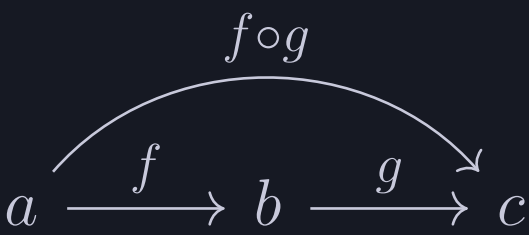
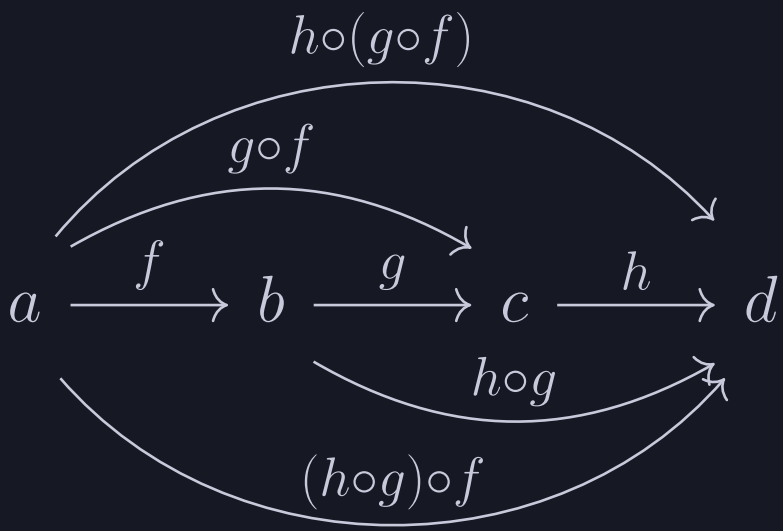


Composition is a property that says that if there is an arrow from  $a$  to  $b$ , and an arrow from  $b$  to  $c$ , there must exist an arrow from  $a$  to  $c$ .



Compositions have the associative property



$$h \circ (g \circ f) = (h \circ g) \circ f$$

And now some Rust code

```
fn main() {
    let s1 = String::from("hello");

    let len = calculate_length(&s1);

    println!("The length of '{}' is {}.", s1, len);
}

fn calculate_length(s: &String) -> usize {
    s.len()
}
```

Let us write  $\Delta z = \Delta x + i\Delta y$ .

We now compute  $f'(z)$  by approaching  $z$  from the horizontal direction ( $\Delta y = 0$ )

$$\begin{aligned} f'(z_0) &= \lim_{\Delta x \rightarrow 0} \frac{f(z + \Delta x) - f(z)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x + iy) - f(x + iy)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{(u(x + \Delta x, y) + iv(x + \Delta x, y)) - (u(x, y) + iv(x, y))}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{u(x + \Delta x, y) - u(x, y)}{\Delta x} + i \frac{v(x + \Delta x, y) - v(x, y)}{\Delta x} \\ &= \frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x} \end{aligned}$$

We now compute  $f'(z)$  by approaching  $z$  from the vertical direction ( $\Delta x = 0$ ).

$$\begin{aligned} f'(z_0) &= \lim_{\Delta y \rightarrow 0} \frac{f(z + \Delta y) - f(z)}{i\Delta y} \\ &= \lim_{\Delta y \rightarrow 0} \frac{f(x + iy + i\Delta y) - f(x + iy)}{i\Delta y} \\ &= \lim_{\Delta y \rightarrow 0} \frac{(u(x, y + \Delta y) + iv(x, y + \Delta y)) - (u(x, y) + iv(x, y))}{i\Delta y} \\ &= \lim_{\Delta y \rightarrow 0} \frac{u(x, y + \Delta y) - u(x, y)}{i\Delta y} + i \frac{v(x, y + \Delta y) - v(x, y)}{i\Delta y} \\ &= \frac{\partial v}{\partial y} - i \frac{\partial u}{\partial y} \end{aligned}$$

We have found two different representations of  $f'(z)$  in terms of the partial derivatives of  $u$  and  $v$ .

$$f'(z) = \frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} - i \frac{\partial u}{\partial y}$$

From this equality we can derive the **Cauchy-Riemann equations**.

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad -\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$$