Composition is a property that says that if there is an arrow from a to b, and an arrow from b to c, there must exist an arrow from a to c.

$$
\begin{tikzcd}
a \arrow[r, "f"] \arrow[rr, bend left, "f \circ g"] & b \arrow[r, "g"] & c
\end{tikzcd}
$$

Compositions have the associative property

$$
\begin{tikzcd}
a \arrow[r, "f"] \arrow[rr, bend left, "g \circ f"] \arrow[rrr, bend left=50, "h \circ (g \circ f)"] \arrow[rrr, bend right=50, "(h \circ g) \circ f"'] & b \arrow[r, "g"] \arrow[rr, bend right, "h \circ g"'] & c \arrow[r, "h"] & d
\end{tikzcd}
$$

$$h \circ (g \circ f) = (h \circ g) \circ f$$

And now some Rust code

```rust
fn main() {
    let s1 = String::from("hello");

    let len = calculate_length(&s1);

    println!("The length of '{}' is {}.", s1, len);
}

fn calculate_length(s: &String) -> usize {
    s.len()
}
```

Let us write $\Delta z = \Delta x + i\Delta y$.

We now compute $f'(z)$ by approaching z from the horizontal direction ($\Delta y = 0$).

$$
\begin{aligned}
f'(z_0) &= \lim_{\Delta x \to 0} \frac{f(z + \Delta x) - f(z)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{f(x + \Delta x + iy) - f(x + iy)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{(u(x + \Delta x, y) + iv(x + \Delta x, y)) - (u(x, y) + iv(x, y))}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{u(x + \Delta x, y) - u(x, y)}{\Delta x} + i\frac{v(x + \Delta x, y) - v(x, y)}{\Delta x} \\
&= \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}
\end{aligned}
$$

We now compute $f'(z)$ by approaching z from the vertical direction ($\Delta x = 0$).

$$
\begin{aligned}
f'(z_0) &= \lim_{\Delta y \to 0} \frac{f(z + \Delta y) - f(z)}{i\Delta y} \\
&= \lim_{\Delta y \to 0} \frac{f(x + iy + i\Delta y) - f(x + iy)}{i\Delta y} \\
&= \lim_{\Delta y \to 0} \frac{(u(x, y + \Delta y) + iv(x, y + \Delta y)) - (u(x, y) + iv(x, y))}{i\Delta y} \\
&= \lim_{\Delta y \to 0} \frac{u(x, y + \Delta y) - u(x, y)}{i\Delta y} + i\frac{v(x, y + \Delta y) - v(x, y)}{i\Delta y} \\
&= \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y}
\end{aligned}
$$

We have found two different representations of $f'(z)$ in terms of the partial derivatives of u and v.

$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y}$$

From this equality we can derive tha **Cauchy-Riemann equations**.

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \qquad -\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$$