

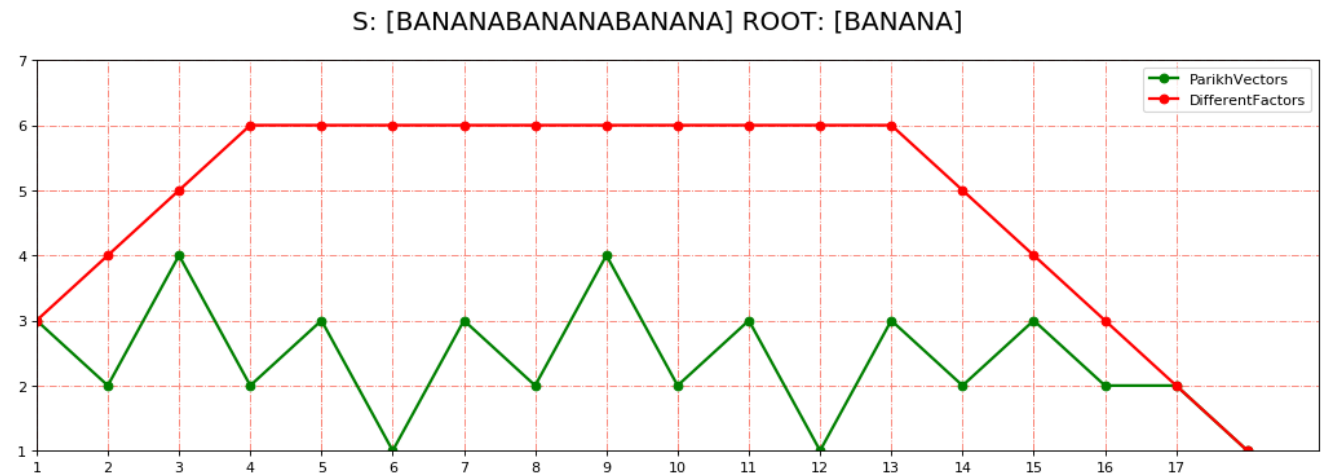
In [1]:

```
from mylibrary import *
```

1° Topic - Basic Plotting

In [2]:

```
myplot("BANANA"*3, "ABN")
```



2° Topic - Basic Conjectures

In [3]:

```
#Inizializzazione variabili
root="abcbcef"
alphabet="abcef"
S=powerword(root, 4)
```

In [4]:

```
%%latex
Given a string S, s.t. S can be written as  $U^k$  with  $U$  primitive  $U \in \Sigma^+$ 
(1) Show that the number of Parikh Vectors( $m \cdot k$ ) is 1  $\forall m \geq 1$  )

 $\Sigma^+$ 

It's trivial to show that if the string  $SS$  can be written as a concatenation of  $U$  (with  $U$  primitive
and  $k$  power of the root ) if we move the window of  $|U|$  size by one position we will still get the same characters in a different order(but same PV), as the different-factors-PVs collapse into one.  $\text{See example 1}$ 
Following the same reasoning, if each time we move the sliding window by one position, we will get, at most, one change (increase or decrease) per PV.
If none of the PV at  $k+1$  collapse, then we will have  $|U|$  as number of distinct Parikh vectors because the substring has been extended in all possible ways  $\text{See example 2}$ 
```

Given a string S, s.t. S can be written as U^k with U primitive $U \in \Sigma^+$ (1) Show that the number of Parikh Vectors($m \cdot k$) is 1 $\forall m \geq 1$) It's trivial to show that if the string SS can be written as a concatenation of U (with U primitive and k power of the root) if we move the window of $|U|$ size by one position we will still get the same characters in a different order(but same PV), as the different-factors-PVs collapse into one. See example 1 Following the same reasoning, if each time we move the sliding window by one position, we will get, at most, one change (increase or decrease) per PV. If none of the PV at $k+1$ collapse, then we will have $|U|$ as number of distinct Parikh vectors because the substring has been extended in all possible ways See example 2

