

## Digital Acquisition and Processing

Week day	Date	Hour	Group	Students Numbers	
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### Step 1

Signal			Acquisition		Apparent frequency
Waveform	Frequency [kHz]	Amplitude [V]	Points	Rate [kHz]	
Sinusoidal	1 kHz	1 V	100 000	100 kHz	1 KHz
Sinusoidal	1 kHz	4 V	100 000	100 kHz	1 KHz
<b>Observations:</b> As expected, when the acquisition rate matches the signal frequency, the FFT outputs a sharp value in the corresponding frequency.					
Sinusoidal	1 kHz	1 V	100 000	100 kHz	1 KHz
Sinusoidal	1 kHz	1 V	999 950	100 kHz	≈1 KHz
<b>Observations:</b> In the second measurement, spectral leakage is observed since the overall acquisition duration does not match an integer number of signal periods.					
Sinusoidal	20 kHz	1 V	100 000	100 kHz	20 kHz
Sinusoidal	40 kHz	1 V	100 000	100 kHz	40 kHz
Sinusoidal	50 kHz	1 V	100 000	100 kHz	50 kHz
Sinusoidal	60 kHz	1 V	100 000	100 kHz	40 kHz
Sinusoidal	100 kHz	1 V	100 000	100 kHz	0 kHz
<b>Observations:</b> Now we sample below the Nyquist frequency, resulting in an inaccurate signal acquisition and hence its spectrum shows apparent frequencies. About the 50 kHz signal, an extra $\pi/4$ phase was added to avoid to sample at the same output ( $\approx 0V$ ) for rising and falling sine wave. For 60 kHz we notice the mirrored fundamental at $60-50 = 40$ kHz. Finally, since the 100 kHz signal matches the sampling frequency, the acquired signal is ideally at the same point (0V) resulting in a reconstructed constant signal.					
Triangle	1 kHz	1 V	100 000	100 kHz	1 (+3+5+...) kHz
Square	1 kHz	1 V	100 000	100 kHz	1 (+3+5+7+...) kHz

Square	15 kHz	1 V	100 000	100 kHz	<b>15 (+ 45) kHz</b>
<b>Observations:</b> Non-sinusoidal signals as triangle and square waves are approximated by summing odd harmonics of the fundamental with power-decreasing amplitudes. This is indeed noticed in this simulation where the fundamental frequency is clearly observed (at 1 kHz and 15 kHz) plus minor frequencies at its odd multiples. In the last case frequencies of the higher harmonics (> 50 kHz) are folded and start being mirrored at 0 Hz. The power decrease is also slower than a triangular wave ( $1/n$ versus $1/n^2$ ).					

## Step 2

Mean value formula, MEAX(x)	$\bar{x} = \sum_{n=0}^{N-1} x_n$
MEAN(X)	$\bar{x} = 1.250 \text{ V}$
MEAN(Y)	
MEAN(Z)	
<b>Observations:</b> The signal is generated with Motor Power Supply = 1V since the generator does not accept 0V (and unbalancing = 0, health = 100%). Y and Z axes are neglected since this is a simulated scenario.	

## Step 3

Root mean square formula, RMS(x)	$RMS(x) = \sqrt{\frac{1}{N} \sum_{i=0}^N x_i^2}$
RMS(X)	$RMS(x) = 1.251 \text{ V}$
RMS(X – MEAN(X))	$RMS(x - \bar{x}) = 0.071 \text{ V}$
<b>Observations:</b> In the second RMS the DC component is subtracted. It captures only on the AC component, that is equivalent to the signal standard deviation, from the vibrations. The two are related by $RMS_{AC+DC} = \sqrt{RMS_{DC}^2 + RMS_{AC}^2}$	

## Step 4

Input range	$\pm 2 \text{ V}$
Sampling rate	1 kHz

<b>Number of points</b>	10000
<b>Observations:</b> By tweaking the motor parameters with the above specifications, we observed that the power voltage affects the main vibration frequency, the unbalancing weight produces an oscillation at lower frequencies whereas a worse gear health lead to additional different vibration frequencies.	

## Step 9

<b>Feature to estimate the rotation speed</b> The rotation speed, directly related to the supply voltage, can be estimated by the most powerful peak at higher frequencies (so besides the constant signal component). At different voltages in the range [1,12] V, this peak appears to be almost linear in power but perfectly linear in its corresponding frequency. The corresponding linear interpolation gives $freq = 14 V - 0.9$ that can be easily inverted to estimate the corresponding voltage input knowing the peak frequency.
<b>Feature to estimate the unbalancing weight</b> Adding a weight to the rotating motor produces an additional periodic force at low frequency, on top of the always present motor vibrations. This frequency is at $\approx 2/3$ Hz for all different values of the weight, so the only way to reverse-estimate it is by looking at its power spectrum. This manifests a linear behavior according $power = 8.2 \times 10^{-4} * weight + 1.3 \times 10^{-4}$ . So once the power of this frequency is known, gauge the unbalancing weight is just a matter of inverting the above expression.
<b>Feature to estimate the gear's health condition</b> Worsening the gear's health condition adds 4 additional frequencies to the signal. This phenomenon is visible for $health \leq 75\%$ when these components have fixed frequencies (at 12V and no weight) of [51.8, 69.4, 92.8, 124.1] Hz, whereas this is neglectable for better conditions. Indeed, we can also affirm that below that threshold the power of these components increases exponentially, up to the worst-case scenario of 0% health. This can be modeled by taking the average of the 4 peaks and use a linear regression for the values in the range [0; 75]%, giving $power = -1 \times 10^{-4} * health + 0.0078$ .

## Step 10

<b>Rotation Speed [V]</b>	<b>Feature: Max Peak Freq. [Hz]</b>	<b>Unbalancing Weight [g]</b>	<b>Feature: Power 3 Hz [W/Hz]</b>	<b>Gear health condition [%]</b>	<b>Feature: Average power of Peaks [W/Hz]</b>
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1	13	0	0.0004	0	0.0086
2	26.9	10	0.0083	5	0.0077
3	40.8	20	0.0166	10	0.0069
4	54.7	30	0.0247	15	0.0061
5	68.5	40	0.0329	20	0.0055
6	82.4	50	0.0412	25	0.0048
7	96.3	60	0.0494	30	0.0042
8	110.2	70	0.0578	35	0.0036
9	124.1	80	0.0661	40	0.0031
10	138.0	90	0.0745	45	0.0027
11	151.9	100	0.0826	50	0.0022
12	165.8			55	0.0017
				60	0.0013
				65	0.0010
				70	0.0007
				75	0.0006

**Observations:**

As already discussed above, these features allow us to build linear models to estimate the experimental setup properties, based only on the output power spectrum.  
One possible algorithm has been proposed on the attached Matlab script.

## Conclusions

In conclusion, we were able to analyze and discuss spectrums for different signals, while dealing also with spectral leakage and Nyquist Theorem. We could also characterize a physical experiment and designed an algorithm to guess its relevant features from the raw acquired data. Although this was successful, we must state that this was a simulated scenario where only certain conditions combinations have been tested and uncertainties have been neglected. Nevertheless, a similar rationale can be extended to more complex real cases.