

Research statement

May 28, 2017

All my current research interests revolve around the theory of factorization systems in $(\infty, 1)$ -category theory.

The present research statement builds on my previous work in higher algebra, and in particular on the topic of “ t -structures as normal torsion theories” initiated in a series of papers [FL15c, FL15a, FL15b] with my advisor D. Fiorenza (La Sapienza, Rome). This constitutes the backbone of my PhD thesis [Lor16] on the topic of t -structures in stable $(\infty, 1)$ -categories.

The results proved so far inspire both practical and abstract questions, related to the general theory of higher-categorical factorization systems, as well as several “concrete” applications of this formalism to (derived) algebraic geometry, stable homotopy theory and noncommutative geometry (an example of this is the quest for a “convenient stable ∞ -category of C^* -algebras”, following [Øst10] and the ∞ -operadic approach outlined in [Mah15]).

My current objective is to pursue this stream of research, expanding the range of application of these techniques. In the following sections I will outline a few ideas, carried on in mutual communication to let the theory and the applications motivate each other. The results obtained so far in the three papers [FL15c, FL15a, FL15b], and subsumed by my thesis [Lor16] are briefly sketched in a preliminary “section zero”.

An overview.

Result 1

In the setting of stable $(\infty, 1)$ -categories the theory of t -structures is subordinated to a flexible and expressive calculus of factorization systems.

This is the central result of [FL15c]: in the setting of stable ∞ -categories a t -structure on (the homotopy category of an ∞ -category) \mathbf{C} is completely determined by a ∞ -categorical factorization system $(\mathcal{E}, \mathcal{M})$ on \mathbf{C} such that

1. the 3-for-2 property holds for both classes \mathcal{E}, \mathcal{M} ; this entails that the subcategories of “cofibrants” $\{X \in \mathbf{C} \mid \begin{bmatrix} 0 \\ \downarrow \\ X \end{bmatrix} \in \mathcal{E}\}$ and “fibrants” $\{Y \in \mathbf{C} \mid \begin{bmatrix} Y \\ \downarrow \\ 0 \end{bmatrix} \in \mathcal{M}\}$ are respectively a coreflective and a reflective subcategory of \mathbf{C} , forming the *aisle* \mathbf{C}_{\geq} and *coaisle* $\mathbf{C}_{<}$ of a t -structure.
2. The reflection $X \rightarrow RX$ is the cofiber of the coreflection $SX \rightarrow X$ (and the coreflection is the fiber of the reflection), in a pullback-pushout diagram

$$\begin{array}{ccc} SX & \longrightarrow & X \\ \downarrow & \lrcorner & \downarrow \\ 0 & \longrightarrow & RX. \end{array}$$

Such factorization systems are called *normal torsion theories* building on the previous work of [CHK85, RT07], where the authors point out (see [RT07, Remark 4.11.(2)]) that the definition of torsion theory “[...] applies, for example, to a triangulated category \mathbf{C} .

Such a category has only weak kernels and weak cokernels and our definition precisely corresponds to torsion theories considered there as pairs \mathcal{F} and \mathcal{T} of colocalizing and localizing subcategories (see [HPS97]).

Result 2

All the classical theory (abelianity of the *heart* $\mathbf{C}_{\geq} \cap \mathbf{C}_{<}[1]$ of a t -structures, the theory of *semiorthogonal decompositions* on derived categories, Postnikov towers in the stable homotopy category. . .) can be expressed in the language of factorization system, giving also elegant new insights.

Our second work [FL15a], which is the natural continuation of [FL15c], outlines some basic consequences of a canonical structure of \mathbb{Z} -poset (that is a set endowed with a monotone action of the group of integers; the algebra of these objects is quite rich of subtleties: see [Bly05]) on the set $\text{ts}(\mathbf{C})$ of t -structures on \mathbf{C} . A t -structure \mathfrak{t} is sent by the generator of \mathbb{Z} to $\mathfrak{t}[1] = (\mathcal{E}[1], \mathcal{M}[1])$ by the *suspension* functor.

Under the “torsio-centric” perspective two apparently disconnected constructions, *Postnikov towers* (induced by the heart of a t -structure) and *semiorthogonal decompositions*, acquire an intrinsic description as, respectively, orbits and fixed points of the \mathbb{Z} -action; it all boils down to specialize to these two extremal particular cases the construction [FL15a, Def. 2.7] of the “tower” $\mathbb{T}(f)$ of a morphism $f: X \rightarrow Y$ with respect to a \mathbb{Z} -equivariant family of t -structures $\mathfrak{t}: J \rightarrow \text{ts}(\mathbf{C})$.

Result 3

The “Grothendieck six functors” formalism becomes clearer in the setting of stable $(\infty, 1)$ -categories; the theory of *recollections* becomes more symmetric and the construction of the t -structure induced by a recollement becomes much more insightful.

