

**MSCI 432 COURSE NOTES  
PRODUCTION AND SERVICE OPERATIONS  
MANAGEMENT**

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# **1 OPERATIONS MANAGEMENT AND DEMAND FORECASTING (I)**

## **1.1 Operations Management**

### **1.1.1 Basic Premise of Supply and Demand**

- As consumers, we decide how much we want to buy and how much we are willing to pay
- Consumers hold the cash and as such are the ultimate decision makers
- This underlies OM as it goes to the heart of the environment in which firms operate

### **1.1.2 Operations as an Aggregate Function**

#### **What is Operations Management?**

- OM is the management of activities and resources that create goods and provide services
- Companies use OM to improve efficiency and effectiveness

#### **Why Study Operations Management?**

- A large percentage of a company's expenses occur in the OM area
- A large number of all jobs are in the OM area
- Activities in all other areas are interrelated with OM activities

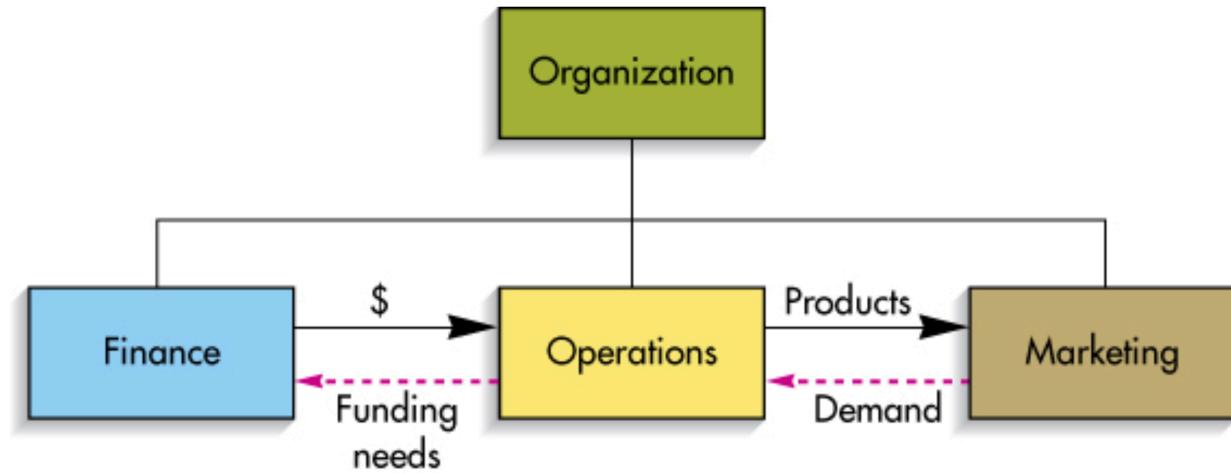
#### **Three Basic Functions:**

1. Operations: Create goods and services
2. Finance: Provide funds and the economic analysis of investment proposals
3. Marketing: Assess customer wants and needs and communicate them to others

#### **Types of OM Decisions:**

- Design (strategic) decisions that are medium to long term and deal with capital equipment and other physical assets (equipment needs, production capacity, etc.)
- Day-to-day operations involve the everyday decisions such as scheduling, packaging, quality control, and labour requirements

### 1.1.3 Operations as it Relates to the Firm



### 1.1.4 What Operations Managers Do

#### Defining the Role of the Operations Manager:

- Core (manufacturing)
- Support (maintenance, accounting, HR, purchasing)
- Managerial (general administration)

#### The Operations Manager's Job:

Management Process	Field of Responsibility
Planning	Capacity, location, make or buy
Organizing	Centralization, specialization, staffing
Controlling	Inventory, quality, motivation
Directing	Scheduling, incentive plans, work orders

#### Establishing Priorities:

- Pareto Phenomenon: A few factors account for a high percentage of the occurrence of some event(s)
- 80/20 Rule: 80% of problems are caused by 20% of the activities

### 1.1.5 The Importance of Collaboration

- A clear and comprehensive systems-based approach for issues is in demand
- Coordination between marketing and operations is essential for success

- For example, changing a package form needs to account for inventory costs and order quantities, existing packaging inventories, new equipment needs, plant layout, etc.

### **1.1.6 Concept of Value Added**

- Difference between the cost of goods and the value of outputs
- For services, the cost of services and the value placed on those services by individuals
- Value of output is determined by the prices that consumers are willing to pay for goods

### **1.1.7 Stakeholder Management**

- “Any group or individual who can affect or is affected by the achievement of an organization’s objective” (Freeman, 1984)
- Stakeholders may include customers, employees, suppliers, financiers, etc.

## **1.2 Demand Forecasting**

- Underlies strategic planning when it comes to plant or service design
- Essential for budgeting and determining capital requirements for both inputs and projects
- Dictates medium-term operations and affects short-term operations

### **1.2.1 Forecast Commonalities**

1. Rely, to some extent, upon past demand and criteria identified as affecting that demand
2. Forecasts of aggregate demand for similar goods is more accurate than forecasts for individual items within a category
3. Increasing forecast horizons introduces greater uncertainty and reduced reliability

### **1.2.2 Requirements for a Useful Forecast**

1. Forecast should be long enough to make it relevant
2. Limitations on accuracy must be clearly stated
3. Forecasting method should be reliable
4. Operations forecasts should be expressed in units

### **1.2.3 How it is Done**

1. Determine why you are forecasting (who wants it and what will they use it for)
2. Assess and state the required levels of detail and accuracy

3. Establish a forecasting horizon
4. Gather historical data
5. Select a forecasting method
6. Complete the forecast
7. Monitor its accuracy

#### **1.2.4 Judgemental Approaches**

- Includes things such as hunches, personal opinions, non-quantitative observations
- Can be tainted by personal bias
- Developed as a non-quantitative analysis of historical data or analysis of subjective data

#### **1.2.5 Quantitative Approaches**

- Utilize hard data from the past (untainted from personal bias)
- Time series models (identifies patterns in data and projects these trends into the future)
- Associative models (describes demand in terms of independent causal variables)

#### **1.2.6 Time Series Modeling**

- A time-ordered sequence of observations taken at regular intervals over a period of time
- Assumes that future values of the series can be estimated from past values

#### **Data Behaviour:**

- Average (level): Horizontal pattern
- Trend: Persistent upward or downward pattern
- Seasonality: Regular wavelike pattern that corresponds with some repeatable event
- Cycles: Lasts more than one year and looks at longer-term patterns
- Irregular Variations: One-time events that tend to skew the data
- Random Variations: Multitude of minor events that combine to affect the data

#### **Methods:**

##### **Naïve Method:**

- Assumes that the value of the data for the last period will be the value of the next period
- Can be applied to average, trend, and seasonal data

Period	Actual	Change	Naïve Forecast
--------	--------	--------	----------------

$t - 1$	50		
$t$	53	+3	
$t + 1$			$53 + 3 = 56$

### Averaging Methods:

- Three types: moving average, weighted moving average, and exponential smoothing
- Requires stable data and can handle random variations but not irregular data points

#### Moving Average Method:

- Average of recent observations are used as the basis for the current forecast
- Choice of the number of data points used for the calculation will affect the sensitivity of the average to the most recent data point

Period	Demand
1	42
2	40
3	43
4	40
5	41

$$F_6 = \frac{43 + 40 + 41}{3} = 41.33$$

$$F_7 = \frac{40 + 41 + 39}{3} = 40.00$$

#### Weighted Moving Average Method:

- Assigns a heavier weight to more recent data points
- Sum of the weights must equal to one

$$F_6 = 0.40(41) + 0.30(40) + 0.20(43) + 0.10(40) = 41.0$$

$$F_7 = 0.40(39) + 0.30(41) + 0.20(40) + 0.10(43) = 40.2$$

#### Exponential Smoothing:

- Current forecast that is based upon the previous period plus a portion of the difference between the actual outcome in the period and the quantity forecast for that period
- Notation:  $F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$ , where  $0 < \alpha < 1$
- For example, if previous forecast was 42 units, previous actual demand was 40 units, and  $\alpha = 0.10$ , the new forecast would be:

$$F_t = 42 + 0.10(40 - 42) = 41.8$$

- Then, if the actual demand turned out to be 43, the next forecast would be:

$$F_{t+1} = 41.8 + 0.10(43 - 41.8) = 41.92$$

### Associative Forecasting:

- If I want to predict ridership from a new train station, what data might I look at?
- 1. Find (predictor) variables associated with ridership at other stations
- 2. *Associated = correlated = as one moves the other moves*
- 3. Create a model that shows the relationship between the predictor variables and the predicted variable (ex. ridership)
- 4. Technique is regression analysis (ex. Simple linear regression with one variable, multiple regression that can be non-linear)
- 5. Test the model to see which variables are most useful in predicting ridership ( $r^2$ )
- 6. Use the model to predict ridership, given the values of the predictor variables

### Associative Models:

- Predictor Variables ( $x$ ): Used to predict values of the variable of interest ( $y$ ) – independent variables
- Linear Regression: Process of finding a straight line that best fits a set of points on a graph – least squares estimation
- Multiple Regression: Models with more than one predictor variables – computations complex, created with computer

### Correlation and Excel:

- Correlation Coefficient ( $r$ ): Measure of strength of the relationship between two variables
  - Ranges from  $-1$  (moves in same direction) to  $+1$  (moves in opposite direction)
- $r^2$  measures proportion of variation in the values of  $y$  that is “explained” by the predictor variables in the regression model
  - Ranges from 0 to 1 (higher values means more useful predictors)

### Linear Regression Assumptions:

- Predictions are being made only within the range of observed values
- Variations around the line are random and normally distributed

### 1.2.7 Forecasting Accuracy

- Definition: A judgement or measurement of how accurate the forecast is to actual results from the demand being forecast (or predicted)
- Presence of random variables such as changes in tastes, preferences, and values of consumers make it almost impossible to create a perfect prediction of future demand

#### Forecasting Error:

$$E_t = A_t - F_t$$

- In words: The error  $E$  for period  $t$  is simply the difference between the actual demand  $A$  and the forecast demand  $F$  for the same period
- **Bias:** Sum of forecast errors, if positive then forecasts are underestimating demand, if negative then forecasts are overestimating demand

#### Three Methods for Measuring Accuracy:

- **Mean Absolute Deviation:**  $MAD = \frac{\sum |e|}{n}$
- **Mean Squared Error:**  $MSE = \frac{\sum (A_i - F_i)^2}{n}$
- **Mean Absolute Percent Error:**  $MAPE = \frac{\sum \left[ \frac{|A_i - F_i|}{A_i} \times 100 \right]}{n}$ 
  - $e$  is the value of individual forecast errors,  $n$  is the number of observations

Calculate MAD, MSE, and MAPE for the following data.

Period	Actual	Forecast	Forecast Error	$(A-F)$  Error	Forecast Error <sup>2</sup>	$\frac{ Error }{Actual} \times 100$
1 .....	217	215	2	2	4	.92%
2 .....	213	216	-3	3	9	1.41
3 .....	216	215	1	1	1	.46
4 .....	210	214	-4	4	16	1.90
5 .....	213	211	2	2	4	.94
6 .....	219	214	5	5	25	2.28
7 .....	216	217	-1	1	1	.46
8 .....	212	216	-4	4	16	1.89
			-2	22	76	10.26%

d

#### SOLUTION

Using the numbers shown in the above table,

$$\begin{aligned} MAD &= \frac{\sum |e|}{n} = \frac{22}{8} = 2.75 \\ MSE &= \frac{\sum e^2}{n} = \frac{76}{8} = 9.5 \\ MAPE &= \frac{\sum \left[ \frac{|e|}{Actual} \times 100 \right]}{n} = \frac{10.26\%}{8} = 1.28\% \end{aligned}$$

### 1.2.8 Dealing with Control Charts

- Objective: Measure the accuracy of the forecasts within a set of prescribed boundaries
- Control limits “ $s$ ” based upon the square root of the mean squared error:  $s = \sqrt{MSE}$

#### Solution Process:

1. Setup the required data table and make the necessary calculations
2. Check average forecast error and comment on its proximity to zero
3. Calculate the square root of the forecast errors
4. Determine the  $2s$  control limits
5. Compare forecast errors to the control limits

Monthly sales of leather jackets at a store for the past 24 months, and forecasts and forecast errors for those months, are shown below. Determine if the forecasting technique is satisfactory using a control chart with  $2s$  limits. Use data from the first eight months to develop the control chart, then evaluate the remaining data with the control chart.

Month	A (Sales)	F (Forecast)	A-F (Error)
1 .....	47	43	4
2 .....	51	44	7
3 .....	54	50	4
4 .....	55	51	4
5 .....	49	54	-5
6 .....	46	48	-2
7 .....	38	46	-8
8 .....	32	44	-12
9 .....	25	35	-10
10 .....	24	26	-2
11 .....	30	25	5
12 .....	35	32	3
13 .....	44	34	10
14 .....	57	50	7
15 .....	60	51	9
16 .....	55	54	1
17 .....	51	55	-4
18 .....	48	51	-3
19 .....	42	50	-8
20 .....	30	43	-13
21 .....	28	38	-10
22 .....	25	27	-2
23 .....	35	27	8
24 .....	38	32	<u>6</u>
			-11

1. Make sure that the average forecast error is approximately zero:

$$\text{Average error} = \frac{\sum \text{errors}}{n} = \frac{-11}{24} = -0.46 \approx 0$$

2. Compute the standard deviation of forecast errors using the first eight months:

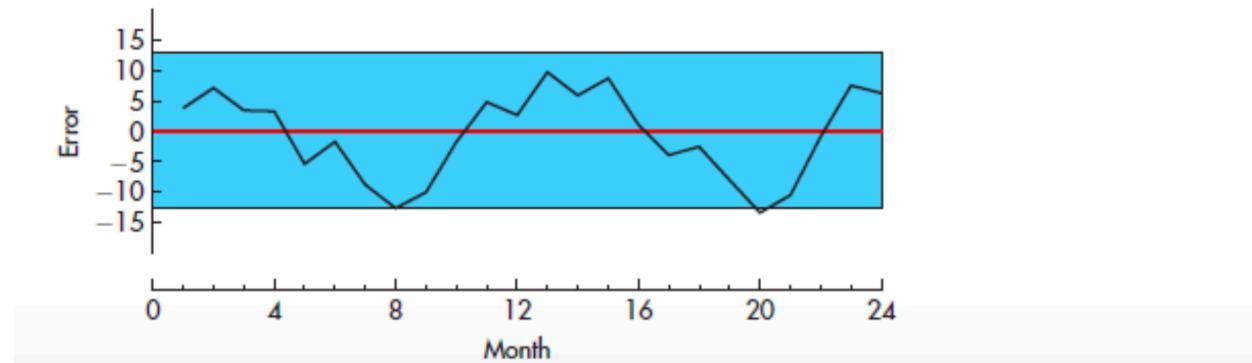
$$s = \sqrt{\text{MSE}} = \sqrt{\frac{\sum e^2}{n}}$$

$$= \sqrt{\frac{\sum 4^2 + 7^2 + 4^2 + 4^2 + (-5)^2 + (-2)^2 + (-8)^2 + (-12)^2}{8}} = 6.46$$

3. Determine  $2s$  control limits:

$$0 \pm 2s = 0 \pm 2(6.46) = -12.92 \text{ to } +12.92$$

- I. Check that all forecast errors are within the control limits. Month 20's forecast error (-13) is just below the lower control limit (-12.92).
- II. Check for nonrandom patterns. Note the runs of positive and negative errors in the following plot. This suggests nonrandomness (and that a better forecasting technique is possible). (We present more on control charts in [Chapter 10](#).)



