**Fosco Loregian**

TAL TECH

May 19 2020

# Past & present positions

- Ph.D. at SISSA - Trieste IT 🇮🇹 (but advisor in Rome, D. Fiorenza)

  Stable homotopy theory, ∞-categories, derived AG

# Past & present positions

- Ph.D. at SISSA - Trieste IT 🇮🇹 (but advisor in Rome, D. Fiorenza)

  Stable homotopy theory, ∞-categories, derived AG

- University of Western Ontario - London CA 🇨🇦

  ∞-categories, derivators

## Past & present positions

- Ph.D. at SISSA - Trieste IT (but advisor in Rome, D. Fiorenza)

  Stable homotopy theory, ∞-categories, derived AG

- University of Western Ontario - London CA

  ∞-categories, derivators

- Masaryk University - Brno CZ

  Accessible categories, derivators, 2-categories

# Past & present positions

- Ph.D. at SISSA - Trieste IT 🇮🇹 (but advisor in Rome, D. Fiorenza)

  Stable homotopy theory, $\infty$-categories, derived AG

- University of Western Ontario - London CA 🇨🇦

  $\infty$-categories, derivators

- Masaryk University - Brno CZ 🇨🇿

  Accessible categories, derivators, 2-categories

- Max Planck Inst. für Math. - Bonn D 🇩🇪

  2-categories, derivators, applied category theory

# Past & present positions

- Ph.D. at SISSA - Trieste IT (but advisor in Rome, D. Fiorenza)

  Stable homotopy theory, ∞-categories, derived AG

- University of Western Ontario - London CA

  ∞-categories, derivators

- Masaryk University - Brno CZ

  Accessible categories, derivators, 2-categories

- Max Planck Inst. für Math. - Bonn D

  2-categories, derivators, applied category theory

- Centro de Matemàtica - Coimbra PT

  2-categories; finishing my first book

## Past & present positions

- Ph.D. at SISSA - Trieste IT 🇮🇹 (but advisor in Rome, D. Fiorenza)

  Stable homotopy theory, ∞-categories, derived AG
- University of Western Ontario - London CA 🇨🇦

  ∞-categories, derivators
- Masaryk University - Brno CZ 🇨🇿

  Accessible categories, derivators, 2-categories
- Max Planck Inst. für Math. - Bonn D 🇩🇪

  2-categories, derivators, applied category theory
- Centro de Matemàtica - Coimbra PT 🇵🇹

  2-categories; finishing my first book
- Tallinna Tehnikaülikooli - Tallinn EE 🇪🇪

  2-categories; functorial semantics; categorical probability theory and its applications

# STABLE HOMOTOPY THEORY

$\infty$-categories: a thickening of the notion of category, suitable for homotopy-coherent mathematics (math.AG, math.AT, math.LO, cs.PL…).

$\infty$-categories: a thickening of the notion of category, suitable for homotopy-coherent mathematics (math.AG, math.AT, math.LO, cs.PL…).

A stable $\infty$-category is an $\infty$-category

- with all finite limits and colimits
- such that a square is cartesian iff cocartesian

- The homotopy category of a stable $\infty$-cat is always triangulated.

A stable $\infty$-category is an $\infty$-category

- with all finite limits and colimits
- such that a square is cartesian iff cocartesian

- The homotopy category of a stable $\infty$-cat is always triangulated.
- Sending an abelian category $\mathcal{A}$ into its derived category has a nice and clear universal property stated in terms of the heart of a canonical $t$-structure.

A stable $\infty$-category is an $\infty$-category

- with all finite limits and colimits
- such that a square is cartesian iff cocartesian

- The homotopy category of a stable $\infty$-cat is always triangulated.
- Sending an abelian category $\mathcal{A}$ into its derived category has a nice and clear universal property stated in terms of the heart of a canonical $t$-structure.
- Stable, rational, $p$-adic, ... homotopy theory become pieces of the commutative algebra of $\infty$-categories.

# Each PhD starts with a question

A t-structure on a triangulated $\mathcal{D}$ is a pair of triangulated subcategories of $\mathcal{D}$ such that every object X lies in a sequence

$$X_{\leq} \to X \to X_{\geq} \to X_{\leq}[1]$$

[FL14] : On stable $\infty$-categories a t-structure is a factorization system $(E, M)$

- such that $E$ and $M$ are 3-for-2 classes

# Each PhD starts with a question

A t-structure on a triangulated $\mathcal{D}$ is a pair of triangulated subcategories of $\mathcal{D}$ such that every object X lies in a sequence

$$X_{\leq} \to X \to X_{\geq} \to X_{\leq}[1]$$

[FL14] : On stable $\infty$-categories a t-structure is a factorization system $(E, M)$

- such that $E$ and $M$ are 3-for-2 classes
- thus the category of $E$-cofibrant objects is coreflective

# Each PhD starts with a question

A t-structure on a triangulated $\mathcal{D}$ is a pair of triangulated subcategories of $\mathcal{D}$ such that every object X lies in a sequence

$$X_{\leq} \to X \to X_{\geq} \to X_{\leq}[1]$$

[FL14] : On stable ∞-categories a t-structure is a factorization system $(E, M)$

- such that $E$ and $M$ are 3-for-2 classes
- thus the category of $E$-cofibrant objects is coreflective
- and the category of $M$-fibrant objects is reflective

## Each PhD starts with a question

A t-structure on a triangulated $\mathcal{D}$ is a pair of triangulated subcategories of $\mathcal{D}$ such that every object X lies in a sequence

$$X_{\leq} \to X \to X_{\geq} \to X_{\leq}[1]$$

[FL14] : On stable $\infty$-categories a t-structure is a factorization system $(E, M)$

- such that $E$ and $M$ are 3-for-2 classes
- thus the category of $E$-cofibrant objects is coreflective
- and the category of $M$-fibrant objects is reflective
- cof/fib replacement = $\pm$ truncation

## Plan: redo $t$-structures (w/ Domenico)

- [FLM15] The set of t-structures has a natural choice of $\mathbb{Z}$-action ($\mathbb{Z}$ = the integers); so, study $\mathbb{Z}$-equivariant monotone maps from a poset $P$ to $TS(\mathcal{C})$. These are called slicings

  apply to: describe Bridgeland stability manifolds [L-PhD], and Postnikov towers on ∞-toposes.

# Plan: redo $t$-structures (w/ Domenico)

- [FLM15] The set of t-structures has a natural choice of $\mathbb{Z}$-action ($\mathbb{Z}$ = the integers); so, study $\mathbb{Z}$-equivariant monotone maps from a poset $P$ to $TS(\mathcal{C})$. These are called slicings

  apply to: describe Bridgeland stability manifolds [L-PhD], and Postnikov towers on $\infty$-toposes.

- [FL15b] Every stratified manifold $(X, \mathfrak{s})$ generates a pair of t-structure that can be glued together

  apply to: recollements, stratified schemes, representation of algebras

## Plan: redo $t$-structures (w/ Domenico)

- The set of slicings on a stable $\infty$-category has a metrizable topology

# Plan: redo $t$-structures (w/ Domenico)

- The set of slicings on a stable $\infty$-category has a metrizable topology
- The space

$$\{J\colon \mathbb{R} \to TS(\mathcal{C}) \mid J \text{ is Sorgenfrey continuous}\}$$

is an interesting set [L-PhD]. Bridgeland: demistified.

# Plan: redo $t$-structures (w/ Domenico)

- The set of slicings on a stable ∞-category has a metrizable topology
- The space

$$\{J\colon \mathbb{R} \to TS(\mathcal{C}) \mid J \text{ is Sorgenfrey continuous}\}$$

is an interesting set [L-PhD]. Bridgeland: demistified.

**Conjecture**

Study

$$\{J\colon \operatorname{Spec}(\mathbb{Z}) \to TS(\mathcal{D}(X_p)) \mid J \text{ is Zariski continuous}\}$$

($X_p$ a variety in positive characteristic) to get something about motivic t-structure.

# Plan: redo $t$-structures (w/ Domenico)

- The set of slicings on a stable ∞-category has a metrizable topology
- The space

$$\{J\colon \mathbb{R} \to TS(\mathcal{C}) \mid J \text{ is Sorgenfrey continuous}\}$$

is an interesting set [L-PhD]. Bridgeland: demistified.

**Conjecture**

Study

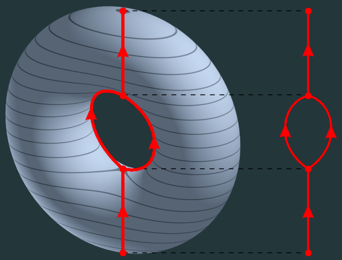
$$\{J\colon \mathrm{Spec}(\mathbb{Z}) \to TS(\mathcal{D}(X_p)) \mid J \text{ is Zariski continuous}\}$$

($X_p$ a variety in positive characteristic) to get something about motivic t-structure.