The results exposed in our third paper [FL15b] do not constitute a continuation of [FL15a], but rather concentrate on a separate problem: given a “recollection” of stable $(\infty, 1)$ -categories, like

$$\begin{array}{ccccc} & \xleftarrow{i_R} & & \xleftarrow{q_R} & \\ \mathbf{D}^0 & \xleftarrow{i} & \mathbf{D} & \xrightarrow{q} & \mathbf{D}^1 \\ & \xleftarrow{i_L} & & \xleftarrow{q_L} & \end{array}$$

where $i_L \dashv i \dashv i_R$ and $q_L \dashv q \dashv q_R$ and suitable “exactness” properties hold (see [FL15b, Def. 3.1]), it is possible to *glue* two t -structures $\mathfrak{t}_0, \mathfrak{t}_1$, respectively on the categories $\mathbf{D}^0, \mathbf{D}^1$ to a t -structure $\mathfrak{t}_0 \mathbin{\forall} \mathfrak{t}_1$ on \mathbf{D} ; this formalism, introduced in [BBD82] is of capital importance in algebraic geometry and in the theory of perverse sheaves, having also applications in intersection homology [Pf01, GM80, GM83] and representation theory [PS88, KW01].

These results shed a new light on classical material and constitute a new viewpoint on it, since until now the literature somewhat neglected an extensive treatment of the algebraic properties of $\mathbin{\forall}: \text{ts}(\mathbf{D}^0) \times \text{ts}(\mathbf{D}^1) \rightarrow \text{ts}(\mathbf{D})$; on the other hand the flexibility and naturality of the stable setting permit such a study:

- It gives a clear “torsio-centric” translation of the construction for $\mathfrak{t}_0 \mathbin{\forall} \mathfrak{t}_1$ given in [BBD82]; the normal torsion theory corresponding to $\mathfrak{t}_0 \mathbin{\forall} \mathfrak{t}_1$ can be characterized as

$(\mathcal{E}_{01}, \mathcal{M}_{01})$ where

$$\begin{aligned}\mathcal{E}_{01} &= \{f \in \text{hom}(\mathbf{D}) \mid q(f) \in \mathcal{E}_1; i_L(f) \in \mathcal{E}_0\} \\ \mathcal{M}_{01} &= \{f \in \text{hom}(\mathbf{D}) \mid q(f) \in \mathcal{M}_1; i_R(f) \in \mathcal{M}_0\}\end{aligned}$$

- It gives a clear explanation of the associativity properties of \mathbb{Y} : why such a property holds for a stratified space X (see [Pfl01, Ban07, Wei94]), and how to generalize some “associativity data” valid in this case to the general situation of a family of gluing data and recollements (see [FL15b, Def. 5.7]), arranged in a diagram such that each square satisfies (a suitable $(\infty, 2)$ -categorical counterpart of) the Beck-Chevalley condition in algebraic geometry.

Plans for future research.

A complete axiomatization of the theory of higher-categorical factorization systems is an urgent objective, since apart from being a topic of independent interest for the category theory of $(\infty, 1)$ -categories, it would clarify certain “natural” constructions on t -structures (restrictions, extensions, meets and joins as exposed in [Bon13], tensoring under the Deligne product of stable $(\infty, 1)$ -categories [Gro10, §5.3]. . .).

Moreover, given that there are several notions of factorization system adapted to various models for higher category theory, this will eventually yield a unified, model-independent description of t -structures arising in stable homotopy theory, homological algebra, algebraic geometry etc.: a t -structure in a DG-category is, for example, the exact counterpart of a normal torsion theory in the world of *enriched* factorization systems (see [Day74, LW14]), whereas a t -structure in a stable model category is a normal torsion theory in the setting of *homotopy* factorization systems (see [Bou77, Joy08]).

This plan looks particularly promising especially because in a recent (May 2017) joint work [LV17] with S. Virili (Murcia) we managed to apply the torsio-centric approach to the setting of *stable derivators*, thus showing a similar “Rosetta stone” theorem in this framework, and completing the nontrivial part of the model-independency statement. Here, we define factorization systems in the 2-category **Der** exploiting their formal description as algebras for a suitable strict 2-monad (a result of independent interest for the theory of this specific 2-category), and prove that when the derivator is stable, a suitable subclass of “coherent” normal torsion theories (the *derivator-normal torsion theories*) correspond bijectively with t -structures on the underlying category of the derivator, and under relatively mild assumptions induce t -structure on *each* category $\mathbf{D}(J)$ and t -exact functors thereof.