## 2 DEMAND FORECASTING (II) AND PRODUCT DESIGN

### 2.1 Demand Forecasting

#### 2.1.1 Trend Adjusted Forecasts (TAF)

- Slightly more complicated version of exponential smoothing:

$$S_t = TAF_t + \alpha(A_t - TAF_t)$$

$$T_t = T_{t-1} + \beta(S_t - S_{t-1} - T_{t-1})$$

$$TAF_{t+1} = S_t + T_t$$

- $S_t$ : Smoothed series at the end of period  $t$
- $T_t$ : Smoothed trend at the end of period  $t$
- $\alpha$  and  $\beta$  are arbitrary smoothing constants
- $A_t$  is the actual data from period  $t$

#### Example 4

Cellphone sales of a company over the last 10 weeks are shown below. The data appear to have a linear trend. Determine the equation of the linear trend and predict the sales of cellphones for weeks 11 and 12. Plot the data and trend line.

Week	Unit Sales
1	700
2	724
3	720
4	728
5	740
6	742
7	758
8	750
9	770
10	775

Using the cellphone data from [Example 4](#) (where it was concluded that the data exhibited a linear trend), use trend-adjusted exponential smoothing to prepare forecasts for periods 5 through 11, with  $\alpha = .4$  and  $\beta = .3$ . Use the first four weeks to estimate starting smoothed series and smoothed trend.

## SOLUTION

[Table 3-1](#) displays the data again in the Actual column. We will use the average of the first four weeks as the starting smoothed series:

$$S_4 = (700 + 724 + 720 + 728)/4 = 718.$$

t (Week)	A <sub>t</sub> (Actual)			
1	700			
2	724	Starting values:		
3	720	$S_4 = (700 + 724 + 720 + 728)/4 = 718, T_4 = (728 - 700)/3 = 9.33$		
4	728	$TAF_5 = 718 + 9.33 = 727.33$		
		$TAF_t + \alpha(A_t - TAF_t) = S_t$	$T_{t-1} + \beta(S_t - S_{t-1} - T_{t-1}) = T_t$	$TAF_{t+1} = S_t + T_t$
5	740	$727.33 + .4(740 - 727.33) = 732.40$	$9.33 + .3(732.40 - 718 - 9.33) = 10.85$	743.25
6	742	$743.25 + .4(742 - 743.25) = 742.75$	$10.85 + .3(742.75 - 732.40 - 10.85) = 10.70$	753.45
7	758	$753.45 + .4(758 - 753.45) = 755.27$	$10.70 + .3(755.27 - 742.75 - 10.70) = 11.25$	766.52
8	750	$766.52 + .4(750 - 766.52) = 759.91$	$11.25 + .3(759.91 - 755.27 - 11.25) = 9.27$	769.18
9	770	$769.18 + .4(770 - 769.18) = 769.51$	$9.27 + .3(769.51 - 759.91 - 9.27) = 9.37$	778.88
10	775	$778.88 + .4(775 - 778.88) = 777.33$	$9.37 + .3(777.33 - 769.51 - 9.37) = 8.90$	786.23

### 2.1.2 Review of Seasonality

- Seasonal data is defined as data that exhibits a repeating pattern that is tied to some regularly occurring event (summer ice cream sales, winter natural gas sales, etc.)
- Expressed as the amount of deviation from the period average, by using the average over the length of the repeating pattern to calculate an adjustment factor
- Seasonality is measured relative to the trend

#### Computing Seasonal Relatives:

- Need to identify the trend in the data in order to compare seasonal values to some average
- Decompose data into smaller segments through use of a centered moving average (CMA)
- Number of periods in CMA is equal to the number of “seasons” portrayed in the data
- Example: Quarterly data, use a 4-period CMA

Period	Historical Value	CMA	Seasonal Relative

1	79		$79/76.67 = 1.03$
2	86	76.67	$86/76.67 = 1.12$
3	65		$65/76.67 = 0.85$

### Steps in Dealing with Seasonal Data:

1. Write out the given data in table form
2. Calculate the appropriate Centered Moving Average (CMA)
3. Use CMA to calculate seasonal relatives
4. Average and adjust the seasonal relatives to the relevant seasonal data
5. Use the “adjusted” averaged seasonal relatives to de-seasonalize the original data
6. Describe the data (does it exhibits a linear trend, upward or downward)
7. Using calculated (GIVEN) Linear Trend Equation, calculate the forecasts for the periods
8. Re-seasonalize forecast using the average seasonal relatives calculated in Step 4

Below are quarterly data for production of ice cream in Canada from quarter 1 of 2006 to quarter 3 of 2009.<sup>2</sup>

- a. Calculate the quarterly relatives using the centred moving average method.
- b. Deseasonalize the data, fit an appropriate model, project it four quarters ahead, and reseasonalize to obtain forecasts for ice cream demand for the 4th quarter of 2009 and Q1 to Q3 of 2010.

a.	Quarter	Production (million litres)	CMA <sub>4</sub>	CMA <sub>2</sub>	Production/CMA <sub>2</sub>
2006	1	66	79.75 78 76.75	78.875 77.375 75.875	$91/78.875 = 1.154$ $66/77.375 = 0.853$ $59/75.875 = 0.778$
	2	96			
	3	91			
	4	66			
2007	1	59	75 73.5 72.5 70.25	74.250 73.000 71.375 69.500	1.226 1.151 0.841 0.791
	2	91			
	3	84			
	4	60			
2008	1	55	68.75 65 62.75 56.75	66.875 63.875 59.750 54.875	1.226 1.221 0.753 0.838
	2	82			
	3	78			
	4	45			
2009	1	46	53		
	2	58			
	3	63			

Year	QUARTER				Total
	1	2	3	4	
2006			1.154	0.853	
2007	0.778	1.226	1.151	0.841	
2008	0.791	1.226	1.221	0.753	
2009	0.838				
Average	0.802	1.226	1.175	0.816	4.019
Adjusted	.802(4/4.019)=0.798	1.226(4/4.019)=1.220	1.170	0.812	4.000

b.

Quarter	Production (million litres)	Seasonal Relatives	Deseasonalized Production
2006 1	66	0.798	66/.798=82.707
2	96	1.220	96/1.220=78.689
3	91	1.170	77.778
4	66	0.812	81.281
2007 1	59	0.798	73.935
2	91	1.220	74.590
3	84	1.170	71.795
4	60	0.812	73.892
2008 1	55	0.798	68.922
2	82	1.220	67.213
3	78	1.170	66.667
4	45	0.812	55.419
2009 1	46	0.798	57.644
2	58	1.220	47.541
3	63	1.170	53.846

The deseasonalized data has a (decreasing) linear trend. Therefore, a linear trend is fitted to the deseasonalized data using regression, and its equation is displayed at the top of the chart above.

Using the regression equation, the trend forecasts for periods 16 to 19 (Q4 of 2009 and Q1 to Q3 of 2010) are:

$$\begin{aligned}Y_{16} &= -2.2575(16) + 86.85 = 50.730 \\Y_{17} &= -2.2575(17) + 86.85 = 48.473 \\Y_{18} &= -2.2575(18) + 86.85 = 46.215 \\Y_{19} &= -2.2575(19) + 86.85 = 43.958\end{aligned}$$

d

The reseasonalized forecasts for the fourth quarter of 2009 and Q1 to Q3 of 2010 are:

$$\begin{aligned}F_{16} &= 50.730(0.812) = 41.193 \\F_{17} &= 48.473(0.798) = 38.681 \\F_{18} &= 46.215(1.220) = 56.382 \\F_{19} &= 43.958(1.170) = 51.431\end{aligned}$$

## 2.2 Product Design

Product Design:

- Determines the form and function
- Is expensive and time consuming
- Governs the level of product quality and hence the level of customer satisfaction
- Contributes to competitive advantage

### 2.2.1 Product Design Process

- Product Approval Committee: Management team
- Core Teams: Members from all aspects of the organization
- Phase Reviews: Milestones
- Structural Development: Structured and explicit project management technique

### 2.2.2 Sequential Steps in New Product Developments

- Idea Generation
- Build a Business Case: Market and Competitive Analysis
- Development: Product attributes we need to meet or exceed customer expectations?
- Testing and Validation
- Launch

### **2.2.3 Sources of New Product Ideas**

- Customers, front-line production workers, suppliers, competitive products, R&D

### **2.2.4 Key Issues in Product Design**

#### **Standardization**

- Every consumer receives the same product
- High volumes of the same product reduce costs through economies of scale
- Can reduce the level of variety in product offerings
- May breed a climate of resistance of product improvements and modifications

**Design for Mass Customization:** Attempts to reach a compromise between the benefits of standardization for the producer and the desire of consumers for variety and customization

**Reliability:** Ability of a product (or part) to perform for its intended use under normal conditions

**Robust Design:** Products that can stand up under a wide range of operational environments

**Legal and Ethical Issues:** Product liability (injuries), ethical issues (sub-standard products)

**Design for the Environment:** Energy efficient, minimization of waste/packaging, etc.

**Concurrent Engineering:** Bringing design engineering staff into direct work with others

**Computer Assisted Design (CAD):** Computerizes design to avoid hand-drawn designs

**Design for Manufacturing and Assembly:** Ease can be incorporated into the design

**Component Commonality:** Occurs when a component can be used in multiple products

### 3 CAPACITY PLANNING AND PROCESS DESIGN/FACILITY LAYOUT

#### 3.1 Strategic Capacity Planning

##### 3.1.1 Overview

- **Capacity:** Upper workload limit that a production unit can accommodate
- **Production Rate:** Measured in terms of the units of production per unit of time
- **Capacity Time Frames:** Long-term (1-5 years), medium-term (next 12 months), short-term (up to 12 weeks)
- **Strategic Capacity Planning:** Systematic determination of facility and equipment needs

##### Importance of Long-Term Capacity

- Capacity directly impacts the ability of the company to meet future demand
- Effective capacity planning optimizes operating costs
- Capacity decisions have a direct impact on initial capital costs
- Capacity involves long-term commitment of resources
- Capacity can directly affect competitiveness

##### Measuring Capacity

- Design Capacity: Max amount of output that can be produced under ideal conditions
- Effective Capacity: Amount of output that can realistically be made given delays, etc.
- Efficiency: Ratio of actual output to effective capacity:

$$\text{Efficiency} = \frac{\text{Actual Output}}{\text{Effective Capacity}}$$

- Utilization: Ratio of actual output to design capacity:

$$\text{Utilization} = \frac{\text{Actual Output (or Uptime)}}{\text{Design Capacity (or Available Time)}}$$

Minco produces silica and magnesia (<http://www.mincoitc.com>). To process silica, eight furnaces are used. Each furnace has 2,200 pounds/hour capacity. Minco works 24 hours a day, 5 days a week. During a typical week, total preventive maintenance is 65 hours and total other downtime is 39 hours. Effective capacity is 1,700,000 pounds and actual production is 1,600,000 pounds per week. Calculate the design capacity, utilization, and efficiency of silica furnaces.

##### SOLUTION

Available hours per week =  $8(24)(5) = 960$

Design capacity per week =  $2,200(960) = 2,112,000$  pounds

Productive hours per week =  $960 - 65 - 39 = 856$

Utilization = uptime hours/available hours =  $856/960 = 0.89$  or 89%

Efficiency = actual output rate/effective capacity =  $1,600,000/2,112,000 = 0.94$  or 94%

Note: Effective capacity = 1,700,000 < utilization × design capacity =  $.89(2,112,000) = 1,879,680$

## Factors Influencing Capacity

Facilities and equipment, products, workers, planning and operations, external factors

### 3.1.2 Effective Capacity Planning

1. Forecast demand for a period of 5 years or more
2. Calculate the capacity required to meet that demand
3. Measure existing in-house capacity and if necessary, begin discussing how additional capacity can be achieved

### 3.1.3 Focusing on Long-Term Demand

1. Identify the trend as expressed over time (growth, decline, cyclical, stable)
2. Consider forecasting technique (regression, judgemental)

### 3.1.4 Basic Capacity Requirement Question

1. Have a reasonably accurate forecast of annual demand
2. Know standard run times for each product on its respective machine
3. Know the number of hours worked per day
4. Know the number of workdays per year

A department works one eight-hour shift, 250 days a year. Three new products with the following annual demands are to be produced in the department. There is no excess capacity, so the department needs to buy new machines. Fortunately, there is a type of machine that can produce all three products. The expected processing times per unit are also given below. How many machines are needed?

Product	Annual Demand	Standard Processing Time per Unit (Hr)	Processing Time Needed (Hr)
#1	400	5.0	2,000
#2	300	8.0	2,400
#3	700	2.0	1,400
			5,800

d

### SOLUTION

Working one eight-hour shift, 250 days a year provides an annual capacity of  $8 \times 250 = 2,000$  hours per year for each machine. We can see that three of these machines would be needed to handle the required volume:

$$\frac{5,800}{2,000 \text{ hours/machine}} = 2.90 \text{ machines (round up to 3)}$$

The size of the facility can be determined by summing the area required for machines, inventory, handling material, and the offices required.

