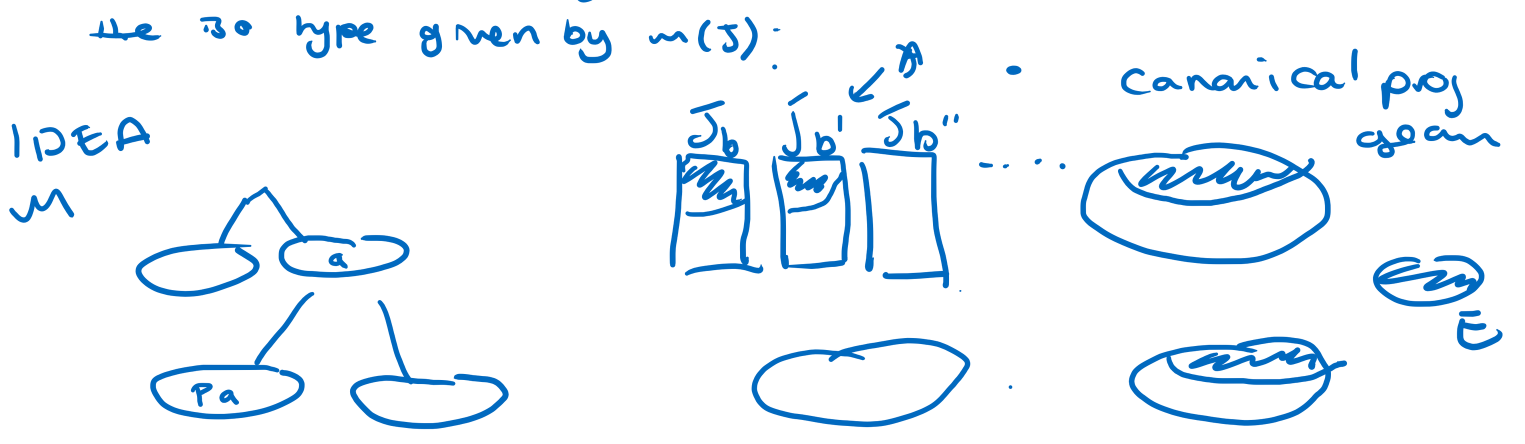
3.1 ENVELOPES. (PART TWO).

THROUGHOUT: M is adequate regular expansion of lie co-ordinalised str.

DEFINITIONS

- J_b a b-def proj geom in M is **canonical** if
  - J_b is fully embedded.
  - $tp(b) = tp(b')$, $b \neq b'$ then $J_b \underset{b\,b'}{\perp} J_{b'}$
- A **standard system of geometries** $J : A \to \{J_a : a \in A\}$ (A: type; J_a: can. proj geom.)
- A **dimension function** m: $m : \left\{\begin{matrix}\text{standard}\\ \text{systems}\end{matrix}\right\} \longrightarrow \left\{\begin{matrix}\text{type} \\ n \in \omega \cup \{\omega\}\end{matrix}\right.$ $\to$ v.s / aff / polar over K.
- A **m-envelope** is a subset E satisfying:
  - E alg-closed (not in M^{eq})
  - For $c \in M \setminus E$ there is a standard system $J : A \to \{J_a : a \in A\}$ and an element $b \in A \cap E$ for which $acl(A, c) \cap J_b \supsetneq acl(E \cap J_b)$
  - For J a standard system defined on A and $b \in A \cap E$, $J_b \cap E$ has the iso type given by $m(J)$.



3.1.3 LEMMA (Existence).

M: ad. reg expansion of Lie co-ordinalised str.

1. Let $E_0 \subseteq M$ be alg closed and suppose for each standard system J with domain A and each $b \in E_0 \cap A$, $J_b \cap E_0$ embeds into a structure of iso type $m(J)$. Then E_0 is contained in a m-envelope.
2. In particular, for any m, m-envelopes exist.

PROOF:

STEP 1: Ok to work with representatives of the equiv. classes of standard systems (3.1.2), call this $\mathcal{J}$.

STEP 2: Take E containing E_0 max alg closed such that:

(*) For $J \in \mathcal{J}$ with domain A, $b \in E \cap A$, J_b embeds into a structure of type $m(J)$.

We need to check (ii) and (iii) for E

(ii) Suppose $c \in M \setminus E$

Aim: Find $J : A \to M^{eq}$, $b \in A \cap E$ such that $acl(E, c) \cap J_b \supsetneq acl(E) \cap J_b$.

Let $E' = acl(E \cup \{c\})$

Then there is some $b \in E' \cap A$ for which J_b does not embed into a structure of type $m(J)$

Either

$\to$ $b \in A \cap E$, $J_b \cap E$ does embed into str. of type $m(J)$.

so $J_b \cap E' \supsetneq J_b \cap E$

so $\underbrace{acl(E, c)}_{E'} \cap J_b \supsetneq acl(E) \cap J_b$.

$\to$ $b \notin A \cap E$?? we show $J_b \cap E = \emptyset \subsetneq J_b \cap E'$

As E is def closed it is a subtree of the co-ordinalising tree $T(E)$

As b is not def over E, J_b is orthogonal to the canonical geometries associated with $T(E)$

Thus by induction $E \cap J_b$ is empty.

(iii) Consider $J : A \to M^{eq}$ std. system.

Aim: $J_b \cap E$ has right iso type $m(J)$.

Let B be an extension of $J_b \cap E$ inside J_b

Our claim $B \subseteq E$

Let $E' = acl(E \cup B)$ we argue E' has property (*) thus $E' = E$.

Suppose $J' : A' \to \{J_{a'} : a' \in A'\}$ is a std system $b' \in A' \cap E'$

Then either

- $J = J'$ and $b = b'$
- J_b and $J_{b'}$ are orthogonal.

If orthogonal $J_{b'} \cap E' = J_{b'} \cap E$ so of the right iso type.

If $J = J'$ and $b = b'$

Any elt of J_b^{eq} alg over E is alg over $J_b \cap E$ 2.3.3

$$J_b \cap E' = J_b \cap acl\big((E \cap J_b) \cup B\big) = J_b \cap acl(B)$$
$$= B \leftarrow \text{chosen to be of right type}$$

So E' has property (*).

In particular envelopes exist: