SRG evolution of two-body interactions in 1D

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Abstract

In this group project, we solve SRG differential flow equation for evolution of spin-independent two-body interactions in one dimension [1].

I. NN INTERACTIONS:

Simple two-boson interactions used to study the SRG evolution of NN interactions:

- one-term separable potential

$$V_{sep}(p,p') = g\, e^{(-p^2/\Lambda^2)}\, e^{(-p'^2/\Lambda^2)} \tag{1}$$

  with parameters $g = -1$ and $\Lambda = 10$.

- Gaussian potential

$$V(p,p') = \sum_{i=1,2} \frac{V_i}{2\pi}\, e^{\left(-\frac{(p-p')^2\alpha_i^2}{4}\right)} \tag{2}$$

  with parameters given in Table I.

	V_1	V_2	α_1	α_2
Model I	12	-12	0.2	0.8
Model II	0	-2	0	0.8

TABLE I. Parameters of the two-body Gaussian potential.

II. STEPS IN THE IMPLEMENTATION PROCESS:

1. Formulation of SRG flow equation - Hadi
2. Coding
   - potential - Sudhanva
   - SRG flow equation - Arbin
   - ODE solver - Linda
   - LS equation for 2B bound state - Sudhanva
3. Preparation of the results - Mamun
   - 2D contour plots for SRG evolution of 2B interaction
   - A table for 2B binding energy as a function of SRG flow parameter
4. Presentation of the results

III. SRG FLOW EQUATION IN 1D:

SRG flow equation for evolution of NN interaction can be written as:

$$\frac{\mathrm{d}H_s}{\mathrm{d}s} = \frac{\mathrm{d}V_s}{\mathrm{d}s} = G_s H_s H_s + H_s H_s G_s - 2H_s G_s H_s, \tag{3}$$

where the generator operator can be chosen to be kinetic energy, i.e. $G_s = T$. The reason for this selection is that T is, in itself, diagonal in momentum space and as we shown by simple example of two state problem, this generator drives the Hamiltonian to the diagonal form, which is usually desirable. Using the kinetic energy operator as generator, the SRG equation can be written as

$$\frac{\mathrm{d}V_s}{\mathrm{d}s} = 2TV_sT + V_sV_sT + TV_sV_s - V_sTT - TTV_s - 2V_STV_s. \tag{4}$$

In order to project the flow equation in momentum space, we define 2B basis states in partial wave representation and in $s-$wave channel as:

$$|p\, l = 0\rangle, \tag{5}$$

where p is the magnitude of the relative 2B momentum $\mathbf{p}$ which is defined as

$$\mathbf{p} = \frac{1}{2}\left(\mathbf{k}_1 - \mathbf{k}_2\right), \tag{6}$$

the momenta $\mathbf{k}_1$ and $\mathbf{k}_2$ are momentum of particles 1 and 2 in the laboratory frame. The 2B basis states are complete as

$$1 = \int_0^\infty \mathrm{d}p\; |p\, l = 0\rangle\langle p\, l = 0| = \int \mathrm{d}p\; |p\, 0\rangle\langle p\, 0|. \tag{7}$$

The representation of SRG flow equation (4) in the introduced basis states in Eq. (5) leads to

$$\begin{aligned}\frac{\mathrm{d}V_s(p,p')}{\mathrm{d}s} = &-\left(p^2 - p'^2\right)^2 V_s(p,p') \\ &+ \left(p^2 + p'^2\right)\int_0^\infty \mathrm{d}p''\, V_s(p,p'')\, V_s(p'',p') \\ &- 2\int_0^\infty \mathrm{d}p'' p''^2\, V_s(p,p'')\, V_s(p'',p'),\end{aligned} \tag{8}$$

Clearly the first term on the right hand side of differential flow equation ensures that the potential is driven to the diagonal.

TABLE II. Binding energies for different values of s.

s	V_β	V_{sep}
0	0.17505661803591174	5.128804871933985
10^{-4}	0.17505661803591246	5.128804873258882
10^{-3}	0.17505661803591293	5.128804873260954
10^{-2}	0.17505661803591257	5.12880487326095

IV. TWO-BODY BOUND STATE

In order to test the SRG evolved 2B interaction, we should check it to be sure 2B observables are preserved. To this aim we solve the homogeneous Lippmann-Schwinger integral equation for 2B bound state to verify the preservation of 2B binding energy in each step of evolution. The bound state of two equal mass particles in momentum space and in partial wave representation and in $s-$wave channel is given by

$$\psi_0(p) = \frac{1}{E - \frac{p^2}{m}} \int_0^\infty dp' \, V_0(p, p') \, \psi_0(p'), \tag{9}$$

where $V_0(p, p')$ is $s-$wave projection of 2B potential and $\psi_0(p)$ is 2B wave function.

V. RESULTS

The value of the potential for various values of s have been tabulated in table (II).

[1] O. Akerlund et al., Eur. Phys. J. A **47**, 122 (2011).