

Extra exercises are marked with a $\star\star$. I DO NOT EXPECT YOU TO ANSWER THEM. I hope they can bring you joy.

Throughout this problem sheet we work with countable theories.

EXERCISE 1. Show that a theory is κ -stable if and only if it is κ -stable for 1-types, i.e.

$$|A| \leq \kappa \Rightarrow |S_1(A)| \leq \kappa.$$

EXERCISE 2. Show that if a theory is κ -stable, then it is logically equivalent to a theory of cardinality $\leq \kappa$.

Definition 1. Let \mathcal{M} be an \mathcal{L} -structure. An infinite $\mathcal{L}(\mathcal{M})$ -definable set D defined by a formula $\phi(x, a)$ is **minimal** if any $\mathcal{L}(\mathcal{M})$ -definable $Y \subseteq D$ is either finite or cofinite in D .

EXERCISE 3. Let T be totally transcendental. Show that for $\mathcal{M} \models T$, there is a minimal formula in \mathcal{M} .

EXERCISE 4. Are the following theories ω -stable? If they are, are they \aleph_1 -categorical? You may use knowledge from previous exercises.

- the theory of the random graph;
- the theory of an equivalence relation with two infinite classes;
- the theory of an infinite dimensional vector space over the finite field \mathbb{F}_q ;
- the theory of (\mathbb{Z}, s) , where s is the successor function $x \mapsto x + 1$;
- the theory of dense linear orders;
- $\star\star$ the theory of the Fraïssé limit of the class of finite trees (in the language with a binary symmetric relation).