Econometrics for Financial Markets

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Abstract

Through the consideration of four Credit Default Swap (CDX North America IG, CDX HY, iTraxx Main, iTraxx Crossover) and three stock indices (SPY, SX5E, IWM), in this paper we aim to present the Credit and Equity Beta coefficient result. The main purpose of this analysis is to underline the differences between the level of volatility of the examined financial instruments. We have developed a simple return and a mark-to-market analysis for every index, focusing, then, on two chosen financial instrument: CDX North America IG and SPY. In the light of that, we have proceeded the study calculating the Beta (β) for the two financial instruments, showing how much an equity value can influence a portfolio more than a CDS.

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1 Financial instruments

A financial instrument is defined as a contract between parties that hold a monetary value. They can either be created, traded, settled, or modified as per the involved parties' requirement. It is to say that any asset that holds capital and can be traded in the market is defined as a financial instrument. The financial instruments can be mainly divided in two macro groups which are **Cash Instruments** and **Derivatives**.

1.1 Cash Instruments

Cash instruments are financial instruments whose values are directly influenced by the condition of the markets. Within cash instruments, we are able to distinguish two types: securities and deposits and loans.

Equity A stock is a financial instrument whose monetary value is traded on the stock market. When purchased or traded, a stock represents ownership of a part of a publicly-traded company on the stock exchange.

Deposits and Loans Both deposits and loans are considered cash instruments because they represent monetary assets that have some sort of contractual agreement between parties.

1.2 Derivatives

A derivative is a contract between two parties which derives its value/price from an underlying asset. The most common types of derivatives are futures, options, forwards and swaps. Originally, underlying corpus is first created and it can consist of one security or a combination of different securities. The value of the derivative is bound to change as the value of the underlying assets keep changing continuously.

In order to proceed with our analysis, among all the possible derivative instruments, we are going to focus on the CDS.

CDS In 1994, JP Morgan created a financial instruments aimed at reducing risk associated with its landing activities. A credit default swap (CDS) is a derivatives instrument that provides insurance against the risk of a default by a particular company. This contract is constituted by three parties: first the issuer of the debt security, second the buyer of the debt security, and then the third party, which is usually an insurance company or a large bank. The third party will sell a CDS to the buyer of

the debt security. Then, the CDS offers insurance to the buyer of the debt security in case the issuer is no longer able to pay (insolvent). In the case of a default, the seller of the CDS is obligated to cover any default related losses and the CDS contract ceases.

2 CDS computation theory

The price of a CDS is defined through the default probability and the recovery rate. Fundamentally, the main focus will be the probability of a credit event.

2.1 Default probability and recovery rate

Firstly, it is possible to highlight two different way of computation of the default probability whether or not there is the presence of the recovery rate. Before defining these two cases, it is important to bear in mind the definition of recovery rate: it represents the extent to which principal and accrued interest on defaulted debt can be recovered, expressed as a percentage of face value. The recovery rate can also be defined as the value of a security when it emerges from default or bankruptcy.

Default probability (No recovery rate) Let's define as p as the probability of default, the investor is indifferent between an expected return of:

$$(1-p)e^{(z+r)t} \tag{1}$$

on the risky corporate bond and

$$e^{zt} \tag{2}$$

Putting these two equation as an equality and solving it for the probability of default, what is obtained will be equal to the implied default probability:

$$p = 1 - e^{-rt} \tag{3}$$

Default probability (With recovery rate) In the other case, when a recovery rate R is included, the expected return to which the investor should be indifferent to, would be equal to:

$$(1-p)e^{(z+r)t} + pRe^{(z+r)t} \tag{4}$$

on the risky corporate bond and

$$e^{zt} \tag{5}$$

Solving in the same ways explained for the no recovery rate case, the output formula will be now:

$$p = \frac{1 - e^{-rt}}{1 - R} \tag{6}$$

2.2 The Credit Triangle

In order to proceed with our analysis, we introduce the Credit Triangle theory. To compute the CDS equilibrium spread, the protection leg and the default leg must be considered as an equality, such as follows:

$$s\frac{1-e^{-(r+h)}T}{r+h} = (1-R)\int_0^T e^{-rt}f(t)dt \tag{7}$$

Doing some math, in the end, what is obtained will be:

$$h^1 = \frac{s}{(1-R)} \tag{8}$$

This credit triangle formula will be included in the Risk Duration definition.

