Time Series and Financial Time Series Project

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Abstract

With the consideration of four different time series, in this paper we aim to present an empirical VaR application through the implementation of two different forecast evaluation methods for each one of them, in order to select the best models. The time series analysed are given by the daily closing stock returns of WTI crude oil, European Brent, Heating Oil and Propane. Their data are fitted with the Generalized AutoRegressive Conditional Heteroskedasticity model (GARCH) and its eGARCH extention, even including a GARCH-MIDAS analysis that considers the Covid-19 deaths macroeconomic variable, for each commodity. The models considered are selected employing the Backtesting and subsequently, the Model Confidence Set procedure to obtain the final choice in the set for forecasting.

Contents

1 Introduction

The Sars-CoV-2 virus, firstly detected in Wuhan in December 2019 and then declared pandemic by the WHO in March 2020, has generated tremendous consequences not only in the health sector but also in those of economy and society. In terms of financial markets and trading activities, it had a crucial impact on the prices volatility.
In economics and finance, forecasting has always represented one of the main tools in order to better understand and manage the market risk. Indeed, considering the described above situation and the consequent impact on the economic system, it is not possible to state for how long these effects will be going on; hence it is even more necessary try to predict the possible future scenarios. Considering the market volatility, one of the instruments that better helps us to do forecasting, is the Value at Risk as a risk measure.
To model the volatility, in the econometrics literature, the most adopted models are the GARCH thanks their ability to catch the main features of financial data. Using these models to compute and predict the VaR, in our analysis we applied a moving window procedure that enables us to forecast it at both 95% and 99% confidence intervals.
By using the Backtesting and MCS methods, we obtain different estimates that, once evaluated, allow us to verify their prediction abilities. The first procedure permits, examining the p-values, to better understand the various model performances for the prediction of the VaR; while the second one, considering the predictive ability computed on the VaR loss function, ranks all the type of model examined. The best possible scenario is that in which the model have passed the Backtesting and got a high position in the MCS ranking.
In order to get better the procedures we are using, we decide to do a brief theoretical background of the models considered and for their error distribution. Going on with the analysis, a more detailed explanation of the VaR, Backtesting and MCS will be given, proceeding then to the application of these onto the four commodities analysed. After that, we realize the main goal of the paper, consisting on the evaluation of the accuracy of the VaR forecast at the two different confidence levels.

2 Value at Risk

The health emergency due to the pandemic situation is strictly related to an economic crisis: several restrictions were needed in order to stem the spread of the virus and the pandemic itself. According to that, the financial markets suffered a demand contraction and high volatility prices. Henceforth, an huge relevance has been given to the Value at Risk measure (VaR) in order to minimise the variance linked to the fluctuations of market indices.

Originally used for capital requirements purposes in United States Stock Exchange, the VaR has become widely adopted nowadays for measuring market risk in trading portfolios, risk management, financial control and reporting. Accurately, considering a portfolio of risky assets, the VaR is a risk measure that estimates the maximum loss that is not exceeded with a given probability, by holding a certain financial asset with respect to a determined confidence interval and a given target horizon. Strictly speaking, it can be defined as a category of probabilistic measures of market risk, or a quantile of the loss distribution.

In probabilistic terms:

$$VaR_{\alpha}(L) = inf\,\{l \in \mathbb{R} : \mathcal{P}(L > l) \leq 1 - \alpha\} = inf\,\{l \in \mathbb{R} : F_L(l) \leq \alpha\} \qquad (1)$$

Where:

- $1 - \alpha$ is the confidence level (typically at 95% or 99%) such that $\alpha \in (0, 1)$;
- $F_L(l) = \mathcal{P}(L \leq l)$ is the distribution function of the loss distribution;

Definition 2.1 (Value at Risk) *Given some confidence level $\alpha \in$ (0,1), the VaR of a portfolio with loss L at the confidence level α is given by the smallest number l such that the probability that the loss L exceeds l is no larger than $1 - \alpha$.*

2.1 Variance - Covariance Method

In view of the VaR, there exist three different methodologies: the historical method and the Monte Carlo method, which refer to a non-parametric approach, and then the variance-covariance method, which instead is known as the parametric approach. According to the aim of this paper, the explained one is the variance-covariance method, which is also the most commonly used in literature.

Firstly introduced by JP Morgan in the 90's, it hypothesises that the revenues or the losses are distributed as a Normal. This implies that considering a Cartesian plain, with the potential losses/revenues on the x-axis and on the y-axis the respective frequencies (which explains the number of times in which the losses and the gains have been verified), it shows a Gaussian distribution (more specifically leptokurtic[1]).

This curve is the result of the following approach: s^2 is the variance of the returns with respect to the mean μ. The variance taken into account corresponds to the volatility of the whole portfolio (so it works for the mean), where the highest value possible can be obtained with $\mu = 0$.

In the light of what said previously, it is possible to state the the VaR graph will correspond to a portion of the area of the curve described above. More specifically, this area will be delimited from below by a variation in the returns and from above, by the portion of the curve that combine the corresponding frequencies. This area must have a calibrated probability equal to the difference between 100% of the probability and the level of confidence; henceforth if it was equal to 95%, the VaR should correspond to at the 5% of the expected event.

[1]It represents a type of Kurtosis distribution characterized by a more edgy curve with heavier tails

3 Models specification

One of the main aims of this research is to obtain different VaRs and in order to do so, we are considering a list of models that are able to capture the fundamental feature of such series. We are firstly analyzing the standard version of the GARCH model and, to focus more on the asymmetry assumption, subsequently the eGARCH.

3.1 ARCH models

The ARCH (AutoRegressive Conditional Heteroskedasticity) represents a statistical model employed to analyse the variance in time series to forecast future volatility. This type of model shows that periods of high volatility are followed by a higher level of volatility and periods of low volatility go along subsequently with a lower level of volatility. The ARCH model acknowledges beyond any volatility clusters that can be seen in financial markets during periods of crisis; moreover, it provides a model that can be used instead of a constant or average to study volatility.

An ARCH(m) process is one for which the variance at time t is conditional on observations at the previous m times, given the following relationship:

$$Var(y_t|y_{t-1},...,y_{t-m}) = \sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + ... + \alpha_m y_{t-m}^2 \tag{2}$$

supposing that y_t is a series (whose variance we are aiming to model), it is conditional on $y_{t-1},...,y_{t-m}$. It is worth noting that the variance at time t is strictly linked to the values of the series to the previous periods, henceforth a large value of y_{t-1}^2 implies a large value of the variance at time t.

3.2 GARCH models

The GARCH model (Generalized AutoRegressive Condition Heteroskedasticity), introduced in 1986 by Bollerslev, is an extension of the ARCH model. This class of models is mostly used for financial data given their ability to catch heteroskedasticity and volatility clustering, which represent key tools for forecasting. The GARCH models assumes that the conditional variance σ_t^2 responds symmetrically to the previous shocks ϵ_{t-1}^2. Furthermore, defining the memories of the GARCH models, the first thing that has to be mentioned is that it is short when not highly parameterized and not only declines it exponentially, but also only an increased number of parameters provides a good fit.

In the light of the above, the GARCH processes are made of an ARCH part with p parameter and another part which ensures the flexibility, that is an autoregressive part with q parameters. The GARCH(p,q) calculates the variance of the error at time p, henceforth the notation is the following:

$$\sigma_t^2 = w + \sum_{j=1}^{p} \alpha_j \epsilon_{t-j}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{3}$$

Having $r_t = \epsilon_t$ and $\epsilon_t | F_{t-1}$ i.i.d. $(0, {\sigma_t}^2)$.

Where $w > 0$ and $\sum_{j=1}^{p} \alpha_j \epsilon_{t-j}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 < 1$ to ensure the weak stationarity. Given that the variance must be always greater than 0, we must impose that both α and β are greater or equal to zero.

To pursue the aim of the research, we consider a specific case of GARCH, the sGARCH, that indicates the standard version of the model in (1,1).

3.2.1 eGARCH model

The exponential GARCH represents an extension of the GARCH model, firstly introduced by Cao and Nelson in 1991, which responds in an asymmetric way to positive and negative news, showing the impact of these positive and negative shocks on the variability. The positive variance is automatically provided due to the restriction of non-negativity of the parameters that are removed considering the logarithm of the variance. According to the previous GARCH assumptions, the notation can be written as:

$$log(\sigma_t^2) = w + \sum_{j=1}^{p} \alpha g(\epsilon_{t-j}) + \sum_{j=1}^{q} \beta_j log(\sigma_{t-j}^2) \tag{4}$$

where, considering the case of eGARCH(1,1), $g(\epsilon_{t-1}) = \theta[\epsilon_t + \gamma(|\epsilon_t| - E(|\epsilon_t|)]$, $\theta\epsilon_t$ determines the sign of the effect of the variance, hence the plausible presence of leverage effect, whilst the second term $\gamma(|\epsilon_t|) - E(|\epsilon_t|)$ is instead the size of the effect. It is possible to observe that by construction the process $g(\epsilon_{t-1})$ allows the conditional variance process to respond asymmetrically to falls and rises in prices.

Evidently, the variance reacts in two different ways, whenever it comes to a positive or negative shock. In fact, the slope ϵ_t is going to have two different values if it is greater or smaller than zero. When the value is greater than zero, it will be equal to $\theta + \gamma$, otherwise $\theta - \gamma$. Pointedly, if $\theta < 1$, a positive increase of the variance will be registered given a negative innovation. Instead, we can notice a symmetry of effect if θ is equal to zero.

In the general GARCH models, the conditional variance is defined as an additive function of ϵ_t, but this specific case it is a multiplicative function of the lagged innovations measured by σ^2. Worth of mentioning, is also the fact that this type of GARCH has no restriction on parameters, so σ_t^2 is stationary and constant overtime.

3.2.2 GARCH-MIDAS

In the light of the purpose of this paper, a worth of interest analysis is obtained through the GARCH-MIDAS model. The model aims to explore the presence and the influence of macroeconomic series onto volatility. Coherently, it is necessary to show the main difference between the financial data, which are high frequency data (such as daily or hourly), and macroeconomic data, which are low frequency data (usually weekly or monthly). To support and elaborate this analysis, in 2004, Ghysels, Santa-Clara and Valkanov have introduced the MIxed Data Sampling where the regressors can have a different level of frequency, among them and with respect to the dependent variable. Following the latter model elaboration, subsequently, Engle, Ghysels and Sohn improved their methodology in order to expand the analysis in the long run, relating it directly to the level of the macroeconomic variables: this represents the elaboration of the GARCH-MIDAS.

The analysis of the model is considered as follows:

$$r_{i,t} = \sqrt{\tau_t g_{i,t} \epsilon_{i,t}} \tag{5}$$

where:

- $r_{i,t}$ is the daily return for the $i - th$ day (with i=1, ...,N_t) of the period t (a week, a month or a quarter; t=1, ..., T);
- τ_t is the long-run component, varying each period t;
- $g_{i,t}$ is the short-run term, varying each day i of the period t (it follows a unit-mean reverting GARCH(1,1))
- $\epsilon_{i,t}$ is an *i.i.d.* error term which has zero mean and unit variance.

The short-run component for the GARCH-MIDAS (parameter "model" set to "GM") when the parameter "skew" is set to YES is:

$$g_{i,t} = (1 - \alpha - \gamma/2 - \beta) + (\alpha + \gamma \cdot I_{(r_{i-1,t}<0)}) \frac{(r_{i-1,t})^2}{\tau_t} + \beta g_{i-1,t}, \tag{6}$$

where $I_{(\cdot)}$ is an indicator function. Whilst, when "skew" is set to NO, the parameter γ disappears.

The long run component τ_t is obtained as one-sided filter of the stationary variable X_t:

$$\tau_t = exp(m + \theta \sum_{j=1}^{K} \delta_j(\omega) X_{t-j}) \tag{7}$$

considering

- m as the intercept;
- θ as the coefficient of interest;
- $\delta_j(\omega)$ as the function weighting the past K realizations of X_t.

4 Distribution specification

To avail the aim of this paper, we are considering five types of distributions in order to model the empirical distributions of the log returns of the variables considered.