With this result in hand, it is clearly visible a deeper pattern: in particular, we formulate the following

Task 1

Is it possible to find a general, model-independent theory of factorization systems in $(\infty, 1)$ -categories, to be applied to the theory of t -structures (or elsewhere)?

A first step in this respect is to describe precisely the relationship between the existing notions of factorization system in different models of $(\infty, 1)$ -categories.

The initial step is a systematic treatment of “comparison results” between models conjectured (or hinted at) in the literature —several insights are interspersed in Joyal’s notes on quasicategories [Joy08].

A possible general framework for this unification program should be a nice framework to do higher category theory in a model independent way: in light of this remark, it seems feasible to adopt Riehl-Verity’s language of ∞ -cosmoi ([RV15a, RV13, RV14, RV15b, RV15c]) heavily relying on the powerful language of enriched category theory.

This could also provide instances of factorization systems in models for $(\infty, 1)$ -categories still lacking an internal definition of a factorization systems (like Segal spaces and Segal categories), in the same vein we are doing with derivators: in this respect, it is remarkable that Riehl-Verity’s theory can handle even some models for (∞, n) -categories with $n \geq 2$, such as Θ_n -spaces.

We hence seek to isolate the properties of ∞ -cosmoi whose objects allow a *calculus of factorization*, expanding the characterization of these objects as *normal pseudoalgebras* for [a suitable analogue of] the squaring monad [KT93] already exploited in [LV17]. These properties should hold for a sufficiently big class of “naturally arising” ∞ -cosmoi, and we plan to investigate the consequences of imposing this stricter definition.

Such a systematic survey will also fill some awkward gaps in the literature; indeed, despite the fact that there is already some literature on factorization systems in $(\infty, 1)$ -categories, there are aspects of the theory that remain somewhat unexplored. As an example, the parts of the theory which rely on the monadic description of Grandis and Tholen’s *algebraic* factorization systems (such as Garner’s algebraic reformulation of the small object argument) are still missing in the $(\infty, 1)$ -categorical setting (even though a flexible and elegant theory of $(\infty, 1)$ -monads already exists in [Lur11]).

A possible objective in this sense is to revisit (and possibly improve, in the sense of aligning it to the canon of model independence) the definition of a *cofibrantly generated model* $(\infty, 1)$ -category, which has been given in [MG14] (with a view towards applications to Goerss-Hopkins obstruction theory).

A good test bench for a general theory of higher factorization systems in undoubtedly a higher theory of homotopical algebra; a realistic goal is to obtain counterparts of the basic theory and a $(\infty, 1)$ -categorical version of the small object argument, pursuing the questions which are likely to arise along the way. This paves the way to the following

Task 2

Is it possible to find substantial applications for the theory outlined in **Task 1**, to algebraic and noncommutative geometry, and algebraic topology?

This track of research has already been explored to some extent, in a series of works by W. Chachólski [Cha96, Cha97] and Farjoun [Far95], in a way that resembles a theory of “unstable t -structures”, and a substantial progress in this direction has been exposed in [ABFJ17] with a proof of a Blakers-Massey theorem in the setting of ∞ -categories. We plan to apply the machinery of “ t -structures as factorizations”, integrating more and more examples from classical homological algebra, representation theory of algebras, and noncommutative geometry. This would provide a unified point of view on the use of t -structures in stable contexts, thereby facilitating cross-fertilization across disciplines, and will provide an immediate test for the theory outlined in the above section.

A promising field of application involves non-commutative geometry and stable $(\infty, 1)$ -categories, since there have been several exchanges between the two disciplines: [Øst10] outlined a zoology of interesting model structures on categories of cubical C^* -algebras, whereas [Mah15] sheds light on some interesting connections between dendroidal sets (a model for ∞ -operads) and higher-dimensional categories of C^* -algebras, and [Del04] gives a proof of the existence on $C^*\text{-Cat}$ (C^* -categories and $*$ -functors) of a cofibrantly generated simplicial symmetric monoidal model structure that is analogous to the “folk” model structure on **Cat**, whose weak equivalences are the equivalences of categories. All these approaches visibly fit in a complicated web of Quillen equivalences giving “the” theory of $(\infty, 1)$ -noncommutative spaces whose theory of $(\infty, 1)$ -factorization systems and geometry are deeply intertwined.

Relevant applications for the theory of factorization systems can also be found in algebraic geometry and differential topology: this observation stems from the fact that we exploit the power of the stability axiom for a $(\infty, 1)$ -category only when we want to prove our “Rosetta stone” theorem. There is, however, still a great deal of information in his unstable version, since the formalism of “towers” described in our [FL15a], adapted to unstable $(\infty, 1)$ -categories, could describe critical values of Morse functions as *pure values* of \mathbb{R} -families of factorization systems (a generalization of the **A-weaves** of [FL15a, Prop. 5]).