2.3 Risk Duration

The Risk Duration (aka Risky DV01) represents the value of risky annuity. The sum of the discount factors used in CDS valuation weighed by their corresponding survival probabilities. The Risky DV01 or Annuity measures the present value of 1bp risky annuity received or paid until the occurrence of a credit event or the expiration of the contract.

For the Risky DV01 formula, assuming annual premium payments frequency for n years CDS, it will be equal to:

$$\sum_{t=1}^{n} DF_t \cdot SP_{0,t} \tag{9}$$

The above formula, which is used for the CDS daily returns computation, requires the estimation of all the default probabilities via bootstrapping[2]. Although, there exists an approximated and rapid way to compute the Risky DV01 avoiding the bootstrapping procedure, this happens when the two following assumption hold:

- the CDS premium is paid continously
- the hazard rate function is constant for each time t

According to the last described procedure, we are now able to formulate a new equation (to do so, we also have to recall the first part of the equation (7), which describes the protection leg):

$$\int_0^T e^{-(r+h)t}dt = \frac{1-e^{-(r+h)T}}{r+h} \tag{10}$$

[1]Hazard rate

[2]The bootstrapping consists in the extrapolation of the zero curve from the swap curve.

2.4 CDS daily returns

Lastly, to pursue the CDS analysis, we might consider the formula for the CDS daily returns as follows:

$$CDSDailyReturn = (s_t - s_{t-1}) * RiskyDV01 \tag{11}$$

2.5 Mark to Market for CDS

Considering a change in the CDS curve, the CDS contracts will show a mark-to-market profit or loss. In general, the mark-to-market (MTM) is an accounting practice that adjusts the value of an asset to reflect its value as determined by current market conditions. The market value is determined based on what a company would get for the asset if it was sold at that point in time.

To determine this value, we have to build the MTM value through the following steps:

- Consider the CDS quotes to construct a new set of default and survival probabilities
- Consider the current set of interest rates in order to build a new set of discounted factor
- Evaluate of the stream of swap payments and default payments through the new curves
- The mark-to-market profit is computed by the difference between the value of the two legs

3 Beta

Beta (β) represents a measure of the volatility –or systematic risk– of a security or portfolio compared to the market as a whole, whose most common application is in the **Capital Asset Pricing Model (CAPM)**. Practically, the Beta value of a given instrument is usually compared with the $S\&P$ 500, which is a reference indicator of the market as a whole. Furthermore, considering a graphical interpretation, the β represents the slope of the line through a regression of data points, where each of these data points stands for an individual stock's returns against those of the market as a whole.

Beta is an effective tool to measure and describe the activity of a security's returns as it responds to swings in the market. It is computed by dividing the covariance of the security's returns and the market's returns by the variance of the market's returns over a specified period. More specifically, the Beta Coefficient formula is:

$$\beta = \frac{Covariance(R_e, R_m)}{Variance(R_m)} \tag{12}$$

- R_e represents the return for the individual stock
- R_m represents the return of the whole market.

The Covariance give as result the changes in the stock's return due to the swing movements of the market return and the variance of the market indicates how far the market's data points spread out from their average value.

In order to let β provide any useful insight, the market that is used as a benchmark should be related to the stock. In order to make sure that a specific stock is being compared to the right benchmark, it should have a high R-squared[3] value in relation to the benchmark.

3.1 Beta values interpretation

<1 When a beta value that is less than 1, this means that the security is theoretically less volatile than the market. Henceforth, following this statement reasoning, if we might include this stock in a portfolio, it would make it less risky than the same portfolio without the stock.

[3]R-squared is a statistical measure that indicates the proportion of the variance for a dependent variable explained by an independent variable(s) in a linear regression model. It explains to what extent the variance of one variable explains the variance of the second variable.

>1 If instead, the computed beta is greater than 1, this implies the contrary: the security's price is more volatile than the market. So, this indicates that adding the stock to a portfolio will increase the portfolio's risk, but may also increase its expected return (the higher the risk, the higher the return).

