Quantum Blackjack

 $\langle \text{Bro} | \text{Ket} \rangle$

1 Classical Game

The game is played with the standard 52 card deck, between the dealer and the player. The main **objective** for the player on each round is to get a count on the cards as close to 21 as possible, without going over 21 (and higher than the dealer) [1]. The values of each cards are the following:

- Pip cards: Their pip value
- Ace: 1 or 11, depending on what the player wants.
- Face cards (King, Queen, Jack): 10

1.1 Betting

The betting <u>must be done before starting each round</u>. The betting is done by using casino chips. After each hand, the possible outcomes of the bet are [2]:

- Lose: The player's bet is given to the dealer.
- Win: The player wins as much as he has bet. If the bet was of $10 \in$, the player keeps those $10 \in$, and also wins another $10 \in$.
- Blackjack: The player wins 1.5 times the bet. If the bet was of 10 €, the player keeps those 10 €, and also wins another 15 €.
- **Push:** There is a draw. The player keeps his bet (he doesn't lose any money).

1.2 Gameplay

First of all the dealer shuffles the cards. Afterwards, the dealer deals one card face up clockwise to each player (in our case to just one player), and then to himself face down. Then deals again, now all cards face up. In the end, players must have two cards face up, and the dealer one card faced up, and the other one down [1].

1.2.1 Natural/Blackjack

A **natural** or **blackjack** happens when after dealing cards one player has 10-valued card and an ace (the count will be 21). The player with said cards automatically wins (except if the dealer also has a blackjack), and wins 1.5 times the bet. When both player and dealer have a blackjack, then there is a Push [2].

1.2.2 Player Turn

After the initial dealing, the player can make the following decisions [1, 2]:

- Hit: Ask for another card, in order to get closer to 21.
- Stand: Not ask for another card.
- Bust: The sum of the cards is over 21. In this case the player loses.

For example, the player may ask for more cards (hit) until the hand is strong enough to go up against the dealer (and then stand), or goes bust.

Note. As seen in [1], there are other actions that the player can make. They can be implemented in future developments.

1.2.3 Dealers turn

After the player has played, the face down card is turned up. Now, the different options are:

- If the count is lower than 17, the dealer takes a card.
- If the count is 17 or more, the dealer must stand.
- If the count is 21 (a blackjack) after turning up the card, then the dealer wins, and every player without a blackjack loses.

1.2.4 Reshuffling

After every bets are settled, the dealer gets every card used in the round and places them at the side. Then the next round can begin. When there are no cards from the deck, the dealer shuffles again and the game continues.

1.2.5 End of the game

The game ends when the dealer or the player loses all their chips.

2 Quantum Game

2.1 Quantum Cards

Instead of classical cards, now the game is played with *Quantum Cards*. Cards will be elements of a state space, and will be denoted by a ket $|\psi\rangle$. This space is generated by a set of states, the *deck base*, which are characterized by the *suit* and the *rank*:

$$|\text{suit, rank}\rangle$$
 with
$$\begin{cases} \text{suit} = \clubsuit, \diamondsuit, \heartsuit, \spadesuit \\ \text{rank} = A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K \end{cases}$$
 (1)

These states are called *deck states*. For example, one can write $|\clubsuit, 9\rangle$. This states are the analogue of the classical cards, as in both of them the suit and the rank are known.

Now, each $|\psi\rangle$ can be written as a linear combination of this states, hence they are a **superposition** of states. For instance, one example of a state could be

$$|\psi\rangle = c_1 |\diamondsuit, 2\rangle + c_2 |\spadesuit, K\rangle$$
 $|c_1|^2 + |c_2|^2 = 1$ (2)

2.2 Deck base measurements

As it can be seen, in general <u>on quantum cards the value of the suit and the rank are not know</u>, as opposed to the classical cards. To play blackjack, one needs to know the rank of the card. Therefore, <u>One must apply the postulates of Quantum Mechanics regarding measurement in order to arrive to a state of the deck base [3].</u>

To do this, one can define a **card operator**, whose eigenvectors are the deck states. In this way, the square modulus of the coefficients that accompany the deck states are the probabilities of measuring said card.

2.3 New rules

Taking into account the new quantum nature of the cards, the classical rules explained in Section 1 must me modified. This section covers these changes. Whatever is not specified here will remain as in the classic rules.

2.3.1 Dealing process

During the dealing process, what is given to player are states $|\psi\rangle$. As the cards are turned up, the player knows the expression of said states. Take also into account that in general the number of the card is not known. Here enters the process of **measurement**, which will be used later on.

Card's given to the dealer are classical cards, or in this context states from the deck base.

2.3.2 Player turn

After the card/states are dealt, players will have the opportunity of making a measurement on each of their cards.

Example 1. Suppose that the player has been given the following cards

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}(|\mathbf{\Phi}, \mathbf{Q}\rangle - |\diamondsuit, 4\rangle) \qquad |\psi_2\rangle = \frac{1}{2}(|\heartsuit, 4\rangle + \sqrt{3}|\mathbf{\Phi}, \mathbf{A}\rangle)$$
 (3)

Then the player knows that:

- \bullet On the first card, there is 50/50 probability of getting a Queen (value of 10) and a 4
- On the second card, there is a probability of 1/4 of obtaining a 4 and 3/4 of obtaining an ace.

Therefore, the player knows that there is a possibility of 3/8 of obtaining a Blackjack. But because of Quantum Mechanics, this is only one possible outcome. With this one can see that the Quantum Blackjack adds another layer of probabilities on the game.

After the measurement on both cards, players can make the decisions explained in Section 1.2.2. In the case of asking for another card, the measurement will also have to be performed.

2.3.3 Producing entanglements

Each player will have a number of tokens to produce entanglements. With them, the player will be able to link states from the first card to the second card. The probabilities will be the ones

from the first or second card, depending on what the player wants. Therefore, when making the measurement on the first card, the deck state of the second one will also be fixed.

Example 2. Consider again the states of the Equation (3). It is desirable to obtain the blackjack, and with the highest probability. Then the player decides to entangle $| \spadesuit, Q \rangle$ with $| \clubsuit, A \rangle$, and $| \diamondsuit, 4 \rangle$ with $| \heartsuit, 4 \rangle$, with the probabilities of the second card. Then the state of both cards will be

$$|\psi_{12}\rangle = \frac{1}{2}(|\diamondsuit, 4; \heartsuit, 4\rangle + \sqrt{3} |\spadesuit, Q; \clubsuit, A\rangle)$$
 (4)

In this way, the player now has a probability of 3/4 obtaining a blackjack, rather than 3/8 if no entanglement was made.

2.3.4 Reshufling

The cards/states that are taken to a side will be the ones that are left after the measurements, i.e., the states to which the state collapses after measurements. The rest of the states will go back to the deck, so that new superpositions can be made.

When there are no more states of the base on the deck, one can build again states with all the deck basis.

References

- [1] Learn to play Blackjack. URL: https://bicyclecards.com/how-to-play/blackjack. (Last visited: 16-9-2025).
- [2] Official Blackjack Rules. URL: https://officialgamerules.org/game-rules/blackjack/. (Last visited: 16-9-2025).
- [3] Claude Cohen Tannoudji, Franck Laloë, and Bernard Diu. Quantum Mechanics, Volume 1: Basic Concepts, Tools, and Applications. 2nd ed. Weinheim: Wiley-Vch, 2019. ISBN: 978-3-527-34553-3.