Some of these ideas can be regarded as the naïve counterpart of the approach that [ABFJ17] follow to modernise [Cha97]’s calculus of *closed classes*, because they all gravitate around the compatibility of a sequence of factorization systems $(\mathcal{E}_i, \mathcal{M}_i)_{i \in I}$ with suitable operations on the set I of indices.

As an additional remark, we note that natural factorization systems on categories of low-dimensional cobordisms have already been studied by J. Abadi, and we believe that our point of view can show that these examples all fit into a general theory of factorization systems on the compact (∞, n) -categories¹ **Cob**(n). A survey about (∞, n) -categories of cobordisms and functorial field theories recently appeared in [CS15]; I plan to profit from the experience of prof. Fiorenza in the field (see [FSV15, FV15]) to understand these topics.

Miscellaneous interests in other fields

- I have recently started thinking about the rôle of concreteness in higher category theory. In [LL17] I prove with I. di Liberti (Masaryk University, Brno) that there is a fairly general method to show that under certain assumptions on a model category \mathcal{M} , its homotopy category $\mathrm{ho}(\mathcal{M})$ cannot be concrete with respect to the universe where it is assumed to be locally small. This result is part of an attempt to understand more deeply the relation between (some parts of) set theory and (some parts of) abstract homotopy theory, and will hopefully be the first step towards a theory of ‘ ∞ -concrete’ $(\infty, 1)$ -categories. The idea in a nutshell is that moving to the theory of $(\infty, 1)$ -categories the notion of concreteness ‘splits’ into a countable spectrum of stronger and stronger n -concreteness conditions, in such a way that being ∞ -concrete is the strongest way in which a $(\infty, 1)$ -category can stray from being concrete.

¹Lurie’s proof of the cobordism hypothesis suggests that a natural setting for these categories **Cob**(n) is the world of Segal-like (∞, n) -categories: in light of this, it is obvious why we are interested in exploiting a full-fledged theory of factorizations in generic (∞, n) -categories.

A rather technical but conceptually deep result proved in [Isb64] is that the notion of concreteness for a category \mathbf{C} is tightly related with a set-theoretical condition of the class of ‘generalized subobjects’ of objects of \mathbf{C} ; under relatively mild assumptions, this is a condition on monomorphisms of \mathbf{C} , and this is the starting point to build an analogy between the (classical, 1-categorical) notion of monic in a 1-category and the fact that this notion breaks into a similar countable spectrum of *n-monic arrow* in a $(\infty, 1)$ -category.

- I have recently started a joint work [GL] with F. Genovese (Hasselt) that tries to clarify the rôle of coends in enriched homotopy theory and $(\infty, 1)$ -categories. A flexible theory of coends is still absent from the currently booming field of higher categories (even though there have been some –highly model-dependent– attempts to draw such a theory, [GHN15]); this is somewhat an unforgivable absence given its undeniable expressive power (of which my [Lor15] has been a rather successful survey).

We concentrate on a further development of a large amount of material, scattered in the enriched-category-theory literature [Shu06, CP97, CP90, Gra80] and developing a sound theory of *homotopy weighted limits*; this paves the way to the definition of “coherent co/ends” as the derived counterparts of the hom-weighted co/limits $\int^A T \cong \text{hom} \otimes T$ and $\int_A T \cong \{\text{hom}, T\}$. More in detail, we offer

- A reasonably model-independent approach to the definition and basic properties of coherent coends (working for quasicategories, simplicial categories, model categories and derivators)
- An homotopy coherent approach to Isbell duality, the theory of Isbell envelopes, and Cauchy completeness of $(\infty, 1)$ -categories (embedded in a suitable bicategory of bimodules/profunctors).
- A sketch of the theory the *model bicategory of profunctors*, which is a toy example of a weak 2-category all whose hom-categories carry “compatible” model structures.

Our approach blends a completely formal-categorical language with the expressive power of homotopical algebraic techniques. The final aim is to build a compact definition of coend in a ∞ -cosmos, echoing [RV15d]: if \mathcal{K} is such a ∞ -cosmos and $A \in \mathcal{K}$ is an object, we consider the 2-cell

$$\begin{array}{ccc} A/a & \longrightarrow & A \\ \downarrow & \nearrow & \downarrow 1 \\ * & \xrightarrow{a} & A \end{array}$$

This gives a diagram $A \rightarrow \mathcal{K}: a \mapsto A/a$, whose universal lax cocone is $A/a \rightarrow \text{tw}(A)^{\text{op}}$, and gives rise to a bifibration $\text{tw}(A) \rightarrow A^{\text{op}} \times A$ that mimicks the twisted arrow category of $A \in \mathbf{Cat}$ and serves to define coends as suitable (higher) colimits.

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