List of Publications

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fosco.loregian@gmail.com
loregianf@math.muni.cz
flore@mpim-bonn.mpg.de

Fosco Loregian
github.com/tetrapharmakon
fosco.loregian

Publications

1 | **Categorical notions of fibration** w/E. Riehl |
1806.06129 | *Expos. Math.* (2019) | doi:10.1016/j.exmath.2019.02.004
Fibrations over a category *B*, introduced to category theory by Grothendieck, encode pseudo-functors $B^\circ \rightsquigarrow$ **Cat**, while the special case of discrete fibrations encode presheaves $B^\circ \to$ **Set**. A two-sided discrete variation encodes functors B° *x A* $\to$ **Set**, which are also known as profunctors from *A* to *B*. By work of Street, all of these fibration notions can be defined internally to an arbitrary 2-category or bicategory. While the two-sided discrete fibrations model profunctors internally to **Cat**, unexpectedly, the dual two-sided codiscrete cofibrations are necessary to model *V*-profunctors internally to *V*-**Cat**.

2 | **Hearts and towers in stable infinity-categories** w/D. Fiorenza, G. Marchetti |
1501.04658 | *Journal of Homotopy and Related Structures* 2019 | doi:10.1007/s40062-019-00237-0
We exploit the equivalence between t-structures and normal torsion theories on a stable ∞-category to show how a few classical topics in the theory of triangulated categories, i.e., the characterization of bounded t-structures in terms of their hearts, their associated cohomology functors, semiorthogonal decompositions, and the theory of tiltings, as well as the more recent notion of Bridgeland's slicings, are all particular instances of a single construction, namely, the tower of a morphism associated with a *J*-slicing of a stable ∞-category **C**, where *J* is a totally ordered set equipped with a monotone $\mathbb{Z}$-action.

3 | **A standard theorem on adjunctions in two variables**
1902.06074 | Preprints of the MPIM, Max-Planck-Institut für Mathematik Preprint Series 2018 (67)
We record an explicit proof of the theorem that lifts a two-variable adjunction to the arrow categories of its domains.

4 | **A Fubini rule for ∞-coends**
1902.06086 | Preprints of the MPIM, Max-Planck-Institut für Mathematik Preprint Series 2018 (68)
We prove a Fubini rule for ∞-co/ends of ∞-functors $F : \mathsf{C}^{\text{op}} \times \mathsf{C} \to \mathsf{D}$. This allows to lay down "integration rules", similar to those in classical co/end calculus, also in the setting of ∞-categories.

5 | **Homotopical Algebra is not concrete** w/I. Di Liberti |
1704.00303 | *Journal of Homotopy and Related Structures* (2017): 1-15 | doi:10.1007/s40062-018-0197-3
We generalize Freyd's well-known result that "homotopy is not concrete", offering a general method to show that under certain assumptions on a model category *M*, its homotopy category $\text{ho}(M)$ cannot be concrete. This result is part of an attempt to understand more deeply the relation between set theory and abstract homotopy theory.

6 | **Sober Ontic Structural Realism and Yoneda lemma**
abstract at the *Triennial conference of the SILFS*, Bologna
A note on why the Yoneda lemma prevents to take too strong a position towards the non-existence of relata (*radical* ontic structural realism posits that only relations exist).

7 | **Coend calculus**
based on 1501.02503v4 | book to appear for Cambridge University Press (2020?)
A survey of the most striking and useful applications of *co/end calculus*. This is a revised version of 1501.02503v4. After having given a series of preliminary definitions, we characterize co/ends as particular co/limits; then we derive a number of results directly from this characterization. The last sections discuss the most interesting examples where co/end calculus serves as a powerful abstract way to do explicit computations in diverse fields like Algebra, Algebraic Topology and Category Theory. The appendices serve to sketch a number of results in theories heavily relying on co/end calculus.

8 | **t-structures are normal torsion theories** w/D. Fiorenza |
1408.7003 | Applied Categorical Structures 24.2 (2016): 181-208 | doi:10.1007/s10485-015-9393-z
We characterize t-structures in stable ∞-categories as suitable quasicategorical factorization systems. More precisely we show that a t-structure $\mathfrak{t}$ on a stable ∞-category **C** is equivalent to a normal torsion theory $\mathbb{F}$ on **C**, i.e. to a factorization system $\mathbb{F} = (E, M)$ where both classes satisfy the 3-for-2 cancellation property, and a certain compatibility with pullbacks/pushouts.

Preprints

1 | **On the unicity of formal category theories** w/I. Di Liberti |
1901.01594v1 | Submitted to TAC, January 2019

We prove an equivalence between cocomplete Yoneda structures and certain proarrow equipments on a 2-category **K**. In order to do this, we recognize the presheaf construction of a cocomplete Yoneda structure as a relative, lax idempotent monad sending each admissible 1-cell $f : A \to B$ to an adjunction $\mathbb{P}_! f \dashv \mathbb{P}^* f$. Each cocomplete Yoneda structure on **K** arises in this way from a relative lax idempotent monad "with enough adjoint 1-cells", whose domain generates the ideal of admissibles, and the Kleisli category of such a monad equips its domain with proarrows. We call these structures "yosegi". Quite often, the presheaf construction associated to a yosegi generates an ambidextrous Yoneda structure; in such a setting there exists a fully formal version of Isbell duality.