### 3.1.5 Major Concerns When Assessing Various Capacity Alternatives

- Design flexibility into system
- Differentiate between new and mature products
- Look at the organization in its entirety
- Prepare to deal with large (chunkier) increases in capacity
- Attempt to smooth out capacity requirements as dictated by consumer demand
- Use of capacity cushions
- Identify the optimal operating level

The first product created by toy company Spin Master, now the third largest toy manufacturer in North America, was Earth Buddy, a grass head.<sup>1</sup> Spin Master was hoping to produce and sell 8,000 Earth Buddies for Mother's Day and had one week to produce them. The Earth Buddy assembly consisted of the following operations: A: fill grass seeds and sawdust in a nylon stocking (average time 90 seconds), B: form ears and nose using elastic bands (average time 48 seconds), C: stick on the eyeglass and eyes (average time 24 seconds; eyeglasses were formed off line), and D: paint the mouth (average time 15 seconds). Finally, Earth Buddies are left to dry overnight and then packed. There are 6 workers doing A, 3 doing B, 2 doing C, and 1 doing D. There is enough space for inventory between operations so that they can work fairly independently. Determine the bottleneck operation and capacity of the "line" for a 40-hour week. Is the line balanced? Can Spin Master meet its production goal?

#### SOLUTION

	Fill	Form	Eyes	Paint
Time per unit per worker (sec)	90	48	24	15
No. of units per minute per worker	$60/90 = 2/3$	$60/48 = 1.25$	$60/24 = 2.5$	$60/15 = 4$
Number of workers	6	3	2	1
Capacity per minute	$6(2/3) = 4$	$3(1.25) = 3.75$	$2(2.5) = 5$	$4(1) = 4$
Capacity per hour	$4(60) = 240$	$3.75(60) = 225$	$5(60) = 300$	$4(60) = 240$
	Bottleneck			

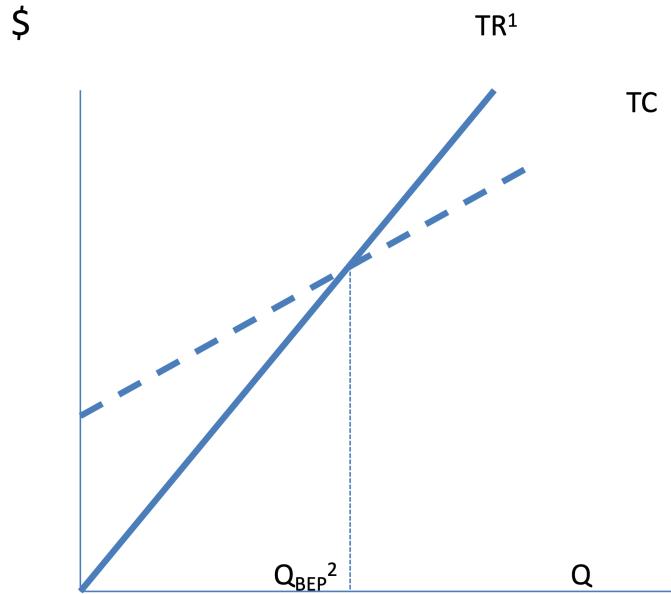
d

Line capacity = 225 units per hour. Yes, the line is fairly balanced because capacity per hour for the operations are all close.

Productive hours required to make 8,000 units =  $8,000/225 = 35.6$  hours.

Yes, Spin Master should be able to meet its target production goal of 8,000 units provided employees work approximately 36 hours (excluding breaks).

### 3.1.6 Relationship Between Total Costs and Total Revenues



<sup>1</sup>  $TR = R \times Q$  where  $R$  is revenue per unit      <sup>2</sup>  $Q_{BEP} = FC/(R-V)$

### 3.1.7 Break-Even Analysis

- Focuses on the relationship between costs, revenues, and volumes
- Goal is to determine the quantity at which the alternative capacity arrangement being considered will provide an economic advantage profit

#### Costs

- Fixed Costs ( $FC$ ) are independent of volume produced
- Variable Costs ( $VC$ ) are entirely dependent upon how much is produced
- Assume that variable costs ( $v$ ) remains constant per unit of output ( $Q$ ) regardless of the volume produced

$$\text{Total Cost (TC)} = FC + VC$$

$$\text{Variable Costs (VC)} = Q \cdot v$$

$$\text{Total Revenue (TR)} = Q \cdot R$$

$$\text{Profit (P)} = TR - TC = (Q \cdot R) - (FC + VC) = (Q \cdot R) - FC - Q \cdot v$$

$$P = Q(R - v) - FC$$

$$Q = \frac{P + FC}{R - v} \Rightarrow Q_{BEP} = \frac{FC}{R - v}$$

The management of a large bakery is contemplating making pies, which will require leasing a new equipment for a monthly cost of \$6,000. Variable cost will be \$2.00 per pie and retail price will be \$7.00 each.

- a. How many pies must be sold per month in order to break even?
- b. What would the profit (loss) be if 1,000 pies are made and sold in a month?
- c. How many pies must be sold to realize a profit of \$4,000 per month?
- d. If demand is expected to be 1,500 pies per month, is this a good investment?

## SOLUTION

$FC = \$6,000$  per month,  $v = \$2$  per pie,  $R = \$7$  per pie  
d

- a.  $Q_{BEP} = \frac{FC}{R - v} = \frac{\$6,000}{\$7 - \$2} = 1,200$  pies/month
- b. For  $Q = 1,000$ ,  $P = Q(R - v) - FC = 1,000 (\$7 - \$2) - \$6,000 = -\$1,000$  (i.e., loss of \$1,000)
- c.  $P = \$4,000$ ; solve for  $Q$  using [formula 5-7](#):  
$$Q = \frac{\$4,000 + \$6,000}{\$7 - \$2} = 2,000$$
 pies
- d. Because  $1,500 > 1,200 = Q_{BEP}$ , this is a good investment.

## 3.2 Process Design and Facility Layout

### 3.2.1 General Overview

**Process Design** and **Plant Layout** have long-term consequences for a company affecting:

- Long-term feasibility of the company
- Ability of the firm to meet its mission statement
- Morale of the workforce who actually operate the equipment and effectively run the plant
- The ability of the firm to meet its long-term strategic goals

### 3.2.2 Process Types

#### Job Shop

- Low volume of highly variable customized goods or services
- Process is intermittent
- Employs skilled workers
- Uses highly flexible equipment that can be used on a wide variety of applications

#### Batch Processing

- This describes a firm that produces a moderate quantity of a good or service
- Process is intermittent (i.e., by batch scheduled)
- Equipment is not as flexible as it is in a job shop
- Workers are skilled but not as skilled as those in a job shop

### **Repetitive Process**

- Marked by higher volumes of more standardized goods
- Lower skilled workforce – this is changing
- Equipment is not very flexible

### **Continuous**

- High volumes of highly standardized output
- Production is continuous and there is not typically counted
- Low skilled workforce

### **3.2.3 Process Considerations – Matching Process to Product Type and Volume**

- **Focused Factory Concept:** High volume shops that specialize in a particular output
- **Hybridization:** Some organizations have been able to combine volume and specialization in their product offerings

### **3.2.4 Automation**

- Using machinery with sensing and control devices that enable it to operate automatically
- Advantages: Low variability, contrasts to labour, strategy necessary for competitiveness
- Disadvantages: Costly, inflexible, can adversely affect worker morale
- Types: Fixed automation, programmable automation, flexible automation

### **3.2.5 Process Design**

- Determines the form and function of how the production of goods and services take place
- Involves the allocation of resources, the sequencing of activities and control mechanisms

**Process:** Determine state of inputs, outline objectives (speed, type, cost, quality, dates, etc.)

**Conceptualize:** *inputs → outputs*, assess alternatives, incremental/hierarchical, flow diagram

**Select:** Establish design of choice, build a prototype (computer model) of the process and test it

**Create:** Specify machinery, determine labour requirements, design placement and work centers

**Implement:** Buy equipment, hire and train workers, initiate trial runs

### 3.2.6 Fail-Safe Service Process

- Customer presence infers that quality and delivery problems are going to have immediate consequences
- This suggests that the service provider has a great incentive to ensure that the process is fail-safe, i.e., not prone to failure or on the positive side, to manage process design

### 3.2.7 Service Process Design

Service process design involves the direct or indirect involvement of the customer or of something belonging to the customer.

### 3.2.8 Basic Layout Types

**Product (Line):** Arranges the process (plant layout) linearly according to the progressive steps by which a product is made.

**Process (Functional):** Arranges production resources together according to the similarity of function.

**Cellular:** Layout in which different machines are arranged in a cell that can process items that have similar processing requirements.

### 3.2.9 Assembly Line Balancing

- Assigning tasks to workstations such that the workstations have approximately equal time requirements
- Cycle time is determined by the desired output rate:

$$Cycle\ Time = \frac{Operating\ Time\ Per\ Day}{Desired\ Output\ Rate}$$

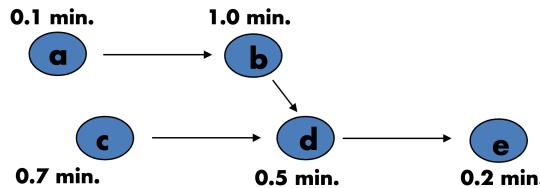
- Number of workstations is determined by the cycle time, the sum of task times, and the ability to bundle activities into appropriately sized workstations:

$$Theoretical\ Number\ of\ Workstations = \frac{Sum\ of\ Task\ Times}{Cycle\ Time}$$

- **Precedence Network:** A diagram that shows the task and their precedence requirements
- **Balancing Rules:** Assign the tasks with the longest time, assign the task with the most followers

A must be done before B, B and C must be done before D, and D must be done before E

Question: Calculate the number of work stations required and the activities done at each work station. Assume a cycle time of 1.0 minutes. 'Assign task with the most followers'



Workstation	Time Remaining	Eligible	Assign Task	Revised Time Remaining	Station Idle Time
1	1.0	a, c	a	0.9	
	0.9	c	c	0.2	
	0.2	none	—		0.2
2	1.0	b	b	0.0	0.0
3	1.0	d	d	0.5	
	0.5	e	e	0.3	0.3
	0.3	—	—		0.5

Two measures of facility layout effectiveness:

$$\text{Percentage Idle Time} = \left\{ \frac{\text{Sum of Idle Times}}{\text{Actual Number of Workstations} \times \text{Cycle Time}} \right\} \times 100$$

$$\text{Efficiency of the Line} = 100 - \text{Percentage Idle Time}$$

In this example:

$$\% \text{ idle time} = \frac{0.5}{3 \times 1.0} \times 100 = 16.7\%$$

$$\text{efficiency} = 100\% - 16.7\% = 83.3\%$$

### Dealing with Variable Task Times

- Task times vary due to worker fatigue, boredom, mechanical failures, shortages, etc.
- Solutions: Reduce variability in the task time themselves, allow for use of buffer inventories, build some idle time into the process itself, cross training workers

### Resolving Bottlenecks

- The task time of the slowest workstation will determine the cycle time for the entire line
- Solutions: Parallelize workstations to decrease process time in half, cross train workers

## **4 QUALITY MANAGEMENT AND STATISTICAL PROCESS CONTROL**

### **4.1 Quality Management**

#### **4.1.1 Quality Overview**

- **Quality:** The ability of a product or service to consistently meet or exceed customer expectations
- Evolution: craftsmanship (pre-industrial revolution), division of labour (industrial revolution), quality assurance (1950s), quality management systems (1970s), TQM/continuous improvement (1980s), Six Sigma/statistical tools (today)
- Dimensions: Performance, aesthetics, special features, conformance, reliability, durability, serviceability (example: quality of car)
- Service: Tangibles, convenience, reliability/consistency, responsiveness, time, assurance, courtesy (example: service of car repair)
- Determinants: Product design, process design, production
- Benefits: Improved quality, better reputation, increased market share, higher profitability
- Costs: Failure (defective), appraisal (inspection, testing), prevention (training, planning)

#### **Deming's 14 Steps to Quality:**

1. Create constancy of purpose toward improvement
2. Management must adopt the TQM philosophy
3. Cease dependence on inspection
4. Minimize total cost
5. Improve constantly and forever
6. Institute training
7. Institute leadership to act on quality issues
8. Drive out fear
9. Work as a team
10. Eliminate exhortations, and fix the system
11. Eliminate work standards and MBO
12. Remove barriers to pride of workmanship
13. Institute education and self-improvement
14. The transformation is everybody's job

#### 4.1.2 Quality Certifications

- ISO 9001: Set of international standards on quality management and quality assurance, critical to international business
- ISO 14000: Family of standards related to environmental management

#### 4.1.3 Hazard Analysis Critical Control Point (HACCP)

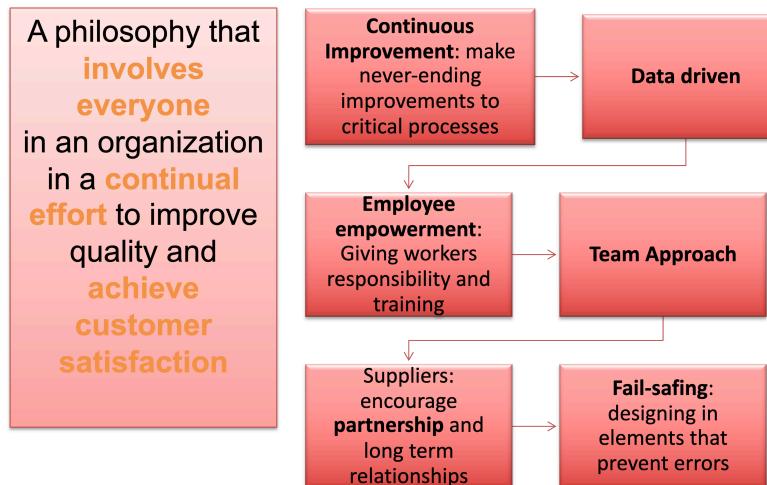
- A quality control system, similar to ISO 9001, designed for food processors
- Main steps: Hazard analysis, determination of the critical control points, creation of the HACCP plan

#### 4.1.4 National Quality Institute (NQI) and Total Quality Management (TQM)

NQI:



TQM:



TQM Approach:

- Find out what the customer wants
- Design a product or service that meets or exceeds customer wants
- Design processes that facilitate doing the job right the first time
- Keep track of results
- Extend these concepts to suppliers

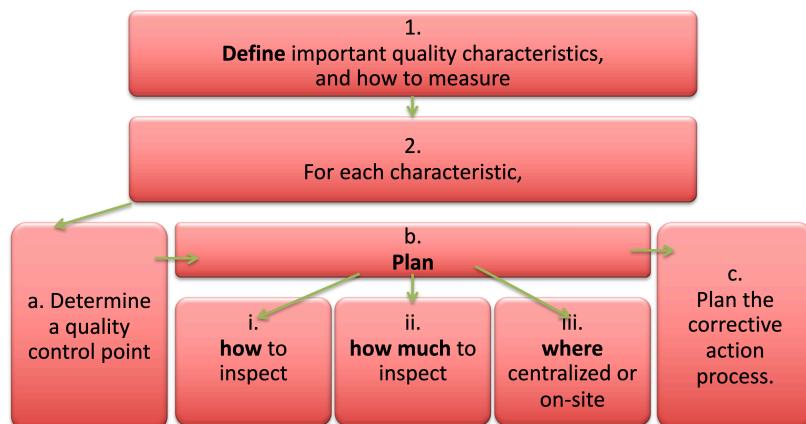
#### 4.1.5 Problem Solving, Process Improvement, and Quality Tools

- Problem solving: Define problem, collect data, analyze problem, generate potential solutions, choose a solution, implement solution, monitor solution
- PDSA cycle: Plan, do, study, act
- Six Sigma: A business process for improving quality, reducing costs, and increasing customer satisfaction ( $\leq 3.4 \text{ defects per million}$ )
- Quality tools: Flowcharts, check sheets, histograms, Pareto charts, scatter diagrams, control charts, cause-and-effect diagrams, run charts
- Methods for generating ideas: Brainstorming, quality circles, interviewing, etc.