In [5]:

```
FIRST(S, alphabet, len(root)).head()
```

The string is: [abcbcefabcbcefabcbcefabcbcef]
The length is: [28]
The root is: [abcbcef]
The period length is: [7]
For the values : [7] ... [14] ... [21] and so on we will always get just one type of PV as the different factors collapse
Number of distinct PV: [1]

Out[5]:

	S_Rotations	PV	index
0	cbcefab	[1, 2, 2, 1, 1]	[2, 9, 16]
1	bcefabcb	[1, 2, 2, 1, 1]	[3, 10, 17]
2	abcbcef	[1, 2, 2, 1, 1]	[0, 7, 14, 21]
3	cefabcb	[1, 2, 2, 1, 1]	[4, 11, 18]
4	efabcbc	[1, 2, 2, 1, 1]	[5, 12, 19]

In [6]:

```
FIRST(S, alphabet, len(root)*2).head()
```

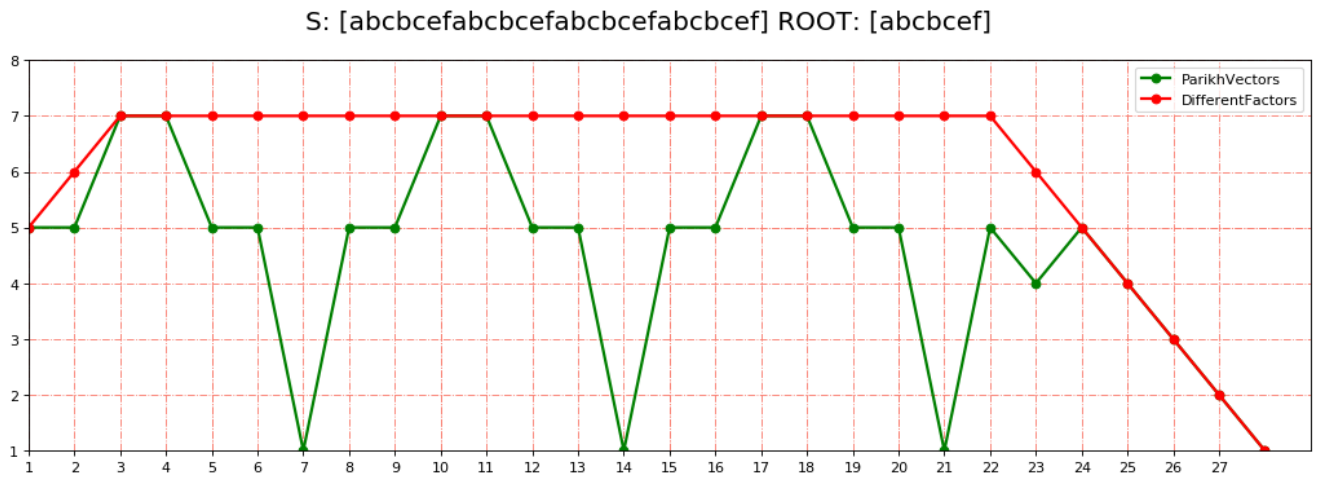
The string is: [abcbcefabcbcefabcbcefabcbcef]
The length is: [28]
The root is: [abcbcef]
The period length is: [7]
For the values : [7] ... [14] ... [21] and so on we will always get just one type of PV as the different factors collapse
Number of distinct PV: [1]

Out[6]:

	S_Rotations	PV	index
0	bcbcefabcbcefa	[2, 4, 4, 2, 2]	[1, 8]
1	abcbcefabcbcef	[2, 4, 4, 2, 2]	[0, 7, 14]
2	cefabcbcefabcb	[2, 4, 4, 2, 2]	[4, 11]
3	fabcbcefabcbce	[2, 4, 4, 2, 2]	[6, 13]
4	bcefabcbcefabcb	[2, 4, 4, 2, 2]	[3, 10]

In [7]:

```
myplot(S, alphabet)
```



3° Topic - Extensibility Table

In [8]:

```
extensibilitytable(S, alphabet)
```

Out[8]:

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	
0	e	-	bc	+	cbc	-	cbce	-	cefab	-	bcbcef	-	cbcefab	-	efabcbce	-	cbcefabcb	-	abcbcefab	-	bcbcefa
1	c	+	fa	-	cef	-	abcb	-	efabc	-	cefab	-	bcefab	-	bcbcefab	-	cefabcbce	-	bcbcefabcb	-	cefabcb
2	a	-	cb	-	abc	-	bcbc	-	bcbce	-	efabcb	-	abcbcef	-	cbcefab	-	fabcbcefa	-	bcefabcbce	-	cbcefab
3	b	-	ab	-	efa	-	efab	-	abcbc	-	bcefab	-	cefabcb	-	abcbcefa	-	bcbcefab	-	cefabcbcef	-	fabcbce
4	f	-	ce	-	fab	-	fab	-	bcefa	-	cbcefa	-	efabcb	-	cefabcb	-	efabcbce	-	cbcefabcb	-	efabcb
5			ef	-	bce	-	cefa	-	cbce	-	abcbce	-	bcbcefa	-	bcefabcb	-	abcbcefab	-	efabcbcefa	-	abcbce
6					bcb	-	bce	-	fabcb	-	fabcb	-	fabcbce	-	fabcbce	-	bcefabcb	-	fabcbcefab	-	bcefab

4° Topic - Strings where #PVs is never equal to the #DFs between small m and big M

In [9]:

```
#First we generate all the words using all
#the permutations of the characters of the
#alphabet. Secondly, we do the calculation
#to cluster the strings accordingly
alphabet="ABC"
length=4
power=2

touch, nottouch=comparison(alphabet, length, power)

#total is length^|alphabet|
```

Words where #PVs is equal to #Dfs at some point

AAAA - AABC - AACB - ABAB - ABBC - ABCA - ABCC - ACAC - ACBA - ACBB - ACCB - BAAC - BABA - BACB -
BACC - BBAC - BBBB - BBAC - BCBA - BCAB - BCBC - BCCA - CAAB - CABB - CABC - CACA - CBAA - CBAC -
CBBA - CBCB - CCAB - CCBA - CCCC

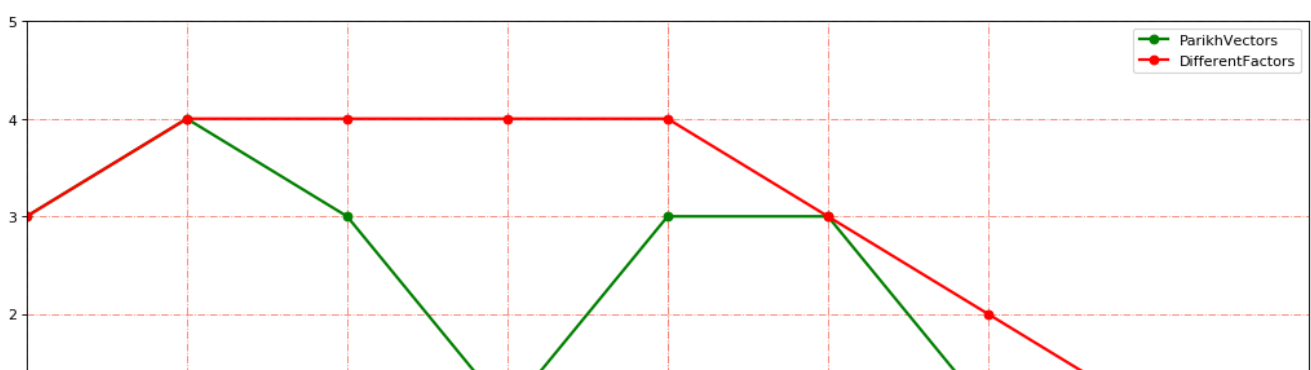
Words where #PVs is NOT equal to #Dfs at any point

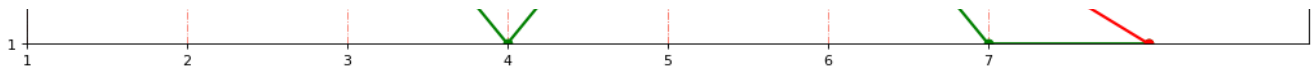
CABA - CCBC - ABAA - BABC - ACAB - CCCB - CBCC - BCCB - BCCC - ABAC - AACA - CAAC - AABA - CAAA -
BCBB - ABCB - BACA - CCBB - AAAB - BBAB - CBAB - CCAC - BAAB - AAAC - CBAC - CBBC - ACCC - AAB -
ABBA - CACA - CACC - BABB - CBBC - BCBA - BCAC - BBAA - ACBC - BBCB - CCAA - BBCC - ACAA - BBBC -
ABBB - CACB - BAAA - ACCA - AACC - BBBA

In [17]:

```
myplot("ABCA"*2, alphabet)
```

S: [ABCAABCA] ROOT: [ABCA]

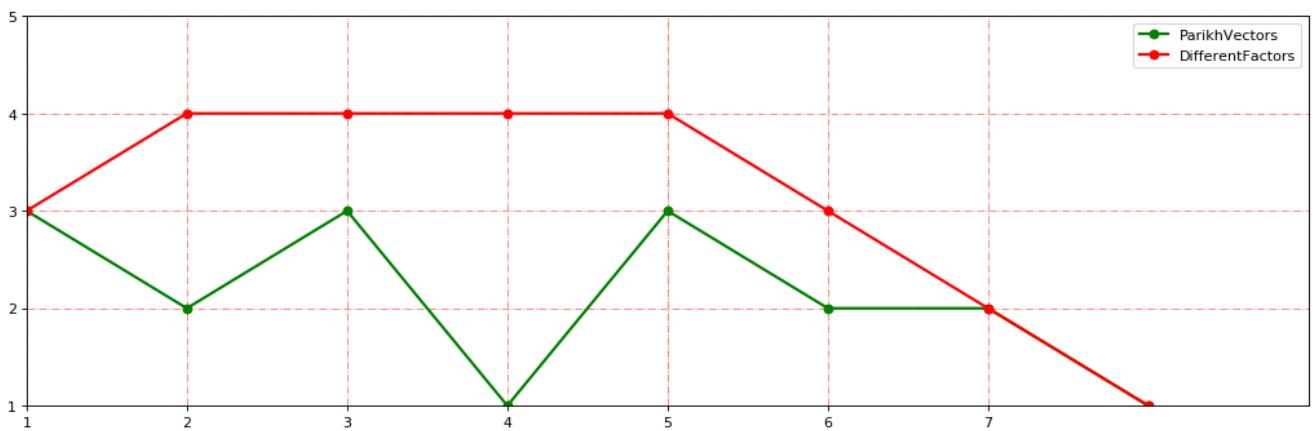




In [18]:

```
myplot("CABA"*2, alphabet)
```

S: [CABACABA] ROOT: [CABA]

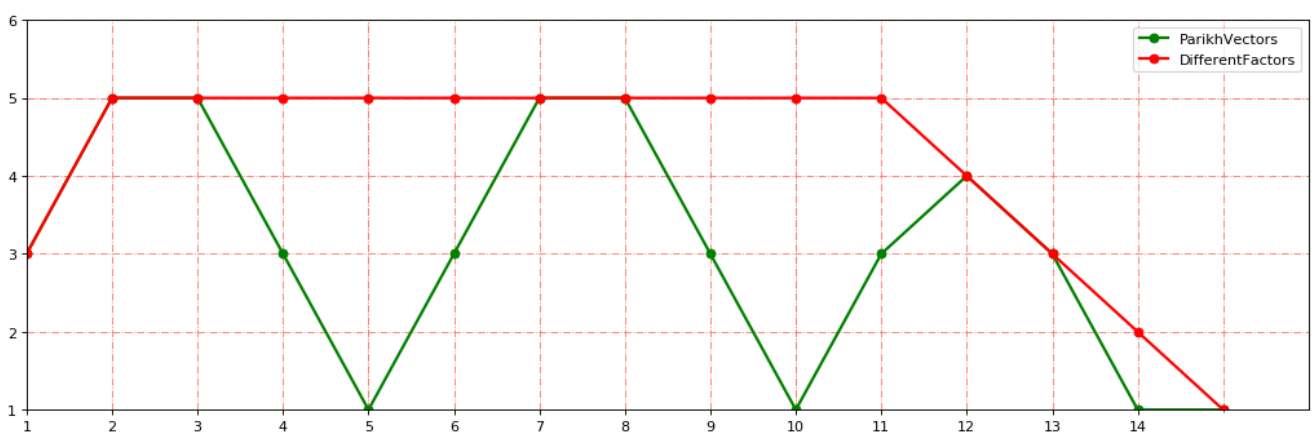


6° Topic - #PVs(mk+i) = |alph(root)| for each i between 1 and x (x is the #repeated char)

In [30]:

```
#REFUTED BY
myplot("ABCCA"*3, "ABC")
```

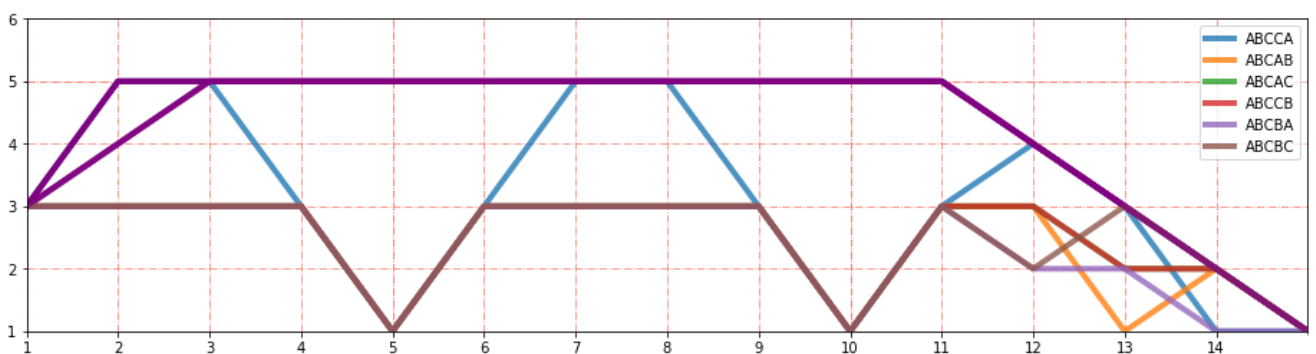
S: [ABCCAABCCAABCCA] ROOT: [ABCCA]



In [29]:

```
L=[ "ABCCA", "ABCCAB", "ABCCAC", "ABCCB", "ABCCBA", "ABCCBC" ]
myplot_list(L, "ABC", 3)
```

DFs in Purple, PVs other colours



7° Topic - Investigate the end

In [33]:

```
ALPHABET="ABC"  
rootsize=4  
power=3  
bias=1  
L=investigate(ALPHABET,rootsize, power, bias )
```

The size of the alphabet is [3]
The lengths of the strings is [12]
The lengths of the roots of the strings is [4]
We investigate position [9]

Words with [1] PVs
AAAA - BBBB - CCCC

Words with [2] PVs
AAAB - AAAC - AABA - AABB - AACA - AACC - ABAA - ABAB - ABBA - ABBB - ACAA - ACAC - ACCA - ACCC -
BAAA - BAAB - BABA - BABB - BBAA - BBAB - BBBA - BBBC - BBCB - BBCC - BCB B - BCBC - BCCB - BCCC -
CAAA - CAAC - CACA - CACC - CBBB - CBBC - CBCB - CBCC - CCAA - CCAC - CCBB - CCBC - CCCA - CCCB

Words with [3] PVs
AABC - AACB - ABAC - ABBC - ABCA - ABCB - ABCC - ACAB - ACBA - ACBB - ACBC - ACCB - BAAC - BABC -
BACA - BACB - BACC - BBAC - BBCA - BCAA - BCAB - BCAC - BCBA - BCCA - CAAB - CABA - CABB - CABC -
CACB - CBAA - CBAB - CBAC - CBBA - CBCA - CCAB - CCBA

In []: