$\mathcal{M}$ is Lie co-ordinatised.

RECAP:

- A weak projective geometry J_b in $\mathcal{M}$ is a canonical projective if
  - J_b fully embedded in $\mathcal{M}$ over b
  - If $tp(b) = tp(b')$, $b \neq b'$ then J_b and $J_{b'}$ are orthogonal. ie $J_b \perp_{b,b'} J_{b'}$

2.5.2 Every co-ordinatising proj geometry is non-orthogonal to a canonical projective geometry.

- A standard system of geometries for $\mathcal{M}$ is a 0-def. function

$$J : A \longrightarrow \{J_a : a \in A\} \subseteq \mathcal{M}^{eq}$$
$$a \longmapsto J_a$$

complete type. ↗ canonical proj geom

- Standard systems $J : A \to \mathcal{M}^{eq}$ and $J' : A' \to \mathcal{M}^{eq}$ are equivalent if there are $a \in A$, $a' \in A'$ such that $J_a \not\perp J_{a'}$, ie 0-linked, $\alpha : a \longrightarrow a'$ where α is a 0-def isomorphism of geometries $J_a \xrightarrow{\alpha} J_{a'}$

3 SMOOTH APPROXIMABILITY

3.1 ENVELOPES.

DEFINITION 3.1.1

Let $\mathcal{M}$ be Lie co-ordinatized.

1. A regular expansion of $\mathcal{M}$ is a structure obtained by adjoining to $\mathcal{M}$ finitely many sets of $\mathcal{M}^{eq}$ with the induced structure.

2. A regular expansion is adequate if it contains a copy of each canonical projective which is non-orthogonal to a co-ordinatising geometry of $\mathcal{M}$.

3. A approximation to a geometry of a given type is a finite or countable dimensional geometry of the same type.

4. A dimension function μ defined on equivalence classes of standard systems of geometry with values isomorphism types of approximations to projective geometries of the given type.

$$J : A \longrightarrow \{J_a : a \in A\}$$
$$\downarrow \mu$$

$\mu(J)$ is an approximation to some J_a, $a \in A$.

μ chooses a dimension

$\sim$ $J : A' \longrightarrow \{J_{a'} : a' \in A'\}$

Have $\alpha : a \longrightarrow a'$

$J_a \longrightarrow J_{a'}$

so $\mu(J) = \mu(J')$

4. If μ is a dimension function then a μ-envelope is a subset E such that:

i) E is algebraically closed in $\mathcal{M}$ (not $\mathcal{M}^{eq}$)

ii) For $c \in \mathcal{M} \setminus E$ there is a standard system $J : A \to \mathcal{M}^{eq}$ and an element $b \in A \cap E$ for which

$$acl^{eq}(E, c) \cap J_b \supsetneq acl^{eq}(E) \cap J_b.$$

iii) For J a standard system of geometries defined on A: and $b \in A \cap E$, $J_b \cap E$ has the isomorphism type given by $\mu(J)$.

Example

$\mathcal{M}$ vector space over k a finite field.

here a μ-envelope is a subspace of appropriate dimension.

5. If μ is a dimension function and E a μ-envelope we write

$$\dim_J(E) = \begin{cases} \mu(J) & \text{if } E \text{ meets the domain of } J. \\ -1 & \text{otherwise.} \end{cases}$$

Lemma 3.1.2. *Let $\mathcal{M}$ be an adequate regular expansion of a Lie coordinatized structure. Suppose that E is algebraically closed, and satisfies (iii) with respect to the standard system of geometries J. Suppose that J' is an equivalent standard system of geometries and that J, J' are in $\mathcal{M}$ (not just $\mathcal{M}^{eq}$). Then E satisfies (iii) with respect to J'.*

iii) For J a standard system of geometries defined on A: and $b \in A \cap E$, $J_b \cap E$ has the isomorphism type given by $\mu(J)$.

Proof:

$E \subseteq \mathcal{M}$ so we can work in $\mathcal{M}$

Condition (iii) for J' say if $b' \in E \cap A'$ $[J' : A' \to \mathcal{M}^{eq}]$

then $E \cap J'_{b'}$ has the structure specified by $\mu(J) = \mu(J')$.

Now we have $\alpha : A' \longrightarrow A$ ↙ iso of geometries so b' corresponds to an element $b \in E \cap A$ Question: why is $\alpha(E) = E$

$b \in \alpha(E) \cap A$

α takes $E \cap J_b$ to $E \cap J'_{b'}$ as it preserves type we have $E \cap J_b = \mu(J) = E \cap J_{b'}$