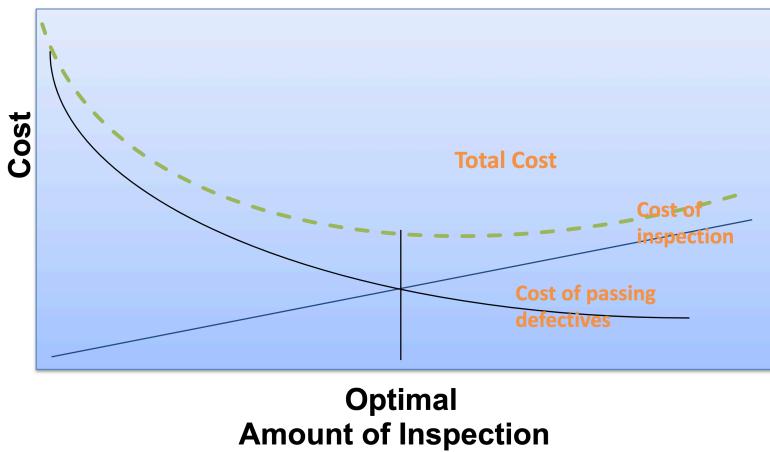
### 4.2 Statistical Quality Control

- **Statistical Quality Control:** Uses statistical techniques and sampling to monitor and test the quality of goods and services (acceptance sampling – accept or reject, statistical process control – if process is operating within acceptable limits)
- Inspection: The appraisal of goods/services against standards

#### 4.2.1 Elements of Statistical Process Control (SPC) Process



## Inspection Costs:



## Statistical Process Control:

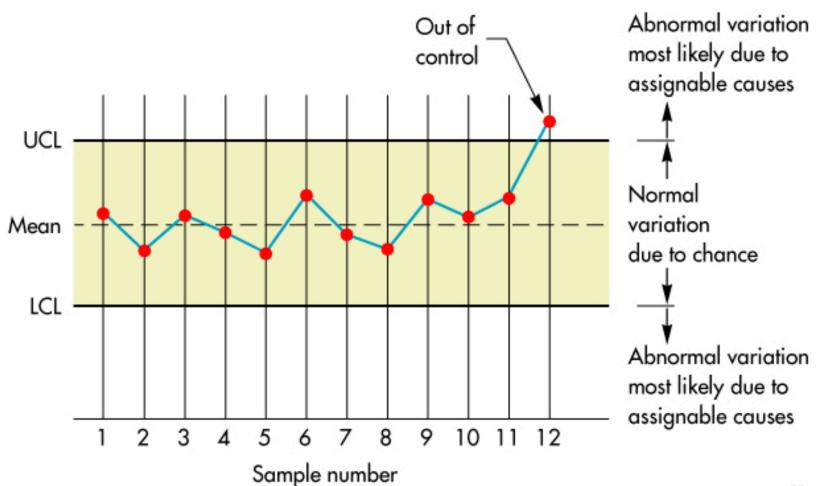
- The quality control steps
- Type of variations
- Control charts
- Designing control charts
- Individual unit and moving range charts
- Control charts for attributes
- Using control charts

**Types of Variations:** Random variation (natural), assignable variation (source can be identified)

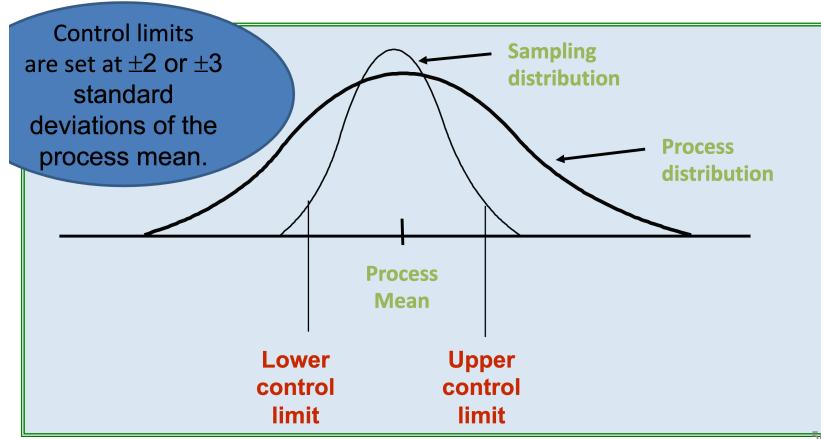
### 4.2.2 Control Charts

**Purpose:** To monitor process output to distinguish between random and assignable variation

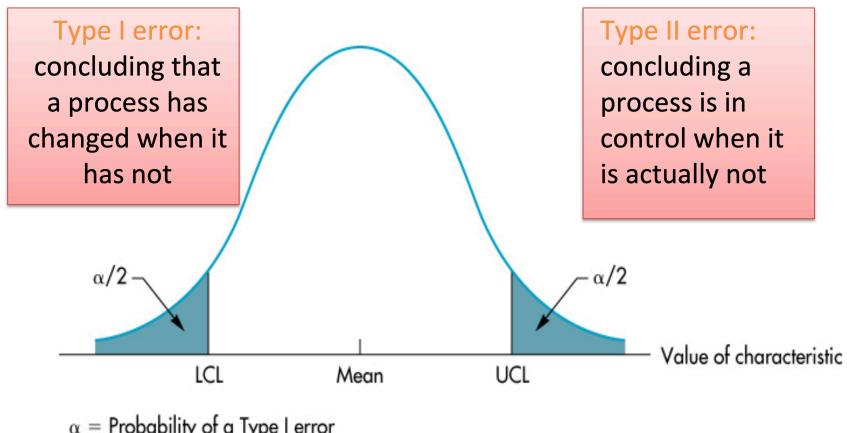
#### Example:



**Control Limits:** The dividing lines between random and non-random deviations from the process mean



### Type I and Type II Errors:



Conclusion		
A c t u a l i l y	In control	Out of control
In control	No Error	Type I error (producers risk)
Out of control	Type II Error (consumers risk)	No error

### Designing Control Charts:

1. Determine a sample size
2. Obtain 20 to 25 samples
3. Establish and graph preliminary control limits

4. Plot sample statistic values on control chart
5. Are any points outside control limits (CL)?
  - a. NO: Assume no assignable cause
  - b. YES: Investigate and correct

**Control Charts for Variables:** Variables generate data that are measured, sample mean/range

### Upper and Lower Control Limits for Sample Mean Chart:

$$UCL_{\bar{x}} = \bar{\bar{X}} + z\sigma_{\bar{x}}$$

$$LCL_{\bar{x}} = \bar{\bar{X}} - z\sigma_{\bar{x}}$$

Where:

$$\sigma_x = \text{Standard deviation of sampling distribution of sample means} = \frac{\sigma}{\sqrt{n}}$$

$\sigma$  = Process standard deviation

$n$  = Sample size

$z$  = Standard normal deviate (usually  $z = 3$ )

$\bar{\bar{X}}$  = Average of sample means (grand mean)

### Example:

A quality inspector took five samples, each with four observations, of the length of a part (in cm). She computed the mean of each sample (see [table](#) below).

- a. Use the following data to obtain Three Sigma (i.e.,  $z = 3$ ) control limits for sample mean. Is the process in control?
- b. If a new sample has values 12.10, 12.19, 12.12, and 12.14, using the sample mean control chart in part a, has the process mean changed (i.e., is there an assignable variation)?

It is known from previous experience that the standard deviation of the process is .02 cm.

	Sample				
	1	2	3	4	5
Observation	1	12.11	12.15	12.09	12.12
	2	12.10	12.12	12.09	12.10
	3	12.11	12.10	12.11	12.08
	4	12.08	12.11	12.15	12.10
	$\bar{x}$	12.10	12.12	12.11	12.10

$$a. \bar{\bar{X}} = \frac{12.10+12.12+12.11+12.10+12.12}{5} = 12.11 \text{ cm}$$

$$UCL_{\bar{x}} = 12.11 + 3 \left( \frac{0.02}{\sqrt{4}} \right) = 12.14$$

$$LCL_{\bar{x}} = 12.11 - 3 \left( \frac{0.02}{\sqrt{4}} \right) = 12.08$$

All 5 sample means fall between the control limits; the process is in control.

$$b. \bar{x}_6 = \frac{12.10+12.19+12.12+12.14}{4} = 12.1375 \text{ cm}$$

Because  $12.08 < 12.1375 < 12.14$ , the process mean has not changed.

### **Upper and Lower Control Limits Alternate Method:**

$$UCL_{\bar{x}} = \bar{\bar{X}} + A_2 \bar{R}$$

$$LCL_{\bar{x}} = \bar{\bar{X}} - A_2 \bar{R}$$

Where:

$A_2$  can be obtained from table below

$\bar{R}$  = Average of sample ranges

Sample range = Maximum value – minimum value in the sample

$\bar{\bar{X}}$  = Average of sample means (grand mean)

# Range Table

Number of Observations In Sample, <i>n</i>	Factor for $\bar{x}$ Charts, <i>A<sub>2</sub></i>	Factors for R-Charts	
		Lower Control Limit, <i>D<sub>3</sub></i>	Upper Control Limit, <i>D<sub>4</sub></i>
2.....	1.88	0	3.27
3.....	1.02	0	2.57
4.....	0.73	0	2.28
5.....	0.58	0	2.11
6.....	0.48	0	2.00
7.....	0.42	0.08	1.92
8.....	0.37	0.14	1.86
9.....	0.34	0.18	1.82
10.....	0.31	0.22	1.78
11.....	0.29	0.26	1.74
12.....	0.27	0.28	1.72
13.....	0.25	0.31	1.69
14.....	0.24	0.33	1.67
15.....	0.22	0.35	1.65
16.....	0.21	0.36	1.64
17.....	0.20	0.38	1.62
18.....	0.19	0.39	1.61
19.....	0.19	0.40	1.60
20.....	0.18	0.41	1.59

Example:

- Twenty samples of  $n = 8$  have been taken of the weight of a part. The average of sample ranges for the 20 samples is .016kg, and the average of sample means is 3kg. Determine three sigma control limits for sample mean of this process.

$$\bar{X} = 3, \bar{R} = 0.016, A_2 = 0.37 \text{ (for } n = 8 \text{ from table)}$$

$$UCL_{\bar{x}} = \bar{X} + A_2 \bar{R} = 3 + 0.37(0.016) = 3.006 \text{ kg}$$

$$LCL_{\bar{x}} = \bar{X} - A_2 \bar{R} = 3 - 0.37(0.016) = 2.994 \text{ kg}$$

### Upper and Lower Control Limits for Sample Range Control Chart:

$$UCL_R = D_4 \bar{R}$$

$$LCL_R = D_3 \bar{R}$$

#### Example:

- Twenty-five samples of  $n=10$  observations have been taken from a milling process. The average of sample ranges is .01 centimetre. Determine upper and lower control limits for sample range.

$$\bar{R} = 0.01 \text{ cm}, D_3 = 0.22, D_4 = 1.78 \text{ (for } n = 10, \text{ from table)}$$

$$UCL_R = D_4 \bar{R} = 1.78(0.01) = 0.0178 \text{ cm}$$

$$LCL_R = D_3 \bar{R} = 0.22(0.01) = 0.0022 \text{ cm}$$

#### Example:

Sample	Observation				
	1	2	3	4	5
1	10.68	10.69	10.78	10.80	10.71
2	10.79	10.86	10.60	10.75	10.78
3	10.78	10.67	10.84	10.79	10.72
4	10.59	10.73	10.81	10.78	10.73
5	10.69	10.71	10.79	10.76	10.67
6	10.75	10.71	10.74	10.72	10.61
7	10.79	10.71	10.69	10.88	10.60
8	10.74	10.78	10.11	10.74	10.75
9	10.77	10.77	10.64	10.64	10.73
10	10.72	10.67	10.71	10.85	10.71
11	10.79	10.82	10.76	10.66	10.71
12	10.62	10.80	10.82	10.87	10.73
13	10.66	10.82	10.89	10.54	10.75
14	10.81	10.75	10.86	10.80	10.70
15	10.66	10.68	10.64	10.75	10.73

Calculate sample means, sample ranges, grand mean, and average of sample ranges:

Sample	Observations					Mean	Range
	1	2	3	4	5		
1	10.68	10.69	10.78	10.80	10.71	10.73	0.12
2	10.79	10.86	10.60	10.75	10.78	10.75	0.26
3	10.78	10.67	10.84	10.79	10.72	10.76	0.17
4	10.59	10.73	10.81	10.78	10.73	10.73	0.22
5	10.69	10.71	10.79	10.76	10.67	10.72	0.12
6	10.75	10.71	10.74	10.72	10.61	10.71	0.14
7	10.79	10.71	10.69	10.88	10.60	10.73	0.27
8	10.74	10.78	10.11	10.74	10.75	10.62	0.67
9	10.77	10.77	10.64	10.64	10.73	10.71	0.13
10	10.72	10.67	10.71	10.85	10.71	10.73	0.18
11	10.79	10.82	10.76	10.66	10.71	10.75	0.16
12	10.62	10.80	10.82	10.87	10.73	10.77	0.25
13	10.66	10.82	10.89	10.54	10.75	10.73	0.35
14	10.81	10.75	10.86	10.80	10.70	10.78	0.16
15	10.66	10.68	10.64	10.75	10.73	10.69	0.10
					<b>Averages</b>	<b>10.73</b>	<b>0.22</b>

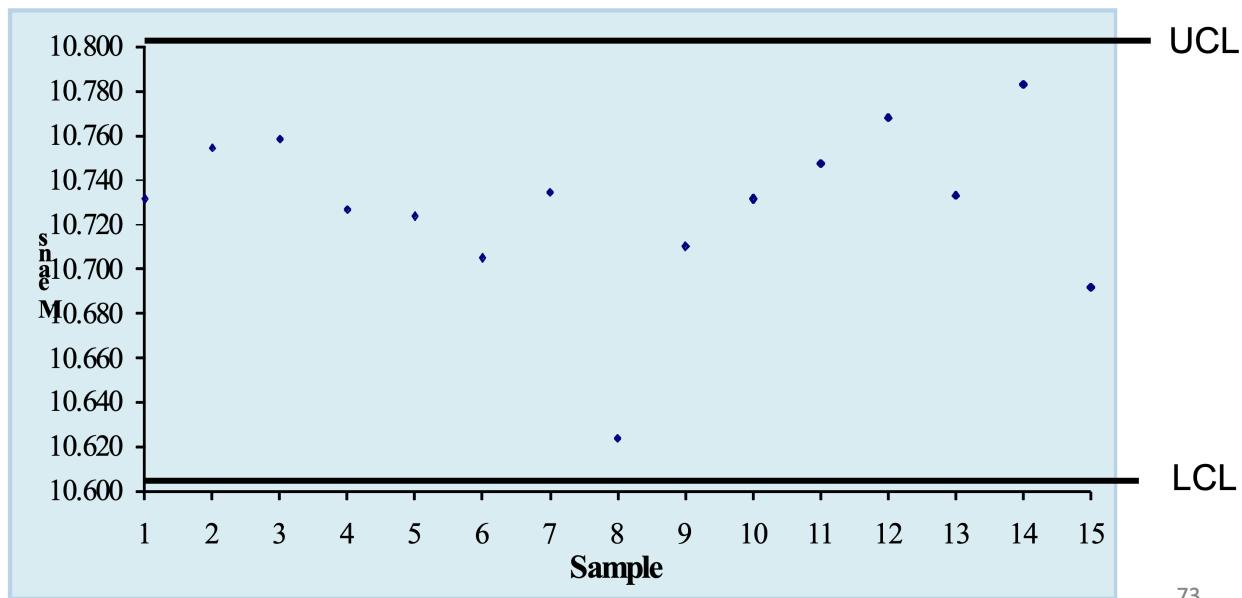
Determine control limits:

From table, with  $n = 5 \rightarrow A_2 = 0.58$

$$UCL_{\bar{x}} = \bar{\bar{X}} + A_2 \bar{R} = 10.728 + 0.58(0.2204) = 10.856$$

$$LCL_{\bar{x}} = \bar{\bar{X}} - A_2 \bar{R} = 10.728 - 0.58(0.2204) = 10.601$$

Create sample mean chart and plot values:



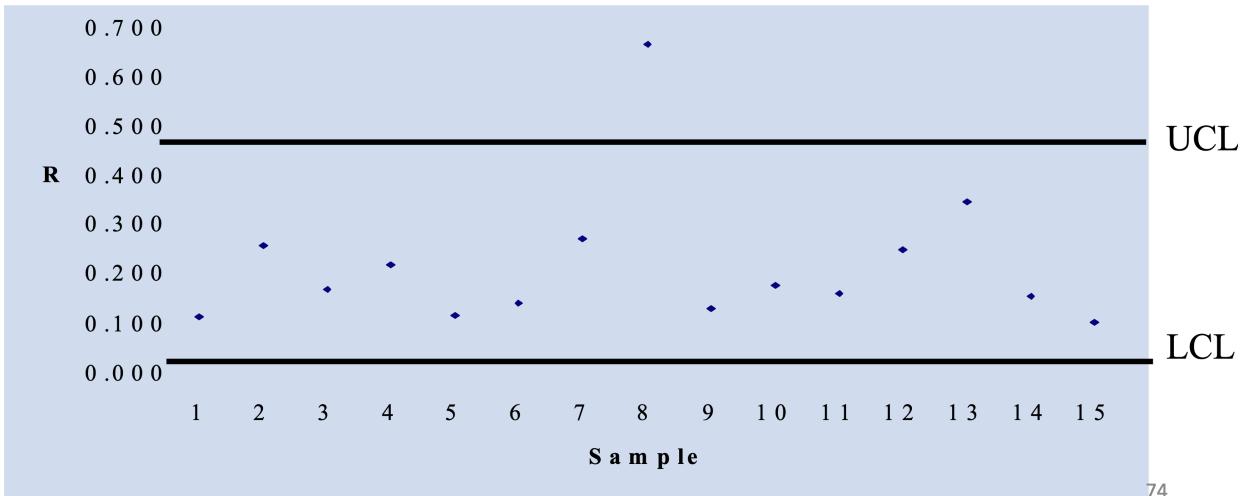
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Create sample range chart and plot values:

From table, with  $n = 5 \rightarrow D_3 = 0, D_4 = 2.11$

$$UCL_R = D_4 \bar{R} = (2.11)(0.2204) = 0.46504$$

$$LCL_R = D_3 \bar{R} = (0)(0.2204) = 0$$



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## 5 SUPPLY CHAIN MANAGEMENT (SCM)

### 5.1 SCM Part 1

#### 5.1.1 SCM

- **Supply Chain:** A sequence of organizations that produce a good or provide a service and get it to the marketplace.
- **Supply Chain Management:** The cooperation, collaboration and coordination of facilities such as manufacturing plants, warehouses, distribution centers; and activities such as production and physical delivery of goods.
- **Value Added:** The creation of a good's value as it moves through production and distribution.

*Suppliers → Manufacturers → Distributors → Retailers → Consumers*

#### 5.1.2 The Need for SCM and Potential Benefits

**Prime Objective of SCM:** To get the product to the marketplace as efficiently (i.e., least costly) as possible to increase the firm's effectiveness in meeting consumer demand.

#### Drivers of Supply Chain Management:

- Ongoing need to improve operations and increase efficiency
- Increased occurrence of outsourcing
- Free trade and increasing globalization
- Increasing levels of e-commerce
- Increasing competition (using lean or JIT)
- Desire to effectively control inventories (Bullwhip effect)

#### 5.1.3 Bullwhip Effect

**Bullwhip Effect:** Demand/order variability gets progressively larger the further back in a supply chain is the company.

- **Causes:** Isolation of manufacturing from the retail sector, errors in forecasting, promotional pricing, trade show effect.
- **Implications:** Strain on manufacturing capacity, rescheduling, increased inventory holdings resulting in increased costs and shelf-life issues.

- **Remedies:** Make product to order, increase inventories.

#### **5.1.4 Strategic and Tactical/Operational Activities in SCM**

**Strategic:** Agreement of goals between supply chain members, desired product characteristics, expected response times to consumer demand, information system requirements, target fill rate.  
**Tactical/Operational:** Forecasting, purchasing/ordering, transportation of material, warehouse and inventory control, scheduling of production and distribution, customer service functions.

#### **5.1.5 Efficient Replenishment Systems**

- **Quick Response:** Just-in-time inventory replenishment (used in retailing).
- **Efficiency Consumer Response (ECR):** Expanded version of quick response initiative which includes some collaboration (used in the grocery industry).
- **Vendor Managed Inventory (VMI):** Agreement for supplier to access and maintain customer's inventory (used in convenience stores).

#### **5.1.6 Information Technology Used in SCM**

- **Distribution Requirements Planning (DRP):** A system for synchronizing replenishment schedules across the supply chain; is a push system (based on forecasts).
- **Management Information System (MIS):** Tools used to gather and access, analyze, and share information; two types are planning software and execution software.
- **Electronic Data Interchange (EDI):** Direct, computer-to-computer transmission of transactions between organizations.
- **RFID Technology:** Used to track goods in supply chain with RFID tags attached to them, transmits product information to receiver, eliminates need for manual counting.

#### **5.1.7 Requirements and Key Steps in Creating Effective SCM**

##### **Requirements:**

- Formation of close relationships based upon trust, co-operation, and agreement.
- Create effective communication channels to facilitate improved co-ordination.
- Supply chain visibility and information sharing, (unexpected) event management, performance measurement.

##### **Key Steps:**

- Develop strategic objectives and tactics
- Integrate and coordinate internal activities
- Coordinate with suppliers and customers
- Coordinate planning and execution across the supply chain
- Form strategic partnerships

### **5.1.8 Reasons for Outsourcing**

- **Outsourcing:** Buying goods or services instead of producing or providing them in-house.
- **Reasons:** Lower cost, gain expertise and knowledge, gain flexibility, increase capacity.

### **5.1.9 CPFR and the Value of Strategic Planning**

- **Collaborative Planning, Forecasting, and Replenishment (CPFR):** Practical test as to the sincerity and effectiveness of a co-operative and effective supply chain.
- Three step process: Planning, initial forecasting, and replenishment

### **5.1.10 SCOR Performance Metrics Model**

**Supply Chain Operations Reference (SCOR) Model:**

Perspective	Metrics
Reliability	Perfect order fulfillment
Responsiveness	Lead time
Agility	Upside flexibility/adaptability
Costs	Supply chain management cost, COGS
Assets management	Cash-to-cash cycle time, return on fixed asset +/- inventory

## **5.2 SCM Part 2**

### **5.2.1 Responsibilities and Activities of Purchasing**

- **Purchasing:** Involves the procurement of raw materials, parts needed for manufacturer, machinery, repair parts for existing machinery, general office and plant supplies and the acquisition of required services; develop and implement purchasing plans.
- **Responsibilities:** Source goods and services, monitor quality of inputs, negotiate and determining prices, maintain database of suppliers and products, cost management, etc.

- **Purchasing Cycle:** Series of steps that begin with a request and ends with paying the supplier. Requisition received, supplier selected, decide how to purchase, monitor orders, receive orders, pay suppliers.

### **5.2.2 Benefits of E-Commerce**

Companies can have a global presence, improve competitiveness and quality, analyze customer interests, collect detailed information, shorten supply chain response times and costs, etc.

### **5.2.3 Value Analysis and Spend Analysis**

- **Value Analysis:** An assessment of the function of a particular manufacturing part to see if it can be modified or replaced with another part that will reduce costs without sacrificing its functionality; select item, identify function, ask questions, evaluate.
- **Price Determination:** Published prices, competitive bidding, and negotiated pricing.
- **Centralized vs. Decentralized Purchasing:** Purchasing handled by one special department vs. individual departments handling their own purchasing.
- **Spend Analysis:** The collection, organization, classification, and analysis of expenditures with the goal of reducing costs, increasing purchasing efficiency, and monitoring supplier performance.

### **5.2.4 Supplier Management and Partnership**

- Choosing suppliers, supplier relationships, supplier partnerships.
- Factors in choosing a supplier: Quality and quality assurance, flexibility, location, price, reputation and stability, lead times and reliability, other customers, service after sale.

### **5.2.5 Logistics and Reverse Logistics**

**Logistics:** The movement and storage (warehousing) of materials, finished products, and information that occur both within the facility and outside of the facility.

**Example:**

$$\begin{aligned}\text{Total Delivery Costs} &= \text{freight costs} + \text{in-transit holding costs} \\ &= \text{freight costs} + H(d)/365\end{aligned}$$

Where:  $H$  is the annual holding costs of the goods shipped  
 $d$  is the number of days in transit

Calculate total **ANNUAL** freight costs, in-transit costs and safety stock costs associated with the shipment of the following:

23,000 units of a good that sells for **\$ 210** each

	Costs (\$/unit)	Time Req'd (days)	Safety Stocks Req'd (weeks)
Air Freight	52	7	2
Ocean Freight	10	35	8

Annual **Holding Costs of 20%** per annum

Air Freight:

$$Freight = \$52 \times 23,000 = \$1,196,000$$

$$In - Transit = \$210 \times 23,000 \times 0.20 \times \frac{7}{365} = \$18,526.03$$

$$Safety Stock = (\$210 + \$52) \times 23,000 \times 0.20 \times \frac{2}{52} = \$46,353.85$$

$$Total = \$1,260,879.88$$

Ocean Freight:

$$Freight = \$10 \times 23,000 = \$230,000$$

$$In - Transit = \$210 \times 23,000 \times 0.20 \times \frac{35}{365} = \$92,630.14$$

$$Safety Stock = (\$210 + \$10) \times 23,000 \times 0.20 \times \frac{8}{52} = \$155,692.31$$

$$Total = \$478,322.45$$

**Reverse Logistics:** The backward flow of goods returned by consumers or retailers.

### 5.2.6 Process and Factors Considered When Selecting a Transportation Mode

- Modes of transportation: Ocean freights, trains, delivery trucks, air freights, etc.
- Speed vs. cost, balance transportation cost with inventory holding costs

**Example:** Selecting a Transportation Mode

**Cost of item = \$1,750.00**

**Holding cost = \$700 per year**

**Alternative A cost \$100 and takes 8 days**

**Alternative B cost \$120 and takes 7 days**

Total delivery cost = transportation (freight) cost + in-transit holding cost

$$\text{In-transit} \quad \text{Holding cost} = \frac{H(d)}{365}$$

where  $H$  = annual holding cost of items being transported

$d$  = duration of transport (in days)

$$\text{For A: } \$100 + \frac{\$700(8)}{365} = \$115.24$$

$$\text{For B: } \$120 + \frac{\$700(7)}{365} = \$133.42$$

Since it costs less to hold the product one day longer than it does for the faster shipping, choose Alternative A.

## 6 INVENTORY MANAGEMENT (I)

### 6.1 Basics

#### 6.1.1 Definition of Inventory

- **Inventory:** Inventory is nothing more than a stock of goods being held in a warehouse or some other storage facility
- For example: inputs, packaging, finished or unfinished goods, spare machinery, supplies
- Inventory Management: Involves some system to plan for and manage these stocks

#### 6.1.2 Why Inventories Demand Respect

Inventories are important because they are worth money:

- Outdated **finished goods** inventories increase waste
- Excessive **input inventories** risk the changes of some goods becoming unsuitable for use
- Inventories tie up funds and will therefore drive-up costs and drive down efficiency

$$ROI = \frac{\text{Profit after taxes}}{\text{Total assets}}$$

#### 6.1.3 Turnover Rate

$$\text{Turnover Rate} = \frac{\text{Annual cost of goods}}{\text{Average inventory value}}$$

## 6.2 Functions and Characteristics of Effective Inventory Management

**Functions:** In-transit inventories, safety stocks, cycle stocks, seasonal stocks, buffer inventories, anticipation inventories

### Characteristics:

- Safe Storage and Inventory Utilization: Warehouse management systems
- Tracking and Inventory Control: Periodic counting, perpetual tracking, two-bin system, bar code systems, radio frequency identification tags
- Forecasting and Lead Times: Purchase lead times
- Estimating Inventory Costs: Carrying costs, ordering costs, shortage costs
- A-B-C Product Classification

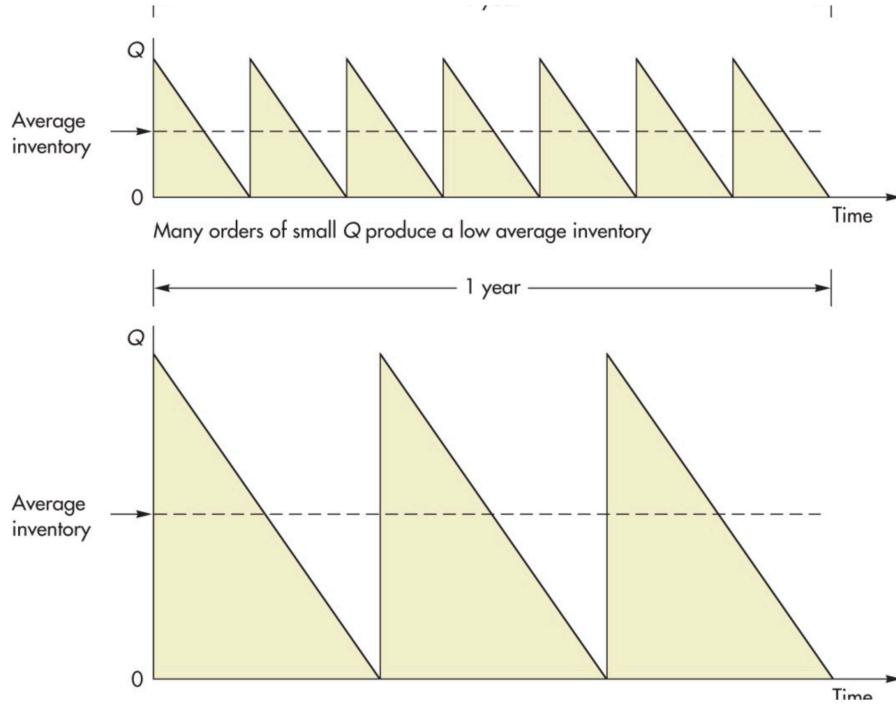
## 6.3 Requirements for Effective Inventory Management

### 6.3.1 Efficient Order Quantity (EOQ)

- Firms want to strike a balance between the costs associated with holding inventory and the costs incurred in ordering product
- For high volume/high value goods there is a double incentive to increase order frequency

**Assumptions:** Single product, annual demand needs are known, constant demand, constant lead time, single delivery per order, no quantity order discounts, no transportation discounts based upon load size

#### Inventory Cycles:



#### Annual Holding Costs:

Assumption of constant rate of use allows to define average inventory as half of the order:

$$\text{Average Inventory} = \frac{Q}{2}, \text{ where } Q \text{ is the order quantity}$$

$$\text{Average Holding Cost} = \frac{Q}{2} \times H, \text{ where } H \text{ is the annual holding cost per unit}$$

#### Annual Ordering Costs:

$$\text{Number of Orders} = \frac{D}{Q}, \text{ where } D \text{ is annual demand and } Q \text{ is order size}$$

$$\text{Annual Ordering Costs} = \frac{D}{Q} \times S, \text{ where } S \text{ is per order process cost}$$

Since annual demand is known, the annual ordering cost will be inversely related to order size

### Total Inventory Control Cost:

- When annual demand ( $D$ ), holding costs ( $H$ ), and per unit order processing costs ( $S$ ) are known, then total inventory holding and ordering costs ( $TC$ ) can be described as follows:

$$TC = \frac{Q}{2} \times H + \frac{D}{Q} \times S$$

- This suggests that optimal total costs can be plotted as the minimum point on a u-shaped total cost curve, where costs increase if order size is too small but also increase if inventory levels are too high:

$$\frac{dTC}{dQ} = \frac{H}{2} - \frac{DS}{Q^2}$$

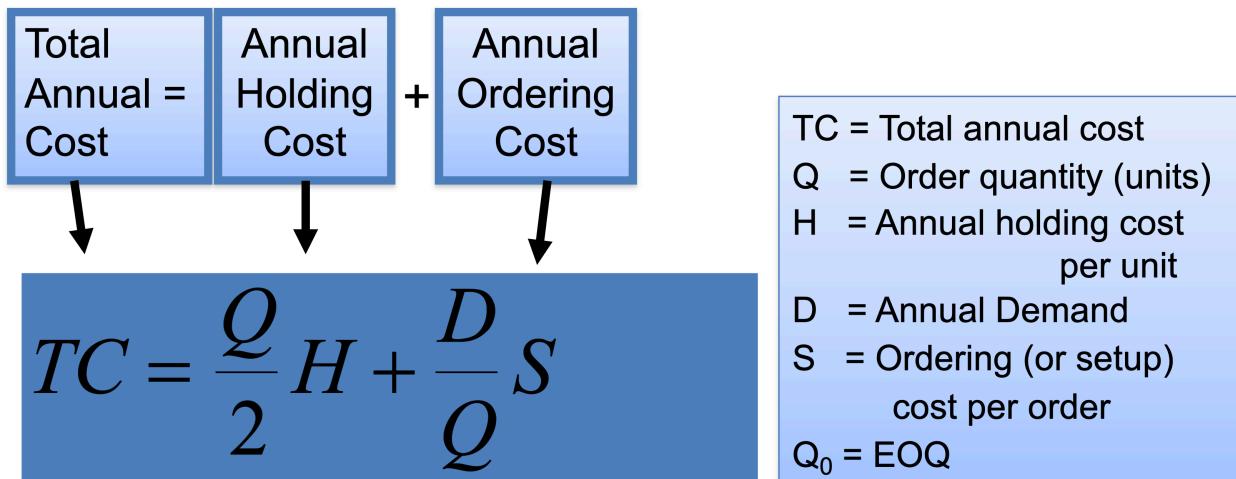
$$\frac{H}{2} - \frac{DS}{Q^2} = 0$$

$$Q = \sqrt{\frac{2DS}{H}} = Q_0$$

- Therefore, if we substitute  $EOQ = Q_0$ , we can solve for minimum costs

$$\text{Length of order cycle (in days)} = \frac{Q_0}{D} \times 365$$

### Summary:



$$Q_0 = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(\text{Annual Demand})(\text{Order or Setup Cost})}{\text{Annual Holding Cost}}}$$

### **EOQ Example 1:**

A tire store expects to sell approximately 100 all-season tires of a certain make, model, and size next year. Annual holding cost is \$16 per tire and ordering cost is \$20 per order. The store operates 360 days a year and has enough storage space in the back.

- a. What is the EOQ?
- b. How many times per year would the store reorder if EOQ units are ordered each time?
- c. What is the length of an order cycle if EOQ units are ordered each time?
- d. What is the total annual inventory control cost if EOQ units are ordered each time?

$$D = 100 \text{ tires per year}$$

$$H = \$16 \text{ per unit per year}$$

$$S = \$20 \text{ per order}$$

- a.  $Q_0 = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(100)(20)}{16}} = 15.8$ , round to 16 units
- b. Number of orders per year:  $\frac{D}{Q_0} = \frac{100 \text{ tires/year}}{16 \text{ tires each time}} = 6.25 \text{ times per year}$
- c. Length of order cycle:  $\frac{Q_0}{D} = \frac{16 \text{ tires}}{100 \text{ tires/year}} = 0.16 \text{ year}$ , which is  $0.16 \times 360$ , or 57.6 workdays or 57.6
- d.  $TC = \text{Annual holding cost} + \text{Annual ordering cost} = \frac{Q_0}{2}H + \frac{D}{Q_0}S$   
$$TC = \frac{16}{2}16 + \frac{100}{16}20 = \$128 + 125 = \$253$$

### **EOD Example 2:**

A distributor of security monitors purchases 3,600 black-and-white monitors a year from the manufacturer at the cost of \$65 each. Ordering cost is \$30 per order and annual holding cost rate is 20 percent of the unit cost. Calculate the optimal order quantity and the total annual cost of ordering and holding the inventory.

$$D = 3,600 \text{ monitors per year}$$

$$S = \$30 \text{ per order}$$

$$i = \text{holding cost rate per year} = 20\%$$

$$R = \text{unit cost} = \$65$$

$$H = iR = 0.20(\$65) = \$13$$

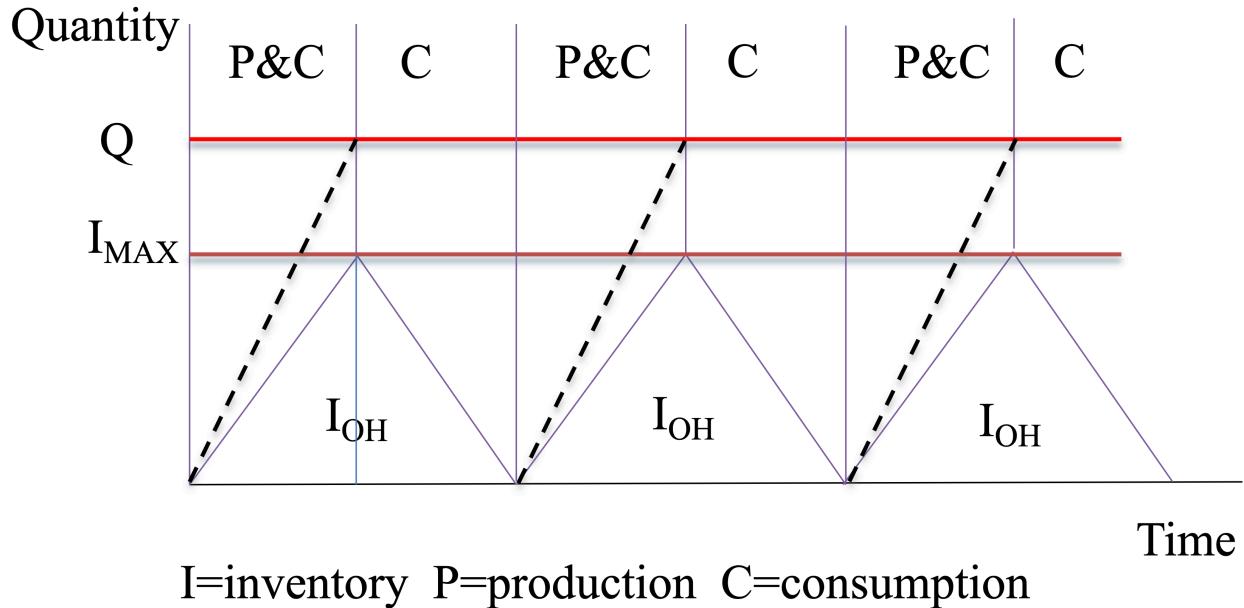
$$Q_0 = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(3,600)(30)}{13}} = 128.9, \text{ round to 129 monitors}$$

$$TC = \frac{Q_0}{2}H + \frac{D}{Q_0}S = \frac{129}{2}13 + \frac{3,600}{129}30 = \$838.50 + \$837.21 = \$1,675.71$$

### 6.3.2 Efficient Production Quantity (EPQ)

- Similar to EOQ as we are still looking at annual inventory cost control
- Rate of production exceeds the rate of consumption
- Since the good is produced in-house the ordering cost associated with EOQ is replaced by a setup cost to change over production equipment

**Inventory Levels:**



**Analysis:**

- Variables

$I_{MAX}$ : maximum inventory level

$Q$ : batch size or production lot size per run

$S$ : setup cost per run (cleanouts, tooling changes, etc.)