**Conjecture**

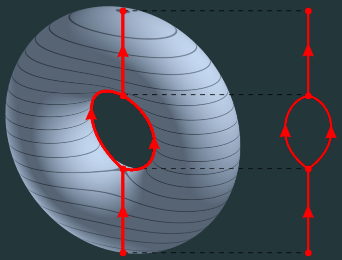
Blakers-Massey in positive characteristic is a theorem about factorization systems.

# Todo: Morse theory is a theory of FS



**Bord**$(n)$ is the free $(\infty,n)$-symmoncat on the point.

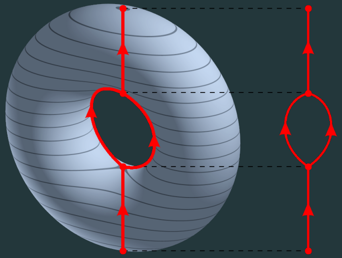
# Todo: Morse theory is a theory of FS



**Bord**$(n)$ is the free $(\infty, n)$-symmoncat on the point.

Tensor functors $Z : Bord(n) \to Vect$ are completely classified.

# Todo: Morse theory is a theory of FS



**Bord**$(n)$ is the free $(\infty,n)$-symmoncat on the point.

Tensor functors
$Z : Bord(n) \to Vect$
are completely classified.

Morse theory is the theory of suitable factorization systems on Bord(n).

critical points of a Morse function correspond to

critical values [L-PhD, Ch.7] of a certain slicing $J : \mathbb{R} \to FS(Bord(n))$.

# DERIVATORS

## The formal category theory of derivators

A derivator is a strict 2-functor

$$\mathbb{D} : \mathbf{Cat}^{\mathsf{op}} \to \mathbf{CAT}$$

satisfying stacky conditions. They form the 2-category **Der**.

They subsume most of ∞-category theory; in particular, their stable homotopy.

# The formal category theory of derivators

A derivator is a strict 2-functor

$$\mathbb{D} : \mathbf{Cat}^{\mathrm{op}} \to \mathbf{CAT}$$

satisfying stacky conditions. They form the 2-category **Der**.

They subsume most of $\infty$-category theory; in particular, their stable homotopy.

[LV17] : A t-structure on a stable derivator is still a certain kind of factorization system;

FS are still strict 2-algebras for the "squaring" 2-monad $(\_)^2$

# The formal category theory of derivators

A derivator is a strict 2-functor

$$\mathbb{D} : \mathbf{Cat}^{\mathrm{op}} \to \mathbf{CAT}$$

satisfying stacky conditions. They form the 2-category **Der**.

They subsume most of ∞-category theory; in particular, their stable homotopy.

[LV17] : A t-structure on a stable derivator is still a certain kind of factorization system;

FS are still strict 2-algebras for the ”squaring” 2-monad $(\_)^2$

[Lor18] : reflective subderivators correspond to reflective factorization systems, and to algebras for idempotent monads

(the formal theory of monads [*S80*] still holds in **Der**)

# The formal category theory of derivators

There is a Yoneda structure[1] on the 2-category of derivators

[1] A 2-categorical device to encode the calculus of pointwise Kan extensions.

# The formal category theory of derivators

There is a Yoneda structure[1] on the 2-category of derivators

- notions of accessible and locally presentable derivator using the theory of LPAO in a Yoneda structure, done in [DLL18];

  categorical logic for derivators (see Prest's treatment of definability for module categories); derivator topos theory?

[1]A 2-categorical device to encode the calculus of pointwise Kan extensions.

# The formal category theory of derivators

There is a Yoneda structure[1] on the 2-category of derivators

- notions of accessible and locally presentable derivator using the theory of LPAO in a Yoneda structure, done in [DLL18];

  categorical logic for derivators (see Prest's treatment of definability for module categories); derivator topos theory?

- adjoint functor theorems for derivators;

  existence of a six-operation calculus. 2-categorical account of Grothendieck duality complicated diagrams (without multicategories)

[1] A 2-categorical device to encode the calculus of pointwise Kan extensions.

# The formal category theory of derivators

There is a Yoneda structure[1] on the 2-category of derivators

- notions of accessible and locally presentable derivator using the theory of LPAO in a Yoneda structure, done in [DLL18];

  categorical logic for derivators (see Prest's treatment of definability for module categories); derivator topos theory?
- adjoint functor theorems for derivators;

  existence of a six-operation calculus. 2-categorical account of Grothendieck duality complicated diagrams (without multicategories)
- profunctors between derivators; fibered derivators;

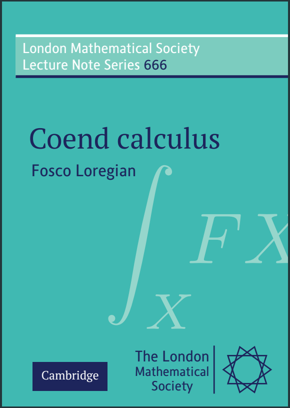
  operads in derivator theory; applications in representation theory of algebras, stable homotopy, ...

