List of Publications

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fosco.loregian@gmail.com
fouche@yoneda.ninja
fosco.loregian@taltech.ee

Fosco Loregian
github.com/tetrapharmakon
fosco.loregian

Publications

1 | Functorial semantics for partial theories w/I. Di Liberti, et al. |
doi:10.1145/3434338 | Proc. ACM Program. Lang. 5, POPL, 2021.
We provide a Lawvere-style definition for partial theories, extending the classical notion of equational theory by allowing partially defined operations. As in the classical case, our definition is syntactic: we use an appropriate class of string diagrams as terms. This allows for equational reasoning about the class of models defined by a partial theory. We demonstrate the expressivity of such equational theories by considering a number of examples, including partial combinatory algebras and cartesian closed categories. Moreover, despite the increase in expressivity of the syntax we retain a well-behaved notion of semantics: we show that our categories of models are precisely locally finitely presentable categories, and that free models exist.

2 | Triangulated factorization systems and *t*-structures w/S. Virili |
1705.08565v3 | *Journal of Algebra* | doi:10.1016/j.jalgebra.2019.12.021
We define triangulated factorization systems on triangulated categories, and prove that a suitable subclass thereof (the normal triangulated torsion theories) corresponds bijectively to t-structures on the same category. This result will then be placed in the framework of derivators regarding a triangulated category as the base of a stable derivator.

3 | Categorical notions of fibration w/E. Riehl |
1806.06129 | *Expos. Math.* (2019) | doi:10.1016/j.exmath.2019.02.004
Fibrations over a category B, introduced to category theory by Grothendieck, encode pseudo-functors $B^{\circ} \rightsquigarrow \mathbf{Cat}$, while the special case of discrete fibrations encode presheaves $B^{\circ} \to \mathbf{Set}$. A two-sided discrete variation encodes functors $B^{\circ} \times A \to \mathbf{Set}$, which are also known as profunctors from A to B. By work of Street, all of these fibration notions can be defined internally to an arbitrary 2-category or bicategory. While the two-sided discrete fibrations model profunctors internally to **Cat**, unexpectedly, the dual two-sided codiscrete cofibrations are necessary to model V-profunctors internally to V-**Cat**.

4 | Hearts and towers in stable infinity-categories w/D. Fiorenza, G. Marchetti |
1501.04658 | *Journal of Homotopy and Related Structures* 2019 | doi:10.1007/s40062-019-00237-0
We exploit the equivalence between t-structures and normal torsion theories on a stable ∞-category to show how a few classical topics in the theory of triangulated categories, i.e., the characterization of bounded t-structures in terms of their hearts, their associated cohomology functors, semiorthogonal decompositions, and the theory of tiltings, as well as the more recent notion of Bridgeland's slicings, are all particular instances of a single construction, namely, the tower of a morphism associated with a J-slicing of a stable ∞-category $\mathbf{C}$, where J is a totally ordered set equipped with a monotone $\mathbb{Z}$-action.

5 | A standard theorem on adjunctions in two variables
1902.06074 | Preprints of the MPIM, 2018 (67)
We record an explicit proof of the theorem that lifts a two-variable adjunction to the arrow categories of its domains.

6 | A Fubini rule for ∞-coends
1902.06086 | Preprints of the MPIM, 2018 (68)
We prove a Fubini rule for ∞-co/ends of ∞-functors $F : \mathrm{C}^{\mathrm{op}} \times \mathrm{C} \to \mathrm{D}$. This allows to lay down "integration rules", similar to those in classical co/end calculus, also in the setting of ∞-categories.

7 | Homotopical Algebra is not concrete w/I. Di Liberti |
1704.00303 | *Journal of Homotopy and Related Structures* (2017): 1-15 | doi:10.1007/s40062-018-0197-3
We generalize Freyd's well-known result that "homotopy is not concrete", offering a general method to show that under certain assumptions on a model category M, its homotopy category $\mathrm{ho}(M)$ cannot be concrete. This result is part of an attempt to understand more deeply the relation between set theory and abstract homotopy theory.

8 | Sober Ontic Structural Realism and Yoneda lemma
abstract at the *Triennial conference of the SILFS*, Bologna
A note on why the Yoneda lemma prevents to take too strong a position towards the non-existence of relata (*radical* ontic structural realism posits that only relations exist).