2 | **Accessibility and presentability in 2-categories** w/I. Di Liberti |
1804.08710v4 | Submitted to JPAA, January 2019

We outline a definition of accessible and presentable objects in a 2-category **K** endowed with a Yoneda structure; this perspective suggests a unified treatment of many "Gabriel-Ulmer like" theorems (like the classical Gabriel-Ulmer representation for locally presentable categories, Giraud theorem, and Gabriel-Popescu theorem), asserting how presentable objects arise as reflections of generating ones. In a 2-category with a Yoneda structure, two non-equivalent definitions of presentability for $A \in$ **K** can in principle be given: in the most interesting, it is generally false that all presheaf objects $\mathbb{P}A$ are presentable; this leads to the definition of a Gabriel-Ulmer structure, i.e. a Yoneda structure rich enough to concoct Gabriel-Ulmer duality and to make this asymmetry disappear. We end the paper with a roundup of examples, involving classical (set-based and enriched), low dimensional and higher dimensional category theory.

3 | **Localization theory for derivators**
1802.08193v1 | Submitted to TAC, March 2018

We outline the theory of reflections for prederivators, derivators and stable derivators. In order to parallel the classical theory valid for categories, we outline how reflections can be equivalently described as categories of fractions, reflective factorization systems, and categories of algebras for idempotent monads. This is a further development of the theory of monads and factorization systems for derivators.

4 | **Factorization systems on (stable) derivators** w/S. Virili |
1705.08565v3 | Submitted to JoA, June 2017

We define triangulated factorization systems on triangulated categories, and prove that a suitable subclass thereof (the normal triangulated torsion theories) corresponds bijectively to t-structures on the same category. This result is then placed in the framework of derivators regarding a triangulated category as the base of a stable derivator. More generally, we define derivator factorization systems in the 2-category PDer, describing them as algebras for a suitable strict 2-monad (this result is of independent interest), and prove that a similar characterization still holds true: for a stable derivator $\mathbb{D}$, a suitable class of derivator factorization systems (the normal derivator torsion theories) correspond bijectively with t-structures on the base $\mathbb{D}(\mathbf{1})$ of the derivator. These two result can be regarded as the triangulated- and derivator- analogues, respectively, of the theorem that says that 't-structures are normal torsion theories' in the setting of stable ∞-categories, showing how the result remains true whatever the chosen model for stable homotopy theory is.

5 | **Recollements in stable ∞-categories** w/D. Fiorenza |
1507.03913v2

We develop the theory of recollements in a stable ∞-categorical setting. In the axiomatization of Beilinson, Bernstein and Deligne, recollement situations provide a generalization of Grothendieck's "six functors" between derived categories. The adjointness relations between functors in a recollement **D**^{0}, **D**, **D**^{1} induce a "recollée" t-structure $\mathtt{t}_0 \uplus \mathtt{t}_1$ on **D** , given t-structures $\mathtt{t}_0, \mathtt{t}_1$ on **D**^{0}, **D**^{1}. Such a classical result, well-known in the setting of triangulated categories, is recasted in the setting of stable ∞-categories and the properties of the associated (∞-categorical) factorization systems are investigated. In the geometric case of a stratified space, various recollements arise, which "interact well" with the combinatorics of the intersections of strata to give a well-defined, associative $\uplus$ operation. From this we deduce a generalized associative property for n-fold gluing $\mathtt{t}_0 \uplus \cdots \uplus \mathtt{t}_n$, valid in any stable ∞-category.

My publication track started in the field of ∞-category theory, and in particular in the setting of *stable* ∞-categories; subsequently, I moved to derivator theory, maintaining an interest in stable homotopy theory, but shifting more and more to a fully category-theoretical setting. During my stay in Brno, from March 2017 to April 2018, I grew a certain interest for accessible and presentable categories, and I started a series of joint works with Ivan Di Liberti, a friend and doctoral student of prof. Rosický. Until now, this collaboration produced three papers, one published on JHRS in 2017. Two more preprints on the theory of accessible and presentable objects in an abstract 2-category, and on the theory of Yoneda structures, await publication. This track of research is motivated by the desire to answer a very specific question: is there a Yoneda structure on the 2-category Der, accounting for the possibility to perform "all" known categorical constructions in it, while at the same time minding of the homotopy-theoretic origin of derivators?

I am currently finishing the draft of a book, to appear during 2020 under Cambridge University Press, based on my note on *coend calculus*. It will appear in LMS' lecture notes series.