<0 Lastly, there may exists beta with negative values. For example, a β of -1 means that the stock is inversely correlated to the market benchmark on a 1:1 basis. This stock could be thought of as an opposite, mirror image of the benchmark's trends (Put Options and inverse ETFs are designed to have negative betas).

3.2 Beta analysis

Even though the coefficient offers some useful information when evaluating a stock, it certainly does have some limitations. It is mainly useful in determining a security's short-term risk and for analyzing volatility to arrive at equity costs when using the CAPM. However, since it is calculated through historical data points, the longer the term of the investment, the less useful it becomes to predict the risk level; that happens because a stock's volatility can change significantly from year to year, depending upon the company's growth stage and other factors. Hence it is unreliable as a stable measure due to the unpredictable swings of the market over time.

A Beta of 1.0 for a stock means that it has been just as volatile as the broader market (i.e., the SP 500 index). If the index moves up or down 1%, so too would the stock, on average. Betas larger than 1.0 indicate greater volatility - so if the beta were 1.5 and the index moved up or down 1%, the stock would have moved 1.5%, on average. Betas less than 1.0 indicate less volatility: if the stock had a beta of 0.5, it would have risen or fallen just half-a-percent as the index moved 1%.

4 Applications

To elaborate the Credit vs Equity Beta analysis, we have been working with four different types of Credit Default Swaps:

- CDX North America IG: A portfolio composed by 125 CDS with rating from BBB to above, its value is defined in basis points;
- CDX North America HY: A portfolio composed by 100 CDS with rating from BBB to below, its value is defined in prices;
- iTraxx Europe Main: A portfolio composed by 125 CDS with rating from BBB to above, its value is defined in basis points. It is listed in the Asian, European and Australian market.
- iTraxx Europe Crossover: A portfolio composed by 70 CDS with rating from BBB to below, its value is defined in BPS and it is listed in the same markets as the Europe Main

These datas have been compared to the daily closing prices of three stock indices ETF: SPY, SX5E and IWM.

SPY The SPY tracks the S&P 500 index, the benchmark U.S. equity index. It is a quite simple index which includes the 500 largest U.S. companies, subject to a basic set of quantitative criteria. It is a market-cap weighted index, and this is to say that larger companies have greater weights. Currently, this mostly highlights mega-cap for tech stocks, such as Apple (AAPL), Microsoft (MSFT), and Google (GOOG).

SX5E The EURO STOXX 50 is a stock index of Eurozone stocks designed by STOXX, an index provider owned by Deutsche Börse Group. The index is composed of 50 stocks from 11 countries in the Eurozone. EURO STOXX 50 represents Eurozone blue-chip companies[4] considered as leaders in their respective sectors. It is composed by fifty of the largest and most liquid stocks. The index futures and options on the EURO STOXX 50, traded on Eurex, are among the most liquid products in Europe and the world.

IWM Lastly, the IWM tracks the Russell 2000 index, the benchmark small-cap U.S. equity index. This is listed in U.S. characterized by U.S.companies ranked 1001st-3000st by market-cap, subject to a basic set of inclusion criteria. The index starts with the 3000 largest companies, and after excluding the 1000 largest, it invests in

[4]A blue-chip company is a multinational firm that has been in operation for a number of years.

the rest. IWM's holdings are mostly less well-known mid-cap and small-cap U.S. companies.

4.1 Analysis

To proceed with the examination, we have considered 751 closing daily stock prices. Through that we have been able to compute the equity simple returns as follows:

$$\frac{p_t - p_{t-1}}{p_{t-1}} \tag{13}$$

The same methodology has been applied to the CDX North America HY which are expressed in prices. For the remaining CDX, we have applied another formulation, considering that they are expressed as bps. Henceforth, we have calculated the spread obtained as the difference between the bps of the current day and the bps of the day before, whose result will be weighted for the riskDV01.

Date	SPY	SX5E	IWM
2015-03-31	-0.008740	-0.008160	-0.003206
2015-04-01	-0.003536	0.004736	0.000402
2015-04-02	0.003597	0.000102	0.001849
2015-04-07	0.004069	0.014387	-0.002006
2015-04-08	0.003377	-0.006923	0.008601

Table 1: Equity simple returns

After that, to determine the risk duration, we have assumed the interest rate to be equal to 4%, the maturity to be equal to five years and the recovery rate is estimated to be equal to 40%. Worth of noting is the h parameter which is included in the riskDV01. This parameter is computed as the ratio between the spread value and 1 - recovery, forming the credit triangle. Through these analysis, we have obtained the MTM for the CDX.

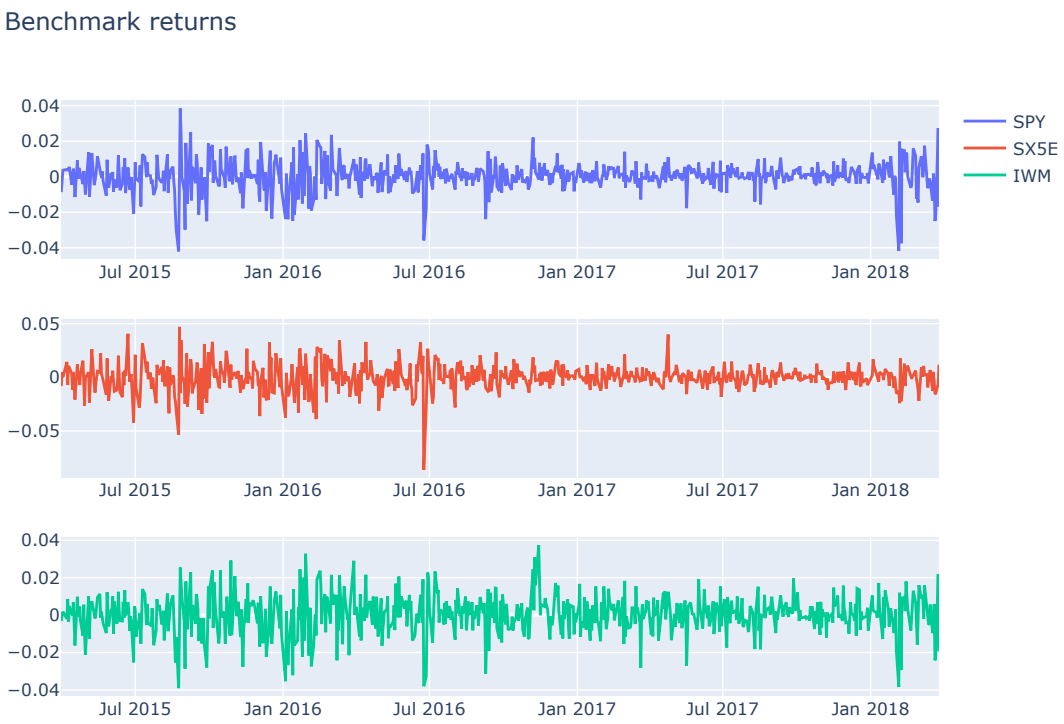

Figure 1: Benchmark Returns

Date	CDX IG	CDX HY	iTraxx Main	iTraxx Crossover
2015-03-31	0.000132	0.000866	0.000177	0.000327
2015-04-01	0.000220	0.000961	-0.000266	0.000900
2015-04-02	0.000220	0.000480	-0.000354	-0.000286
2015-04-07	0.001057	0.004127	0.000753	0.002459
2015-04-08	-0.000088	-0.000382	-0.000089	0.000820
2015-04-09	-0.000044	0.000861	0.000443	0.001026

Table 2: CDS simple returns

As the main goal of analysis was that of computing the Beta coefficient for credit and equity, specifically between SPY and IG, this implies the will of defining beta as

a volatility indicator. To do so, we have applied the β formula which expects, in this specific case, the value of the covariance between SPY and CDX IG over the CDX IG variance (for the denominator, it is important to recall that the variance, for the general formula, must be calculated for the less volatile asset taken in consideration). Concluding the examination, what we have obtained is that the β obtained for the selected portfolio is equal to 8,35. This result means that the SPY is eight times more volatile than the CDX IG.

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