Do massive neutron stars end as invisible dark energy objects?

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ABSTRACT

Astronomical observations reveal a gap in the mass spectrum of relativistic objects: neither black holes nor neutron stars having masses in the range of 2 - 5 M_{\odot} have ever been observed.

Based on the solution of the TOV equation modified to include a universal scalar field \mathcal{H} , we argue that all moderate and massive neutron stars should end invisible dark energy objects (DEOs).

Triggered by the \mathcal{H} -baryonic matter interaction, a phase transition from normal compressible nuclear matter into an incompressible quark-superfluid is shown to occur at roughly 3 times the nuclear density. At the transition front, the scalar field is set to inject energy at the maximum possible rate via a non-local interaction potential $V_{\phi} = a_0 r^2 + b_0$. This energy creates a global confining bag, inside which a sea of freely moving quarks is formed in line with the asymptotic freedom of quantum chromodynamics. The transition front, r_f , creeps from inside-to-outside to reach the surface of the object on the scale of Gyrs or even shorter, depending on its initial compactness. Having r_f reached R_{\star} , then the total injected dark energy via V_{ϕ} turns NSs into invisible DEOs.

While this may provide an explanation for the absence of stellar BHs with $M_{BH} \leq 5 M_{\odot}$ and NSs with $M_{NS} \geq 2 M_{\odot}$, it also suggests that DEOs might have hidden connection to dark matter and dark energy in cosmology.

Keywords: Relativity: general, black hole physics — neutron stars — superfluidity — QCD — dark energy — dark matter

TURBULENT SUPERFLUIDITY IN NEUTRON STARS

The interiors of pulsars and NSs most likely are made of superfluids governed by triangular lattice of quantized vortices as prescribed by the Onsager-Feynman equation: $\oint \mathbf{v} \cdot \mathbf{d}l = \frac{2\pi\hbar}{m} N. \ \mathbf{v}, \mathbf{d}l, \ \hbar, m \text{ here denote the velocity field,}$ the vector of line-element, the reduced Planck constant and the mass of the superfluid particle pair, respectively.

Accordingly, the core of the Crab pulsar, should have approximately $N_n = 8.6 \times 10^{17}$ neutron and $N_p \approx 10^{30}$ protonvortices (Fig. 1). Let the evolution of the number density of vortex lines, n_v , obey the following advection-diffusion equation:

$$\frac{\partial n_v}{\partial t} + \nabla \cdot n_v \mathbf{u}_f = \nu_t \triangle n_v, \tag{1}$$

where t, \mathbf{u}_f , ν_t denote the transport velocity at the cylindrical radius $r=r_f$ and dissipative coefficient in the lo-

cal frame of reference, respectively. When $\nu_t = 0$, then the radial component of \mathbf{u}_f in cylindrical coordinates reads: $u_f^{max} \approx -(\dot{\Omega}/\Omega) r > 0$. In the case of the Crab; this implies that approximately 10⁶ neutron vortices must be expulsed/annihilated each second, and therefore the object should switch off after 10^6 up to 10^{13} yr, depending on the underlying mechanism of heat transport (see Baym 1995; Link 2012, and the references therein). On the other hand, recent numerical calculations of superfluids reveal generation of large amplitude Kelvin waves that turn superfluids turbulent (see Baranghi 2008; Baggaley & Laurie 2014; Dix 2014, and the references therein). It is therefore unlikely that trillions of Kilometer-long neutron and protonsvortices inside pulsars and NSs would behave differently. In this case, u_f should be replaced by a mean turbulent velocity $\langle u_f \rangle^t$ with u_f^{max} being an upper limit¹. As the number of vortex lines decreases with time due to emission

¹ The rotational energy associated with the outward-transported vortex lines from the central regions are turbulently re-distributed in the outer shells and should not necessary suffer a complete annihilation.

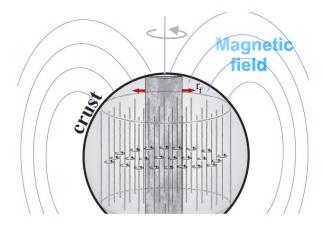


Figure 1. A magnetized neutron star with a superfluid core threaded by billions of vortex lines and magnetic flux tubes.

of magnetic dipole radiation and therefore the separation between them increases non-linearly, it is reasonable to associate a time-dependent turbulent length scale $\ell_t(t)$, which covers the two limiting cases: $\ell_t(t=0) = \ell_0 \approx 10^{-3} \, \mathrm{cm}$ and $\ell_t(t=\infty) = \ell_\infty = R_\star$. This yields the geometrical mean $\langle \ell_t \rangle = \sqrt{\ell_0 \ell_\infty} \approx \mathcal{O}(10) \, cm$. Putting terms together and using $\nu_{tur} = \langle \ell_t \rangle \langle u_f \rangle^t$ to describe the effective turbulent viscosity, we obtain an upper limit for the global diffusion time scale: $\tau_{diff} = R_{NS}^2/\nu_{tur} = \mathcal{O}(10^9)$ yr. Similarly, a comparable time scale for the Ohmic diffusion in this turbulent medium can be constructed as well. This is in line with observations, which reveal that most isolated luminous NSs known are younger than 10⁹ yr (see Espinoza 2011, and the references therein). Assuming quantized vortices in NSs to obey a triangular lattice distribution, then the very central region would be the first to be evacuated from vortex lines and all other removable energies that do not contribute significantly to the pressure. Hence the radius of this region, r_f , would creep outwards with an average velocity: $\dot{r}_f \sim R_{\star}/\tau_{diff} \approx 10^{-10}$ cm/s. The nuclear matter inside r_f would be in the lowest possible energy state, which, as argued here, must be the incompressible quark-superfluid phase. Having r_f reached R_{\star} , then the NS turns invisible. In analogy with normal massive luminous stars, massive and highly compact NSs appear to also switch-off earlier than their less massive counterparts (Hujeirat 2016). For alternative models explaining the above-mentioned mass gap see Belczynski (2012), Chapline (2014) and the references therein.

2 THE ONSET OF INCOMPRESSIBILITIY

Modeling the internal structure of cold NSs while constraining their masses and radii to observations, would require their central densities to inevitably be much higher than the nuclear density $-\rho_0$: a density regime in which all EOSs become rather uncertain and mostly acausal (see Hempel et al. 2011, and the references therein). This however can be viewed as a consequence of the considerable reduction of the compressibility of the nuclear matter at r=0. To clarify this argument: Assume that the energy density and the pressure at r=0 have reached the critical state, at which the particle

involved communicate with each other at the the maximum possible speed, e.g. the speed of light. This corresponds to the EOS: $p=\mathcal{E}$. In this case, the the chemical potential equation reads:

$$\mu = \frac{\partial \mathcal{E}}{\partial n} = \frac{P + \mathcal{E}}{n} = \frac{2\mathcal{E}}{n}, \text{ whose solution is: } \mathcal{E} = a \ n^2.$$

Let the fast communicating particles occupy the finite central volume $dV_c = 4\pi \int_0^{\epsilon} r^2 dr$, where ϵ is an arbitrary small radius. The particles involved practically form a fluid portion that cannot accept compression anymore, as otherwise the causality condition would be violated. The number density here would saturate around n_{cr} and yields a maximum local pressure $P_{cr} = a n_{cr}^2$. With this n_{cr} and P_{cr} , the fluid portion inside dV_c is practically incompressible. On the other hand, the energy inside dV_c is uniform and the involved particles share the same energy, i.e. $\mathcal{E} = \mathcal{E}_0 \times n$, where $\mathcal{E}_0 = a \ n_{cr}$. But as $\mathcal{E}/n = const.$ then local pressure P_L must vanish. This means that the validity of calculating the pressure from the chemical potential alone $(P = n^2 \frac{\partial}{\partial n} (\mathcal{E}/n))$ breaks down. As a consequence, using this formula in this regime would give rise to unrealistically high central energy densities and most likely would violate causality. Moreover, the regularity condition imposed on the pressure at r=0 enforces the supranuclear dense fluid inside dV_c to also be nearly incompressible. To explain this point: since the gradient of the pressure vanishes at r=0, the RHS of the TOV-equation (see Eq. 9) must vanish as well. This is feasible, if the enclosed mass becomes vanishingly small i.e. $m(r) = 4\pi \int_0^{\epsilon} \mathcal{E}r^2 dr \ll 1$ for $\epsilon \ll 1$. On such small length scales and, in the absence of local or exotic feeding mechanisms, gravity alone cannot enforce \mathcal{E} to increase faster than 1/r as $r \to 0$. Therefore the spatial variation of \mathcal{E} inside dV_c remains limited, which means that $m(r) \approx \mathcal{E}_0 r^3$ and therefore the formation of the density plateau around r = 0 becomes inevitable. Under these conditions, computing the local pressure P_L from the chemical potential alone would yield an unrealistic $P_L(\leq 0)$. The usual adopted strategy to escape this pressure-deficiency is to enforce an unfounded inward-increase of \mathcal{E} as $r \to 0$, resulting therefore in unreasonably large central densities.

3 THE ONSET OF QUARK-SUPERFLUIDITY

Another possible solution, which we propose here, runs as follows:

- The nuclear matter at r=0 indeed reaches the compressibility limit and can be well-described by the stiffest EOS $P=\mathcal{E}$.
- A pure incompressible nuclear matter has a constant chemical potential and therefore the validity of computing the local pressure from the chemical potential alone breaks down. In such flows a non-local pressure P_{NL} for controlling the dynamics of the nuclear fluid is required.
- The transition from compressible into pure-incompressible fluid phase might be provoked by the onset of a scalar field matter interaction, which become active, once a critical density, n_{cr} , is surpassed. The interaction potential, V_{ϕ} , generates a non-local negative pressure

 P_{NL} , which is capable of supporting the fluid-configuration against its own self-gravity.

