Research statement

October 9, 2017

All my current research interests revolve around the theory of factorization systems in $(\infty, 1)$-category theory.

The present research statement builds on my previous work in higher algebra, and in particular on the topic of "t-structures as normal torsion theories" initiated in a series of papers [**FL16**, **FL15a**, **FL15b**] with my advisor D. Fiorenza (La Sapienza, Rome). This constitutes the backbone of my PhD thesis [**Lor16**] on the topic of t-structures in stable $(\infty, 1)$-categories.

The results proved so far inspire both practical and abstract questions, related to the general theory of higher-categorical factorization systems, as well as several "concrete" applications of this formalism to (derived) algebraic geometry, stable homotopy theory and noncommutative geometry (an example of this is the quest for a "convenient stable ∞-category of C*-algebras", following [**Øst10**] and the ∞-operadic approach outlined in [**Mah15**]).

My current objective is to pursue this stream of research, expanding the range of application of these techniques. In the following sections I will outline a few ideas, carried on in mutual communication to let the theory and the applications motivate each other. The results obtained so far in the three papers [**FL16**, **FL15a**, **FL15b**], and subsumed by my thesis [**Lor16**] are briefly sketched in a preliminary "section zero".

An overview.

Result 1

In the setting of stable $(\infty, 1)$-categories the theory of t-structures is subordinated to a flexible and expressive calculus of factorization systems.

This is the central result of [**FL16**]: in the setting of stable ∞-categories a t-structure on (the homotopy category of an ∞-category) $\mathbf{C}$ is completely determined by a ∞-*categorical factorization system* $(\mathcal{E}, \mathcal{M})$ on $\mathbf{C}$ such that

1. the 3-for-2 property holds for both classes $\mathcal{E}, \mathcal{M}$; this entails that the subcategories of "cofibrants" $\left\{X \in \mathbf{C} \mid \left[\begin{smallmatrix} 0 \\ \downarrow \\ X \end{smallmatrix}\right] \in \mathcal{E}\right\}$ and "fibrants" $\left\{Y \in \mathbf{C} \mid \left[\begin{smallmatrix} Y \\ \downarrow \\ 0 \end{smallmatrix}\right] \in \mathcal{M}\right\}$ are respectively a coreflective and a reflective subcategory of $\mathbf{C}$, forming the *aisle* $\mathbf{C}_{\geq}$ and *coaisle* $\mathbf{C}_{<}$ of a t-structure.
2. The reflection $X \to RX$ is the cofiber of the coreflection $SX \to X$ (and the coreflection is the fiber of the reflection), in a pullback-pushout diagram

$$\begin{tikzcd} SX \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner"] & X \arrow[d] \\ 0 \arrow[r] & RX. \end{tikzcd}$$

Such factorization systems are called *normal torsion theories* building on the previous work of [**CHK85**, **RT07**], where the authors point out (see [**RT07**, Remark **4.11.(2)**]) that the definition of torsion theory "[...] applies, for example, to a triangulated category $\mathbf{C}$.

Such a category has only weak kernels and weak cokernels and our definition precisely corresponds to torsion theories considered there as pairs $\mathcal{F}$ and $\mathcal{T}$ of colocalizing and localizing subcategories (see [**HPS97**])".

Result 2

All the classical theory (abelianity of the *heart* $\mathbf{C}_{\geq} \cap \mathbf{C}_{<}[1]$ of a t-structures, the theory of *semiorthogonal decompositions* on derived categories, Postnikov towers in the stable homotopy category...) can be expressed in the language of factorization system, giving also elegant new insights.

Our second work [**FL15a**], which is the natural continuation of [**FL16**], outlines some basic consequences of a canonical structure of $\mathbb{Z}$-*poset* (that is a set endowed with a monotone action of the group of integers; the algebra of these objects is quite rich of subtleties: see [**Bly05**]) on the set $\mathrm{TS}(\mathbf{C})$ of t-structures on $\mathbf{C}$. A t-structure $\mathfrak{t}$ is sent by the generator of $\mathbb{Z}$ to $\mathfrak{t}[1] = (\mathcal{E}[1], \mathcal{M}[1])$ by the *suspension* functor.

Under the "torsio-centric" perspective two apparently disconnected constructions, *Postnikov towers* (induced by the heart of a t-structure) and *semiorthogonal decompositions*, acquire an intrinsic description as, respectively, orbits and fixed points of the $\mathbb{Z}$-action; it all boils down to specialize to these two extremal particular cases the construction [**FL15a**, Def. 2.7] of the "tower" $\text{♖}(f)$ of a morphism $f\colon X \to Y$ with respect to a $\mathbb{Z}$-equivariant family of t-structures $\mathfrak{t}\colon J \to \mathrm{TS}(\mathbf{C})$.