$D$ : annual demand

$p$ : production rate per day

$d$ : demand rate (consumption) per day

$H$ : holding cost

- Equations

$$\text{Optimal batch size} = Q = Q_0 = \sqrt{\frac{2DS}{H}} \times \sqrt{\frac{p}{p-d}}$$

$$I_{MAX} = \left(\frac{Q}{p}\right)(p - d)$$

$$Average\ inventory\ level = \frac{I_{MAX}}{2}$$

$$Annual\ holding\ cost = \frac{I_{MAX}}{2} \times H$$

$$Annual\ setup\ costs = \frac{D}{Q} \times S$$

$$Total\ inventory\ control\ costs = \frac{I_{MAX}}{2} \times H + \frac{D}{Q} \times S$$

$$Production\ length = \frac{Q}{p}$$

$$Number\ of\ runs\ per\ year = \frac{D}{Q}$$

$$Cycle\ length\ between\ production\ runs = \frac{Q}{d}$$

**Summary:**

$$TC = \begin{pmatrix} \text{Annual} \\ \text{Holding} \\ \text{Cost} \end{pmatrix} + \begin{pmatrix} \text{Annual} \\ \text{Setup} \\ \text{Cost} \end{pmatrix} = \left(\frac{I_{max}}{2}\right)H + \left(\frac{D}{Q}\right)S$$

$$Cycle\ length = \frac{Q}{d}; \text{Run\ length} = \frac{Q}{p}; I_{max} = \frac{Q}{p}(p - d)$$

**TC** = Total annual cost  
**Q** = Order quantity (units)  
**H** = Annual holding cost  
           per unit  
**D** = Annual Demand  
**S** = Ordering (or setup)  
           cost per order

$$Q_0 = \sqrt{\frac{2DS}{H} \left( \frac{p}{p-d} \right)}$$

**$Q_0$**  = Optimal run or order quantity  
**p** = Production rate  
**d** = Usage or demand rate  
 **$I_{max}$**  = Maximum inventory level

**Example:**

A toy manufacturer uses 48,000 rubber wheels per year for its popular dump truck series. The company can make its own wheels at a rate of 800 per day. The toy trucks are assembled uniformly over the entire year. Holding cost is \$1 per wheel per year. Setup cost for a production run of wheels is \$45. The company operates 240 days per year. Determine the:

- Optimal production run quantity.
- Minimum total annual cost for holding and setup.
- Cycle length for the optimal production run quantity.
- Production run length for the optimal production run quantity.

$$D = 48,000 \text{ wheels per year}$$

$$S = \$45 \text{ per production run}$$

$$H = \$1 \text{ per wheel per year}$$

$$p = 800 \text{ wheels per day}$$

$$d = 48,000 \text{ wheels per } 240 \text{ days, or } 200 \text{ wheels per day}$$

$$\text{a. } Q_0 = \sqrt{\frac{2DS}{H}} \times \sqrt{\frac{p}{p-d}} = \sqrt{\frac{2(48,000)(45)}{1}} \times \sqrt{\frac{800}{800-200}} = 2,400 \text{ wheels}$$

$$\text{b. } TC_{min} = \frac{I_{MAX}}{2} \times H + \frac{D}{Q_0} \times S$$

$$I_{MAX} = \left(\frac{Q}{p}\right)(p - d) = \left(\frac{2,400}{800}\right)(800 - 200) = 1,800 \text{ wheels}$$

$$TC_{min} = \frac{1,800}{2} \times \$1 + \frac{48,000}{2,400} \times \$45 = \$900 + \$900 = \$1,800$$

$$\text{c. } \text{Cycle length} = \frac{Q}{d} = \frac{2,400 \text{ wheels}}{200 \text{ wheels per day}} = 12 \text{ days}$$

$$\text{d. } \text{Production run length} = \frac{Q}{p} = \frac{2,400 \text{ wheels}}{800 \text{ wheels per day}} = 3 \text{ days}$$

### 6.3.3 EOQ with Quantity Discounts

- Quantity Discounts:** Price discounts used as an incentive for purchasers to order in large quantities
- Objective: To compare the benefits of purchase price savings for buying in quantity as opposed to the increase in annual holding costs of maintaining larger average inventories

#### Revised Total Cost Simplification:

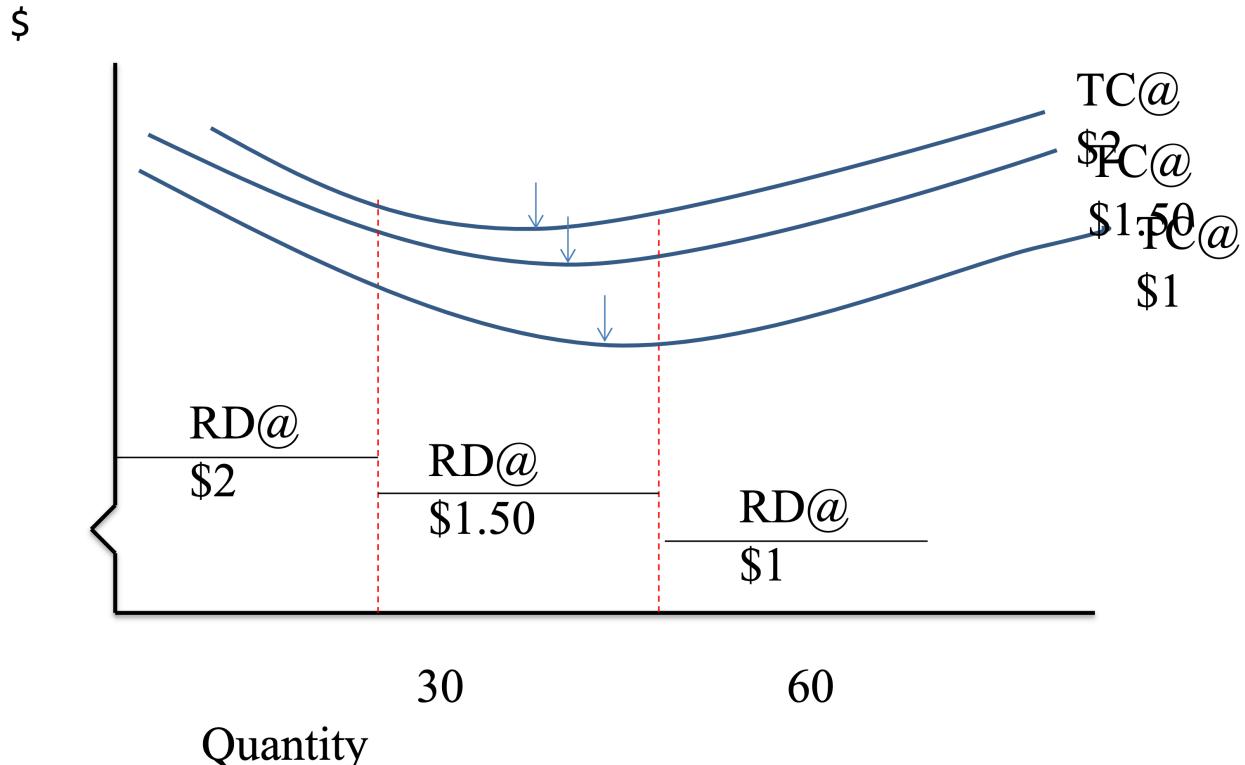
- Without quantity discounts, the efficient order quantity was selected to minimize annual holding and orders cost, such that:

$$\frac{Q}{2} \times H \approx \frac{D}{Q} \times S$$

- With quantity discounts we want to select an order quantity that takes the purchase price into account such that total cost can now be defined as:

$$TC = \frac{Q}{2} \times H + \frac{D}{Q} \times S + RD, \text{ where } R \text{ is the per unit price}$$

### Total Cost Curve with Quantity Discounts:



### Process of Calculating EOQ (Discounts):

- Finding an initial feasible EOQ:** Calculate the EOQ for each price level starting with the lowest price level, until you find a feasible EOQ
- Refining your choice:** If the EOQ for the lowest price unit is feasible, you are done. If it is not, then compare the **Total Costs** for each price break larger than the feasible EOQ with the **Total Cost** of the feasible EOQ

### Example:

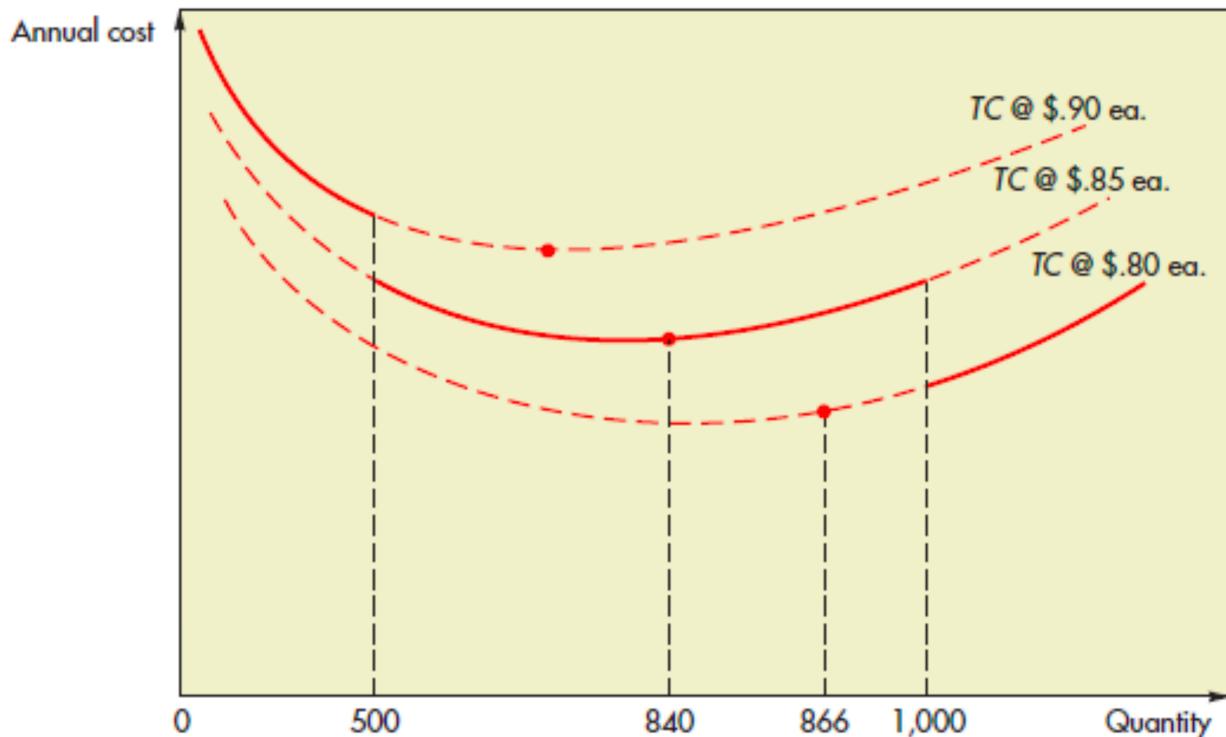
A factory uses 4,000 light bulbs per year. Light bulbs are priced as follows:

1 – 499	.90
500 – 999	.85
>1,000	.80

It costs \$ 30 to prepare a purchase order, receive the goods and pay for them

The firm incurs a 40% interest rate charge to fund operations

- Determine the optimal order size
- Calculate the total annual costs



$$D = 4,000 \text{ light bulbs per year}, S = \$30, H = 0.40R, R = \text{unit price}$$

- Find the EOQ for each price, starting with the lowest, until you locate a feasible EOQ:

$$EOQ_{0.80} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(4,000)(30)}{0.40(0.80)}} = 866 \text{ light bulbs}$$

Because an order size of 866 light bulbs is fewer than 1,000 units, 866 is not feasible for the unit price of \$0.80 per light bulb. Next, try \$0.85 per unit:

$$EOQ_{0.85} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(4,000)(30)}{0.40(0.85)}} = 840 \text{ light bulbs}$$

This EOQ is feasible; it falls in the \$0.85 per light bulb quantity range (500 to 999).

- b) Now calculate the total cost for 840 order size, and compare it to the total cost of the break quantity larger than 840, i.e., 1,000 units:

$$TC = \frac{Q}{2}H + \frac{D}{Q}S + RD$$

$$TC_{840} = \frac{840}{2}(0.34) + \frac{4,000}{840}(30) + 0.85(4,000) = \$3,686$$

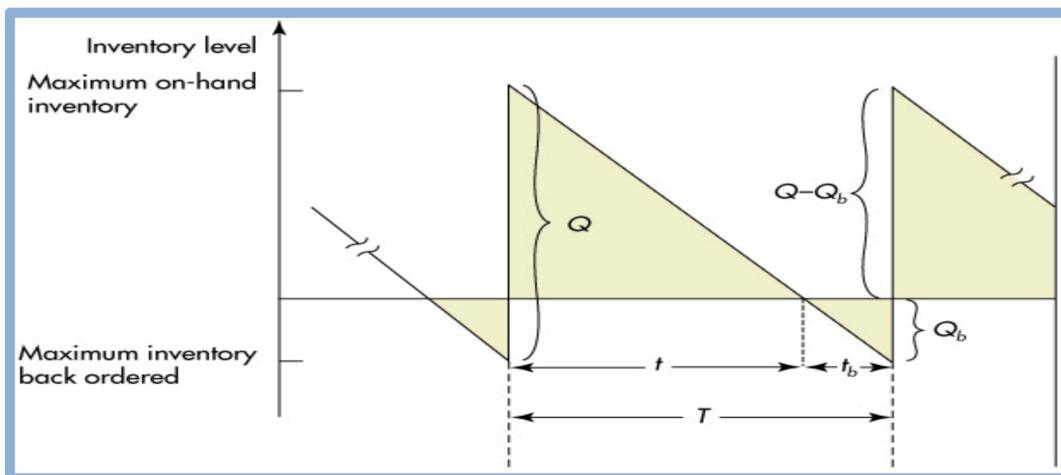
$$TC_{1000} = \frac{1,000}{2}(0.32) + \frac{4,000}{1,000}(30) + 0.80(4,000) = \$3,480$$

Because  $\$3,480 < \$3,686$ , the minimum cost order quantity is 1,000 light bulbs.