4.1 Normal distribution

The Normal distribution, also known as Gaussian, is the most common distribution function for independent, randomly generated, variables. It is defined as symmetric probability distribution characterized by two parameters: the mean, which indicates the maximum of the curve and about which the graph will be always symmetric; and the standard deviation, that defines the amount of the dispersion away from the mean. The general form of its probability density function is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{z-\mu}{\sigma})^2} \tag{8}$$

According to the general definition, a small standard deviation, compared with the mean, produces a steep graph, while a large standard deviation, does exactly the contrary. Furthermore, the Gaussian distribution has 0 skewness, given its mean being equal to 0, and kurtosis equal to 3, that implies the absence of the excess kurtosis.

4.1.1 Skew Normal distribution

Firstly introduced by Leonard and O'Hagan in 1976, the Skew Normal distribution is a continous probability distribution that generalises the Gaussian in order to allow the non-zero skewness. It includes an additional shape parameter γ which skews the Normal distribution to the left or right, but when $\gamma = 0$, we obtain precisely a Gaussian. Given that the only part of the Normal distrubution equations that is changed is the skew, it can be written as:

$$f(x) = 2\phi(x)\Phi(\gamma x) \tag{9}$$

where $\phi(x)$ is the probability density function of the standard Normal distribution and $\Phi(x)$ represents its distribution function.

4.2 Student-t distribution

In probability and statistics, a Student t distribution is a family of distributions that look almost identical to the Normal curve, but with heavier tails. Due to its structure, it is able to catch the leptokurtosis phenomena (widely known and found in returns distributions). The Student t depends on the ν parameter which defines the degrees of freedom, in fact, the larger the sample size, the more the t distribution will look like a Gaussian given the Central Limit Theorem (CLT). The density function with $\nu > 2$ is the following:

$$f(x) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\pi(\nu-2)}\Gamma(\frac{\nu}{2})}\Big(1 + (\frac{x^2}{(\nu-2)})\Big)^{-\frac{\nu+1}{2}} \tag{10}$$

Comparing once again it with a Normal distribution, it is possible to notice that it also has 0 skewness but what changes is the value of the excess of kurtosis, that in this specific case exists and it is $\frac{6}{(\nu-4)}$ for any value of $\nu > 4$.

4.2.1 Skew-Student-t distribution

To model the skewness in order to obtain a more flexible probability distribution, this type represents a valid option. The skew t-densities emulate the symmetric t-densities in forming a mathematical sequence tending to the normal density; besides, it can take in consideration the leptokurtic properties and all the possible skewness. It is defined as:

$$f(x) = \begin{cases} \frac{1}{\sigma}K(v)[1 + \frac{1}{v}(\frac{x-\mu}{2\gamma\sigma})^2]^{-\frac{(v+1)}{2}} & \text{for } x \leq \mu \\ \frac{1}{\sigma}K(v)[1 + \frac{1}{v}(\frac{x-\mu}{2(1-\gamma)\sigma})^2]^{-\frac{(v+1)}{2}} & \text{for } x > \mu \end{cases} \tag{11}$$

defining $\gamma \in (0,1)$ as the skewness parameter, μ and σ as the location and scale parameters, respectively, $2 < \nu < \infty$ and $K(\nu) \equiv \Gamma((v+1)/2)/[\sqrt{\pi v}\Gamma(v/2)]$. By letting:

$$\gamma = \frac{1-\lambda}{2} \tag{12}$$

$$\sigma = \frac{1}{b}\sqrt{(v-2)/v} \tag{13}$$

and:

$$\mu = -\frac{a}{b} \tag{14}$$

4.3 Generalized Error distribution

The Generalized Error distributions are part of a symmetric family of distributions used in mathematical modeling, considering when the errors do not follow a Normal distribution. Its density function depends on three parameters: μ which is the mean as parameter of position; σ, parameter of scale, that describes the dispersion of the distribution; and κ that describes the structure. Additionally, there is a difference in the tails, when comparing it to a Normal distribution. If $\kappa < 2$ the tails are heavier, and if $\kappa > 2$, otherwise. The density function is the following:

$$f(x) = \frac{\kappa \epsilon^{-0.5\left|\frac{x-\mu}{\sigma}\right|^{\kappa}}}{2^{1+\kappa^{-1}} \sigma \Gamma(\kappa^{-1})} \tag{15}$$

5 Backtesting procedure

The aim of the backtesting analysis is to evaluate the correctness of the estimated VaR, specifically considering how efficiently a strategy or a model have performed. In the ideal backtest scenario, sample data should have been chosen from a relevant time period of a duration that includes all the variety of market conditions. It allows to judge whether the results of the backtest represent a coincidence or sound trading. The sample of stocks in the historical data set must be faithfully representative, given that, including only data from historical stocks that are still around today, will produce artificially high returns in backtesting. For this reason, we must observe and include also those companies that went bankrupt or sold or also liquidated.

The structure of this procedure is developed splitting the sample of the observations in two parts: S as the in-Sample period length (for which the parameters of the model are estimated) and H as the out-of-Sample period length (which tests the ability of forecasting). The $h-steps$ ahead prediction of the return distribution is obtained at the time $S+t$ accordingly to its VaR level: this happens following the estimation of the model parameters over the in-sample period. Successivly, the next steps will be repeated using a rolling window that will proceed with other $h-steps$ ahead that are going to eliminate the previous observations, in a recursive way until the end of all the observations.

After the VaR estimation, the forecast adequacy is evaluated through the backtesting which will compare its results to the true returns observed in the $out-of-sample$. In a more detailed way, the hit function can be defined as:

$$I_{S+h(\alpha)} = \begin{cases} 1 & if x_{S+h} < VaR_{1-\alpha} \\ 0 & if x_{S+h} \geq VaR_{1-\alpha} \end{cases} \tag{16}$$

Following this reasoning, it is possible to note that the function will be 1 in case of the violation of the VaR, that is when x_{S+h} has a lower value than the estimated VaR; when it is 0, we are considering the opposite case. Furthermore, it has to be claimed that the majority of the backtesting methods are based on hypothesis testing: it is verified when the violations over the $out-of-sample$ period differs in a significative way from the picked confidence level of the VaR. Basically, the estimation is correct only if the number of variations will coincide with those of that level of α. To follow the above mentioned reasoning, we are enumerating three different tests that help to examine the prediction ability. The tests computed are the following:

1. The unconditional coverage (UC): It verifies if the violations follow the α level of the VaR, this refers to the probability of the hit function equal to 1 is equal

to α: $P(I_{S+h} = 1) = \alpha$. The UC test performs as a likelihood ratio test for the null, that is to say that it will be distributed as χ^2 with 2 d.o.f. The null hypothesis considers a correct number of VaR violations.

2. The conditional coverage (CC): This test proves if the numbers of exceeds is correct and their independence, which is $I_{S+h} \sim i.i.d.$ Bernoulli (α). As said before, under the null it will be distributed as χ^2 with 2 d.o.f.

3. The Dynamic Quantile test (DQ): It jointly tests for UC and CC and it is more powerful of all the listed previous methods. It follows a regression based approach in which a linear regression model is developed on a set of explanatory variables that includes the lagged values of the hit variable. The aim is to check the independence of all the regressors. It also follows a χ^2 distribution but, it has as many d.o.f. as the number of lagged violations we consider.

6 Model Confidence Set procedure

The continuous and enormous number of developed models for analysis raises the question of providing a statistical method or procedure that determines the "best" models with respect to a given criterium but, what happens sometimes, is that a model selection issue can appear when comparing procedures does not deliver an unique result. For this reason, recently, researchers are focusing on the development of new testing procedures that are able to identify the "best fitting" models. Among those multiple–testing procedures, the Model Confidence Set procedure (MCS), introduced for the first time by Hansen, Lunde, and Nason in 2003 consists of a sequence of statistic tests that allows the construction of a set of "superior" models, more specifically the "Superior Set Models" (SSM), where the null hypothesis of equal predictive ability, also known as EPA, is not rejected at certain confidence level α. The EPA is calculated for an arbitrary loss function that satisfies general weak stationarity conditions, in our case specifically, it computes an asymmetric VaR loss function to contrast the capacity of different GARCH specification to predict a notable loss.

The Model Confidence Set procedure begins considering an initial set of m competing models M_0 and results in a smaller set of superior models denoted by $\hat{M}^*_{1-\alpha}$. The scenario we strive for is that of when the final set consists in a single model. The EPA hypothesis is tested at every step and, if the null hypothesis is accepted, then the procedure stops and the SSM is created; otherwise, the EPA will be tested again subsequently to the elimination of worst model. There is the asymmetric VaR loss function, defined as follows:

$$l(y_t, VaR^\tau_T) = (\tau - d^\tau_t)(y_t - VaR^\tau_T) \tag{17}$$

where VaR^τ_t is the VaR at time t at the level τ, $d^\tau_t = 1(y_t, VaR^\tau_t)$ acts for the τ level of the quantile loss function. Subsequently, d_i. represents the average loss between the model i and the initial set of m competing models. The equation is the following:

$$d_{i.,t} = (m-1)^{-1} \sum_{j \in M/\{i\}} d_{ij,t} \tag{18}$$

for $i \in M_0$

Furthermore, considering the EPA, the null hypothesis can be constructed in the following manner:

$$H_0 : E[d_{i.}] = 0 \tag{19}$$

for $i \in M_0$

This hypothesis is tested by the statistics as:

$$t_{i.} = \frac{\bar{d_{i.}}}{\sqrt{V\hat{a}R}}(\bar{d_{i.}}) \tag{20}$$

supposing that $\bar{d_{i.}} = m^{-1}\sum_{j\in M/\{i\}}\bar{d_{ij}}$ represents the average loss between model i and the average loss given by the other values in the set. Then, $V\hat{a}R(\bar{d_{i.}})$ is the estimation of $Va\bar{R(}d_{i.})$

To conclude, the MCS procedure tests this hypothesis by using a adequate statistic, where it defines a specific elimination rule that excludes at each step the rejection of the null hypothesis H_0 until the best one is allowed in all models in the SSM:

$$T_{max,M} = maxt_i \tag{21}$$

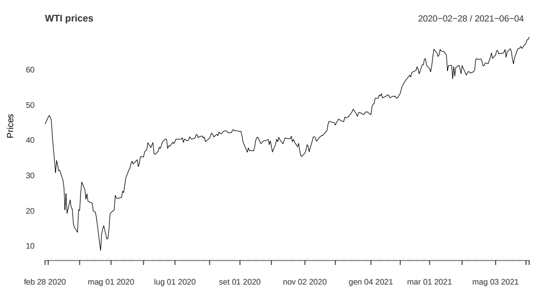
for $i\in M_0$

7 Applications

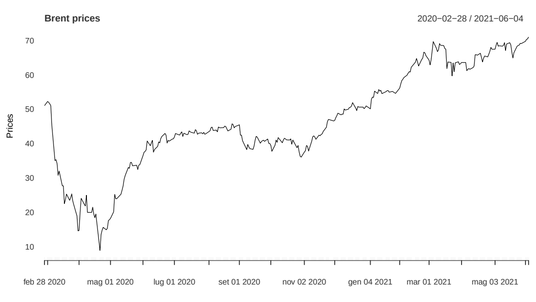
According to the several procedures described above, we are given a set of data composed by six different commodities, of which we are analyzing: WTI crude oil, European Brent, Heating Oil and Propane. After examining the main characteristics of the latter, the mentioned models will be fitted with five errors distributions (Normal, Student t, Skew Normal, Skew Student t and GED). Subsequently, we have inserted a low frequency variable that concerns Covid-19 deaths in USA in a period that goes from March 2^{nd} 2020 to June 14^{th} 2021, and have estimated again the previous GARCH models. Furthermore, through a rolling window procedure with and without the MIDAS method, we have predicted the VaR at 95% and at 99% confidence levels. Following that, the results coming from Backtesting and the MCS procedures will be discussed in order to select the best models.