Result 3

The "Grothendieck six functors" formalism becomes clearer in the setting of stable $(\infty,1)$-categories; the theory of *recollements* becomes more symmetric and the construction of the t-structure induced by a recollement becomes much more insightful.

The results exposed in our third paper [**FL15b**] do not constitute a continuation of [**FL15a**], but rather concentrate on a separate problem: given a "recollement" of stable $(\infty,1)$-categories, like

$$\begin{tikzcd} \mathbf{D}^0 \arrow[r, "i" description] & \mathbf{D} \arrow[l, bend right, "i_R"'] \arrow[l, bend left, "i_L"] \arrow[r, "q" description] & \mathbf{D}^1 \arrow[l, bend right, "q_R"'] \arrow[l, bend left, "q_L"] \end{tikzcd}$$

where $i_L \dashv i \dashv i_R$ and $q_L \dashv q \dashv q_R$ and suitable "exactness" properties hold (see [**FL15b**, Def. 3.1]), it is possible to *glue* two t-structures $\mathfrak{t}_0, \mathfrak{t}_1$, respectively on the categories $\mathbf{D}^0, \mathbf{D}^1$ to a t-structure $\mathfrak{t}_0 \yen \mathfrak{t}_1$ on $\mathbf{D}$; this formalism, introduced in [**BBD82**] is of capital importance in algebraic geometry and in the theory of perverse sheaves, having also applications in intersection homology [**Pfl01, GM80, GM83**] and representation theory [**PS88, KW01**].

These results shed a new light on classical material and constitute a new viewpoint on it, since until now the literature somewhat neglected an extensive treatment of the algebraic properties of $\yen\colon \mathrm{TS}(\mathbf{D}^0) \times \mathrm{TS}(\mathbf{D}^1) \to \mathrm{TS}(\mathbf{D})$; on the other hand the flexibility and naturality of the stable setting permit such a study:

- It gives a clear "torsio-centric" translation of the construction for $\mathfrak{t}_0 \yen \mathfrak{t}_1$ given in [**BBD82**]; the normal torsion theory corresponding to $\mathfrak{t}_0 \yen \mathfrak{t}_1$ can be characterized as

$(\mathcal{E}_{01}, \mathcal{M}_{01})$ where

$$\begin{aligned}\mathcal{E}_{01} &= \left\{f \in \hom(\mathbf{D}) \mid q(f) \in \mathcal{E}_1;\ i_L(f) \in \mathcal{E}_0\right\}\\ \mathcal{M}_{01} &= \left\{f \in \hom(\mathbf{D}) \mid q(f) \in \mathcal{M}_1;\ i_R(f) \in \mathcal{M}_0\right\}\end{aligned}$$

- It gives a clear explanation of the associativity properties of ¥: why such a property holds for a stratified space X (see [**Pfl01, Ban07, Wei94**]), and how to generalize some "associativity data" valid in this case to the general situation of a family of gluing data and recollements (see [**FL15b**, Def. 5.7]), arranged in a diagram such that each square satisfies (a suitable $(\infty, 2)$-categorical counterpart of) the Beck-Chevalley condition in algebraic geometry.

Plans for future research.

A complete axiomatization of the theory of higher-categorical factorization systems is an urgent objective, since apart from being a topic of independent interest for the category theory of $(\infty, 1)$-categories, it would clarify certain "natural" constructions on t-structures (restrictions, extensions, meets and joins as exposed in [**Bon13**], tensoring under the Deligne product of stable $(\infty, 1)$-categories [**Gro10**, §5.3]. . .).

Moreover, given that there are several notions of factorization system adapted to various models for higher category theory, this will eventually yield a unified, model-independent description of t-structures arising in stable homotopy theory, homological algebra, algebraic geometry etc.: a t-structure in a DG-category is, for example, the exact counterpart of a normal torsion theory in the world of *enriched* factorization systems (see [**Day74, LW14**]), whereas a t-structure in a stable model category is a normal torsion theory in the setting of *homotopy* factorization systems (see [**Bou77, Joy08**]).

This plan looks particularly promising especially because in a recent (May 2017) joint work [**LV17**] with S. Virili (Murcia) we managed to apply the torsio-centric approach to the setting of *stable derivators*, thus showing a similar "Rosetta stone" theorem in this framework, and completing the nontrivial part of the model-independency statement. Here, we define factorization systems in the 2-category **Der** exploiting their formal description as algebras for a suitable strict 2-monad (a result of independent interest for the theory of this specific 2-category), and prove that when the derivator is stable, a suitable subclass of "coherent" normal torsion theories (the *derivator-normal torsion theories*) correspond bijectively with t-structures on the underlying category of the derivator, and under relatively mild assumptions induce t-structure on *each* category $\mathbf{D}(J)$ and t-exact functors thereof.

With this result in hand, it is clearly visible a deeper pattern: in particular, we formulate the following

Task 1

Is it possible to find a general, model-independent theory of factorization systems in $(\infty, 1)$-categories, to be applied to the theory of t-structures (or elsewhere)?

A first step in this respect is to describe precisely the relationship between the existing notions of factorization system in different models of $(\infty, 1)$-categories.

The initial step is a systematic treatment of "comparison results" between models conjectured (or hinted at) in the literature —several insights are interspersed in Joyal's notes on quasicategories [**Joy08**].

A possible general framework for this unification program should be a nice framework to do higher category theory in a model independent way: in light of this remark, it seems feasible to adopt Riehl-Verity's language of ∞-cosmoi ([**RV15a**, **RV13**, **RV14**, **RV15b**, **RV15c**]) heavily relying on the powerful language of enriched category theory.

This could also provide instances of factorization systems in models for $(\infty, 1)$-categories still lacking an internal definition of a factorization systems (like Segal spaces and Segal categories), in the same vein we are doing with derivators: in this respect, it is remarkable that Riehl-Verity's theory can handle even some models for (∞, n)-categories with $n \geq 2$, such as Θ_n-spaces.

We hence seek to isolate the properties of ∞-cosmoi whose objects allow a *calculus of factorization*, expanding the characterization of these objects as *normal pseudoalgebras* for [a suitable analogue of] the squaring monad [**KT93**] already exploited in [**LV17**]. These properties should hold for a sufficiently big class of "naturally arising" ∞-cosmoi, and we plan to investigate the consequences of imposing this stricter definition.

Such a systematic survey will also fill some awkward gaps in the literature; indeed, despite the fact that there is already some literature on factorization systems in $(\infty, 1)$-categories, there are aspects of the theory that remain somewhat unexplored. As an example, the parts of the theory which rely on the monadic description of Grandis and Tholen's *algebraic* factorization systems (such as Garner's algebraic reformulation of the small object argument) are still missing in the $(\infty, 1)$-categorical setting (even though a flexible and elegant theory of $(\infty, 1)$-monads already exists in [**Lur11**]).

A possible objective in this sense is to revisit (and possibly improve, in the sense of aligning it to the canon of model independence) the definition of a *cofibrantly generated model* $(\infty, 1)$-*category*, which has been given in [**MG14**] (with a view towards applications to Goerss-Hopkins obstruction theory).

A good test bench for a general theory of higher factorization systems in undoubtedly a higher theory of homotopical algebra; a realistic goal is to obtain counterparts of the basic theory and a $(\infty, 1)$-categorical version of the small object argument, pursuing the questions which are likely to arise along the way. This paves the way to the following

Task 2

Is it possible to find substantial applications for the theory outlined in **Task 1**, to algebraic and noncommutative geometry, and algebraic topology?

This track of research has already been explored to some extent, in a series of works by W. Chachólski [**Cha96**, **Cha97**] and Farjoun [**Far95**], in a way that resembles a theory of "unstable t-structures", and a substantial progress in this direction has been exposed in [**ABFJ17**] with a proof of a Blakers-Massey theorem in the setting of ∞-categories. We plan to apply the machinery of "t-structures as factorizations", integrating more and more examples from classical homological algebra, representation theory of algebras, and noncommutative geometry. This would provide a unified point of view on the use of t-structures in stable contexts, thereby facilitating cross-fertilization across disciplines, and will provide an immediate test for the theory outlined in the above section.

A promising field of application involves non-commutative geometry and stable $(\infty, 1)$-categories, since there have been several exchanges between the two disciplines: [**Øst10**] outlined a zoology of interesting model structures on categories of cubical C*-algebras, whereas [**Mah15**] sheds light on some interesting connections between dendroidal sets (a model for ∞-operads) and higher-dimensional categories of C*-algebras, and [**Del04**] gives a proof of the existence on C*-**Cat** (*C*-categories* and $*$-functors) of a cofibrantly generated simplicial symmetric monoidal model structure that is analogous to the "folk" model structure on **Cat**, whose weak equivalences are the equivalences of categories. All these approaches visibly fit in a complicated web of Quillen equivalences giving "the" theory of $(\infty, 1)$-noncommutative spaces whose theory of $(\infty, 1)$-factorization systems and geometry are deeply intertwined.

Relevant applications for the theory of factorization systems can also be found in algebraic geometry and differential topology: this observation stems from the fact that we exploit the power of the stability axiom for a $(\infty, 1)$-category only when we want to prove our "Rosetta stone" theorem. There is, however, still a great deal of information in his unstable version, since the formalism of "towers" described in our [**FL15a**], adapted to unstable $(\infty, 1)$-categories, could describe critical values of Morse functions as *pure values* of $\mathbb{R}$-families of factorization systems (a generalization of the **A**-*weaves* of [**FL15a**, Prop. 5]).

Some of these ideas can be regarded as the naïve counterpart of the approach that [**ABFJ17**] follow to modernise [**Cha97**]'s calculus of *closed classes*, because they all gravitate around the compatibility of a sequence of factorization systems $(\mathcal{E}_i, \mathcal{M}_i)_{i\in I}$ with suitable operations on the set I of indices.

As an additional remark, we note that natural factorization systems on categories of low-dimensional cobordisms have already been studied by J. Abadi, and we believe that our point of view can show that these examples all fit into a general theory of factorization systems on the compact (∞, n)-categories[1] $\mathbf{Cob}(n)$. A survey about (∞, n)-categories of cobordisms and functorial field theories recently appeared in [**CS15**]; I plan to profit from the experience of prof. Fiorenza in the field (see [**FSV15, FV15**]) to understand these topics.

Miscellaneous interests in other fields

- I have recently started thinking about the rôle of concreteness in higher category theory. In a joint work with I. di Liberti (Masaryk University, Brno) [**LL17**] I propose a fairly general method to show that under certain assumptions on a model category $\mathcal{M}$, its homotopy category $\mathrm{ho}(\mathcal{M})$ cannot be concrete with respect to the universe where it is assumed to be locally small. This result will hopefully be the first step towards a theory of '∞-concrete' $(\infty, 1)$-categories. Moving to the theory of $(\infty, 1)$-categories, the notion of concreteness 'splits' into a countable spectrum of stronger and stronger n-concreteness conditions, in such a way that being ∞-*concrete* is the strongest way in which a $(\infty, 1)$-category can stray from being concrete.
  A rather technical but conceptually deep result proved in [**Isb64**] is that the notion of concreteness for a category **C** is tightly related with a set-theoretical condition of the

[1] Lurie's proof of the cobordism hypothesis suggests that a natural setting for these categories $\mathbf{Cob}(n)$ is the world of Segal-like (∞, n)-categories: in light of this, it is obvious why we are interested in exploiting a full-fledged theory of factorizations in generic (∞, n)-categories.

class of ‘generalized subobjects’ of objects of **C**; under relatively mild assumptions, this is a condition on monomorphisms of **C**, and this is the starting point to build an analogy between the (classical, 1-categorical) notion of monic in a 1-category and the fact that this notion breaks into a similar countable spectrum of *n-monic arrow* in a $(\infty, 1)$-category.

- I have recently started a joint work [GL] with F. Genovese (UHasselt) that tries to clarify the rôle of co/end calculus in enriched homotopy theory and $(\infty, 1)$-categories. A flexible theory to manipulate co/ends is still absent from the currently booming field of higher category theory (even though there have been some –highly model-dependent– attempts like [GHN15]); this is somewhat an unforgivable absence, given the undeniable expressive power of this calculus (of which my [Lor15] has been a rather successful survey).
  A large amount of material is scattered in the literature about enriched category theory [Shu06, CP97, CP90, Gra80], tackling a sound theory of *homotopy weighted limits*; this paves the way to the definition of “coherent co/ends” as the derived counterparts of the hom-weighted co/limits $\int^A T \cong \hom \otimes T$ and $\int_A T \cong \{\hom, T\}$. These different approaches can be synthesized, and after this one gets
  - a reasonably model-independent approach to the definition and basic properties of coherent coends (working for quasicategories, simplicial categories, model categories and derivators);
  - an homotopy coherent and formal-categorical approach to Isbell duality, the theory of Isbell envelopes, and Cauchy completeness of $(\infty, 1)$-categories (embedded in a suitable bicategory of profunctors);
  - a sketch of the theory the *model bicategory of profunctors*, which is a toy example of a *model 2-category*: a bicategory all whose hom-categories carry “compatible” model structures.

  Our approach blends a completely formal-categorical language with the expressive power of homotopical-algebraic techniques. The final aim is to build a compact definition of coend in a ∞-cosmos, echoing [RV15d]: if $\mathcal{K}$ is such a ∞-cosmos and $A \in \mathcal{K}$ is an object, we consider the 2-cell

$$\begin{tikzcd} A/a \arrow[r] \arrow[d] & A \arrow[d, "1"] \arrow[dl, phantom, "\swarrow"] \\ * \arrow[r, "a"'] & A \end{tikzcd}$$

  This gives a diagram $A \to \mathcal{K} : a \mapsto A/a$, whose universal lax cocone is $A/a \to \textsc{tw}(A)^{\mathrm{op}}$, and gives rise to a bifibration $\textsc{tw}(A) \to A^{\mathrm{op}} \times A$ that mimicks the twisted arrow category of $A \in$ **Cat** and serves to define coends as suitable (higher) colimits.

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