ESERCITAZIONE II–OUTPUT REGULATION

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1. Problem statement and motivation

We consider the problem of controlling an inverted pendulum on a cart. The system is composed of a mass m connected to a cart of mass M via a connecting rod of length l. We assume that the cart can be actuated via a force f and that the mass m can be actuated via a torque τ applied at the joint. A schematic representation of the system is shown in Figure 1. We assume that the connecting rod is weightless. Our goal is the design of a feedback controller ensuring the tracking of reference trajectories for the cart position and the rod angle. To this end, we make use of output regulation theory.

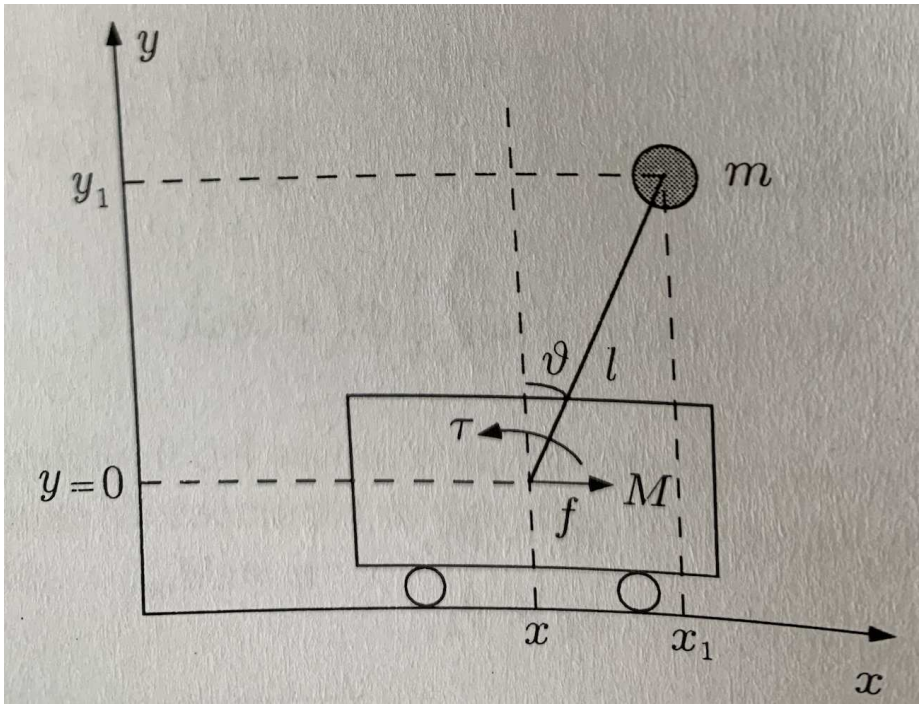

Figure 1. Inverted pendulum on a cart

2. Modeling

The system has two degrees of freedom, i.e., the position of the cart on the rail x and the angle of the rod with the vertical axis ϑ. Therefore, we take as state

$$\mathbf{x} = (x, \dot{x}, \vartheta, \dot{\vartheta})$$

and consider the following linearized model around the origin:

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{(M+m)Mgl}{MJ_p} & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 & 0 \\ \frac{1}{M} & -\frac{ml}{MJ_p} \\ 0 & 0 \\ -\frac{ml}{MJ_p} & \frac{m+M}{MJ_p} \end{bmatrix} \begin{bmatrix} f \\ \tau \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \mathbf{x}$$

where $J_p = ml^2$ and g is the gravity constant. In particular, the following numerical values are considered for the physical parameters

$$M = 5, m = 0.5, l = 1$$

3. Control objetive

Assuming that both the states x and ϑ can be measured, we want to design a feedback controller ensuring the tracking of the following reference input

$$y_d(t) = \begin{bmatrix} A\sin(t) \\ B \end{bmatrix}$$

where A and B can be selected arbitrarily by the user.