9 | Coend calculus
based on 1501.02503v4 | book to appear for Cambridge University Press (2020?)
A survey of the most striking and useful applications of *co/end calculus*. This is a revised version of 1501.02503v4. After having given a series of preliminary definitions, we characterize co/ends as particular co/limits; then we derive a number of results directly from this characterization. The last sections discuss the most interesting examples where co/end calculus serves as a powerful abstract way to do explicit computations in diverse fields like Algebra, Algebraic Topology and Category Theory. The appendices serve to sketch a number of results in theories heavily relying on co/end calculus.

10 | t-structures are normal torsion theories w/D. Fiorenza |
1408.7003 | Applied Categorical Structures 24.2 (2016): 181-208 | doi:10.1007/s10485-015-9393-z
We characterize t-structures in stable ∞-categories as suitable quasicategorical factorization systems. More precisely we show that a t-structure $\mathtt{t}$ on a stable ∞-category **C** is equivalent to a normal torsion theory $\mathbb{F}$ on **C**, i.e. to a factorization system $\mathbb{F} = (E, M)$ where both classes satisfy the 3-for-2 cancellation property, and a certain compatibility with pullbacks/pushouts.

Preprints

1 | Differential 2-rigs w/F. Genovese |

We explore the notion of a category with coproducts $\cup$ and a monoidal structure $\otimes$ distributing over it, endowed with an endo-functor ∂ which is 'linear and Leibniz'. Such ∂ can be legitimately called a *derivation* on $\mathcal{C}$, that appears as the categorification of a differential ring. We explore a number of possible special examples: the theory is paricularly well-behaved when $\mathcal{C}$ is semicartesian (i.e., its $\otimes$-monoidal unit is the terminal object); interesting examples arise when $\mathcal{C} = [\mathcal{A}, \mathbf{Set}]$ is a presheaf topos endowed with a distributive convolution; we discuss the 'chain rule' in the category of functors $[\mathbf{Fin}, \mathbf{Set}]$, obtained when a substitution of species $F \diamond G$ is regarded as composition $T_F \circ T_G$ of the associated finitary functors.

2 | Nets with Mana: A Framework for Chemical Reaction Modelling w/F. Genovese |

We use categorical methods to define a new flavor of Petri nets which could be useful in modelling chemical reactions.

3 | A Categorical Semantics for Bounded Petri Nets w/F. Genovese, D. Palombi |

We provide a categorical semantics for bounded Petri nets, both in the collective- and individual-token philosophy. In both cases, we describe the process of bounding a net internally, by just constructing new categories of executions of a net using comonads, and externally, using lax-monoidal-lax functors. Our external semantics is non-local, meaning that tokens are endowed with properties that say something about the global state of the net. We then prove, in both cases, that the internal and external constructions are equivalent, by using machinery built on top of the Grothendieck construction. The individual-token case is harder, as it requires a more explicit reliance on abstract methods.

4 | Rosen's no-go theorem for regular categories
2012.11648
The famous biologist Robert Rosen argued for an intrinsic difference between biological and artificial life, supporting the claim that 'living systems are not mechanisms'. This result, understood as the claim that life-like mechanisms are non-computable, can be phrased as the non-existence of an equivalence between a category of 'static'/analytic elements and a category of 'variable'/synthetic elements. The property of a system of being synthetic, understood as being the gluing of 'variable families' of *analytica*, must imply that the latter class of objects does not retain sufficient information in order to describe said variability; we contribute to this thesis with an argument rooted in elementary category theory. Seen as such, Rosen's 'proof' that no living system can be a mechanism arises from a tension between two contrapuntal needs: on one side, the necessity to consider (synthetically) variable families of systems; on the other, the necessity to describe a syntheticum via an universally chosen analyticum.

5 | **Functorial Erkennen** w/D. Dentamaro |

`http://philsci-archive.pitt.edu/18519/`

We outline a 'formal theory of scientific theories' rooted in the theory of profunctors; the category-theoretic asset stresses the fact that the scope of scientific knowledge is to build 'meaningful connections' (i.e. well-behaved adjunctions) between a linguistic object (a 'theoretical category' $\mathcal{T}$) and the world $\mathcal{W}$ said language ought to describe. Such a world is often unfathomable, and thus we can only resort to a smaller fragment of it in our analysis: this is the 'observational category' $\mathcal{O} \subseteq \mathcal{W}$. From this we build the category $[\mathcal{O}^{\text{op}}, \text{Set}]$ of all possible displacements of observational terms $\mathcal{O}$. The self-duality of the bicategory of profunctors accounts for the fact that theoretical and observational terms can exchange their rôle without substantial changes in the resulting predictive-descriptive theory; this provides evidence for the idea that their separation is a mere linguistic convention; to every profunctor $\mathfrak{R}$ linking $\mathcal{T}$ and $\mathcal{O}$ one can associate an object $\mathcal{O} \uplus_{\mathfrak{R}} \mathcal{T}$ obtained *glueing* together the two categories and accounting for the mutual relations subsumed by $\mathfrak{R}$. Under mild assumptions, such an arrangement of functors, profunctors, and gluings provides a categorical interpretation for the 'Ramseyfication' operation, in a very explicit sense: in a scientific theory, if a computation entails a certain behaviour for the system the theory describes, then saturating its theoretical variables with actual observed terms, we obtain the entailment *in the world*.

6 | **Coends of higher arity** w/T. de Oliveira Santos |

2011.13881

We specialise a recently introduced notion of generalised dinaturality for functors $T : (\mathcal{C}^{\text{op}})^p \times \mathcal{C}^q \to \mathcal{D}$ to the case where the domain (resp., codomain) is constant, obtaining notions of ends (resp., coends) of higher arity, dubbed herein (p, q)-ends (resp., (p, q)-coends). While higher arity co/ends are particular instances of "totally symmetrised" (ordinary) co/ends, they serve an important technical role in the study of a number of new categorical phenomena, which may be broadly classified as two new variants of category theory. The first of these, weighted category theory, consists of the study of weighted variants of the classical notions and construction found in ordinary category theory, besides that of a limit. This leads to a host of varied and rich notions, such as weighted Kan extensions, weighted adjunctions, and weighted ends. The second, diagonal category theory, proceeds in a different (albeit related) direction, in which one replaces universality with respect to natural transformations with universality with respect to dinatural transformations, mimicking the passage from limits to ends. In doing so, one again encounters a number of new interesting notions, among which one similarly finds diagonal Kan extensions, diagonal adjunctions, and diagonal ends.

7 | **Categorical Ontology I** w/D. Dentamaro |

`http://philsci-archive.pitt.edu/17136/`

The present paper approaches ontology and metaontology through mathematics, and more precisely through category theory. We exploit the theory of *elementary toposes* to claim that a satisfying "theory of existence", and more at large ontology itself, can both be obtained through category theory. In this perspective, an *ontology* is a mathematical object: it is a category, the universe of discourse in which our mathematics (intended at large, as a theory of knowledge) can be deployed. The *internal language* that all categories possess prescribes the modes of existence for the objects of a fixed ontology/category. This approach resembles, but is more general than, fuzzy logics, as most choices of E and thus of Ω_E yield nonclassical, many-valued logics. Framed this way, ontology suddenly becomes more mathematical: a solid corpus of techniques can be used to backup philosophical intuition with a useful, modular language, suitable for a practical foundation. As both a test-bench for our theory, and a literary *divertissement*, we propose a possible category-theoretic solution of Borges' famous paradoxes of Tlön's "nine copper coins", and of other seemingly paradoxical construction in his literary work. We then delve into the topic with some vistas on our future works.

8 | **Profunctor optics, a categorical update** w/B. Clarke, et al. |

2001.07488

Profunctor optics are bidirectional data accessors that capture data transformation patterns such as accessing subfields or iterating over containers. They are modular, meaning that we can construct accessors for complex structures by combining simpler ones. Profunctor optics have been studied only using **Sets** as the enriching category and in the non-mixed case. However, functional programming languages are arguably better described by enriched categories and we have found that some structures in the literature are actually mixed optics. Our work generalizes a classic result by Pastro and Street on Tambara theory and uses it to describe mixed V-enriched profunctor optics and to endow them with V-category structure. We provide some original families of optics and derivations, including an elementary one for traversals that solves an open problem posed by Milewski. Finally, we discuss a Haskell implementation.