[1] A 2-categorical device to encode the calculus of pointwise Kan extensions.

# COENDS AND DG-STUFF

# Coends

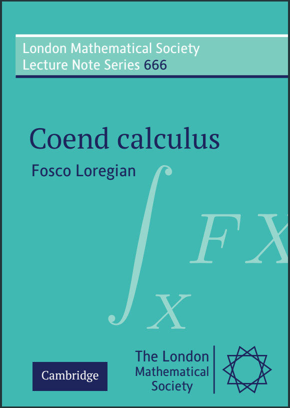
I have written a book [L20] on coend calculus, soon to appear under Cambridge LNSs:




- Coends $\int_C T$ are universal objects associated to $T : C^{op} \times C \to D$, treated as integrals (a “Fubini rule” is valid).

# Coends

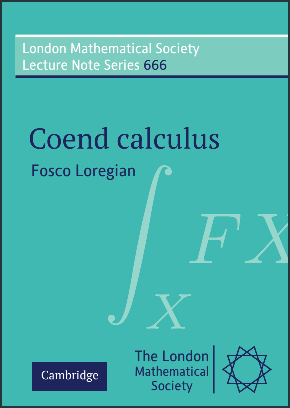
I have written a book [L20] on coend calculus, soon to appear under Cambridge LNSs:




- Coends $\int_C T$ are universal objects associated to $T : C^{op} \times C \to D$, treated as integrals (a "Fubini rule" is valid).
- applications in (monoidal) category theory, algebraic topology, universal algebra, algebraic geometry, categorical logic, representation theory (see ch.7 for an application to DG-categories), functional programming…

# Coends

I have written a book [L20] on coend calculus, soon to appear under Cambridge LNSs:




- Coends $\int_C T$ are universal objects associated to $T : C^{op} \times C \to D$, treated as integrals (a "Fubini rule" is valid).
- applications in (monoidal) category theory, algebraic topology, universal algebra, algebraic geometry, categorical logic, representation theory (see ch.7 for an application to DG-categories), functional programming…
- The book is being extensively cited (45 citations on Scholar May 19, 2020)

# DG-stuff

In [L20, 7.2.2]

For example: if $\mathcal{A}$ is any dg-category its identity profunctor $\mathcal{A} \rightsquigarrow \mathcal{A}$ is a functor $\mathcal{A}^{\mathrm{op}} \boxtimes \mathcal{A} \to \mathrm{Ch}(\mathbb{Z})$, so that the coherent end

$$\oint_A \mathcal{A}(A, A) \tag{7.82}$$

i.e. the object of derived natural transformations of the identity functor $\mathrm{id}_{\mathcal{A}}$, recovers the *Hochschild complex* of $\mathcal{A}$. Then, if $\mathcal{A}$ is an associative algebra regarded as a one-object dg-category concentrated in degree zero, the object $H^n(\int_* A)$ is the *Hochschild cohomology* of $A$, understood in the classical sense of, say, [Pie82, Ch. 11].

# DG-stuff

In [L20, 7.2.2]

For example: if $\mathcal{A}$ is any dg-category its identity profunctor $\mathcal{A} \rightsquigarrow \mathcal{A}$ is a functor $\mathcal{A}^{\mathrm{op}} \boxtimes \mathcal{A} \to \mathrm{Ch}(\mathbb{Z})$, so that the coherent end

$$\oint_A \mathcal{A}(A, A) \tag{7.82}$$

i.e. the object of derived natural transformations of the identity functor $\mathrm{id}_{\mathcal{A}}$, recovers the *Hochschild complex* of $\mathcal{A}$. Then, if $\mathcal{A}$ is an associative algebra regarded as a one-object dg-category concentrated in degree zero, the object $H^n(\int_* A)$ is the *Hochschild cohomology* of $A$, understood in the classical sense of, say, [Pie82, Ch. 11].

Applications to Kuznetsov-Lunts categorical resolutions of singularities: a smooth DG-category is a $\mathcal{D}$ such that its identity profunctor $h : \mathcal{D} \rightsquigarrow \mathcal{D}$ is a perfect object.