7.1 Descriptive statistics

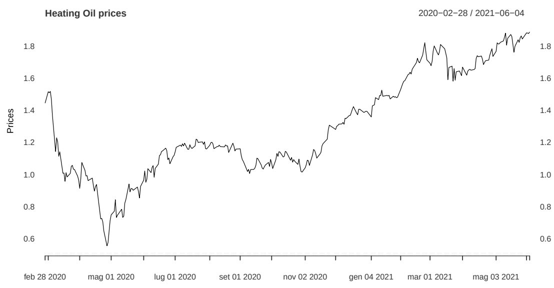
In our analysis, developed from February 28^{th} 2020 to June 4^{th} 2021, the number of daily observations, originally equal to 328, with the subsequently elimination of all the missing values, has been reduced to 316. Graphically, as we can see in Figure 1, the earliest representation of the prices of the commodities analyzed, appears to be similar for all of them. What can be easily seen is that the lowest points have been registered in March-April 2020, which correspond to the first spread of the epidemic. The several prices fluctuations have implied that this worldwide emergency generated a reduction of supply, with consequent an increasing trend in the demand for most of the basic needs. Although, the decrease in the supply does not represent the only reason of this trajectory: with the advent of the vaccine and a better organization of sanitary restrictions, it has been quite easier to face the pandemic with respect to its beginning era. As follows, an higher price level can be detected for all the four commodities analyzed in the very first quarter of 2021.



(a) WTI



(b) Brent



(c) Heating Oil

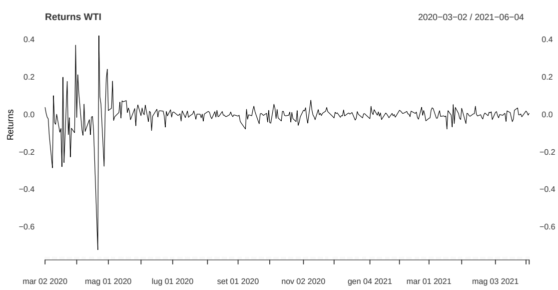
(d) Propane

Figure 1: Time Series Prices

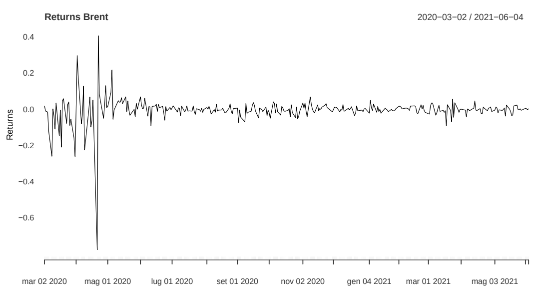
As can be seen from the previous plots, the prices are not stationary in the mean, henceforth they must be converted into stationary series in order to proceed with our analysis. To do so, we had to model the data, transforming them into daily returns, which are obtained by computing the first order differences of the logarithms. More specifically, the logarithms reduce the volatility and the differences are taken to remove the trend. Here follows the returns equation:

$$r_t = [log_{(p_t)} - log_{(p_{t-1})}] \tag{22}$$

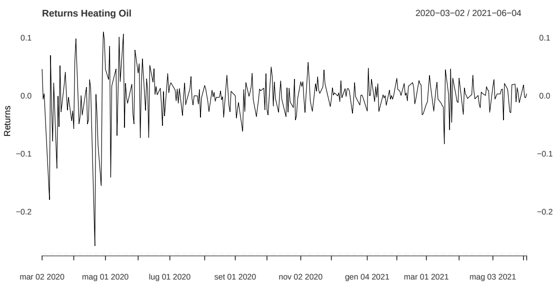
According to the latter definition, it can be noted that the graphs representing the returns correspond exactly to those that represent the stationarity of the series. The representation of the returns is reported below:



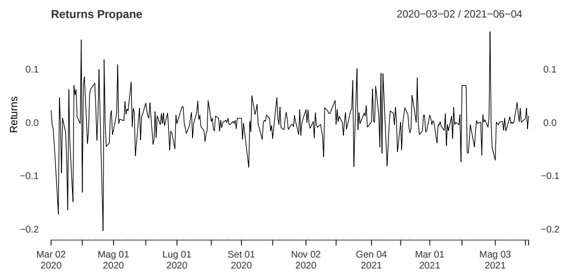
(a) WTI



(b) Brent



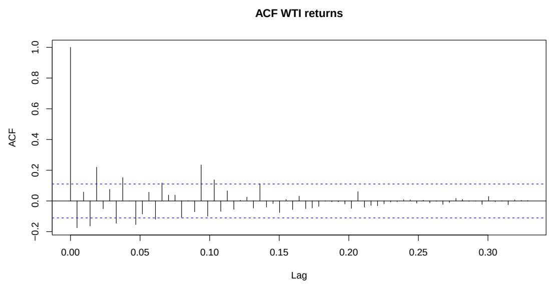
(c) Heating Oil



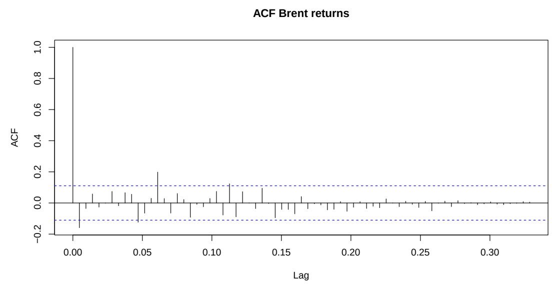
(d) Propane

Figure 2: Returns

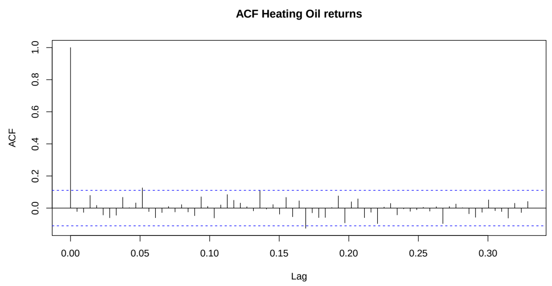
Studying the ACF and PACF of returns with lag 70, it can be noted that some of the observations go beyond the threshold, even though there is no autocorrelation.



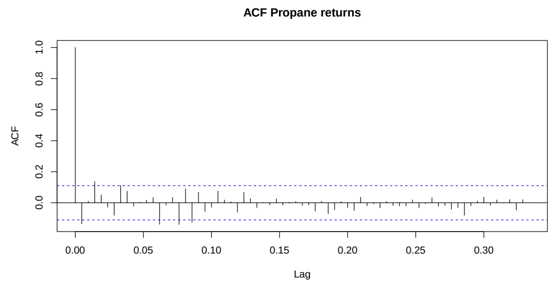
(a) WTI



(b) Brent

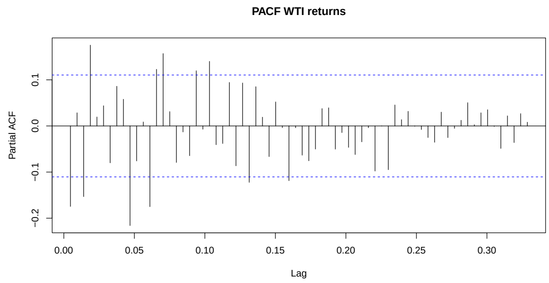


(c) Heating Oil

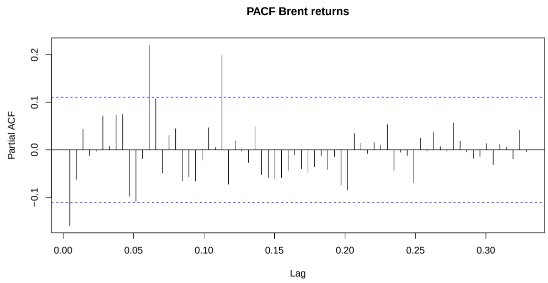


(d) Propane

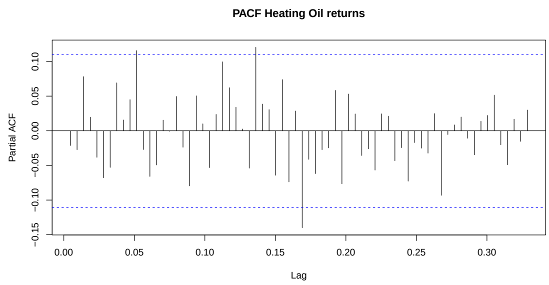
Figure 3: Returns ACFs



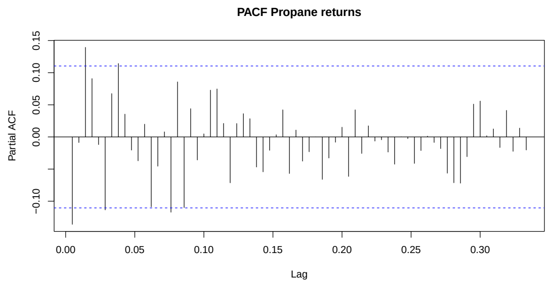
(a) WTI



(b) Brent



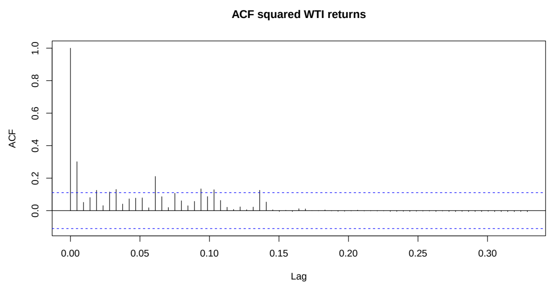
(c) Heating Oil



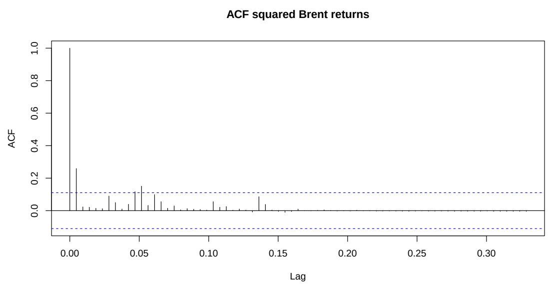
(d) Propane

Figure 4: Returns PACFs

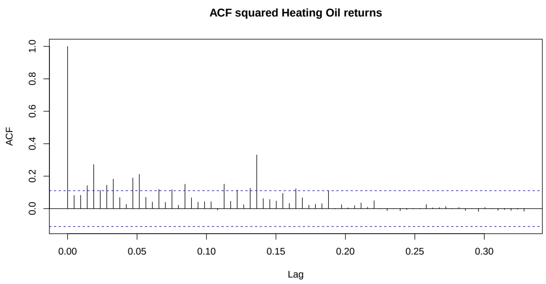
In each commodities plot, the values crossing the blue range are considered to be statistically significant, and this does not represent a problem given that it is one of the main characteristics of financial data. After the lag 35, the ACFs decrease and go to zero, which imply the absence of correlation among the results; hence the behavior is mainly the same for all of them.



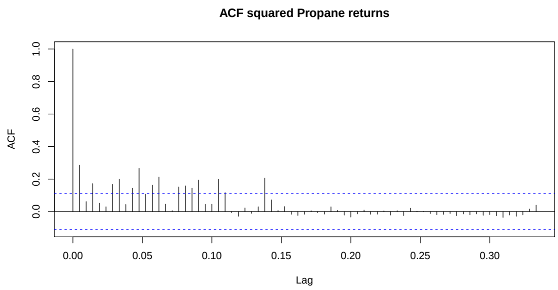
(a) WTI



(b) Brent



(c) Heating Oil



(d) Propane

Figure 5: ACF on squared returns

Comparing the above correlograms, the squared Brent series is the one which presents a lower level of correlation, in particular with respect to the Heating oil and Propane ones. The latter two plots show some volatility cluster which indicates that the ACFs go to zero slowly.

Table 1: Stationarity test

Tests	WTI	Brent	Heating Oil	Propane
ADF test	0.01*	0.01*	0.01*	0.01*
Box-Pierce test	1.82e-10*	4.67e-3*	0.7925	0.0539
Box-Pierce test on squared	1.46e-7*	0.102	< 2.2e-16*	< 2.2e-16*

*All the tests have been performed at 5% confidence levels.

According to what previously mentioned, all the series we are working with are stationary: this has been stated through the Augmented Dickey-Fuller test which highlights the rejection of the null hypothesis (*p-value* < 0.05) that confirms the stationary of the series. Following the stationarity analysis, the Box-Pierce test (with lag = 35) shows that the hypothesis not rejected is that of the Heating Oil and Propane, whilst the non rejected one in the Box-Pierce test on squared returns is that of the Brent (red highlighted values). The other results state that the rejected values are still correlated until lag 35.