9 | **On the unicity of formal category theories** w/I. Di Liberti |

1901.01594v1

We prove an equivalence between cocomplete Yoneda structures and certain proarrow equipments on a 2-category **K**. In order to do this, we recognize the presheaf construction of a cocomplete Yoneda structure as a relative, lax idempotent monad sending each admissible 1-cell $f : A \to B$ to an adjunction $\mathbb{P}_! f \dashv \mathbb{P}^* f$. Each cocomplete Yoneda structure on K arises in this way from a relative lax idempotent monad "with enough adjoint 1-cells", whose domain generates the ideal of admissibles, and the Kleisli category of such a monad equips its domain with proarrows. We call these structures "yosegi". Quite often, the presheaf construction associated to a yosegi generates an ambidextrous Yoneda structure; in such a setting there exists a fully formal version of Isbell duality.

10 | Accessibility and presentability in 2-categories w/I. Di Liberti |

1804.08710v4

We outline a definition of accessible and presentable objects in a 2-category K endowed with a Yoneda structure; this perspective suggests a unified treatment of many "Gabriel-Ulmer like" theorems (like the classical Gabriel-Ulmer representation for locally presentable categories, Giraud theorem, and Gabriel-Popescu theorem), asserting how presentable objects arise as reflections of generating ones. In a 2-category with a Yoneda structure, two non-equivalent definitions of presentability for $A \in \mathsf{K}$ can in principle be given: in the most interesting, it is generally false that all presheaf objects $\mathbb{P}A$ are presentable; this leads to the definition of a Gabriel-Ulmer structure, i.e. a Yoneda structure rich enough to concoct Gabriel-Ulmer duality and to make this asymmetry disappear. We end the paper with a roundup of examples, involving classical (set-based and enriched), low dimensional and higher dimensional category theory.

11 | Localization theory for derivators

1802.08193v1

We outline the theory of reflections for prederivators, derivators and stable derivators. In order to parallel the classical theory valid for categories, we outline how reflections can be equivalently described as categories of fractions, reflective factorization systems, and categories of algebras for idempotent monads. This is a further development of the theory of monads and factorization systems for derivators.

12 | Recollements in stable ∞-categories w/D. Fiorenza |

1507.03913v2

We develop the theory of recollements in a stable ∞-categorical setting. In the axiomatization of Beilinson, Bernstein and Deligne, recollement situations provide a generalization of Grothendieck's "six functors" between derived categories. The adjointness relations between functors in a recollement $\mathsf{D}^0, \mathsf{D}, \mathsf{D}^1$ induce a "recollée" t-structure $\mathtt{t}_0 \uplus \mathtt{t}_1$ on D, given t-structures $\mathtt{t}_0, \mathtt{t}_1$ on $\mathsf{D}^0, \mathsf{D}^1$. Such a classical result, well-known in the setting of triangulated categories, is recasted in the setting of stable ∞-categories and the properties of the associated (∞-categorical) factorization systems are investigated. In the geometric case of a stratified space, various recollements arise, which "interact well" with the combinatorics of the intersections of strata to give a well-defined, associative $\uplus$ operation. From this we deduce a generalized associative property for n-fold gluing $\mathtt{t}_0 \uplus \cdots \uplus \mathtt{t}_n$, valid in any stable ∞-category.

My publication track started in the field of ∞-category theory, and in particular in the setting of *stable* ∞-categories; subsequently, I moved to derivator theory, maintaining an interest in stable homotopy theory, but shifting more and more to a fully category-theoretical setting. During my stay in Brno, from March 2017 to April 2018, I grew a certain interest for accessible and presentable categories, and I started a series of joint works with Ivan Di Liberti, a friend and doctoral student of prof. Rosický. Until now, this collaboration produced three papers, one published on JHRS in 2017. Two more preprints on the theory of accessible and presentable objects in an abstract 2-category, and on the theory of Yoneda structures, await publication. This track of research is motivated by the desire to answer a very specific question: is there a Yoneda structure on the 2-category Der, accounting for the possibility to perform "all" known categorical constructions in it, while at the same time minding of the homotopy-theoretic origin of derivators?

I am currently finishing the draft of a book, to appear during 2020 under Cambridge University Press, based on my note on *coend calculus*. It will appear in LMS' lecture notes series.