# TEACHING AND ORGANIZATIONAL ACTIVITIES

# Teaching and…

- 2015 A short course on model categories @unipv;

## Teaching and…

- 2015 A short course on model categories @unipv;
- 2016 “Elements of Finite Mathematics” @uwo (mostly statistics to kinesiologists).

## Teaching and...

- 2015 A short course on model categories @unipv;
- 2016 “Elements of Finite Mathematics” @uwo (mostly statistics to kinesiologists).
- 2016 Advisor of a BSc thesis @unibo, ”Elementary aspects of adjoint functors”. I enjoyed it!

# Teaching and...

- 2015 A short course on model categories @unipv;
- 2016 “Elements of Finite Mathematics” @uwo (mostly statistics to kinesiologists).
- 2016 Advisor of a BSc thesis @unibo, ”Elementary aspects of adjoint functors”. I enjoyed it!
- 2018 A short course on 2-category theory @unipd: monoidal and enriched, categories, the calculus of coends and Kan extensions, bicategories, monads...

# Teaching and...

- 2015 A short course on model categories @unipv;
- 2016 “Elements of Finite Mathematics” @uwo (mostly statistics to kinesiologists).
- 2016 Advisor of a BSc thesis @unibo, ”Elementary aspects of adjoint functors”. I enjoyed it!
- 2018 A short course on 2-category theory @unipd: monoidal and enriched, categories, the calculus of coends and Kan extensions, bicategories, monads...
- 2020 Category theory course Teacher @taltech. Mentoring activity for MSc students interested in category theory in CS.

# Organisation of conferences

- 2015 and 2019 Attendee and speaker at the Kan Seminar I

  a webinar on category theory

# Organisation of conferences

- 2015 and 2019 Attendee and speaker at the Kan Seminar I

  a webinar on category theory

- and Applied Category Theory 2019

  (a webinar on applied category theory, from which the paper [MLR+20] stemmed)

# Organisation of conferences

- 2015 and 2019 Attendee and speaker at the Kan Seminar I

  a webinar on category theory

- and Applied Category Theory 2019

  (a webinar on applied category theory, from which the paper [MLR+20] stemmed)

- 2018 Organiser of the 103rd PSSL

  Peripathetic Seminar on Sheaves and Logic, Brno.

# Organisation of conferences

- 2015 and 2019 Attendee and speaker at the Kan Seminar I

  a webinar on category theory

- and Applied Category Theory 2019

  (a webinar on applied category theory, from which the paper [MLR+20] stemmed)

- 2018 Organiser of the 103rd PSSL

  Peripathetic Seminar on Sheaves and Logic, Brno.

- 2019 and 2020 Among the organisers of ItaCa

  Italian Category theorists) in Milan and soon on zoom, due to COVID19.

# Organisation of conferences

- 2015 and 2019 Attendee and speaker at the Kan Seminar I

  a webinar on category theory

- and Applied Category Theory 2019

  (a webinar on applied category theory, from which the paper [MLR+20] stemmed)

- 2018 Organiser of the 103rd PSSL

  Peripathetic Seminar on Sheaves and Logic, Brno.

- 2019 and 2020 Among the organisers of ItaCa

  Italian Category theorists) in Milan and soon on zoom, due to COVID19.

- Reviewer for zbMath and AMS.

## Miscellaneous projects

- Homotopy theory and set theory: [DL18]

  no homotopy category of a model category is “concrete”; what about $\infty$-categories?

# Miscellaneous projects

- Homotopy theory and set theory: [DL18]

  no homotopy category of a model category is “concrete”; what about $\infty$-categories?

- Functional programming and type theory:

  HoTT, linear types, proof-checkers, categorical algebra in relational database architecture; natural language processing using category theory…

# Miscellaneous projects

- Homotopy theory and set theory: [DL18]

  no homotopy category of a model category is "concrete"; what about $\infty$-categories?

- Functional programming and type theory:

  HoTT, linear types, proof-checkers, categorical algebra in relational database architecture; natural language processing using category theory…

- Categorical logic and foundations of mathematics;

  functorial semantics *à la* Lawvere, but sprinkled with operads and multicategories.

## Miscellaneous projects

- Homotopy theory and set theory: [DL18]

  no homotopy category of a model category is "concrete"; what about $\infty$-categories?

- Functional programming and type theory:

  HoTT, linear types, proof-checkers, categorical algebra in relational database architecture; natural language processing using category theory…

- Categorical logic and foundations of mathematics;

  functorial semantics *à la* Lawvere, but sprinkled with operads and multicategories.

- more in detail, "2-semantics" of algebraic theories:

  profunctorial PROPs and theories, categorical algebra of cartesian bicategories…

Reach me out at my web page: