DEEC
DEPARTAMENTO DE ENGENHARIA ELECTROTÉCNICA E DE COMPUTADORES
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Big Data Measuring Systems

LAB 1 Short Report: Uncertainty

Week day	Date	Hour	Group	Students Numbers	
Tuesday	24/3			Paolo Frazzetto	94942

Step 1

Uncertainty propagation law	$u_c(y) = \sqrt{\sum_{i=1}^{M} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{M-1}\sum_{j=i+1}^{M} \frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j} u(x_i, x_j)}$

Step 2

$u(\hat{m})$	$\sqrt{\sum_{i=1}^{V} \left(\frac{\partial \hat{m}}{\partial v_i}\right)^2 u^2(v_i)} = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial \hat{m}}{\partial x_i}\right)^2 u^2(x_i) + \sum_{i=1}^{N} \left(\frac{\partial \hat{m}}{\partial y_i}\right)^2 u^2(y_i)}$
$\frac{\partial \hat{m}}{x_i}$	$= \frac{(y_i - \bar{y})(\sum(x_i - \bar{x})^2) - 2\left(\sum(x_i - \bar{x})(y_i - \bar{y})\right)(x_i - \bar{x})}{(\sum_{i=1}^{N}(x_i - \bar{x})^2)^2}$

$\frac{\partial \hat{m}}{y_i}$	$= \frac{x_i - \bar{x}}{\sum_i (x_i - \bar{x})^2}$
$u(\hat{b})$	$= \sqrt{\sum_{i=1}^{N} \left(-\bar{x}\frac{\partial \hat{m}}{\partial x_i} - \frac{\hat{m}}{N}\right)^2 u(x)^2 + \left(\frac{1}{N}\right)^2 u(y)^2}$

Step 2

ΔN	**10**	**25**	**50**	**75**	**100**	**150**	**200**	**500**
$u(\hat{m})$	0.022	0.015	0.011	0.009	0.008	0.006	0.005	0.003
$u(\hat{b})$	0.064	0.040	0.028	0.023	0.020	0.016	0.014	0.009
$\Delta u(x)$	**0.001**	**0.01**	**0.05**	**0.1**	**0.5**	**1**	**5**	**10**
$u(\hat{m})$	0.003	0.003	0.005	0.008	0.033	0.062	0.158	0.193
$u(\hat{b})$	0.001	0.002	0.010	0.020	0.100	0.200	1.003	2.043
$\Delta u(y)$	**0.001**	**0.01**	**0.05**	**0.1**	**0.5**	**1**	**5**	**10**
$u(\hat{m})$	0.19	0.20	0.18	0.20	0.21	0.20	0.23	0.25
$u(\hat{b})$	2.01	2.05	2.00	2.02	2.01	2.00	2.00	2.08

Observations:
As expected for the central limit theorem, increasing N lead to a general smaller uncertainty. u(x) has a larger impact on the parameter's errors, since the datapoints are more spread and both intercept and slope can vary significantly depending on a specific sample. In fact, the linear least squares method assumes negligible uncertainty on the dependent variable. On the

other hand, u(y) does not affect so significantly the uncertainties since, even for larger variances, it is still normally distributed.

Step 3

Multimeter Accuracy: 0.1%

Luxmeter Accuracy: 4% rdg + 8 dgts

Maximum error (uniform distribution) to uncertainty		$u(x) = \frac{\epsilon_x}{\sqrt{3}}$						
Luminosity	**[Lux]**	651	940	1450	1932	2640	3420	3885
Maximum error	**[Lux]**	34	46	66	85	114	145	163
Voltage	**[V]**	-0.540	-0.402	-0.260	-0.076	0.021	0.151	0.230
Maximum Error	**[V]**	0.00054	0.00040	0.00026	0.00008	0.00002	0.00015	0.00023
$\hat{m}$		$= 0.000228 \pm 0.000006 \text{ V/Lux}$						
$\hat{b}$		$= -0.612 \pm 0.009 \text{ V}$						

Step 4

Luminosity as a function of the sensor measurement	$x_i = \frac{y_i - \hat{b}}{\hat{m}}$
Uncertainty as a function of the sensor measurement	$u(x_i) = \sqrt{u(y_i)^2 + \left(\frac{1}{N}\right)^2 u(b)^2 + \left(\frac{x_i - \bar{x}}{\sum_i (x_i - \bar{x})^2}\right)^2 u(m)^2}$

Step 5

Temperature effect on the LDR sensor	Semiconductors are sensible to temperature, therefore even a small variation can play a relevant role. In this setting, increasing temperature diminishes the LDR resistance, causing biased luminosity measurements.

Uncertainty as a function of the sensor measurement	The LDR datasheet states that the device working range spans $-30 \sim +70^{o}C$ but additional information about the temperature response are not provided. Once modeled, this unknown effect can be incorporated into the uncertainty: $$u(x_i) = \sqrt{\text{previous terms} + \left(\frac{\partial x_i}{\partial \, \mathrm{T}}\right)^2 u(\mathrm{T})^2}$$
Observations: There are various models to describe the relation between Resistance and Temperature of semiconductors, relating the ions of the semiconductor with the thermal energy of the charge carriers. At room temperature, we can assume that this will not affect the calibration and reproducibility of the experiment, whereas it may be relevant at different temperature conditions.	

Conclusions

The laboratory experiment was carried out successfully. The measurements and their uncertainties have been combined to obtain a simple linear calibration model that relates the sensor output with the physical quantity.