

EXERCISE 1. Let T be an arbitrary theory and $\mathcal{M} \models T$ be κ -saturated and strongly κ -homogeneous. Suppose that $p \in S(M)$ forks over A with $|A| \leq \kappa$ and \cdot . Then p has $\geq \kappa$ many $\text{Aut}(M/A)$ -conjugates.

EXERCISE 2. Let T be stable. Show there is an ordinal α such that if $U(p) \geq \alpha$, then $U(p) = \infty$. [Hint: use local character.]

Theorem 1 (Equivalents to superstability). *Let T be stable. The following are equivalent:*

1. T is superstable;
2. Every type has ordinal-valued U -rank;
3. For every finite tuple a and every B there is some finite $C \subseteq B$ such that $a \downarrow_C B$.

EXERCISE 3. Prove the implication (3) \Rightarrow (1) in Theorem 1.

ZOMBIE EXERCISE 4. Consider a stable theory T . Let $p \in S_x(B)$ and let $A \subseteq B$. Show that the following are equivalent:

- p does not fork over A ;
- there is a global type extending p which is $\text{acl}^{eq}(A)$ -invariant;
- there is a global type extending p which is $\text{acl}^{eq}(A)$ -definable.