Table 2: Stationarity test with different lags

Tests	WTI	Brent	Heating Oil	Propane
Box-Pierce test	0.053222	0.08031	-	-
Box-Pierce test on squared	0.5809	-	0.05637	0.05315

All the tests have been performed at 5% confidence levels.

To show what happens with lags of higher values, we have computed again both the Box-Pierce tests. The results obtained in Table 2 show the absence of correlation and its stationarity, henceforth the null hypothesis can not be rejected. In particular, the commodities are analyzed with different lags, which are associated to each one of them as follows:

- WTI: lag 100 for its returns and lag 100 for its squared returns
- Brent: lag 50 for its returns
- Heating Oil: lag 150 for its squared returns
- Propane: lag 190 for its squared returns

Moreover, the missing values are those reported in Table 1 since they are already uncorrelated.

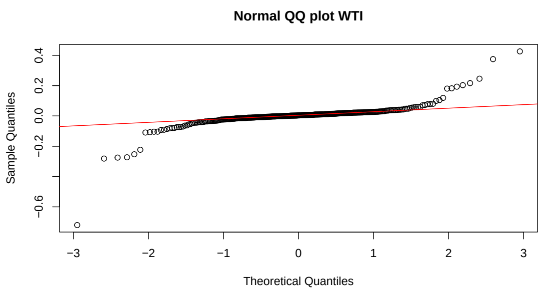
Analysing the main basic statistics in Table 3, it can be claimed that all the mean values are approximately equal to zero, as a confirmation of the zero mean assumption of returns. Moreover, from the assumption of the stylized facts for the financial data, we can state that the distributions are leptokurtic and negative skewed[2]. This evidence obtained allows us to say that the distribution for the series considered is not the Normal one, which requires a Kurtosis index equal to 3 and the skewness index equal to 0.

[2]The negative skewness indicates that the curve is moved more to the right and the longest tail is moved more to the left. Furthermore, in terms of probability distribution, the center of the distribution corresponds to the mode and not to the mean, as happens in the Kurtosis.

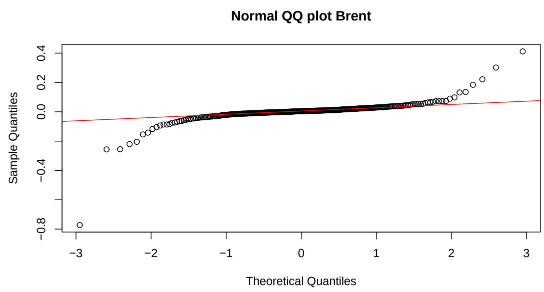
Table 3: Basic statistic

	WTI	Brent	Heating Oil	Propane
Minimum	-0.7202	-0.7726	-0.2570	-0.2007
Maximum	0.4258	0.4120	0.1118	0.1717
1st Quantile	-0.0114	-0.0096	-0.0126	-0.0116
Mean	0.0013	0.0010	0.0008	0.0027
3rd Quantile	0.0201	0.0203	0.0179	0.0180
Median	0.0035	0.0041	0.0027	0.0025
St. Dev.	0.0746	0.0693	0.0372	0.0403
Skewness	-2.2322	-3.8796	-1.6209	-0.4813
Kurtosis	33.7946	53.8053	9.9344	5.5690

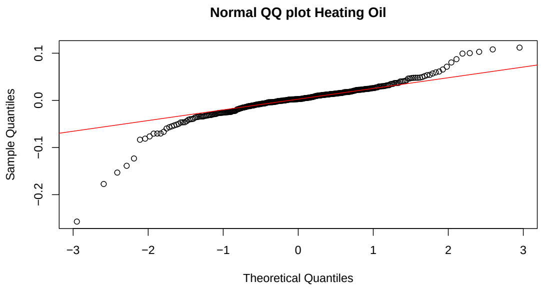
The Q-Q plots allow to understand the nature of our distributions, confronting them with a theoretical distribution. Here, the red line is the representation of the theoretical one while the black and thicker one represents our distribution. Accordingly to what said before, the distributions we obtain are exactly those of a heavy tail distribution, consistently with the leptokurtosis previously noted, that is characterized by heavy tails. In particular, it is possible to say that the clouds of the series differ a lot from the extremes of a Normal distribution, hence it can not cover the values analyzed.



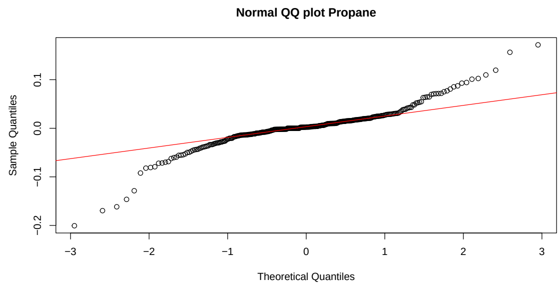
(a) WTI



(b) Brent



(c) Heating Oil



(d) Propane

Figure 6: Returns QQ-plots

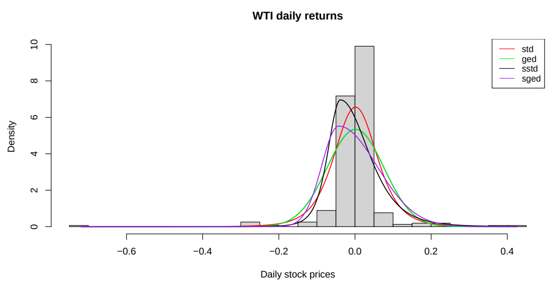
As a further confirmation, we have computed two normality tests: Jarque-Bera and Shapiro (which performs only with small samples, up to 3000), where both of them consider as the null hypothesis the normality of the residuals. The non rejection of the null hypothesis takes in consideration a p-value>0.05, which is not present in any of the results calculated in the Table 4, confirming again the non normality of the distributions in our analysis.

Table 4: Normality tests

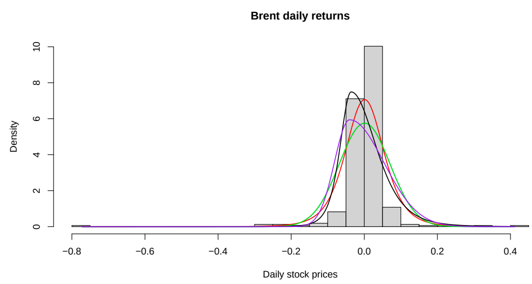
	WTI	Brent	Heating Oil	Propane
Jarque-Bera test	$< 2.2e\text{-}16$	$< 2.2e\text{-}16$	$< 2.2e\text{-}16$	$< 2.2e\text{-}16$
Shapiro test	2.2e-16	2.2e-16	9.4e-16	2.302e-14

Examining the different error distributions, the Figure 7 shows the following ones: Normal, Student t, GED, Skew-Student t, Skew Normal. In all the four figures it can be noted that the Normal distribution has been overlapped by the GED. Furthermore, among them the Skew-Student t represents, for the WTI and Brent, the distribution that better captures the data, given that it covers the density of the returns more than the others; in the other hand, for the Heating Oil and Propane, the same can be stated with the Student t.

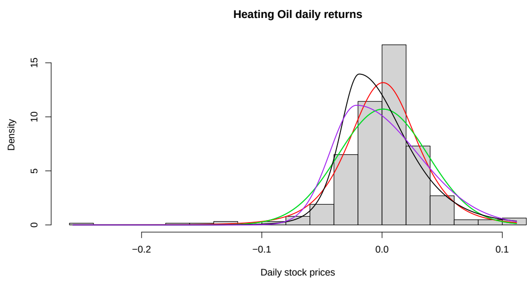
Coherently, the negative shocks have an higher impact compared to the positive ones, highlighted by the presence of the leverage effect. This effect stems from the fact that losses have a greater influence on future volatilities, in fact the distribution of losses has a heavier tail than the distribution of gains. Subsequently, considering the correlation between the volatility of the time series and its value in the past, it represents the presence of the ARCH effects which demand the use of models that ponder the heteroskedasticity.



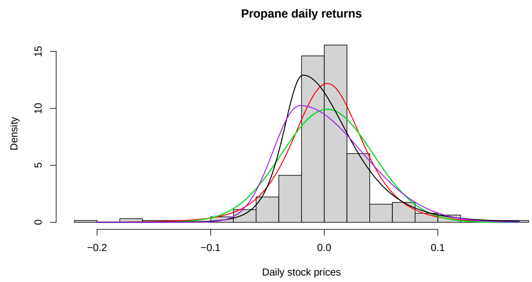
(a) WTI



(b) Brent



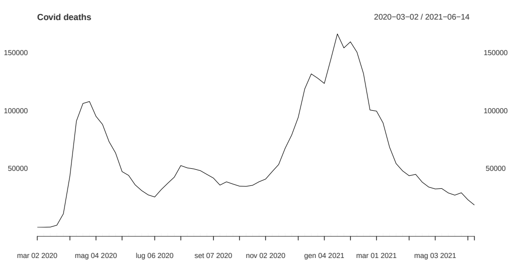
(c) Heating Oil



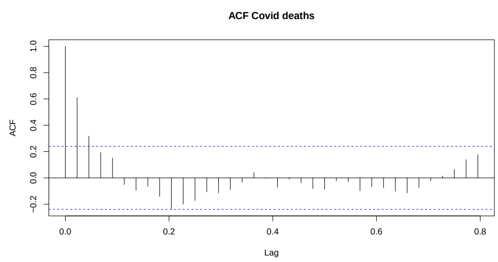
(d) Propane

Figure 7: Histograms with various distributions

In order to analyze the fluctuations of the commodities together with the deaths data in USA, we had to check previously the stationarity of the latter values. As can be seen in Figure 8a, there is an irregular evolution that implies the non stationarity of the series. After that, we have computed, at lag 35, the first order differences so that the series would become stationary. The stationarity has also been checked computing the Dickey-Fuller test, whose p-value equal to 0.0351, that allows us to reject the H_0.



(a) Covid deaths



(b) ACF stationary Covid deaths

7.2 Fitting models

Coherently with one of the main reasons of the paper, which requires focusing on the study of the VaR, we need to use models able to catch the volatility clustering and heteroskedasticity; thus, GARCH models suit perfectly. In the following analysis, all the data will be fitted with the GARCH(1,1) considering five error distributions: Normal, Skew-Normal, Student t, Skew-Student t and GED[3].

Developing the analysis on the returns as dependent variable, the parameters considered are those of the different GARCH analysis. The most important explanatory variables are α and β which capture the conditional variance behaviours, specifically, α for the ARCH and β for the GARCH. Furthermore, γ stands for the asymmetry parameter that represents an important value to take in consideration in our specific analysis given the possible presence of negative effects on the returns. As characteristic of the financial data, the values of μ should always be approximately around zero because the unconditional expected value of returns is equal to zero; it can be proved throughout the analysis. In the eGARCH and GARCH we also evaluate the constant parameter ω, while in the GARCH-MIDAS, this parameter is no longer present. As far as concern the latter models, focusing on θ, it is related to the Covid deaths macroeconomic variable, explaining then its influence on the dependent variable. Finally, the constant m provides the intercept of the long run component (τ)[4].

WTI When considering the results after having applied the different GARCH models, it is possible to state that the parameters α, β and ω show p-values that have as a consequence the rejection of the null hypothesis given their significance. The intercept of the models is statistically significant in most of the cases, but it is not true only in the sstd error distributions. Furthermore, the asymmetric parameter (γ) is also always strongly significant, which highlights the negative asymmetry studied in the previous graphs. With the addition of the weekly variable, some changes in the parameters are worth of noting. The models without the skewness present that all their parameters are statistically significant. In fact, among these, it can be easily pointed out that α is the parameter that varies the most, on the other hand when examining the values that consider the asymmetry, the parameter loses significance, even arriving to almost the total loss of significance in the GARCH-snorm. Going on with the analysis, the only value of θ that is statistically significant is obtained in the GM-norm computation. Lastly, m always assumes a negative value which implies a negative value in the variance independently from the values assumed by α and β.

[3]See Data Appendix

[4]See Equation (7)

Brent In this specific case, the path followed by the Brent is almost the same as the one of the WTI. The α, β and γ are all strongly significant, which allows us to assume that the GARCH models applied performed in a proper way. For the μ parameter, the only p-values not statistically significant are those of the sstd error distribution for both the models analyzed, contrary to the ω that shows non significant values in the sGARCH-std and sGARCH-ged. In the MIDAS examination, worth of mentioning is that every value of θ, m and γ is not statistically significant. Additionally, the β parameter is always statistically significant under a strong evidence, but it is not true for α, that shows a non significant value in the GM-sstd.

Heating Oil Here, worth of noting is that α, β and γ are all significant. Considering the μ parameter, it is still significant, but for the GARCH-sstd. In the MIDAS analysis, the parameter β is always strongly significant; the same happens for α except for the GM-sstd. Similarly to the previous cases, most of the values of θ are not statistically significant: this implies that the Covid parameter is not able to explain the independent variable.

Propane For this commodity, when evaluating the μ parameter, the only statistically significant values are those of GARCH-std, GARCH-ged and eGARCH-ged. The β parameter, the asymmetry one, the shape and the skew are all statistically significant. Subsequently, some changes are made more evident when studying the case that includes the MIDAS, infact, the μ parameter for the GARCH-snorm is not significant and the only value that implies the non significance for α is that of the GM-sstd. On the other hand, the values of γ and β kept the significance highlighted in the models without MIDAS.

7.3 Backtesting

One of the main purposes of the research is to verify if and how the influence of the addition of the weekly frequency variable, concerning the Covid-19 deaths in the USA, generates a salient change in the estimation of the VaR or not. The VaR has been computed at 95% and 99% confidence levels. The total data have been divided into in-sample and out-sample evidence, pointedly, the in-sample value is equal to 165 and out-sample 150. In our analysis, given the forecast length set to 150, we went for a moving window and a refit equal to 5: this implies that every five observations, a new value is estimated. The purpose of the backtesting is to compare ex-post the estimated values, that specifically correspond to the VaR estimation, to those that have been actually realized, that correspond to the 150 values of the out of sample. The p-values of the estimations obtained, reject the null at 0.01 and 0.05 respectively to every α value. The tests calculated can pass to the MCS procedure only if they pass two out of three of the typoligies of the backtesting procedure: Kupiec, Christoffersen and Dynamic Quantile (DQ).

Confidence level	95%			99%		
	LR.uc	LR.cc	DQ	LR.uc	LR.cc	DQ
sGARCH-norm	0.853	0.662	0.677	0.696	0.902	0.849
sGARCH-std	0.853	0.662	0.712	0.696	0.902	0.722
sGARCH-snorm	0.850	0.731	0.379	0.696	0.902	0.723
sGARCH-sstd	0.561	0.685	0.450	0.696	0.902	0.480
sGARCH-ged	0.853	0.662	0.676	0.696	0.902	0.774
eGARCH-norm	0.372	0.591	0.114	0.089	0.049	5.98e-12
eGARCH-std	0.219	0.447	0.114	0.279	0.523	0.105
eGARCH-snorm	0.561	0.657	0.639	0.662	0.903	0.984
eGARCH-sstd	0.561	0.657	0.653	0.662	0.903	0.980
eGARCH-ged	0.372	0.591	0.433	0.279	0.523	0.081
GM-snorm	0.561	0.657	0.060	0.662	0.903	0.965
GM-norm	0.152	0.320	0.654	0.662	0.903	0.965
GM-sstd	0.152	0.320	0.612	0.696	0.902	0.712
GM-std	0.850	0.695	0.655	0.662	0.903	0.964

Table 5: Backtesting WTI

Examining the backtesting on WTI, it can be noted that every model passes the test at both the 95% and 99% confidence level, even though the eGARCH-norm fails the Dynamic Quantile test at 99%. Moreover, with the inclusion of the phenomenon represented by the θ variable, it is possible to state that all the models are allowed into the MCS procedure.

Confidence level	95%			99%		
	LR.uc	LR.cc	DQ	LR.uc	LR.cc	DQ
sGARCH-norm	0.561	0.685	0.829	0.696	0.902	0.620
sGARCH-std	0.850	0.731	0.893	0.662	0.903	0.952
sGARCH-snorm	0.320	0.513	0.801	0.662	0.903	0.959
sGARCH-sstd	0.561	0.657	0.857	0.662	0.903	0.937
sGARCH-ged	0.850	0.731	0.893	0.662	0.903	0.956
eGARCH-norm	0.585	0.208	0.018	0.696	0.902	0.700
eGARCH-std	0.372	0.232	0.037	0.279	0.523	0.196
eGARCH-snorm	0.850	0.588	0.230	0.279	0.523	0.073
eGARCH-sstd	0.850	0.588	0.317	0.279	0.061	2.72e-13
GM-snorm	0.563	0.152	0.535	0.662	0.903	0.973
GM-norm	0.015	0.051	0.543	0.662	0.903	0.973
GM-sstd	0.002	0.010	0.204	0.662	0.903	0.977
GM-std	0.015	0.051	0.358	0.662	0.903	0.980

Table 6: Backtesting Brent

In the European Brent analysis, all the models have been accepted for the MCS procedure, although the eGARCH-norm, eGARCH-snorm and eGARCH-ged at 95% and the eGARCH-sstd at 99% do not pass the Dynamic Quantile test. Indeed, it is possible to state that every model taken in consideration are able to compute a good VaR estimation. In the MIDAS analysis at 95%, the GM-sstd does not pass two out of three test of the backtesting, thus only the remaining ones are admitted into the final procedure. Whilst, at 99% every model has been taken in consideration for the MCS.

Confidence level	95%			99%		
	LR.uc	LR.cc	DQ	LR.uc	LR.cc	DQ
sGARCH-norm	0.853	0.714	0.303	0.696	0.902	0.785
sGARCH-std	0.585	0.674	0.369	0.696	0.902	0.781
sGARCH-snorm	0.850	0.588	0.356	0.696	0.902	0.780
sGARCH-sstd	0.850	0.588	0.377	0.662	0.903	0.974
sGARCH-ged	0.853	0.714	0.328	0.696	0.902	0.766
eGARCH-norm	0.372	0.591	0.012	0.005	0.001	9.71e-15
eGARCH-std	0.585	0.720	0.049	0.089	0.211	0.004
eGARCH-snorm	0.585	0.720	0.004	0.005	0.010	4.62e-16
eGARCH-sstd	0.585	0.720	0.026	0.005	0.010	1.47e-13
eGARCH-ged	0.372	0.618	0.007	0.005	0.010	4.64e-14
GM-snorm	0.151	0.320	0.689	0.696	0.902	0.981
GM-norm	0.056	0.152	0.503	0.696	0.902	0.852
GM-sstd	0.151	0.320	0.604	0.662	0.903	0.996
GM-std	0.056	0.152	0.558	0.662	0.903	0.984

Table 7: Backtesting Heating Oil

Contrary to what seen in WTI and Brent, the Heating Oil Backtesting procedure highlights that at 99% confidence interval, a part from the eGARCH-std, that all the eGARCH models have failed the tests. At 95% there is no model that has been rejected, even though all the Dynamic Quantile test of the eGARCH models have not passed. Converse to what happens in the analysis without the macroeconomic variable,here there is no model that do not pass the backtesting evaluation.

Confidence level	95%			99%		
	LR.uc	LR.cc	DQ	LR.uc	LR.cc	DQ
sGARCH-norm	0.853	0.624	0.108	0.005	0.016	2.61e-12
sGARCH-std	0.120	0.297	0.010	0.279	0.523	0.005
sGARCH-snorm	0.853	0.624	0.039	0.005	0.016	1.49e-12
sGARCH-sstd	0.585	0.483	0.107	0.279	0.523	0.005
sGARCH-ged	0.585	0.483	0.022	0.279	0.523	0.009
eGARCH-norm	0.585	0.483	0.008	0.001	0.003	4.44e-20
eGARCH-std	0.120	0.295	0.007	0.279	0.523	0.009
eGARCH-snorm	0.372	0.327	0.019	0.005	0.016	2.71e-14
eGARCH-sstd	0.585	0.483	0.038	0.696	0.902	0.789
eGARCH-ged	0.372	0.352	0.017	0.089	0.211	0.000
GM-snorm	0.853	0.624	0.478	0.279	0.523	0.273
GM-norm	0.561	0.657	0.500	0.279	0.523	0.230
GM-sstd	0.585	0.483	0.069	0.696	0.902	0.434
GM-std	0.152	0.320	0.400	0.662	0.903	0.780

Table 8: Backtesting Propane

Lastly, as pointed out in the table, although some statistics reject the null hypothesis in the Dynamic Quantile test at 95%, all the models can be ranked by the MCS procedure. Conversely, at 99% confidence level, four out of ten models have not been admitted to the final procedure. This difference is not obtained into the examination with the MIDAS, given that neither at 95% nor at 99% there is a rejection of the H_0.

7.4 Model Confidence Set results

Subsequently to the computation of the backtesting results, in this section the Model Confidence Set has listed a rank between all the models for each series. The examination has been developed for both the case with and without the inclusion of the Covid-19 deaths variable, in order to state which model performs at the best for the VaR estimation. The parameters considered are the t_i test statistic, the p-value and the VaR losses. Furthermore, we decided to highlight the difference between those model that have not passed the backtesting (in red) and those that have been precisely eliminated by the MCS procedure (in blue).

WTI	Rank	t_i	P-value	Loss	Rank	t_i	P-value	Loss
Confidence level	95%				99%			
sGARCH-norm	7	0.3891	0.980	0.0026958	6	0.3841	0.886	0.0009112
sGARCH-std	6	0.3284	0.990	0.0026932	9	1.0572	0.491	0.0009751
sGARCH-snorm	9	1.4841	0.331	0.0028035	7	0.9503	0.561	0.0009772
sGARCH-sstd	10	1.6888	0.234	0.0028289	-	-	-	-
sGARCH-ged	8	0.4578	0.960	0.0027012	8	1.0104	0.521	0.0009583
eGARCH-norm	1	-1.4132	1.000	0.0025216	3	-0.8063	1.000	0.0007951
eGARCH-std	4	-0.9698	1.000	0.0025705	4	-0.7118	1.000	0.0008138
eGARCH-snorm	3	-0.9937	1.000	0.0025642	1	-1.4037	1.000	0.0007501
eGARCH-sstd	5	0.3106	0.993	0.0026987	5	0.0009	1.000	0.0008718
eGARCH-ged	2	-1.1710	1.000	0.0025602	2	-1.0117	1.000	0.0007930
GM-snorm	-	-	-	-	-	-	-	-
GM-norm	-	-	-	-	-	-	-	-
GM-sstd	1	-0.8877	1.000	0.0028722	-	-	-	-
GM-std	2	0.8877	0.375	0.0029462	1	-2.7041	1.000	0.0009395

Table 9: Table Superior Set of Models WTI

According to the results computed by the analysis above, at 95% considering the rank and the lowest values of the losses, the best models obtained are the eGARCH, where the 1^{st} ranked is the eGARCH-norm, that henceforth is more precise when estimating. At the 99%, the latter model passes from being ranked 1^{st} to be ranked 3^{rd} due to the higher precision of this evaluation, where now the best model considered is the eGARCH-snorm. Thus, even though the change of the best model considered, regarding the p-values at 99% analysis, the best models remain the same: the eGARCH. Some difference can be noted in the GARCH-MIDAS analysis where the models in which the error term follows a Normal distribution have been deleted by the MCS, hence the best models computed are those with a Student-t error distribution; where the ideal scenario is obtained at 99% with the GM-std.

Brent	Rank	t_i	P-value	Loss	Rank	t_i	P-value	Loss
Confidence level	95%				99%			
sGARCH-norm	3	-1.036	1.000	0.0025639	4	0.456	0.845	0.0008647
sGARCH-std	6	-0.589	1.000	0.0025709	-	-	-	-
sGARCH-snorm	7	0.306	0.999	0.0026324	6	1.578	0.216	0.0009009
sGARCH-sstd	8	0.360	0.997	0.0026416	-	-	-	-
sGARCH-ged	2	-1.256	1.000	0.0025526	-	-	-	-
eGARCH-norm	4	-0.826	1.000	0.0025650	1	-3.058	1.000	0.0007950
eGARCH-std	5	-0.599	1.000	0.0025807	3	-0.084	0.999	0.0008484
eGARCH-snorm	9	1.402	0.444	0.0027082	5	0.659	0.743	0.0008653
eGARCH-sstd	10	1.643	0.291	0.0028001	-	-	-	-
eGARCH-ged	1	-1.268	1.000	0.0025530	2	-1.830	1.000	0.0008058
GM-snorm	2	0.862	0.388	0.0034075	1	-0.669	1.000	0.0011225
GM-norm	1	-0.862	1.000	0.0034027	2	-0.635	1.000	0.0011228
GM-sstd	-	-	-	-	-	-	-	-
GM-std	-	-	-	-	3	0.652	0.551	0.0001203

Table 10: Table Superior Set of Models Brent

What that catches the eye is that at 95% in the first analysis is that the 9^{th} and 10^{th} ranked models are those which present the lowest p-values compared to the others, but in spite of that they have still passed the final procedure. At 99% the MCS procedure deletes four models, of which the 10^{th} at 95%, but an interesting change has been computed into the 9^{th} at 95% that at 99% is ranked 5^{th}, hence we have an improvement of the model. In the MIDAS case, contrary to the previous one of the WTI commodity, the best models considered are those with a Normal error distribution, both at 95% and 99%.

Heating Oil	Rank	t_i	P-value	Loss	Rank	t_i	P-value	Loss
Confidence level	95%				99%			
sGARCH-norm	3	-1.4797	1.000	0.0024632	1	-1.0104	1.000	0.0007874
sGARCH-std	1	-1.9119	1.000	0.0024502	2	-0.9812	1.000	0.0008071
sGARCH-snorm	5	-0.4403	1.000	0.0025155	3	-0.1131	1.000	0.0008095
sGARCH-sstd	4	-0.8357	1.000	0.0024924	5	1.3814	0.267	0.0008260
sGARCH-ged	2	-1.5935	1.000	0.0024601	4	0.3380	0.868	0.0008105
eGARCH-norm	-	-	-	-	-	-	-	-
eGARCH-std	6	1.1105	0.343	0.0027110	-	-	-	-
eGARCH-snorm	-	-	-	-	-	-	-	-
eGARCH-sstd	-	-	-	-	-	-	-	-
eGARCH-ged	7	1.2574	0.278	0.0027391	-	-	-	-
GM-snorm	-	-	-	-	1	-0.6783	1.000	0.0007587
GM-norm	-	-	-	-	-	-	-	-
GM-sstd	2	0.2395	0.823	0.0025864	-	-	-	-
GM-std	1	-0.2395	1.000	0.0025740	2	0.6783	0.545	0.0008454

Table 11: Table Superior Set of Models Heating Oil

As pointed out in the Table 11, the best models obtained at 95% are those of the sGARCH. This has been afterwards confirmed at 99% with the elimination of the eGARCH-std, that together with the previous model that have not passed the back-testing, entails the total elimination of the eGARCH models: the major confirmation of the best models is given by the values of the losses that diminish at 99%. The GM-snorm at 95% do not pass the MCS procedure, but at 99% represents the best model among them.

Propane	Rank	t_i	P-value	Loss	Rank	t_i	P-value	Loss
Confidence level	95%				99%			
sGARCH-norm	6	0.2653	1.000	0.0038277	-	-	-	-
sGARCH-std	8	0.3844	0.998	0.0038347	6	1.6287	0.264	0.0011736
sGARCH-snorm	10	0.9687	0.843	0.0039563	-	-	-	-
sGARCH-sstd	7	0.2924	1.000	0.0038322	4	-0.3619	1.000	0.0010973
sGARCH-ged	4	-0.3607	1.000	0.0037899	5	1.3641	0.416	0.0011830
eGARCH-norm	9	0.5085	0.992	0.0038524	-	-	-	-
eGARCH-std	1	-1.5648	1.000	0.0037190	3	-0.4604	1.000	0.0010900
eGARCH-snorm	5	0.0571	1.000	0.0038140	-	-	-	-
eGARCH-sstd	3	-0.8471	1.000	0.0037492	2	0.8451	1.000	0.0010519
eGARCH-ged	2	-1.4519	1.000	0.0037372	1	-3.6866	1.000	0.0010433
GM-snorm	2	-0.4547	1.000	0.0037668	2	-0.6688	1.000	0.0011228
GM-norm	4	0.9767	0.490	0.0039931	4	1.3512	0.3026	0.0012758
GM-sstd	3	-0.3145	1.000	0.0037747	3	-0.0126	1.000	0.0011579
GM-std	1	-0.7594	1.000	0.0037331	1	-1.4573	1.000	0.0010780

Table 12: Table Superior Set of Models Propane

Considering the results computed, we are able to say that the models, with and without the θ parameter, at 95% have a strong behaviour; hence it is possible to state that the models considered are adequate to forecast through this series. At 99% case, some models, such as eGARCH-ged and eGARCH-sstd have improved their ranking, but the same statement can not be made in the MIDAS analysis where the rank has been confirmed, without any improvement.

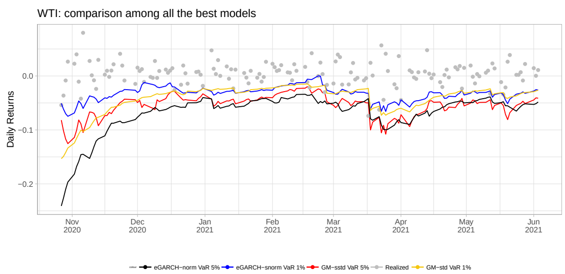
8 Conclusions

It is surely uncertain for how long the effects of the Covid-19 pandemic, begun in February 2020, will be protracted; but it results anyways worth of nothing the financial-economic impact until nowadays. For this extent, we aimed to elaborate an analysis of models to put in place an appropriate VaR forecasting for four commodities: the WTI, the European Brent, the Heating Oil and the Propane. Given that the aim is to find the best model for each commodity that can predict the VaR with and without the implementation of the Covid-19 macroeconomic variable, here we highlight the results of the examination, to better understand the weight associated to the phenomenon, checking the level of the influence of the pandemic in the markets. The goal we intend to pursue in this paper is to evaluate how the time series are conditioned on this phenomenon, given the commodities taken in consideration. Detecting the closing prices and focusing on their trend, it is clear how the complex historic period has produced instabilities on those ones. Hence, to go on with our analysis, it has been made necessary to model the data.

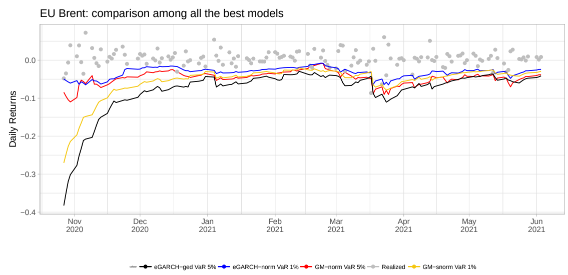
At beginning of the analysis, graphically and quantitative speaking, different statistic tests have been carried out, such as ADF, Box-Pierce, Shapiro, Jarque-Bera, in order to certify some typical characteristics of the financial data as the presence of negative skewness and the excess of kurtosis (leptokurtosis). Since the type of data, it has been considered adequate to elaborate two different analysis where for both of them the best fit models are the GARCH. For the first type of analysis we computed two different GARCH versions for each commodity: the sGARCH and eGARCH; while for the second analysis we went for only the standard version of the GARCH-MIDAS. After the models estimation, the backtesting procedure has allowed us to decide which, between the examinated models, have a better capacity to estimate the VaR both at 95% and 99%. In the WTI time series, the best models are the eGARCH, specifically the eGARCH-norm at 95% and the eGARCH-snorm at 99%.

In the WTI, the backtesting shows at 95% them capacity for all the models to correctly estimate the VaR, both with and without MIDAS method; at 99%, instead, every model appears to be adequate but for the eGARCH-norm that fails both the Christoffersen and Dyniamic Quantile tests. The MCS procedure conferms that the best model are those of the eGARCH (at 95% the eGARCH-snorm); the GARCH-MIDAS with the Student t distribution perform best, even in the case of absence of skewness. In the Brent, all the models pass the backtesting phase for every confidence level considered, with and without the MIDAS regressor. The GM-sstd model, nevertheless, has not been admitted to the MCS. Contrary to what happens in the WTI, in the MIDAS case, the best models have a Normal error distribution. For what concerns the Heating Oil, the backtesting procedure excludes all the eGARCH models at 99%. This result has been confirmed by the MCS, that states that the best models are the sGARCH, whichever the considered distribution is. Some disharmonious results are obtained with the implementation of the Covid variable, as can be seen at 95% that the best model is the GM-std. The best one at 99% is instead the GM-snorm, that contrarly had been excluded at 95% previously. Lastly, in the Propane case, taking in cosideration the macroeconomic variable, all the models are satisfactory. Nevertheless, worth of noting is the difference among the general case and the case that provides for the Covid variable: pointedly, in the first case, at 99% every model that considers a Normal error distribution is excluded by the backtesting, as a confirmaton of the non adequacy for the VaR estimation. Baring the models that do not pass the backtesting, the remaining ones are suitable for the VaR estimation. All the GARCH-MIDAS models at 95% and at 99%, having a loss value approximable to zero, confirm that the Covid variable consideration has a huge importance in our analysis.

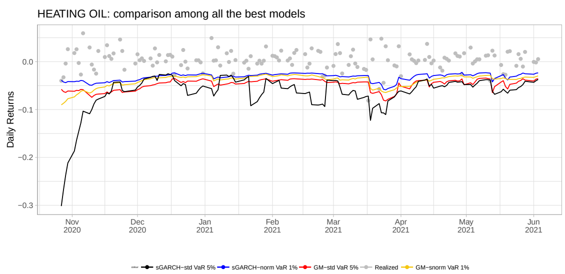
The following graphical representation show the best models for each time series of the commodities, with each confidence level regarding the VaR estimation.



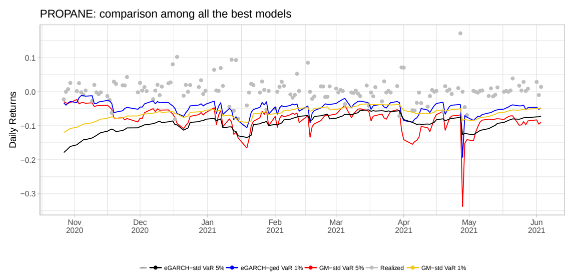
(a) WTI



(b) Brent



(c) Heating Oil



(d) Propane

Figure 9: Best models, GARCH and GARCH-MIDAS at 95% and 99%

9 Data appendix

WTI	μ	ω	α	β	γ	shape	skew
sGARCH-norm	0.003275 (0.011)	0.000014 (0.026)	0.150733 (0.000)	0.848267 (0.000)	-	-	-
sGARCH-snorm	0.002627 (0.036)	0.00017 (0.017)	0.176912 (0.000)	0.819529 (0.000)	-	-	0.707942 (0.000)
sGARCH-std	0.003953 (0.001)	0.00052 (0.034)	0.223173 (0.009)	0.759737 (0.000)	-	3.715914 (0.000)	-
sGARCH-sstd	0.002220 (0.084)	0.00049 (0.025)	0.240875 (0.003)	0.747028 (0.000)	-	0.749373 (0.000)	4.123717 (0.000)
sGARCH-ged	0.003794 (0.000)	0.00029 (0.047)	0.179018 (0.002)	0.805960 (0.000)	-	1.066573 (0.000)	-
eGARCH-norm	0.002688 (0.000)	-0.106773 (0.000)	-0.212401 (0.000)	0.985741 (0.000)	0.185088 (0.000)	-	-
eGARCH-snorm	0.001973 (0.001)	0.156775 (0.000)	-0.210447 (0.000)	0.978175 (0.000)	0.209329 (0.000)	-	0.693449 (0.000)
eGARCH-std	0.003188 (0.006)	-0.209647 (0.000)	-0.189464 (0.000)	0.972997 (0.000)	0.199528 (0.001)	4.683379 (0.000)	-
eGARCH-sstd	0.001612 (0.203)	-0.218877 (0.000)	-0.189568 (0.000)	0.969760 (0.000)	0.243368 (0.000)	5.160014 (0.001)	0.731475 (0.000)
eGARCH-ged	0.003462 (0.017)	-0.165574 (0.031)	-0.199167 (0.000)	0.979854 (0.000)	0.179516 (0.003)	1.173289 (0.000)	-

Table 13: Estimated parameters WTI

WTI	m	α	β	γ	θ	shape
GM-snorm	-3.9364 (0.532)	0.01022 (0.751)	0.8327 (0.000)	0.1283 (0.814)	0.0001 (0.627)	-
GM-norm	-3.8027 (0.026)	0.1849 (0.000)	0.8141 (0.000)	-	0.0001 (0.001)	-
GM-sstd	-4.6832 (0.000)	0.0001 (0.999)	0.8635 (0.000)	0.2699 (0.158)	0.0000 (0.065)	3.7119 (0.001)
GM-std	-4.5690 (0.000)	0.1619 (0.005)	0.8359 (0.000)	-	0.0000 (0.064)	2.8974 (0.000)

Table 14: Estimated parameters GARCH MIDAS WTI

Brent	μ	ω	α	β	γ	shape	skew
sGARCH-norm	0.003714 (0.005)	0.000020 (0.039)	0.188145 (0.000)	0.810855 (0.000)	-	-	-
sGARCH-snorm	0.003082 (0.026)	0.000026 (0.015)	0.180630 (0.000)	0.801803 (0.000)	-	-	0.765844 (0.000)
sGARCH-std	0.003908 (0.001)	0.000049 (0.062)	0.219063 (0.018)	0.779115 (0.000)	-	3.376906 (0.000)	-
sGARCH-sstd	0.002417 (0.086)	0.000052 (0.047)	0.206756 (0.014)	0.776632 (0.000)	-	3.559393 (0.000)	0.838838 (0.000)
sGARCH-ged	0.004186 (0.000)	0.000033 (0.062)	0.193026 (0.004)	0.789753 (0.000)	-	1.040031 (0.000)	-
eGARCH-norm	0.002797 (0.000)	-0.122481 (0.047)	-0.155459 (0.000)	0.981779 (0.000)	0.244027 (0.000)	-	-
eGARCH-snorm	0.002314 (0.002)	-0.153391 (0.006)	-0.146196 (0.000)	0.977501 (0.000)	0.228554 (0.000)	-	0.780739 (0.000)
eGARCH-std	0.003478 (0.002)	-0.196046 (0.003)	-0.131856 (0.004)	0.973520 (0.000)	0.241589 (0.006)	3.926944 (0.000)	-
eGARCH-sstd	0.002242 (0.102)	-0.212377 (0.001)	-0.127320 (0.004)	0.970291 (0.000)	0.246058 (0.003)	4.086784 (0.000)	0.848239 (0.000)
eGARCH-ged	0.003786 (0.000)	-0.169039 (0.018)	-0.134299 (0.002)	0.978297 (0.000)	0.229845 (0.006)	1.108426 (0.000)	-

Table 15: Estimated parameters Brent

Brent	m	α	β	γ	θ	shape
GM-snorm	-3.6545 (0.223)	0.1984 (0.024)	0.8239 (0.000)	-0.0465 (0.787)	0.0001 (0.155)	-
GM-norm	-3.6584 (0.170)	0.1724 (0.000)	0.8266 (0.000)	-	0.0001 (0.065)	-
GM-sstd	-3.1737 (0.178)	0.0368 (0.794)	0.8637 (0.000)	0.1953 (0.410)	0.0000 (0.696)	2.5547 (0.006)
GM-std	-3.0292 (0.111)	0.1560 (0.007)	0.8422 (0.000)	-	0.0000 (0.725)	2.6058 (0.002)

Table 16: Estimated parameters GARCH-MIDAS Brent

Heating oil	μ	ω	α	β	γ	shape	skew
sGARCH-norm	0.003172 (0.015)	0.000042 (0.171)	0.235875 (0.034)	0.738802 (0.000)	-	-	-
sGARCH-snorm	0.002716 (0.041)	0.000032 (0.092)	0.172701 (0.014)	0.792231 (0.000)	-	-	0.787846 (0.000)
sGARCH-std	0.003025 (0.019)	0.000025 (0.099)	0.148641 (0.008)	0.826593 (0.000)	-	6.972681 (0.005)	-
sGARCH-sstd	0.002323 (0.087)	0.000027 (0.079)	0.142992 (0.008)	0.825114 (0.000)	-	8.139463 (0.018)	0.835068 (0.000)
sGARCH-ged	0.002808 (0.015)	0.000028 (0.144)	0.174977 (0.020)	0.802100 (0.000)	-	1.361271 (0.000)	-
eGARCH-norm	0.001809 (0.007)	-0.332339 (0.012)	-0.159054 (0.000)	0.953828 (0.000)	0.279186 (0.001)	-	-
eGARCH-snorm	0.001648 (0.022)	-0.331622 (0.013)	-0.146342 (0.001)	0.954078 (0.000)	0.251112 (0.000)	-	0.804695 (0.000)
eGARCH-std	0.00246 (0.005)	-0.27176 (0.003)	-0.12121 (0.006)	0.96353 (0.000)	0.24354 (0.001)	9.65129 (0.047)	-
eGARCH-sstd	0.001658 (0.005)	-0.313285 (0.001)	-0.127839 (0.004)	0.957159 (0.000)	0.240753 (0.001)	12.74197 (0.153)	0.828682 (0.000)
eGARCH-ged	0.002298 (0.008)	-0.295567 (0.024)	-0.136301 (0.005)	0.960400 (0.000)	0.263146 (0.002)	1.488735 (0.000)	-

Table 17: Estimated Heating Oil

Heating oil	m	α	β	γ	θ	shape
GM-snorm	-3.9014 (0.051)	0.1886 (0.000)	0.7582 (0.000)	0.1028 (0.789)	-0.0001 (0.064)	-
GM-norm	-4.6209 (0.001)	0.1391 (0.000)	0.8597 (0.000)	-	0.0000 (0.426)	-
GM-sstd	-5.0262 (0.000)	0.0643 (0.544)	0.8608 (0.000)	0.1444 (0.479)	0.0000 (0.038)	5.2068 (0.137)
GM-std	-5.7040 (0.000)	0.1359 (0.000)	0.8578 (0.000)	-	0.0000 (0.359)	5.1333 (0.007)

Table 18: Estimated parameters GARCH-MIDAS Heating oil

Propane	μ	ω	α	β	γ	shape	skew
sGARCH-norm	0.003047 (0.064)	0.000047 (0.015)	0.129356 (0.001)	0.841283 (0.000)	-	-	-
sGARCH-snorm	0.002959 (0.082)	0.000046 (0.018)	0.127753 (0.001)	0.843341 (0.000)	-	-	0.989305 (0.000)
sGARCH-std	0.002692 (0.030)	0.000131 (0.143)	0.301982 (0.028)	0.697018 (0.000)	-	2.919932 (0.002)	-
sGARCH-sstd	0.001920 (0.220)	0.000123 (0.146)	0.291815 (0.026)	0.707185 (0.000)	-	2.941375 (0.000)	0.947909 (0.000)
sGARCH-ged	0.002230 (0.000)	0.000075 (0.156)	0.204871 (0.053)	0.764421 (0.000)	-	0.849288 (0.000)	-
eGARCH-norm	0.001115 (0.512)	-0.157950 (0.024)	-0.085127 (0.010)	0.972816 (0.000)	0.247746 (0.000)	-	-
eGARCH-snorm	0.000737 (0.681)	-0.149939 (0.030)	-0.087270 (0.009)	0.973861 (0.000)	0.245083 (0.000)	-	0.963695 (0.000)
eGARCH-std	0.002451 (0.053)	-0.630488 (0.144)	-0.037245 (0.659)	0.900460 (0.000)	0.566161 (0.014)	2.649159 (0.000)	-
eGARCH-sstd	0.001453 (0.372)	-0.582523 (0.173)	-0.051199 (0.554)	0.907232 (0.000)	0.566997 (0.018)	2.640657 (0.000)	0.934062 (0.000)
eGARCH-ged	0.002053 (0.000)	-0.281220 (0.209)	-0.051134 (0.335)	0.959152 (0.000)	0.312590 (0.007)	0.854942 (0.000)	-

Table 19: Estimated Propane

Propane	m	α	β	γ	θ	shape
GM-snorm	-4.5752 (0.067)	0.2710 (0.041)	0.6490 (0.000)	0.1492 (0.566)	0.0000 (0.122)	-
GM-norm	-4.7169 (0.003)	0.3470 (0.000)	0.6466 (0.000)	-	0.0000 (0.069)	-
GM-sstd	-3.9123 (0.005)	0.0321 (0.932)	0.8605 (0.000)	0.2128 (0.536)	0.0000 (0.203)	3.6405 (0.249)
GM-std	-5.1804 (0.000)	0.0679 (0.007)	0.9307 (0.000)	-	0.0000 (0.300)	2.8974 (0.000)