### 6.3.4 EOQ with Planned Shortages

#### Assumptions:

- High costs of some goods may encourage suppliers to allow shortages (backorders) if the market is willing and demand can wait
- Backorders will incur shortage costs with these charges proportional to the amount of time that product is backordered



$T$ : length of the order cycle

$t$ : the time during the order cycle when inventory is non-negative

**Variables:**

$Q$ : the quantity that you want to order (demand you want to fill)

$Q_b$ : the quantity that you want to backorder (demand that you are willing to defer until the next order arrives)

**Equations:**

$$TC = \text{annual ordering costs} + \text{annual holding costs} + \text{annual backorder costs}$$

$$TC = \frac{D}{Q} \times S + \left[ \frac{(Q - Q_b)^2}{2Q} \right] \times H + \frac{Q_b^2}{2Q} \times B$$

$$Q = \sqrt{\frac{2DS}{H} \times \frac{H+B}{B}}$$

$$Q_b = Q \times \frac{H}{H+B}$$

**Example:**

Annual demand for a particular model of a refrigerator at an appliance store is 50 units. The holding cost per unit per year is \$200. The back-order cost per unit per year is estimated to be \$500. Ordering cost from the manufacturer is \$10 per order. Determine the optimal order quantity and the back-order quantity per order cycle.

$$D = 50 \text{ units per year}$$

$$H = \$200 \text{ per unit per year}$$

$$B = \$500 \text{ per unit per year}$$

$$S = \$10 \text{ per order}$$

$$Q = \sqrt{\frac{2DS}{H} \times \frac{H+B}{B}} = \sqrt{\frac{2(50)(10)}{200} \times \frac{200+500}{500}} = 2.65, \text{ round to 3 units}$$

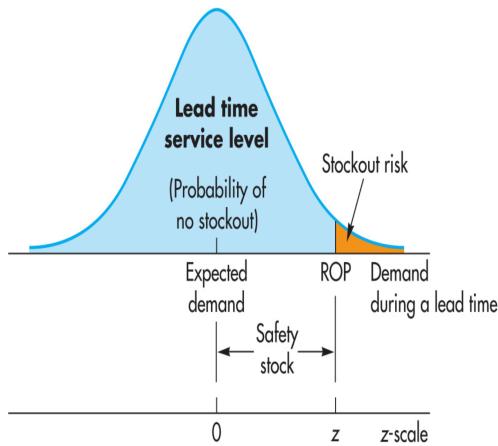
$$Q_b = Q \times \frac{H}{H+B} = 3 \times \frac{200}{200+500} = 0.86, \text{ round to 1}$$

## 7 INVENTORY MANAGEMENT (II)

### 7.1 Fixed Order Quantity – Reorder Point Model

#### Reorder Point:

- Definition: The inventory level that triggers a replacement order.
- $ROP = d \times LT$  with demand  $d$  and lead time  $LT$  being expressed in similar units of time (day, week or month).
- If demand is not constant, then a case can be made for holding extra inventory to act as insurance against a shortage – **Safety Stock**.
  - ROP inventory is now equal to  $(d \times LT) + Safety\ Stock$ .



#### Safety Stocks and Service Levels:

- Lead Time Service Levels (probability that demand  $\leq$  supply during the lead time):

$$Lead\ time\ service\ level = 1 - \frac{number\ short}{lead\ time\ demand}$$

- Annual Service Level (annual fill rate):

$$Annual\ service\ level = 1 - \frac{number\ short}{order\ cycle\ demand}$$

- Both of these service levels assume demand is normally distributed during a lead time:

- $Safety\ stock = z \cdot \sigma_{dLT}$
- If stockout risk =  $1 - lead\ time\ service\ level$ ,  $\rightarrow$  then lead time service level =  $1 - stockout\ risk$
- Then look up this probability on the Standard Normal Distribution Table to get z

### 7.1.1 ROP Using Lead Time Service Level

- When lead time demand data is known:

$$ROP = d \cdot LT + z \cdot \sigma_{dLT}$$

- When only demand is variable:

$$ROP = \bar{d} \cdot LT + z\sqrt{LT} \cdot \sigma_d$$

- When both demand and lead time are variable:

$$ROP = \bar{d} \cdot \bar{LT} + z\sqrt{\bar{LT}\sigma_d^2 + \bar{d}^2\sigma_{LT}^2}$$

### 7.1.2 ROP Using Annual Service Level

- Step 1: Calculate the standardized expected number of units that you will be short during the order cycle  $E(z)$  where:

$$E(z) = \frac{Q(1 - SL_{ANNUAL})}{\sigma_{dLT}}$$

$$SL_{ANNUAL} = \frac{\# \text{ orders filled}}{\# \text{ orders taken}} = \text{fill rate}$$

- Step 2: Convert  $E(z)$  number to its “z” equivalent using Lead Time Service Table
- Step 3: Substitute this “z” value into the calculation of ROP where:

$$ROP = \text{expected demand during order lead time} + z \cdot \sigma_{dLT}$$

## 7.2 Fixed Order Interval – Order Up to Level Model

### 7.2.1 Definition

**Definition:** This model is used when orders are placed at fixed intervals with the amount of the order changed adjusted to bring inventory up to a pre-determined level.

**Setting Up an Order:** Determine how often you will order and then how much you will order.

### 7.2.2 Equations

#### Order Interval Notation:

- $OI$ : order interval (fraction of a year)
- $S$ : fixed order processing costs
- $s$ : variable ordering costs (per SKU)
- $n$ : number of SKUs purchased from the supplier

- $R_j$ : unit cost per  $SKU_j$
- $i$ : annual holding cost rate
- $D_j$ : annual demand of  $SKU_j$

### Step 1: Determining Order Interval

$$Optimal\ order\ interval = \sqrt{\frac{2(S + ns)}{i \sum D_j R_j}}$$

$$Total\ annual\ inventory\ costs = \frac{\sum (D_j \cdot OI) R_j}{2} \cdot i + (S + ns) \frac{1}{OI}$$

where  $Q_j = D_j \cdot OI$

### Step 2: Determining Order Up to Level

- Definition: An up to inventory level is the amount of good that should be sufficient to last until the next order arrives.
- The order quantity is the difference between the order up to level ( $I_{MAX}$ ) and the quantity of the good still on hand at the time of order placement.

$$I_{MAX} = \bar{d}(OI + LT) + z \cdot \sigma_d \sqrt{OI + LT}$$

$$Q(\text{order quantity}) = I_{MAX} - \text{inventory position}$$

### 7.2.3 Examples

The manager of a hardware store has determined, from historical records, that demand for a given type and size of bagged cement during its lead time could be described by a Normal distribution that has a mean of 50 bags and a standard deviation of 5 bags. Answer these questions, assuming that the manager is willing to accept a stock-out risk of no more than 3 percent during a lead time:

- What value of  $z$  is appropriate?
- How many bags of safety stock should be held?
- What reorder point should be used?

*Expected demand during a lead time = 50 bags,  $\sigma_{dLT} = 5$  bags, Stockout risk = 3%*

- Service level of  $1 - 0.03 = 0.97$ , obtain value of  $z = 1.88$  (Table 12-3 on Learn)
- Safety stock =  $z \cdot \sigma_{dLT} = 1.88(5) = 9.40$  bags*
- $ROP = Expected\ demand + Safety\ stock = 50 + 9.40 = 59.40$  round to 59 bags*

A restaurant uses an average of 50 jars of a special sauce each week. Weekly usage of sauce has a standard deviation of 3 jars. The manager is willing to accept no more than a 10 percent risk of stock-out during a lead time, which is 2 weeks. Assume the distribution of usage is Normal.

- Which of the above formulas is appropriate for this situation? Why?
- Determine the value of  $z$ .
- Determine the ROP.

$$\bar{d} = 50 \text{ jars per week}, \sigma_d = 3 \text{ jars per week}, LT = 2 \text{ weeks}, \text{stockout risk} = 10\%$$

$$\text{service level} = 1 - \text{stockout risk} = 1 - 0.10 = 0.90$$

- Because only demand is variable (has a standard deviation):  $ROP = \bar{d} \cdot LT + z\sqrt{LT} \cdot \sigma_d$
- Using a service level of 0.90, we obtain  $z = 1.28$  from the Table 12-3.
- $ROP = \bar{d} \cdot LT + z\sqrt{LT} \cdot \sigma_d = 50(2) + 1.28\sqrt{2}(3) = 105.43$  round to 105 jars

An inventory item has order quantity  $Q = 250$  units, expected demand during a lead time = 50 units, and standard deviation of demand during a lead time  $\sigma_{dLT} = 16$  units. Determine the ROP if the desired annual service level is (a) 0.997, and (b) 0.98.

$$a. E(z) = \frac{Q(1-SL_{annual})}{\sigma_{dLT}} = \frac{250(1-0.997)}{16} = 0.047$$

From Table 12-3,  $E(z) = 0.047$  falls between  $E(z) = 0.048$  with  $z = 1.28$  and  $E(z) = 0.044$  with  $z = 1.32$ . Therefore, using interpolation,  $z = 1.29$ . Finally:

$$ROP = 50 + 1.29(16) = 70.64 \text{ round to 71 units.}$$

$$b. E(z) = \frac{250(1-0.98)}{16} = 0.3125$$

From Table 12-3,  $E(z) = 0.3125$  falls between  $E(z) = 0.324$  with  $z = 0.16$  and  $E(z) = 0.307$  with  $z = 0.20$ . Therefore, using interpolation,  $z = 0.19$ . Finally:

$$ROP = 50 + 0.19(16) = 53.04 \text{ round to 53 units.}$$

Three parts are purchased from the same supplier. The basic cost of placing an order for one of the SKUs is \$1.50. The inclusion of each additional SKU costs \$.50 more. The annual holding cost rate is 24 percent of unit cost. The annual demand (in units) and unit prices are given below. Assume that the buying company works 250 days a year. Calculate the optimal order interval.

SKU	Demand/Year	Price/Unit
1	12,000	.50
2	8,000	.30
3	700	.10

$$S = \$1.50 - \$0.50 = \$1.00, s = \$0.50, i = 0.24$$

$$OI^* = \sqrt{\frac{2(1.00 + 3(0.50))}{0.24(12,000 \times 0.50 + 8,000 \times 0.30 + 700 \times 0.10)}} = 0.0496 \text{ years} = 12.4 \text{ days}$$

Round to 12 days.

Given the following information for a SKU, determine its order up to level and amount to order (i.e., order quantity).

$$\bar{d} = 30 \text{ units per day}, \text{Desired service level} = 99 \text{ percent}$$

$$\sigma_d = 3 \text{ units per day}, \text{Inventory position at reorder time} = 71 \text{ units}$$

$$LT = 2 \text{ days}, OI = 7 \text{ days}$$

$$z = 2.33 \text{ for 99 percent desired service level from Table 12-3}$$

$$I_{max} = \bar{d}(OI + LT) + z\sigma_d\sqrt{OI + LT} = 30(7 + 2) + 2.33(3)\sqrt{7 + 2} = 291 \text{ units}$$

$$\text{Amount to order} = I_{max} - \text{inventory position} = 291 - 71 = 220 \text{ units}$$

## 7.3 Single Period Model

### 7.3.1 Characteristics

- Goods typically cannot be held over to the next period, and when they can they incur significant price reductions
- Objective is to choose an order quantity that will minimize shortage and excess costs.
- Described by examining shortage costs and the costs of having excess goods.

### 7.3.2 Costs

Shortage Costs:

$$C_{SHOR TAGE} = C_S = \text{Revenue per unit} - \text{Cost per unit}$$

Excess Costs:

$$C_{EXCESS} = C_E = \text{Cost per unit} - \text{Salvage value}$$

### 7.3.3 Continuous Stocking Levels

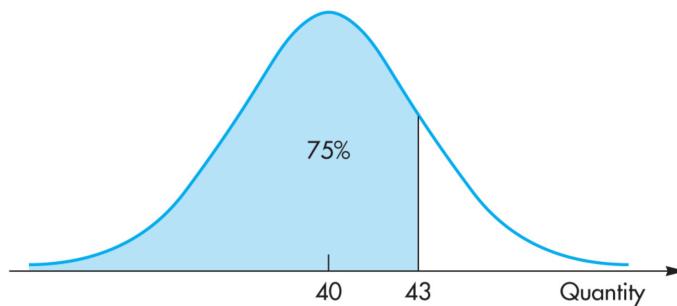
- The service level ( $SL$ ) is the probability that demand will not be greater than the quantity of goods on hand:

$$SL = P(D \leq Q) = \frac{C_S}{C_E + C_S}$$

- Uniform Demand** distributions will have an optimal order quantity that is equal to the sum of the minimum order quantity plus an amount that is equal to the product of the  $SL$  and the difference between minimum and maximum order quantity levels.

#### Normal Distribution of Demand:

- Optimal  $Q = \mu + z\sigma$  where  $\mu$  is the mean of demand and  $\sigma$  is the standard deviation of demand (both of which will be given).
- Solution process is to use  $C_S$  and  $C_E$  values to determine the service level and then use that service level to find the corresponding  $z$ -value and then apply the optimal equation.



### 7.3.4 Example: Continuous Stocking Levels

A cafeteria buys muffins daily. Demand is **approximately normal with a mean of 40 and standard deviation of 5 muffins per day**. The cafeteria pays \$0.20 per muffin and charges \$0.80 per muffin. Find the optimal stocking level.

If service level is 0.75, then from Table 12-3,  $z = 0.675$

$$\text{Optimal} = 40 + 0.675(5) = 43 \text{ muffins}$$

A cafeteria buys muffins daily. Demand **varies uniformly between 30 and 50 muffins per day**. The cafeteria pays \$0.20 per muffin and charges \$0.80 per muffin. Unsold muffins are discarded at the end of the day. Find the service level.

$$C_E = \text{Cost per unit} - \text{Salvage per unit} = \$0.20 - \$0.00 = \$0.20 \text{ per unit}$$

$$C_s = \text{Revenue per unit} - \text{Cost per unit} = \$0.80 - \$0.20 = \$0.60 \text{ per unit}$$

$$SL = \frac{0.60}{0.20 + 0.60} = 0.75$$

$$\text{Optimal} = \min + SL(\max - \min) = 30 + 0.75(50 - 30) = 45 \text{ muffins}$$



### 7.3.5 Discrete Stocking Levels

- In this case, the goal is to choose a **stocking level** that will **just** exceed demand.
- Service level is calculated as it was before with  $SL = \frac{C_S}{C_E+C_S}$  where the results are compared to the cumulative probability levels.

### 7.3.6 Example: Discrete Stocking Levels

Historical records on the use of a spare part for an old press (shown in the following table) are used to estimate usage for the spare part in a similar but new press. Stock-out cost involves downtime expenses and special ordering costs. These average \$4,200 per unit short. The spare part costs \$1,200 each, and an unused part has \$400 salvage value. Determine the optimal stocking level for the spare part.

Number of Spares Used	Probability	Cumulative Probability
0.....	.20	.20
1.....	.40	.60
2.....	.30	.90
3.....	.10	1.00
4 or more.....	<u>.00</u>	<u>1.00</u>

$$C_S = \$4,200, C_E = \$1,200 - \$400 = \$800, SL = \frac{C_S}{C_E+C_S} = \frac{4,200}{4,200+800} = 0.84$$

The cumulative probability column indicates the fraction of time that demand was equal to or less than some amount. For example, demand  $\leq$  one spare occurred 60 percent of the time, or demand  $\leq$  two spares occurred 90 percent of the time. Go down the table until cumulative probabilities just exceed service level of 84 percent. This is 0.90 which relates to two spares.