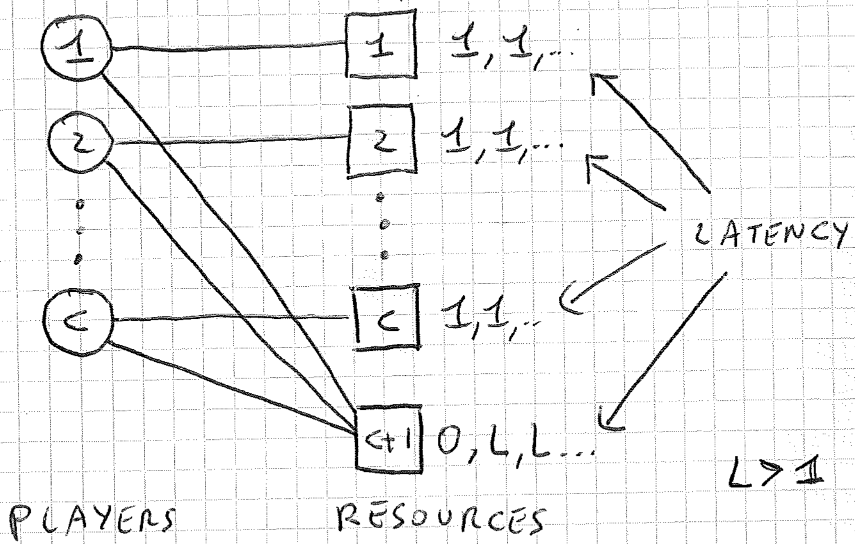
PROBLEM 3 (PART 1)

CONSIDER THIS CONGESTION GAME:



EACH PLAYER i CAN CHOOSE RESOURCE $\boxed{i}$ OR $\boxed{c+1}$

$$\boxed{\text{PNE} \iff \text{EXACTLY ONE PLAYER CHOOSES } \boxed{c+1}}$$

s^* PNE $\Rightarrow \Phi(s^*) = m - 1$

(EXACTLY ONE CHOOSES $\boxed{c+1}$)

(RESOURCE $\boxed{c+1}$ HAS LATENCY 0, ALL OTHERS LATENCY 1)

s NONE CHOOSES $\boxed{c+1}$ $\Rightarrow \Phi(s) = m$

(LATENCY IS 1 ON RESOURCES DIFFERENT FROM $\boxed{c+1}$)

AND WE HAVE ONE PLAYER IN EACH RESOURCE

s TWO OR MORE CHOOSE $\boxed{c+1}$ $\Rightarrow \Phi(s) = m - k + \underbrace{(k-1)L}_{\text{FROM RESOURCE } \boxed{c+1}}$

WHERE $k \geq 2$ IS THE NUMBER OF PLAYERS CHOOSING $\boxed{c+1}$

AND $L > 1$ IMPLIES $\Phi(s) > \Phi(s^*)$ IN THIS LAST CASE

DETAILS ON POTENTIAL CALCULATIONS:

$$\Phi(s) = \sum_r \underbrace{\sum_{e=1}^{M_r(s)} d_r(e)}_{\Phi_r(s)}$$

$\Phi_r(s) = 0$ IF NONE CHOOSES r $(M_r(s)=0)$

$\Phi_r(s) = 1$ FOR ALL $r \neq c+1$ WITH ONE PLAYER

$$\Phi_{c+1}(s) = \underbrace{0 + L + \cdots + L}_{M_{c+1}(s)}$$

WHAT WE CALLED K ABOVE

CONCLUSION:

MINIMUM FOR POTENTIAL IS

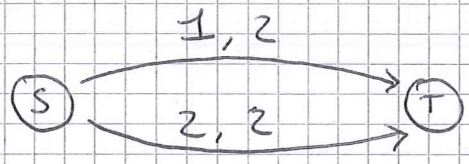
$\Phi(s^*) = m-1$ WHICH

CORRESPONDS TO ALL STATES WITH EXACTLY ONE PLAYER CHOOSING $\boxed{c+1}$.

THERE ARE EXACTLY c SUCH STATES (WHICH PLAYER CHOOSES $\boxed{c+1}$).

PROBLEM 3 (PART 2)

CONSIDER THIS CONGESTION GAME:



$M = 2$ PLAYERS (CHOOSE UPPPER OR LOWER LINK)

K = # PLAYERS IN UPPER LINK (IN STATE s YOU CONSIDER)

$$K = 2 \Rightarrow \Phi(s) = \boxed{1+2}, \quad SC = 2+2$$

$$K = 1 \Rightarrow \Phi(s) = \boxed{1+2}, \quad SC = 1+2$$

$$K = 0 \Rightarrow \Phi(s) = 2+2, \quad SC = 2+2$$

SO $K=2$ IS A STATE MINIMIZING GLOBALLY THE POTENTIAL, BUT NOT MINIMIZING THE SOCIAL COST

THE OTHER MINIMUM FOR THE POTENTIAL (PNE) IS $K = \underline{1}$
(ACTUALLY, THESE ARE TWO STATES)
WHICH IS ALSO THE OPTIMUM FOR THE SOCIAL COST
(THUS MIN COST PNE)