Table 20: Estimated parameters GARCH-MIDAS Propane

9.1 R code

```
Energy_data_ <- read_excel("Energy data .xls")

Data <- na.omit(Energy_data_)
Data1 <- Data[, c(1,2)]
sum(is.na(Data1))
Data2 <- Data1[-36, ]

dataset <- Data2

rm(Data, Data1, Data2, Energy_data_)

### WTI ###

WTI <- dataset
WTI_ts <- ts(WTI, start = c(2020,2), frequency = 213)
ts.WTI <- WTI_ts[ ,2]
plot(ts.WTI, ylab = "WTI")
acf(ts.WTI) #no stationary
pacf(ts.WTI)

logWTI_ts <- log(ts.WTI)
diffWTI_ts <- diff(logWTI_ts)

plot(diffWTI_ts)
M <- mean(diffWTI_ts)
abline(h = M, col = "red")
acf(diffWTI_ts) #stationary
pacf(diffWTI_ts)

pdf("WTI.pdf")
plot(ts.WTI, ylab = "WTI")
acf(ts.WTI, main = "")
pacf(ts.WTI, main = "")
acf(diffWTI_ts, main = "")
pacf(diffWTI_ts, main = "")
dev.off()

adf.test(ts.WTI, alternative ="stationary")
adf.test(diffWTI_ts, alternative = "stationary")

logWTI_ts <- log(ts.WTI) #logprices
diffWTI_ts <- diff(logWTI_ts) #difference of order 1 (log) = the daily returns series
plot.ts(diffWTI_ts) #stationarity and daily returns
retWTI <- diffWTI_ts
sq_retWTI <- retWTI^2
plot(retWTI, main = "WTI returns", ylab = "Returns")
plot(sq_retWTI, main = "WTI squared returns", ylab = "Returns")
```

```r
# ACF and PACF
par(mfrow = c(4,2))
acf(retWTI)
acf(retWTI, lag.max = 70)
pacf(retWTI)
pacf(retWTI, lag.max = 70)

acf(sq_retWTI)
acf(sq_retWTI, lag.max = 70)
pacf(sq_retWTI)
pacf(sq_retWTI, lag.max = 70)

par(mfrow = c(1,1))

#QQ - PLOT
qqnorm(y = retWTI, main = "Normal QQ plot WTI",
    xlab = "Theoretical Quantiles", ylab = "Sample Quantiles")
qqline(retWTI, distribution = qnorm, col = "red")

# STATISTICAL ANALYSIS (CI = 95%)
basicStats(retWTI)

# KURTOSIS TEST
kurtosis(sq_retWTI) #coherently with the previous result, we note that the values exceed
#positively 0 and confirm the leptokurtic distribution

# BOX-PIERCE TEST Ho: no correlation
Box.test(retWTI, lag = 35, type = "Box-Pierce") #We need to reject the null.
#For this reason we can conclude that within the 35th lag there's correlation
Box.test(retWTI, lag = 100, type = "Box-Pierce") #We cannot reject the null
#because crossing the 100th lag, there's no correlation among residuals/returns

Box.test(sq_retWTI, lag = 35, type = "Box-Pierce") #We reject the null
Box.test(sq_retWTI, lag = 100, type = "Box-Pierce") #After the 100th lag, there's no correlation

# SHAPIRO-WILK TEST (HO = normally distributed)
shapiro.test(retWTI) #As we said before, the data are not normally distributed,
#therefore we reject the null

# HISTOGRAMS
hist(retWTI, xlab = "Daily stock prices", prob = T,
    main = "WTI daily returns", freq = F, breaks = 20)
#shifted distribution different from the Normal one

xfit <- seq(min(retWTI), max(retWTI), length = 1000)
yfit <- dnorm(xfit, mean = M, sd = sd(retWTI)) #normal distribution
lines(xfit, yfit, type = "l", col = "blue", lwd = 1.5)

y1fit <- dstd(xfit, mean = M, sd = sd(retWTI)) #student-t distribution
```

```
lines(xfit, y1fit, type = "l", col = "red", lwd = 1.5)

y2fit <- dged(xfit, mean = M, sd = sd(retWTI)) #generalized error distribution
lines(xfit, y2fit, type = "l", col = "green", lwd = 1.5)

y3fit <- dsstd(xfit, mean = M, sd = sd(retWTI)) #skew student-t distribution
lines(xfit, y3fit, type = "l", col = "black", lwd = 1.5)

y4fit <- dsnorm(xfit, mean = M, sd = sd(retWTI)) #skew normal distribution
lines(xfit, y4fit, type = "l", col = "purple", lwd = 1.5)

legend("topright", legend = c("std", "ged", "sstd", "snorm"),
    col = c("red", "green", "black", "purple"), lty = 1, cex = 0.9)

# FITTING MODELS
models <- c("sGARCH", "eGARCH")
distributions <- c("norm", "std", "snorm", "sstd", "ged")
m.d_bind <- list()
m <- c()
d <- c()
for (m in models) {
  for (d in distributions) {
    m.d_bind[[paste(m, d, sep = "-" )]] <-
      ugarchspec(mean.model = list(armaOrder = c(0, 0)),
                 variance.model = list(model = m, garchOrder = c(1, 1)),
                 distribution.model = d)
  }
}

spec.model <- names(m.d_bind); spec.model

fitmod <- list()
for (s in spec.model) {
  fitmod[[s]] <- ugarchfit(spec = m.d_bind[[s]], data = retWTI)
}
fitmod

model_1 <- fitmod$`eGARCH-sstd`
plot(model_1, which = "all")
plot(model_1, which = 2)

res1 <- model_1@fit$residuals
res1

plot(res1)
abline(h=0, col = "red")

acf(res1)
acf(res1, lag.max = 50) # after lag 29, the residuals seems uncorrelated
acf(res1, lag.max = 100)
```

```r
pacf(res1, lag.max = 50)
pacf(res1, lag.max = 100)

jarque.bera.test(res1) # Ho: normality
Box.test(res1, lag = 30, type = "Ljung-Box") # Ho: no correlations

dates <- as.Date(WTI$...1)
dates_1 <- dates[-1]
dates <- dates_1
rm(dates_1)

retWTI_xts <- xts(retWTI, order.by = dates)

# PERFORMING VaR (GARCH models)
VaR_test <- list()
for (s in spec.model) {
  VaR_test[[s]] <- ugarchroll(spec = m.d_bind[[s]], data = retWTI_xts, forecast.length = 150,
     refit.every = 5, refit.window = "moving",
        calculate.VaR = T)
}

VaR_test

plot(VaR_test$`eGARCH-sstd`, which = 4, VaR.alpha = 0.05)
plot(VaR_test$`eGARCH-sstd`, which = 4, VaR.alpha = 0.01)

# BACKTESTING PROCEDURE GARCH (VaR)

#Report gives information about the VaR backtest with VaR 5%
BT95 <- list()
for (s in spec.model) {
  BT95[[s]] <- as.data.frame(report(VaR_test[[s]], type = "VaR",
    VaR.alpha = 0.05, conf.level = 0.95))
}

#Report gives information about the VaR backtest with VaR 1%
BT99 <- list()
for (s in spec.model) {
    BT99[[s]] <- as.data.frame(report(VaR_test[[s]], type = "VaR",
        VaR.alpha = 0.01, conf.level = 0.99))
}

# DQ test at confidence level 95%
DQ5 <- list()
for (s in spec.model) {
  DQ5[[s]] <- DQtest(retWTI, VaR = VaR_test[[s]]@forecast$VaR$`alpha(5%)`, VaR_level = 0.95)
}

# DQ test at confidence level 99%
DQ <- list()
```

```
for (s in spec.model) {
  DQ[[s]] <- DQtest(retWTI, VaR = VaR_test[[s]]@forecast$VaR$`alpha(1%)`, VaR_level = 0.99)
}

# MCS PROCEDURE
# Loss function with VaR confidence level 95%
Loss5 <- do.call(cbind, lapply(spec.model, function(s) {
  LossVaR(tau = 0.05, realized = VaR_test[[s]]@forecast[["VaR"]][["realized"]],
          evaluated = VaR_test[[s]]@forecast[["VaR"]][["alpha(5%)"]])
}))

colnames(Loss5) <- spec.model

# Superior Set of Models 95%

SSM5 <- MCSprocedure(Loss = Loss5, alpha = 0.2, B = 5000, statistic = "Tmax")
SSM5
# Loss function with VaR confidence level 99%
succ.test <- list("sGARCH-norm", "sGARCH-std", "sGARCH-snorm", "sGARCH-sstd",
            "sGARCH-ged", "eGARCH-std", "eGARCH-snorm", "eGARCH-sstd", "eGARCH-ged")

Loss <- do.call(cbind, lapply(succ.test, function(s) {
       LossVaR(tau = 0.01, realized = VaR_test[[s]]@forecast[["VaR"]][["realized"]],
                                evaluated = VaR_test[[s]]@forecast[["VaR"]][["alpha(1%)"]])
}))

colnames(Loss) <- succ.test

# Superior Set of Models 99%

SSM <- MCSprocedure(Loss = Loss, alpha = 0.2, B = 5000, statistic = "Tmax")
SSM

# Now, we add the weekly data of Covid deaths in US
data_Covid <- read_excel("Covid deaths.xlsx")

dates1 <- as.Date(data_Covid$...1)
Covid_ts <- ts(data_Covid$`Deaths USA`, start = c(2020,3), frequency = 44)
plot(Covid_ts)
Covid_xts <- xts(Covid_ts, order.by = dates1)
diffCovid.ts <-diff(Covid_ts)
acf(diffCovid.ts)
adf.test(diffCovid.ts, alternative = "stationary")

Date <- as.Date(dates)
retWTI_TS <- ts(retWTI)
WTI_weekly_sum <- apply.weekly(retWTI_TS, sum)
WTI_weekly <- as.xts(coredata(WTI_weekly_sum), order.by = Date, by = "week",
            length.out = length(WTI_weekly_sum))
```

```
Deaths <- ts(data_Covid$`Deaths USA`)
Date1 <- as.Date(data_Covid$...1)
death_weekly_sum <- apply.weekly(Deaths, sum)
death_weekly <- as.xts(coredata(death_weekly_sum), order.by = Date1, by = "week",
            length.out = length(death_weekly_sum))

mat <- mv_into_mat(WTI_weekly["2020-04-06/"], diff(death_weekly), K = 4, type = "weekly")
mat

# ESTIMATION STRATEGY FOR GARCH-MIDAS MODELs
GM_model <- list()
distr1 <- c("norm","std")
skewn <- c("YES", "NO")
di <- c()
sk <- c()
for (di in distr1) {
  for(sk in skewn) {
    GM_model[[paste(di, sk,  sep = "-")]] <-
        ugmfit(model = "GM", skew = sk, distribution = di,
           daily_ret = retWTI_xts["2020-04-06/"], mv_m = mat,
           K = 4, out_of_sample = 150)
  }
}

summary.rumidas(GM_model$`norm-YES`)
GM_model$`norm-YES`$inf_criteria

summary.rumidas(GM_model$`std-YES`)
GM_model$`std-YES`$inf_criteria

summary.rumidas(GM_model$`norm-NO`)
GM_model$`norm-NO`$inf_criteria

summary.rumidas(GM_model$`std-NO`)
GM_model$`std-NO`$inf_criteria

# VaR
var_snorm <- qnorm(0.05) * GM_model$`norm-YES`$est_vol_oos
var_sstd <- qstd(0.05) * GM_model$`std-YES`$est_vol_oos
var_norm_no <- qnorm(0.05) * GM_model$`norm-NO`$est_vol_oos
var_std_no <- qstd(0.05) * GM_model$`std-NO`$est_vol_oos

var_snorm99 <- qnorm(0.01) * GM_model$`norm-YES`$est_vol_oos
var_sstd99 <- qstd(0.01) * GM_model$`std-YES`$est_vol_oos
var_norm_no99 <- qnorm(0.01) * GM_model$`norm-NO`$est_vol_oos
var_std_no99 <- qnorm(0.01) * GM_model$`std-NO`$est_vol_oos

# Backtesting VaR
VaRTest(alpha = 0.05, actual = as.numeric(retWTI_xts[166:315]),
        VaR = as.numeric(var_snorm), conf.level = 0.95)
```

```r
VaRTest(alpha = 0.05, actual = as.numeric(retWTI_xts[166:315]),
        VaR = as.numeric(var_sstd), conf.level = 0.95)
VaRTest(alpha = 0.05, actual = as.numeric(retWTI_xts[166:315]),
        VaR = as.numeric(var_norm_no), conf.level = 0.95)
VaRTest(alpha = 0.05, actual = as.numeric(retWTI_xts[166:315]),
        VaR = as.numeric(var_std_no), conf.level = 0.95)
VaRTest(alpha = 0.01, actual = as.numeric(retWTI_xts[166:315]),
        VaR = as.numeric(var_snorm99), conf.level = 0.99)
VaRTest(alpha = 0.01, actual = as.numeric(retWTI_xts[166:315]),
        VaR = as.numeric(var_sstd99), conf.level = 0.99)
VaRTest(alpha = 0.01, actual = as.numeric(retWTI_xts[166:315]),
        VaR = as.numeric(var_norm_no99), conf.level = 0.99)
VaRTest(alpha = 0.01, actual = as.numeric(retWTI_xts[166:315]),
        VaR = as.numeric(var_std_no99), conf.level = 0.99)

# DQ test
DQtest(retWTI_xts, as.numeric(var_snorm), VaR_level = 0.95)
DQtest(retWTI_xts, as.numeric(var_sstd), VaR_level = 0.95)
DQtest(retWTI_xts, as.numeric(var_norm_no), VaR_level = 0.95)
DQtest(retWTI_xts, as.numeric(var_std_no), VaR_level = 0.95)
DQtest(retWTI_xts, as.numeric(var_snorm99), VaR_level = 0.99)
DQtest(retWTI_xts, as.numeric(var_sstd99), VaR_level = 0.99)
DQtest(retWTI_xts, as.numeric(var_norm_no99), VaR_level = 0.99)
DQtest(retWTI_xts, as.numeric(var_std_no99), VaR_level = 0.99)

specifications5 <- c("midas_snorm", "midas_sstd", "midas_norm_no", "midas_std_no")
var.comp5 <- list(midas_snorm = var_snorm, midas_sstd = var_sstd,
    midas_norm_no = var_norm_no, midas_std_no = var_std_no)

# Loss function with VaR confidence level 95%
midas_Loss5 <- do.call(cbind, lapply(specifications5,
    function(s) LossVaR(tau = 0.05, realized = retWTI_xts[166:315], evaluated = var.comp5[[s]])))
colnames(midas_Loss5) <- specifications5

# Superior Set of Models 95%
SSM_midas5 <- MCSprocedure(Loss = midas_Loss5, alpha = 0.2, B = 5000, statistic = "Tmax")

# Loss function with VaR confidence level 99%
specifications <- c("midas_snorm", "midas_norm_no", "midas_std_no")
midas_succ.test <- list(midas_snorm = var_snorm99,
    midas_norm_no = var_norm_no99, midas_std_no = var_std_no99)

midas_Loss <- do.call(cbind, lapply(specifications,
    function(s) LossVaR(tau = 0.01, realized = retWTI_xts[166:315],
        evaluated = midas_succ.test[[s]])))
colnames(midas_Loss) <- specifications

# Superior Set of Models 99%
SSM_midas <- MCSprocedure(Loss = midas_Loss, alpha = 0.2, B = 5000, statistic = "Tmax")
```

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