• The onset of interaction has a run-away character: V_{ϕ} injects dark energy, which in turn enforces the transition front to creep from inside-to-outside to abruptly terminate at the surface of the object.

Indeed, beyond ρ_0 , short-range repulsive interactions between particles mediated by the exchange of vector mesons most likely will dominate the dynamics of nuclear matter and would enhance the asymptotic convergence of the EOSs towards $P \to \mathcal{E} \sim n_b^2$ (see Haensel et al. 2007; Camenzind 2007, and the references therein). The chemical potential here $\mu(\doteq (\mathcal{E}+p)/n_b)$ increases linearly with the number density of the baryonic matter n_b . This regime is classified here as H-State and depicted in red-color in Fig. (2).

Recalling that central densities, ρ_c , in NSs increase with their masses, but upper-bounded by $\rho_c \leq 12, 5 \times \rho_0$ to fit the observed mass function (see Lattimer 2011, and the references therein), we conclude that the linear correlation $\mu \sim n_b$ must terminate at a certain critical density n_{cr} , where μ attains a global maximum (Baym & Chin 1976).

On the other hand, in an ever expanding universe, the eternal-state of matter should be the one at which the internal energy reaches a global minimum in spacetime (zero-temperature, zero-entropy and where Gibbs energy per baryon is lowest; henceforth the L-State). Taking into account that $\mu(r)e^{V(r)}=const.$ inside the object together with the a posteriori results $e^{V}\ll 1$ (see Fig. 5), we conclude that $\mu\sim\mathcal{E}/n_b=const.$ and therefore the local pressure $P_L=n_b^2\frac{\partial}{\partial n}(\frac{\mathcal{E}}{n_b})$ must vanish as well. In this case a non-local pressure P_{NL} must be generated in order to oppose self-collapse² of NSs into BHs with $M\leqslant 5\,M_{\odot}$.

If the transition layer between the H and L-states is of finite width in the n-space, then $d\mu/dn$ here may be positive, negative and/or discontinuous.

However, the case $d\mu/dn>0$ should be excluded, as it implies that the eternal state of matter would be more energetic than the H-State, which is a contradiction by constrcution. Similarly, the case $d\mu/dn<0$ is forbidden as it would violate energy conservation (; $d\mu/dn<0\Leftrightarrow dP/dn<0\Leftrightarrow$ adding more particles yields a smaller pressure). Moreover, let us re-write the TOV equation in terms of μ :

$$\frac{d\mu}{dr} = -\frac{G}{c^2r^2}(\frac{d\mathcal{E}}{d\mu})(\frac{d\mu}{dn})(\frac{m+4\pi r^3P}{1-r_s/r}). \tag{2}$$

Obviously, as $\mu > 0$, a negative $d\mu/dn$ would destabilize the hydrostatic equilibrium, unless external sources are included, e.g. bag energy and/or external fields.

Therefore, although a first order phase transition may not be completely excluded, a crossover phase transition into an incompressible superfluid phase with $\mu = \mu(n = n_{cr}) = const.$ would be more likely. Here, μ and P on both sides of the transition front are equal and, with the help of an external field, both $(\mathcal{E}/n)^+$ and $(\mathcal{E}/n)^-$ across the front can be made

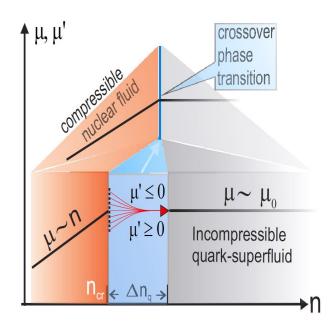


Figure 2. A schematic description of the chemical potential μ , $\mu'(\dot{=}\partial\mu/\partial n)$ versus the number density n at the centers of NSs. At a critical central density, n_{cr} , the universal scalar field \mathcal{H} is set to provoke a crossover phase transition from compressible nuclear fluid into incompressible quark-superfluid. The transition front creeps from inside-to-outside to reach the surface of the object on the scale of Gyrs.

even continuous (Fig. 2)).

In the present study, the simultaneous occurrence of the onset of \mathcal{H} -baryonic matter interaction with the crossover phase transition is necessary in order to generate a non-local pressure with $\nabla P_{NL} < 0$ capable of opposing compression exerted by the surrounding curved spacetime. In the regime $\rho > \rho_0$, such a pressure may nicely resemble a non-local bag energy of quarks in the continuum.

