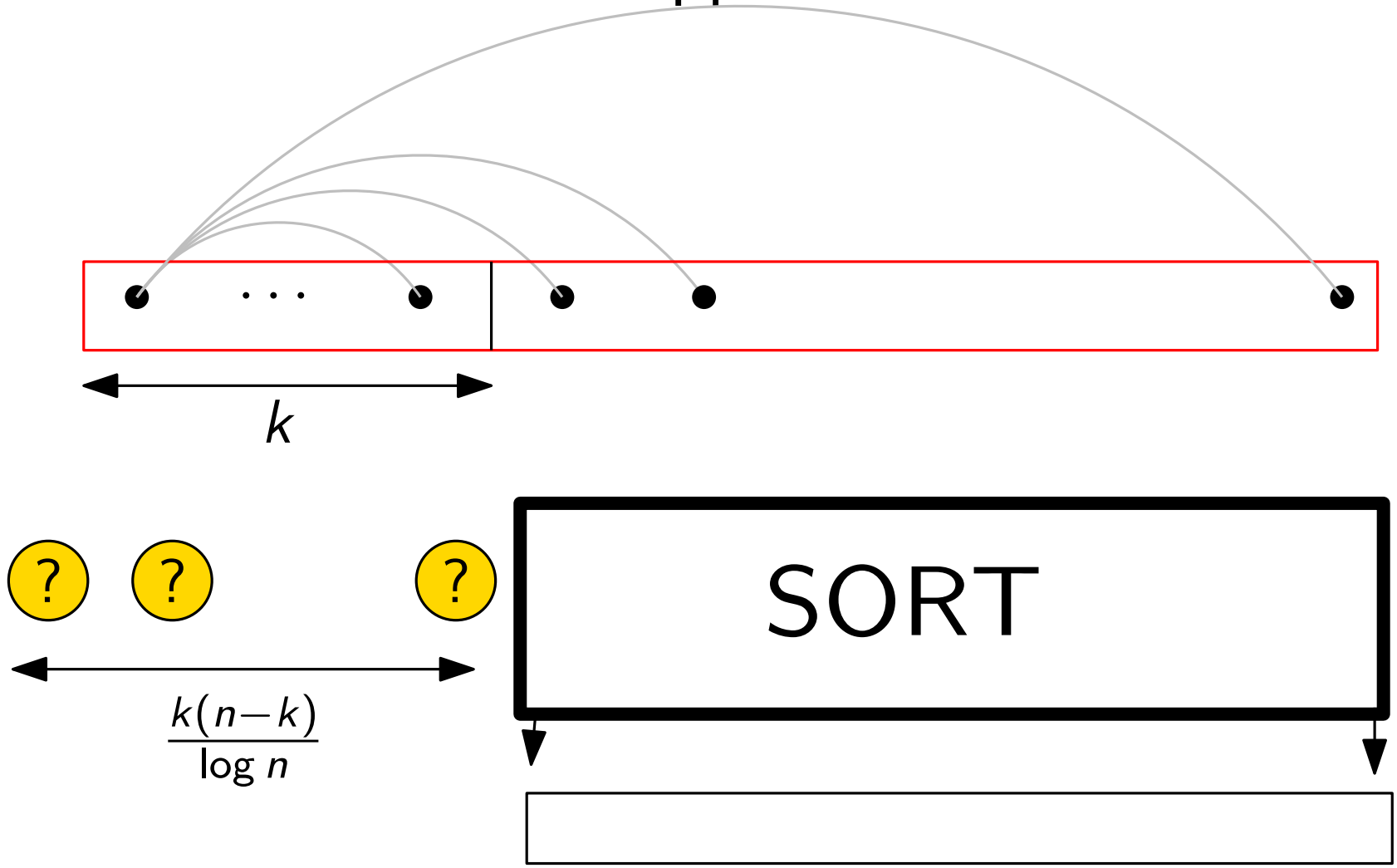
One random bit

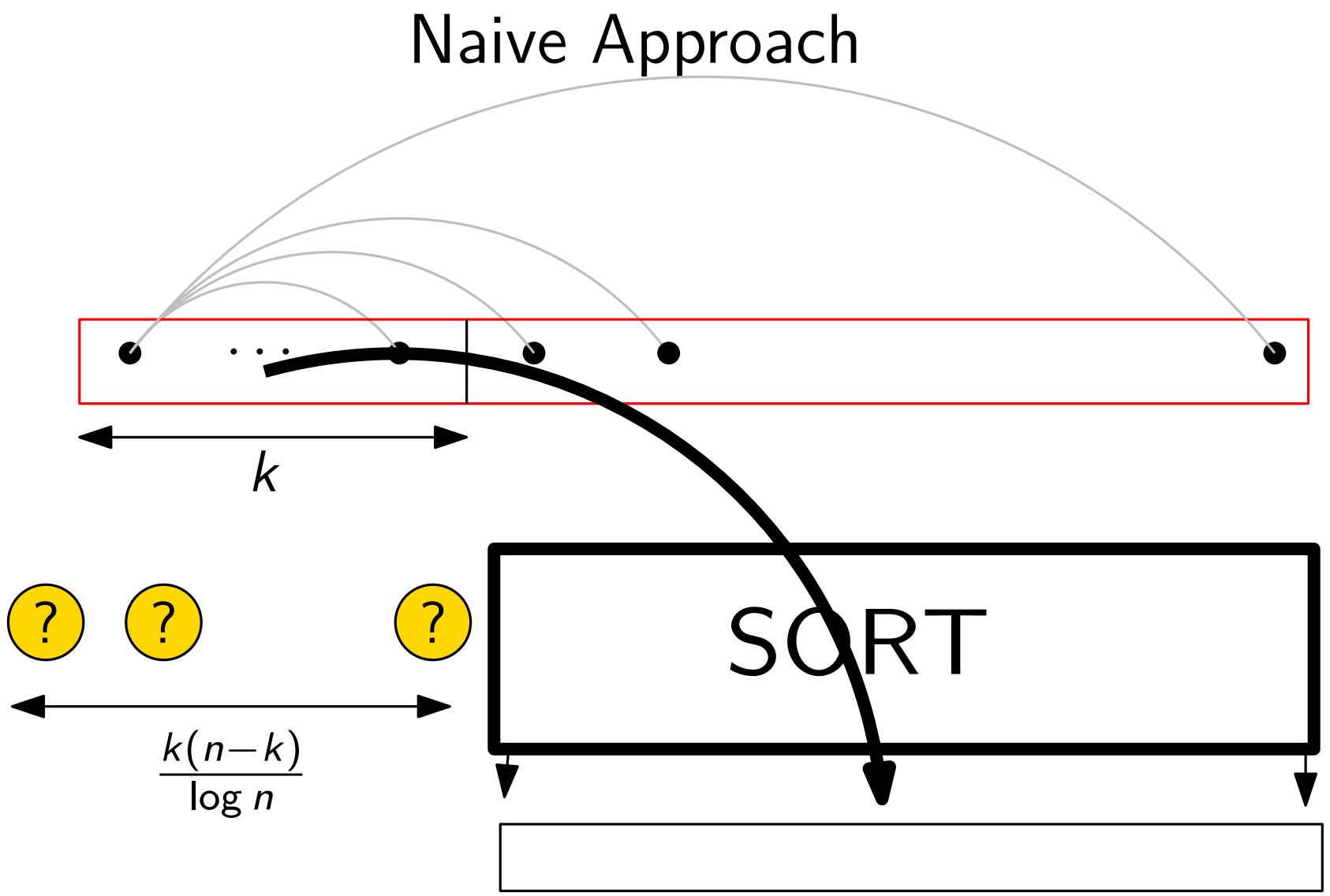
Naive Approach





Naive Approach



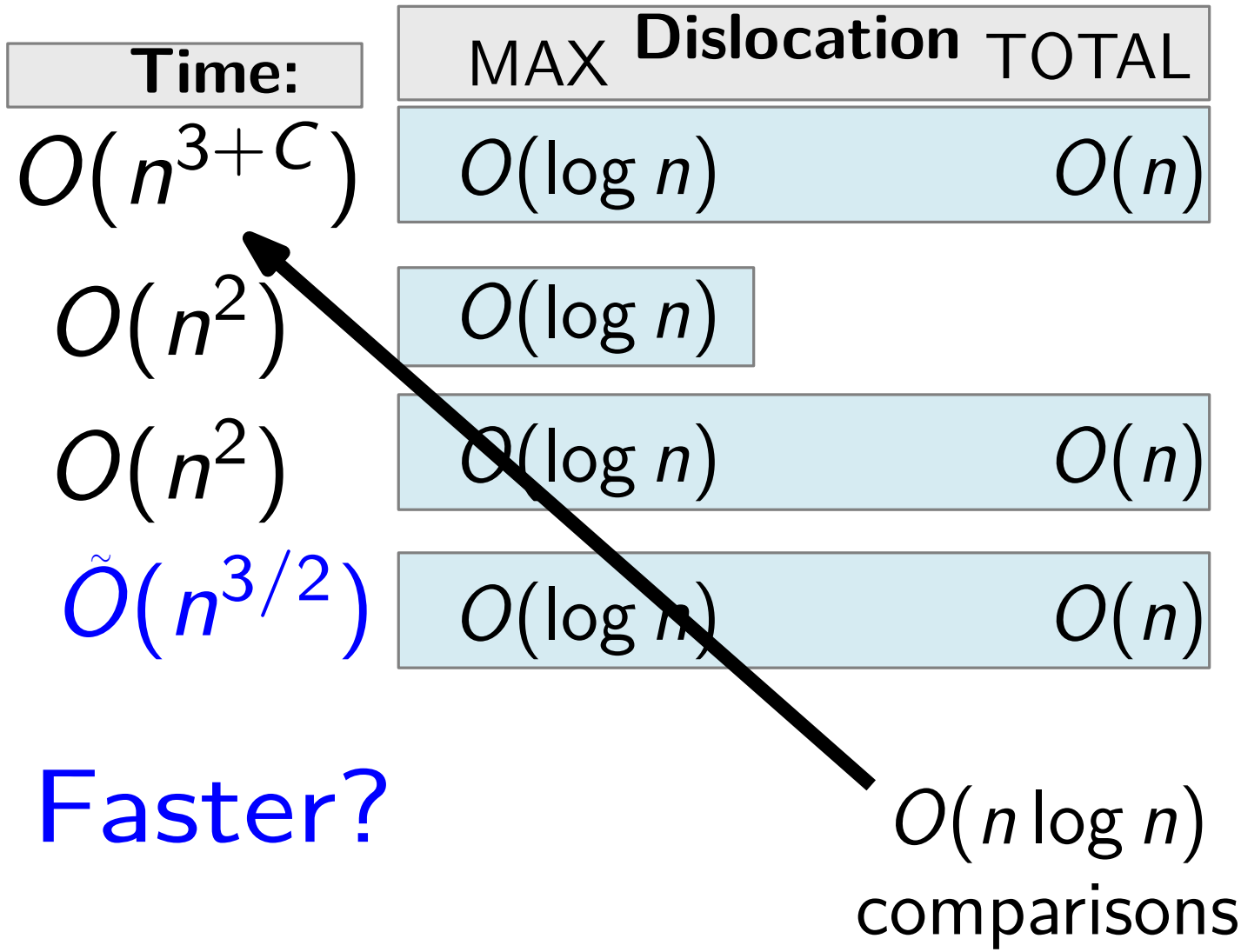

Naive Approach
k
?
?
?
SORT
$\frac{k(n-k)}{\log n}$
REINSERT

Open Questions

Time:	Dislocation MAX	TOTAL
$O(n^{3+C})$	$O(\log n)$	$O(n)$
$O(n^2)$	$O(\log n)$	
$O(n^2)$	$O(\log n)$	$O(n)$
$O(n^{3/2})$	$O(\log n)$	$O(n)$

Braverman & Mossel (SODA'08)
Klein, Penninger, Sohler, Woodruff (ESA'11)
Geissmann, Leucci, Liu, Penna (ISAAC'17)

Open Questions



Braverman & Mossel (SODA'08)
Klein, Penninger, Sohler, Woodruff (ESA'11)
Geissmann, Leucci, Liu, Penna (ISAAC'17)

Open Questions

Time:	Dislocation MAX	TOTAL	
$O(n^{3+C})$	$O(\log n)$	$O(n)$	$p < 1/16$
$O(n^2)$	$O(\log n)$		
$O(n^2)$	$O(\log n)$	$O(n)$	
$O(n^{3/2})$	$O(\log n)$	$O(n)$	

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Open Questions

Time:	Dislocation MAX	TOTAL	
$O(n^{3+C})$	$O(\log n)$	$O(n)$	$p < 1/16$
$O(n^2)$	$O(\log n)$		
$O(n^2)$	$O(\log n)$	$O(n)$	
$O(n^{3/2})$	$O(\log n)$	$O(n)$	

Any $p < 1/2$?

Braverman & Mossel (SODA'08)
Klein, Penninger, Sohler, Woodruff (ESA'11)
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Open Questions

Time:	Dislocation MAX	TOTAL	
$O(n^{3+c})$	$O(\log n)$	$O(n)$	$p < 1/16$
$O(n^2)$	$O(\log n)$		
$O(n^2)$	$O(\log n)$	$O(n)$	
$O(n^{3/2})$	$O(\log n)$	$O(n)$	

Any $p < 1/2$?

$p < 1/2$

Braverman & Mossel (SODA'08)
Klein, Penninger, Sohler, Woodruff (ESA'11)
Geissmann, Leucci, Liu, Penna (ISAAC'17)

Open Questions

Time:	Dislocation MAX	TOTAL
$O(n^{3+C})$	$O(\log n)$	$O(n)$
$O(n^2)$	$O(\log n)$	
$O(n^2)$	$O(\log n)$	$O(n)$
$O(n^{3/2})$	$O(\log n)$	$O(n)$

Other error models?

Braverman & Mossel (SODA'08)
Klein, Penninger, Sohler, Woodruff (ESA'11)
Geissmann, Leucci, Liu, Penna (ISAAC'17)

Tahnk You