Fosco Loregian

May 19 2020

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  2-categories; finishing my first book
- Tallinna Tehnikaülikooli - Tallinn EE 🇪🇪

  2-categories; functorial semantics; categorical probability theory and its applications

STABLE HOMOTOPY THEORY

∞-categories: a thickening of the notion of category, suitable for homotopy-coherent mathematics (math.AG, math.AT, math.LO, cs.PL…).

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Turns out some parts of Mathematics are easier if stated in these terms:

- homological algebra : the scary part of algebraic topology
  higher algebra: the linear algebra of ∞-categories
- 1-topos theory : a synthetic type theory
  ∞-topos theory: a synthetic homotopy theory of homotopy types

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- with all finite limits and colimits
- such that a square is cartesian iff cocartesian

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- The homotopy category of a stable ∞-cat is always triangulated.
- Sending an abelian category $\mathcal{A}$ into its derived category has a nice and clear universal property stated in terms of the heart of a canonical t-structure.
- Stable, rational, p-adic, ... homotopy theory become pieces of the commutative algebra of ∞-categories.

Each PhD starts with a question

A t-structure on a triangulated $\mathcal{D}$ is a pair of triangulated subcategories of $\mathcal{D}$ such that every object X lies in a sequence

$$X_{\leq} \to X \to X_{\geq} \to X_{\leq}[1]$$

[FL14] : On stable ∞-categories a t-structure is a factorization system (E, M)

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- and the category of M-fibrant objects is reflective
- cof/fib replacement = $\pm$ truncation

Plan: redo t-structures (w/ Domenico)

- [FLM15] The set of t-structures has a natural choice of $\mathbb{Z}$-action ($\mathbb{Z}$ = the integers); so, study $\mathbb{Z}$-equivariant monotone maps from a poset P to $TS(\mathcal{C})$. These are called slicings

  apply to: describe Bridgeland stability manifolds [L-PhD], and Postnikov towers on ∞-toposes.

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  apply to: describe Bridgeland stability manifolds [L-PhD], and Postnikov towers on ∞-toposes.
- [FL15b] Every stratified manifold $(X, \mathfrak{s})$ generates a pair of t-structure that can be glued together

  apply to: recollements, stratified schemes, representation of algebras

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Blakers-Massey in positive characteristic is a theorem about factorization systems.

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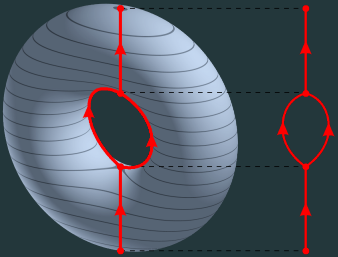

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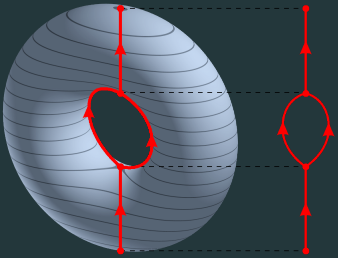

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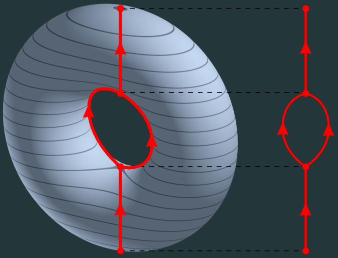

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Morse theory is the theory of suitable factorization systems on Bord(n).

critical points of a Morse function correspond to

critical values [L-PhD, Ch.7] of a certain slicing $J : \mathbb{R} \to FS(Bord(n))$.

DERIVATORS

The formal category theory of derivators

A derivator is a strict 2-functor

$$\mathbb{D} : \mathbf{Cat}^{\mathrm{op}} \to \mathbf{CAT}$$

satisfying stacky conditions. They form the 2-category **Der**.

They subsume most of ∞-category theory; in particular, their stable homotopy.

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[Lor18] : reflective subderivators correspond to reflective factorization systems, and to algebras for idempotent monads

(the formal theory of monads [*S80*] still holds in **Der**)

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[1] A 2-categorical device to encode the calculus of pointwise Kan extensions.

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- notions of accessible and locally presentable derivator using the theory of LPAO in a Yoneda structure, done in [DLL18];

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- profunctors between derivators; fibered derivators;

  operads in derivator theory; applications in representation theory of algebras, stable homotopy, ...

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COENDS AND DG-STUFF

Coends

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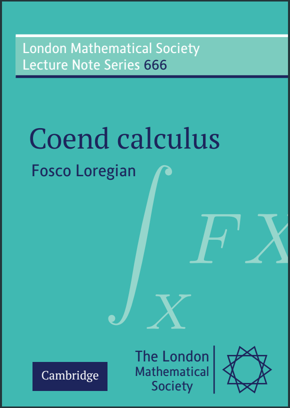

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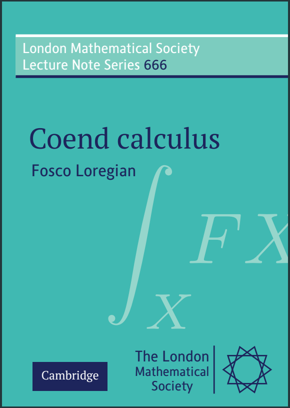

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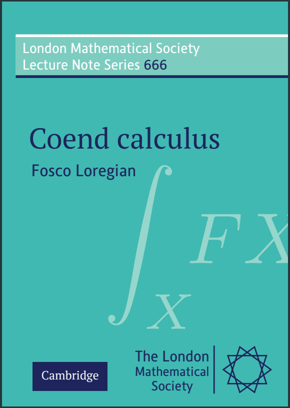

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- They find application in (monoidal) category theory, algebraic topology, algebraic geometry, categorical logic, representation theory (see ch.7 for an application to DG-categories), functional programming…
- The book is being extensively cited (45 citations on Scholar May 18, 2020)

DG-stuff

In [L20, 7.2.2]

For example: if $\mathcal{A}$ is any dg-category its identity profunctor $\mathcal{A} \rightsquigarrow \mathcal{A}$ is a functor $\mathcal{A}^{\mathrm{op}} \boxtimes \mathcal{A} \to \mathrm{Ch}(\mathbb{Z})$, so that the coherent end

$$\oint_A \mathcal{A}(A, A) \tag{7.82}$$

i.e. the object of derived natural transformations of the identity functor $\mathrm{id}_{\mathcal{A}}$, recovers the *Hochschild complex* of $\mathcal{A}$. Then, if $\mathcal{A}$ is an associative algebra regarded as a one-object dg-category concentrated in degree zero, the object $H^n(\int_* A)$ is the *Hochschild cohomology* of A, understood in the classical sense of, say, [Pie82, Ch. 11].

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Applications to Kuznetsov-Lunts categorical resolutions of singularities: a smooth DG-category is a $\mathcal{D}$ such that its identity profunctor $h : \mathcal{D} \rightsquigarrow \mathcal{D}$ is a perfect object (read as: a variety is smooth if the diagonal map $\Delta : X \to X \times X$ is smooth)

TEACHING AND ORGANIZATIONAL ACTIVITIES

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- 2020 Category theory course Teacher @taltech. Mentoring activity for MSc students interested in category theory in CS.

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- Reviewer for zbMath and AMS.

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- more in detail, "2-semantics" of algebraic theories:

  profunctorial PROPs and theories, categorical algebra of cartesian bicategories…

Reach me out at my web page:

> A human being should be able to change a diaper, plan an invasion, butcher a hog, conn a ship, design a building, write a sonnet, balance accounts, build a wall, set a bone, comfort the dying, take orders, give orders, cooperate, act alone, solve equations, analyze a new problem, pitch manure, program a computer, cook a tasty meal, fight efficiently, die gallantly. Specialization is for insects.
>
> R. Heinlein