In the presence of \mathcal{H} , the chemical potential per particle at r=0 would be upper-limited by the energy required for quark-deconfinement. In this case, the corresponding Gibbs function reads:

$$f(n) = \frac{\mathcal{E}_b + \mathcal{E}_\phi}{n} - 0.939 \text{ GeV}$$
 (3)

Based on our test calculations, an interaction potential obeying a power law distribution of the type: $V_{\phi}(r) = a_0 r^2 + b_0$ turns out to be optimal for maximizing the compactness of the compact object, i.e, $\alpha_s(\doteq r_s/r) \rightarrow 1$, where r_s corresponds to the dynamical Schwarzschild radius (Fig. 5).

Subtituting $\mathcal{E}_b = a_0 n^2$ and $V_{\phi}(r)$ in Eq. (3) at r = 0, then f(n) reduces to:

$$f(n) = a_0 n + \frac{b_0}{n} - 0.9396. \tag{4}$$

The Gibbs function here may accept several minima at $n_{min} = (b_0/a_0)^{1/2}$, though $f(n = n_{min})$ doesn't necessary vanish. However f(n) > 0 and f(n) < 0 should be excluded, as they are energetically unfavorable for smooth crossover phase transitions to occur.

On the other hand, by varying a_0 and b_0 , a set of true minima could be found. One way to constrain b_0 is to relate

 $^{^2}$ An incompressible fluid with $\mathcal{E}=const.$ has a negative local pressure. Therefore an acausal non-local pressure is necessary for stabilizing the configuration.

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it to the canonical energy scale characterizing the effective coupling of quarks, i.e. $b_0 = 0.221$ (see Bethke 2007, and the references therein). Indeed, as shown in Fig (3), f(n) attains a zero-minimum at $n \approx 3 \ n_0$ for $a_0 = 1.0$.

The question to be addressed here is whether the abovementioned localized analysis would apply for the whole object as well?

Indeed, the injected dark energy, $E_{\phi}(=4\pi\int\mathcal{E}_{\phi}r^2dr)$ via $V_{\phi}(r)$ enforces the spacetime embedding the whole object to be increasingly curved, thereby maximizing the compression of the fluid in front of r_f up to the critical limit and sets r_f into an outward motion. The enclosed dark energy E_{ϕ} via $V_{\phi}(r)$ grows with radius as r_f^5 , i.e. faster than the growth of the baryonic mass, thereby enabling the object to reach a maximum compactness precisely at $r=R_{\star}$. Note that the cases with $E_{\phi}>r_f^5$ and $E_{\phi}< r_f^5$ should be excluded. In the former case, the resulting objects must have collapsed into BHs with $M<5\,M_{\odot}$, which have not been observed. The latter case is not supported by observation either as the surfaces of these massive NSs would continue to be dominated by a normal luminous matter.

Behind r_f , a sea of freely moving quarks is formed, though globally confined by the strongly curved spacetime surrounding the object, which acts as a global confining bag for the quarks. Note that, unlike the constant bag energy model of quarks, where the enclosed deconfinement energy scales linearly with the number of 3-quarks flavors A, the injected dark energy in the present model scales as $A^{5/3}$. This extraenergy may be viewed as a mechanism for further enhancing the gluon like-field embedding the quark-continuum.

We may examine the conditions of coupling of particles in this pure quark-sea by setting $\Lambda=b_0$ and taking N=3 to be the number of quark flavors in the effective quark-gluon coupling constant:

$$\alpha_q = \frac{\pi}{9} \frac{1}{\ln(Q^2/\Lambda^2)}. (5)$$

Relating Q to Fermi momentum and use $n = n_{cr} = 3 n_0$ to infer the Fermi wave number k_F , we obtain $\alpha_q \approx 0.199$.

However, noting that the sea of quarks is incompressible in which communication between particles is mediated with the speed of light, we conclude that the value of α_q should attain its true minimum, which is expected to be much smaller than 0.199. Nevertheless, the present value of α_q still ensures that quarks are in the safe energy regime, where they move almost freely in line with the asymptotic freedom of quantum chromodynamics -QCD (see Bethke 2007, and the references therein).

4 GOVERNING EQUATIONS AND SOLUTION METHOD

Our investigation here is based on numerical solving the TOV equation modified to include scalar fields (\mathcal{H}) . The modified stress energy tensor reads:

$$T_{\mu\nu}^{mod} = T_{\mu\nu}^0 + T_{\mu\nu}^\phi. \tag{6}$$

The superscripts "0" and " ϕ " correspond to baryonic and scalar field tensors:

$$T^0_{\mu\nu} = -P^0 g_{\mu\nu} + (P^0 + \mathcal{E}^0) U_\mu U_\nu$$
 and

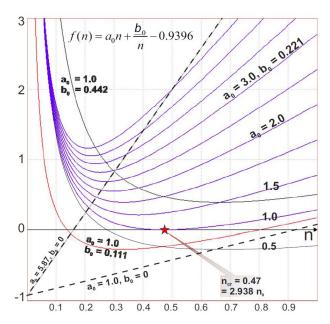


Figure 3. The modified Gibbs function f(n) versus baryonic number density n (in units of n_0) is shown for various values of a_0 and b_0 . Obviously, $a_0 = 1$ and $b_0 = 0.221$ appear to be the most appropriate parameters that are compatible with QCD. The value $b_0 = 0.221$ corresponds to the canonical energy scale characterizing the effective coupling of quarks inside individual hadrons. The Gibbs function f(n) here attains a zero-minimum at $n = 2.938 \, n_0$, at which \mathcal{H} is set to provoke a phase transition into the INQSF state.

$$T^{\phi}_{\mu\nu} = (\partial_{\mu}\phi)(\partial_{\nu}\phi) - g_{\mu\nu} \left[\frac{1}{2}(\partial_{\sigma}\phi)(\partial^{\sigma}\phi) - V(\phi)\right]. \tag{7}$$

 U_{μ} here is the 4-velocity, the subindices μ , ν run from 0 to 3 and $g_{\mu\nu}$ is a background metric of the form:

$$g_{\mu\nu} = e^{2V} dt^2 - e^{2\lambda} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d^2\varphi^2,$$
 (8)

where V, λ are functions of the radius.

Assuming the configuration to be in hydrostatic equilibrium, then the GR field equations, $G_{\mu\nu}=-8\pi G T_{\mu\nu}$ reduce into the generalized TOV equations:

$$\frac{dP}{dr} = -\frac{G}{c^4 r^2} [\mathcal{E} + P][m(r) + 4\pi r^3 P]/(1 - r_s/r),\tag{9}$$

where $m(r)=4\pi\int\mathcal{E}r^2\,dr$ is the total enclosed mass: $\mathcal{E}=\mathcal{E}^0+\mathcal{E}^\phi,\ P=P^0+P^\phi,$ and where $\mathcal{E}^\phi=\frac{1}{2}\dot{\phi}^2+V(\phi)+\frac{1}{2}(\nabla\phi)^2,\ P^\phi=\frac{1}{2}\dot{\phi}^2-V(\phi)-\frac{1}{6}(\nabla\phi)^2.$ $V(\phi)$ here denotes the interaction potential of the scalar field with the baryonic matter, i.e., the rate at which dark energy is injected into the system and $\dot{\phi}$ is the time-derivative of ϕ .

Our reference object is a NS with 1.44 \mathcal{M}_{\odot} with a radius $R_{\star} = 2 \times R_S$, where R_S is the Schwarzschild radius. ϕ is assumed to be spatially and temporarily constant, whereas V_{ϕ} is set to obey the power-law distribution: $V_{\phi} = a_0 r^{\Gamma} + b_0$. a_0 and b_0 are constant parameters that are chosen so to fulfill the a posteriori requirement: $R_{\star} = R_S + \epsilon$, for $\epsilon \ll 1$. In most of the cases considered here, b_0 is set to be identical to the canonical energy scale at which mometum transfer between quarks saturates, i.e., $b_0 = 0.221 \text{GeV}$. The

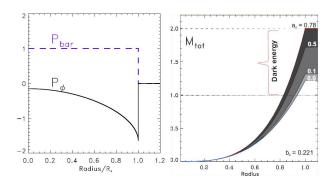


Figure 4. The radial distributions of the baryonic pressure (P_{bar}) and negative pressure (P_{ϕ}) inside an incompressible quark-superfluid core (left). The enclosed mass of the baryonic matter and the gradual mass-enhancement due to dark energy is shown for different values of a_0 (right).

fluid in the post transition phase is governed by the EOS: $P^0 = \mathcal{E}^0 = \rho_{cr}c^2 = const.$

For a given central density, the solution procedure adopted here is based on integrating the equations for the pressure, enclosed mass and pseudo-gravitational potential from inside-to-outside, using either the first order Euler or fourth order Runge-Kutte integration methods.

5 RESULTS & DISCUSSIONS

The here-presented model of DEOs is motivated by the following three unresolved theoretical and observational problems in the astrophysics of NSs:

- Why neither NSs nor BHs have ever been observed in the mass-range 2 5 M_{\odot} .
- Most sophisticated EOS used to model the internal structure of NSs are based on central densities that are far beyond the nuclear density: a density regime of great uncertainty.
- How NSs end their life in an ever expanding universe and whether there is a hidden connection between the missing massive NSs and dark matter on the one hand and with dark energy in the universe on the other hand.

In this paper we argue that the formation of DEOs may provide answers to these unresolved problems. This scenario could be summarized as follows:

- (i) The very central regions of NSs are made of superfluid nuclear matter and that these would be the first to be evacuated from vortex lines and all other removable energies that do not contribute significantly to the pressure. The nuclear fluid here is governed by the stiff EOS: $P = \mathcal{E} = a_0 \, n^2$.
- (ii) In order to escape collapse into a BH with $M < 5~M_{\odot}$, the chemical potential μ in the very central regions cannot grow indefinitely, and it must terminate at a certain critical value, n_{cr} , where the fluid is set to undergo a phase transition.

Based on minimum energy consideration, a crossover phase transition into an incompressible quark-superfluid has been shown to be energetically a favorable transition.

(iii) We have shown that in the presence of a universal

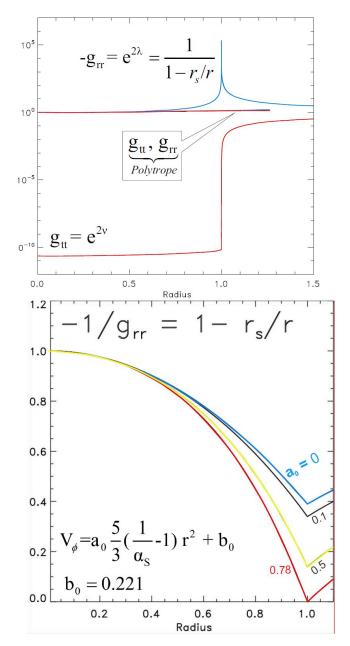


Figure 5. In the top panel we show the radial distributions of the metric coefficients g_{rr} and g_{tt} inside a NS (; $P_L = \mathcal{K}\rho^{\gamma}$ and $P_{\phi} = 0$) and inside a DEO (; $P_L = const.$ and $P_{\phi} = -V_{\phi}$). Obviously, normal models of NSs have larger radii and considerably less compact than their DEO-counterparts, which can be inferred from the very limited spacial variations of g_{rr} and g_{tt} . In the lower panel, the compactness of a typical DEO, expressed in terms of $-1/g_{rr}$ is shown for different values of a_0 . The object turns invisible if V_{ϕ} is calculated using $a_0 = 0.78$ and $b_0 = 0.221$.

scalar field \mathcal{H} , the injected dark energy is capable of provoking a phase transition into INQSF that roughly occurs at $n \approx 3~n_0$. The action of the injected energy is equivalent to generating a gluon-like field, or enhance the available gluon-field through forming a global energy bag in the continuum, inside which quarks move almost free in line with the asymptotic freedom of quantum chromodynamics.

(iv) We have shown that the transition front creeps from inside-to-outside on the scale of Gyrs, forming a sea of

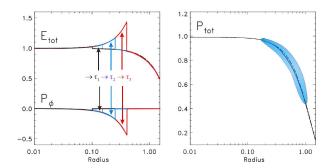


Figure 6. The profiles of the total energy density E_{tot} , the pressure P_{ϕ} induced by \mathcal{H} and the combined pressure P_{tot} versus radius are shown for different evolutionary epochs $\tau_1 < \tau_2 < \tau_3$. Inside $r_f \colon P_L = 0$ and $P_{\phi} = -V_{\phi}$, whereas outside $r_f \colon P_L = \mathcal{K}\rho^{\gamma}$ and $P_{\phi} = 0$. In each epoch, the object has an INQSF-core overlayed by a shell of normal compressible matter obeying a polytropic EOS. Obviously, the object appear to comfortably adjust itself to the mass-redistribution inside r_f , where matter is converted into INQSF.

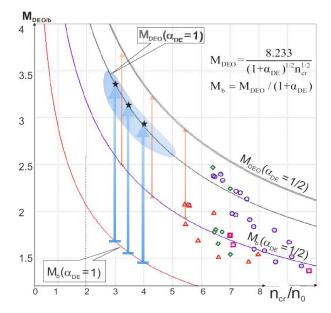


Figure 7. Upper mass limit of DEOs versus critical density n_{cr} (in units of n_0) is shown. The \mathcal{H} -baryon interaction is set to occur at n_{cr} , which in turn provokes the phase transition into the INQSF state. The most probable mass-regime of DEOs is marked here as a blue region. Accordingly, the progenitor of a DEO with $3.36\,M_{\odot}$ should be a NS of $1.68\,M_{\odot},$ provided it has an initial compactness $\alpha_S=1/(1+\alpha_{DE})=1/2$ and $n_{cr}=3\,n_0$. Similarly, a Hulse-Taylor type pulsar would end as a DEO of $2.91 M_{\odot}$, if its initial compactness is $\alpha_S = 1/2$ and if $n_{cr} = 4 n_0$. On the other hand, moderate and massive NSs with initial compactness $\alpha_S \geqslant 2/3$, i.e., $\alpha_{DE} \leqslant 1/3$, need less dark energy to become invisible DEOs, but require unreasonably high n_{cr} for the onset of H-matter interaction. NSs falling in this category are to be compared with the colored small cycles and triangles, which show the approximate locations of various NS-models as depicted in Fig. (4) of Lattimer & Prakash (2011).

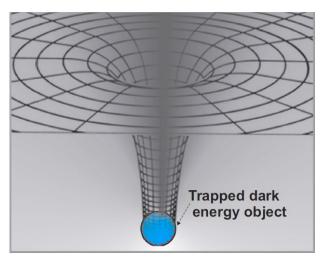


Figure 8. A schematic description of a DEO, inside which spacetime is fairly flat, but becomes extra-ordinary curved across their surfaces. As the binding energy inside DEOs vanishes, they are astoundingly similar to ultra-giant hadrons trapped inside a strongly curved spacetime, which render them invisible.

quarks behind the front. Indeed, the very slow outwards propagation of r_f grants the NS ample of time to stably react to all possible conditions associated with the phase transition, including a global re-distribution of mass inside r_f (Fig. 6).

- (v) We have shown that an interaction potential of the type $V_{\phi} = a_0 r^2 + b_0$ is capable of maximizing the compactness of the object (Fig. 5).
- (vi) Having r_f reached the surface of the NSs, these object become DEOs. Their interiors are made solely of IN-QSFs with constant chemical potential. The spacetime inside DEOs has been identified to be fairly flat, whereas it promptly becomes extra-ordinary curved across their surfaces (Fig. 5). Inside DEOs, the nuclear fluid has a vanishing binding energy and therefore mimicking the configuration of an ultra-giant hadron trapped in a strongly curved spacetime.
- (vii) According to the here-presented scenario, all visible pulsars and NSs must contain incompressible quark-superfluid cores supported and confined by a dark energy component which is induced by a scalar field of universal origin. The gravitational significance of the injected dark energy in these cores depends strongly on their evolutionary phase and in particular on their ages and initial compactness. Accordingly, young NSs should be less massive than old ones, and the very old NSs should turn invisible by now. To quantify the mass-enhancement by \mathcal{H} , let M_b be the mass of the NS at its birth and M_ϕ being the mass enhancement due to \mathcal{H} . Requiring $R_\star > R_S$, then the following inequality holds:

$$(1 + \alpha_{DE}) \leqslant (\frac{3\rho_{cr}}{32\pi})^{1/3} \frac{c^2}{GM_b^{2/3}},$$
 (10)

or equivalently,

$$1 \leqslant \frac{E_{tot}}{E_b} \leqslant 2.06 \frac{\rho_{15}^{1/3}}{M_{b/1.44}^{2/3}},\tag{11}$$

where $\alpha_{DE} \doteq M_{\phi}/M_{b}$. E_{tot} , ρ_{15} , $M_{1.44}$ denote the total

energy, the density in units of 10^{15} g/cc and the baryoinc mass of the NS in units of $1.44\,M_\odot$, respectively.

Thus, NSs are born with $E_{tot} = E_b$, and by interacting with \mathcal{H} , they become more massive and more compact to finally reach $R_{\star} = R_S + \epsilon$ at the end of their luminous phase, which would last for approximately 10^9 yr or less, depending on their initial compactness. Thus, a NS with initial compactness $\alpha_s = 1/2$ will have to double its mass to become a DEO (Fig. 4 and Fig. 7).

According to the present scenario, the Hulse-Taylor pulsar should have an INQSF core, though the dark energy component is gravitationally insignificant due to its young age, and therefore the size of its INQSF-core must be still small. Assuming the baryon mass of the pulsar to remain constant as it evolve on the cosmological time scale, then the pulsar will turn into invisible-DEO in roughly one Gyr. This would imply that the onset of \mathcal{H} -baryon interaction should occur at roughly four times the nuclear density, which is in the range of the here-predicated critical density (Fig. 3). On the other hand, the extra-mass resembles the lower energy limit required for deconfining the sea of quarks, i.e., the energy needed for generating a see of quark antiquark pairs. Similar to quarks in hadrons, the sea of quarks inside DEOs can never be observed as free objects in the sky.

Recalling that the effective potential of the gluon-field inside individual hadrons is on the average predicted to increases with radius as $r^{\Gamma(>1)}$ and that the spatial variation of the coefficient g_{rr} of the Schwarzschild metric on comparable length scales is negligibly small $(dg_{rr}/dl \ll 10^{-19})$, we conclude that gluon-fields do not accept stratification by gravitational fields.

Therefore as E_{ϕ} in the present DEO-models is dominant and increases with radius, the sea of quarks inside DEOs is in a purely incompressible state and cannot accept stratification (see g_{tt} in Fig. 5). In such gravitationally bounded incompressible fluid-configurations, not only that $\mu = \mathcal{E} = const.$, but the classical repulsive pressure $P_L(\stackrel{.}{=} n_b^2 \frac{\partial}{\partial n}(\frac{\mathcal{E}}{n_b}))$ must vanish also and should be replaced by a non-local pressure, P_{NL} , in order to avoid the formation of BHs with $M < 5 M_{\odot}$.

Unlike EOSs in compressible normal plasmas, classical EOSs in incompressible superfluids are non-local. In the latter case, constructing a communicator that merely depends on local exchange of information generally would not be sufficient for efficiently coupling different/remote parts of the fluid in a physically consistent manner. A relevant example is the solution of the TOV-equation for classical incompressible fluids ($\mathcal{E}=const.$). In this case, the pressure depends, not only on the global compactness of the object, but it becomes even acausal whenever the global compactness is enhanced.

This is similar to the case when solving the incompressible Navier-Stokes equations, where an additional Laplacian operator for describing the spatial variation of a non-local scalar field is constructed to generate a pseudo-pressure (; actually a Lagrangian multiplier), which, again, does not respect causality (Hujeirat & Thielemann 2009).

Indeed, DEOs made of incompressible quark-superfluids would be stable also against mass-enhancement from outside. Let a certain amount of baryonic matter, $\delta \mathcal{M}_b$, be

added to the object from outside. Then the relative increase of R_{\star} compared to R_{S} scales as: $\frac{\delta R_{\star}}{\delta R_{S}} \simeq \frac{\rho_{cr}}{\tilde{\rho}_{new}}$, where $\tilde{\rho}_{new}$ is the average density of the newly settled matter. Unless $\tilde{\rho}_{new} > \rho_{cr}$, which is forbidden under normal astrophysical conditions, the star would react stably. However, in the case of super-Eddington accretion or merger, the newly settled matter must first decelerate, compressed and subsequently becomes virially hot, giving rise therefore to $\tilde{\rho}_{new} \ll \rho_{cr}$. On the other hand, such events would lower the confinement stress at the surface and would turn the quantum jump of the energy density at R_{\star} , which falls abruptly from approximately $\mathcal{E} \approx 10^{36}$ erg/cc at R_{\star} down to zero outside it, into an extra-ordinary steep pressure gradient in the continuum. While such actions would smooth the strong curvature of spacetime across R_{\star} , they would enable DEOs to eject quark matter into space with ultra-relativistic speeds, which is forbidden. Nonetheless, even if this would occur instantly, then the corresponding time scale τ_d would be of order Λ_j/c , where Λ_j is the jump width in centimeters and c is the speed of light. Relating Λ_j to the average spacing between two arbitrary particles ($\sim n^{-1/3}$), this yields $\tau_d \approx 10^{-24} \,\mathrm{s}$, which is many orders of magnitude shorter than any known thermal relaxation time scale between arbitrary luminous particles.

Although electromagnetic activities and jets have not been observed in dark matter halos, they are typical events for systems containing black holes. Recalling that supermasive GBECs are dynamically unstable (Hujeirat 2012), our results here address the following two possibilities:

- If the onset of \mathcal{H} -baryon interaction indeed occurs at n_{cr} , then the majority of the first generation of stars and the massive stars formed in the subsequent early epochs must have ended as pulsars and NSs, rather than collapsing into stellar BHs with $M \leq 5 \times M_{\odot}$. In this case, dark matter halos most likely should be DEO-rich clusters. These clusters must have been extraordinary luminous in the early universe, but became inactive and dark after the nuclear matter in the interiors of NSs converted into the INQSF-phase, subsequently sweeping away all sorts of luminous matter in their surroundings due to their inability to accrete normal matter. The enormous surface stress confining the sea of quarks in the interiors of DEOs render their surfaces impenetrable for normal matter, hence these objects behave as non-interacting objects.
- The average repulsive forces governing clusters of DEOs most likely would enforce approaching luminous matter to deviate from face-to-face collisions and therefore stay inactive, though n-body and SPH-numerical calculations are needed here to verify this argument.

Finally we note that, similar to the gluon field confining and governing the dynamics of almost massless quarks in hadrons, the $\mathcal{H}-$ induced energy enhancement of the gluon-like field in DEOs cannot surpass the limit, beyond which they collapse to form BHs with $M<5~M_{\odot}$. Moreover, the enclosed dark energy injected via V_{ϕ} scales as r^5 : this outlines an upper limit for the increase of confining energy with radius in DEOs, beyond which they undergo a self-collapse. However, whether this limit applies for the potential of the gluon field inside hadrons is not clear at the moment and demands further investigations.

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In a subsequent article, we discuss the compatibility and physically consistency of the here-presented internal structures of DEOs with the bi-metric formulation of spacetime in general relativity proposed by Rosen (1977).

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