Extra exercises are marked with a $\star\star$. I DO NOT EXPECT YOU TO ANSWER THEM. I hope they can bring you joy.

EXERCISE 1. Prove the following: let $\mathcal{M}$ be ω-saturated. Suppose that $\phi \in \mathcal{L}(M)$ is minimal in $\mathcal{M}$. Then ϕ is strongly minimal.

EXERCISE 2. Show Neumann's Lemma: Let $A, B \subseteq \mathbb{M}$ and $(c_1, \dots, c_n)$ a sequence of elements not algebraic over A. Show that $\mathrm{tp}(c_1, \dots, c_n / A)$ has a realisation which is disjoint from B.

EXERCISE 3. Show that $\mathrm{acl}(A)$ is the intersection of all models containing A.

EXERCISE 4. (a) Consider the theory of $(\mathbb{Z}, s)$, the integers with the successor operation $s(x) = x + 1$. This theory has quantifier elimination. What is algebraic closure in this theory? Is this theory minimal? is it strongly minimal?

(b) Consider the theory of $(\mathbb{N}, <)$. This theory has quantifier elimination if we add a function symbol for the successor and a constant symbol for 0. What does algebraic closure look like in this theory? Show that $x = x$ is minimal in $(\mathbb{N}, <)$, but not strongly minimal?;

$\star\star$ **EXERCISE 5.** Consider the theory of Presburger arithmetic, i.e. of $(\mathbb{Z}; +, -, <, 0, 1)$. This theory has quantifier elimination after adding for each $n \in \mathbb{N}$ a predicate P_n expressing divisibility by n. What does algebraic closure look like in this theory? What about definable closure (where a is definable over A if there is some $\mathcal{L}(A)$-formula which is true only of a)?

Definition 1. A set of definable subsets of $\mathbb{M}$ in the variable x, $I \subseteq \mathrm{Def}_x(\mathbb{M})$ is an **ideal** if it contains $\emptyset$, and it is closed under (definable) subsets and finite unions.

EXERCISE 6. Prove the following:
Let $I \subseteq \mathrm{Def}_x(\mathbb{M})$ be an ideal. Let $\pi(x)$ be a partial type over A (closed under conjunctions) such that $p(\mathbb{M})$ is not contained in any set in I. Then, for every $B \supseteq A$, there is a type $q \in S(B)$ extending p and such that $q(\mathbb{M})$ is not contained in any set in I.