Astro Calculations

SOME CALCULATIONS ABOUT ASTRO WORLD

The Newton equation

$$p^2 = kr^3$$

$$P = T = \frac{2\pi r}{v}$$

$$\frac{4\pi^2 r^2}{v^2} = kr^3$$

$$\frac{m4\pi^2}{kr^2} = m\frac{v^2}{r}$$

$$F = \frac{4\pi m}{kr^2}$$

$$F = \frac{4\pi M}{k' r^2}$$

$$\frac{4\pi m}{kr^2} = \frac{4\pi M}{k' r^2}$$

$$k = \frac{k''}{M}$$

$$k' = \frac{k''}{m}$$

$$F = \frac{GMm}{r^2}$$

The Newton equation

$$\oint_C \vec{f} \cdot \vec{dl} = 0$$

$$\vec{dl} = dr\vec{u_r} + rd\theta\vec{u_\theta}$$

$$\int_{r_0}^{r_1} f dl = F(r_1) - F(r_0)$$

$$H = E_k + U$$

The Newton equation

$$\vec{f} = -\frac{m_1 m_2 G}{r^2}\vec{u_r}$$

$$\int_{\infty}^{r_1} \vec{f} \cdot \vec{dl}$$

$$\int_{r}^{\infty} \frac{m1m2G}{r^2} dr = -\frac{m_1 m_2 G}{r}$$

$$H = E_k + U$$

$$E_k = \frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2$$

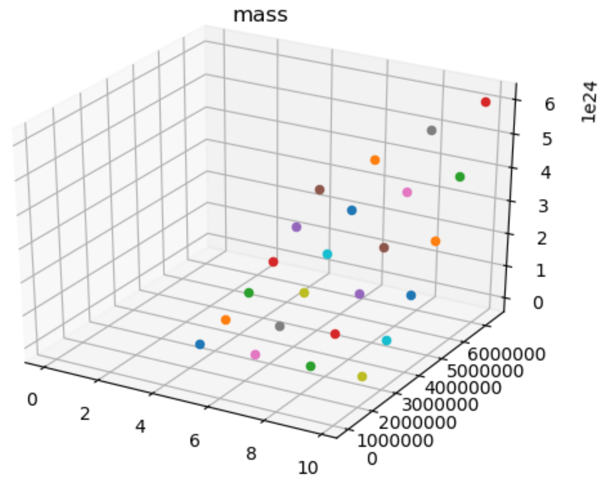
Mass of planet

$$\frac{M_t m G}{r^2} = mg$$

$$M_t = \frac{gr^2}{G}$$

$$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$$

```
g    ,   r     ,      m
4.0 1400000 1.1754122938530735e+23
4.0 2400000 3.454272863568216e+23
4.0 3400000 6.932533733133433e+23
4.0 4400000 1.1610194902548726e+24
4.0 5400000 1.7487256371814093e+24
4.0 6400000 2.4563718140929535e+24
6.0 1400000 1.7631184407796102e+23
6.0 2400000 5.181409295352324e+23
6.0 3400000 1.039880059970015e+24
6.0 4400000 1.7415292353823088e+24
6.0 5400000 2.623088455772114e+24
6.0 6400000 3.68455772113943e+24
8.0 1400000 2.350824587706147e+23
8.0 2400000 6.908545727136432e+23
8.0 3400000 1.3865067466266867e+24
8.0 4400000 2.322038980509745e+24
8.0 5400000 3.497451274362818e+24
8.0 6400000 4.912743628185907e+24
9.81 1400000 2.882698650674662e+23
9.81 2400000 8.47160419790105e+23
9.81 3400000 1.7002038980509746e+24
9.81 4400000 2.847400299850075e+24
9.81 5400000 4.288749625187406e+24
9.81 6400000 6.024251874062968e+24
```



The Two body problem

Two body problem

Runge Kutta 4 Numerical method

$$-\vec{F} = m_1\ddot{\vec{r_1}}$$

$$\vec{F} = m_2\ddot{\vec{r_1}}$$

$$m_1\ddot{\vec{r_1}} = -m_2\ddot{\vec{r_2}}$$

$$m_1\ddot{\vec{r_1}} + m_2\ddot{\vec{r_2}} = 0$$

$$M_{tot}\ddot{\vec{r_{cm}}} = 0$$

$$\ddot{\vec{r_2}} = \frac{1}{m_2}\vec{F}$$

$$\ddot{\vec{r_1}} = -\frac{1}{m_1}\vec{F}$$

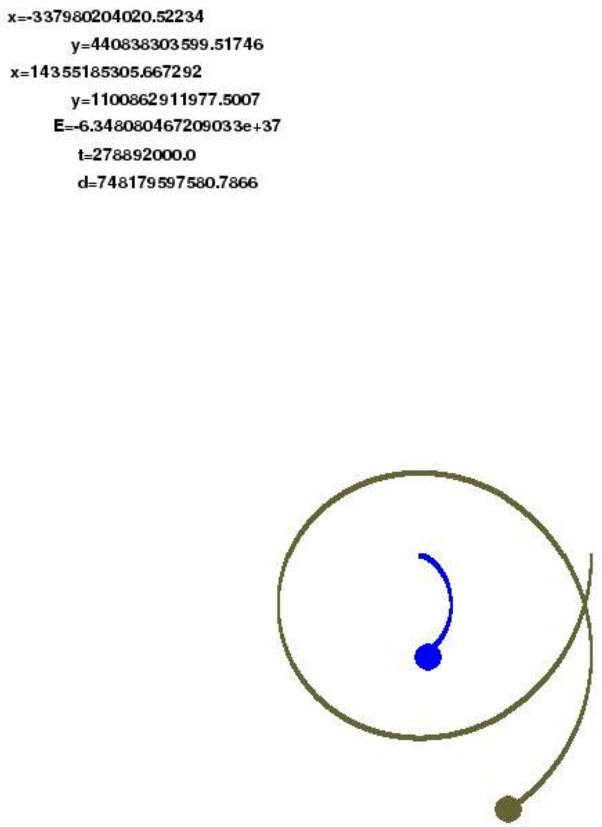
$$\ddot{\vec{r_2}} - \ddot{\vec{r}}_1 = \vec{F}(\frac{1}{m_2} + \frac{1}{m_1})$$

$$\vec{r} = \vec{r_2} - \vec{r_1}$$

$$\vec{F} = -\frac{Gm_1m_2}{r^2}\vec{e_r}$$

$$\mu\ddot{\vec{r}} = -\frac{Gm_1m_2}{r^2}\vec{e_r}$$

Two body problem



$$\mu = \frac{(m_1 m_2)}{m_1 + m_2}$$

$$\mu \ddot{x} = \frac{-m_1 m_2 G}{d^3} x_n$$

$$x_n = c_{n1} \cos(\sqrt{\frac{k}{d^3 \mu}} t) + c_{n2} \sin(\sqrt{\frac{k}{d^3 \mu}} t)$$

$$k = G m_1 m_2$$

$$c_{11} = c_{22} = d$$

$$x_1 = r \cos(\sqrt{\frac{k}{d^3 \mu}} t)$$

$$y_1 = r \sin(\sqrt{\frac{k}{d^3 \mu}} t)$$

$$x_2 = x_1 - d \cos(\sqrt{\frac{k}{d^3 \mu}} t)$$

$$y_2 = y_1 - d \cos(\sqrt{\frac{k}{d^3 \mu}} t)$$

$$\omega = \sqrt{\frac{k}{d^3 \mu}}$$

$$m_1 + m_2 = \frac{\omega^2 d^3}{G}$$

Two body problem

Mass 1 (Sun masses)	2
Speedx 1 m/s	0
Speedy 1 m/s	0
Mass 2 (Sun masses)	0.2
Speedx 2 m/s	0
Speedy 2 m/s	19753.49
Distance Au	5
Delta t s	9000
Calculate	Masses = 3.9780000000000004e+30Kg,3.978000000000001e+29Kg

$$T = 238176000.0s$$

$$d = 5Au$$

$$\omega = \frac{2\pi}{T}$$

$$\omega = 2.6380430048281884e - 08\frac{rad}{s}$$

$$m1 + m2 = \frac{\omega^2 d^3}{G} = 4.3664064250822195e + 30Kg$$

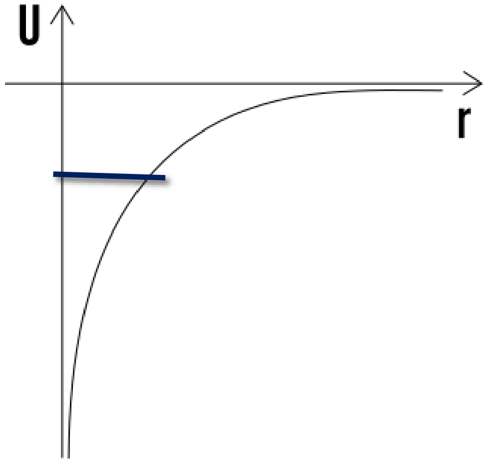
Theoretical

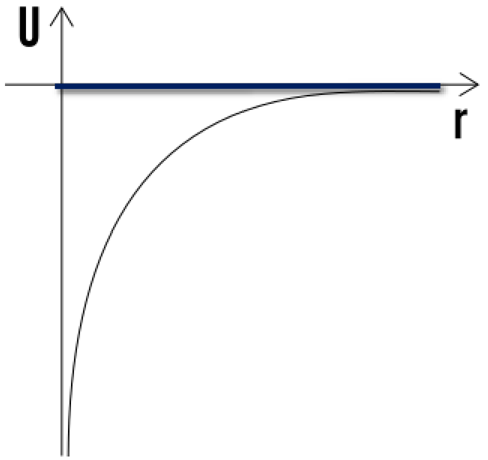
$$(m1 + m2) = 4.3758000000000006e + 30Kg$$

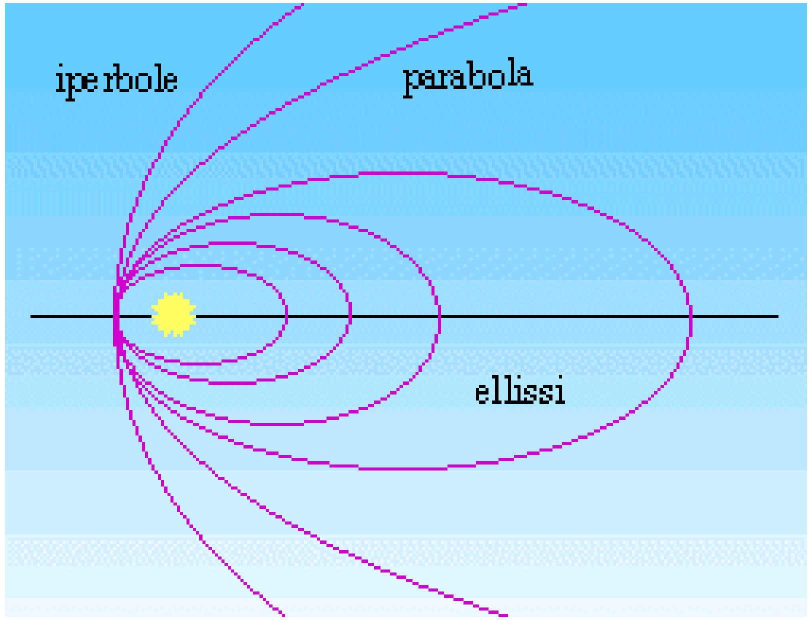
Percentage error

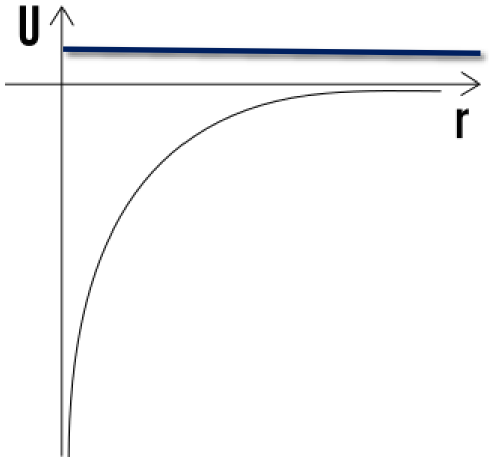
$$\frac{(4.36640642e + 30Kg - 4.375800e + 30kg)}{4.366406425e + 30Kg} = 0.0021- > 0.21\%$$

The Keplero Problem









The Keplero Problem

$$\vec{F} = \frac{GmM}{r^2}\hat{u_r} \tag{1.32}$$

$$\frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2) - \frac{GmM}{r} = E \tag{1.33}$$

$$mr^2\dot{\theta} = L \tag{1.34}$$

$$c = \frac{L}{m}, k = GM, E' = \frac{E}{m} \tag{1.35}$$

$$\dot{r} = \frac{dr}{d\theta}\dot{\theta}, \dot{\theta} = \frac{c}{r^2}, \dot{r} = \frac{c}{r^2}\frac{dr}{d\theta} \tag{1.36}$$

$$(\frac{d\frac{1}{r}}{d\theta})^2 = \frac{2E'}{c^2} + \frac{2k}{c^2}u - u^2 \tag{1.37}$$

$$u = \frac{1}{r} \tag{1.38}$$

$$(\frac{du}{d\theta})^2 = \frac{2E'}{c^2} + \frac{2k}{c^2}u - u^2 \tag{1.39}$$

$$(\frac{du}{d\theta})^2 = \frac{2E'}{c^2} + \frac{2k}{c^2}u - (u - \frac{k}{c^2})^2 = q^2 - (u - \frac{k}{c^2})^2 \tag{1.40}$$

$$q^2 = \frac{2E'}{c^2} + \frac{k^2}{c^4} \tag{1.41}$$

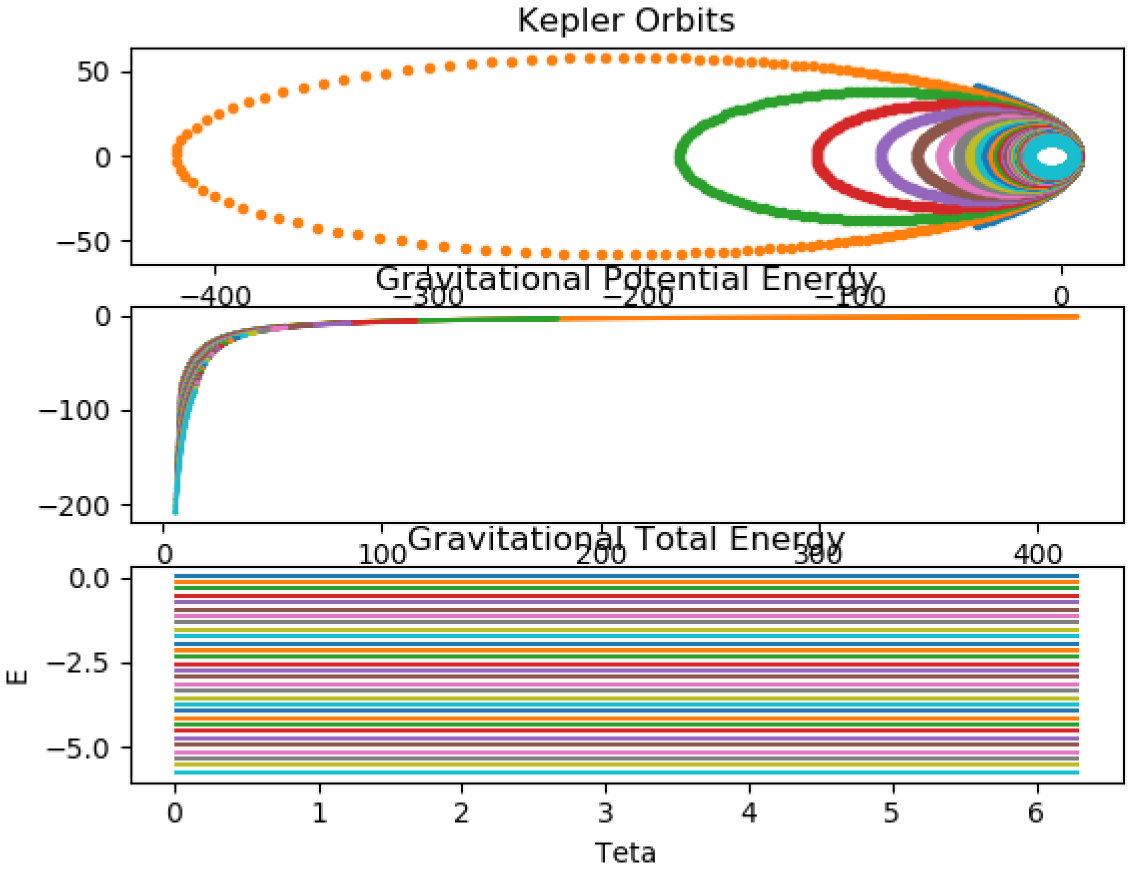
$$\frac{du}{d\theta} = \sqrt{q^2 - (u - \frac{k}{c})^2} \tag{1.42}$$

$$\frac{du}{\sqrt{1 - \frac{1}{q^2}(u - \frac{k}{c})^2}} = qd\theta \tag{1.43}$$

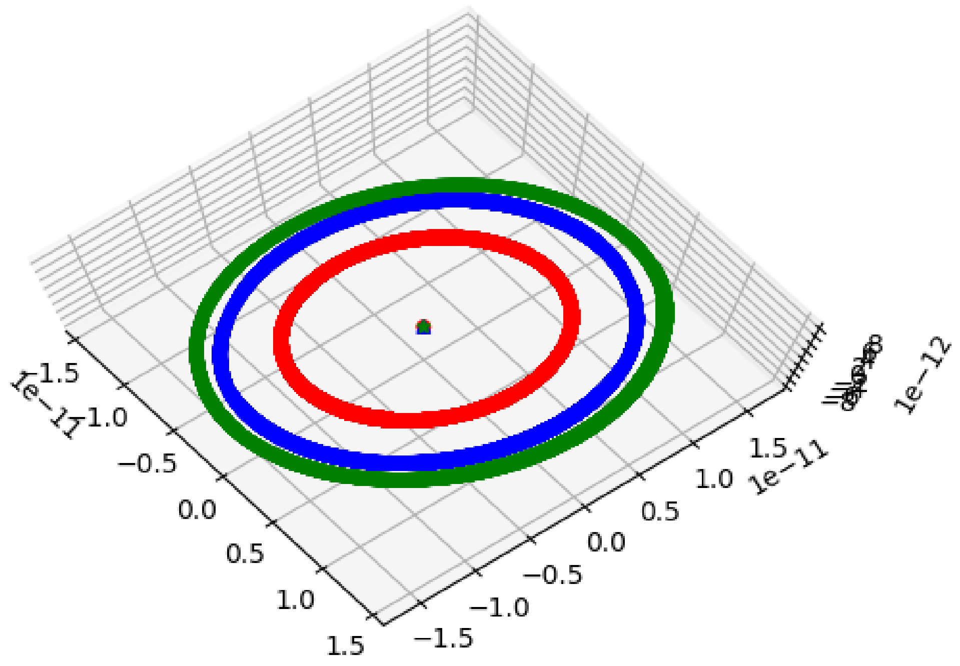
$$u = \frac{1}{r} = \frac{k}{c^2} + qcos(\theta - \theta_0) \tag{1.44}$$

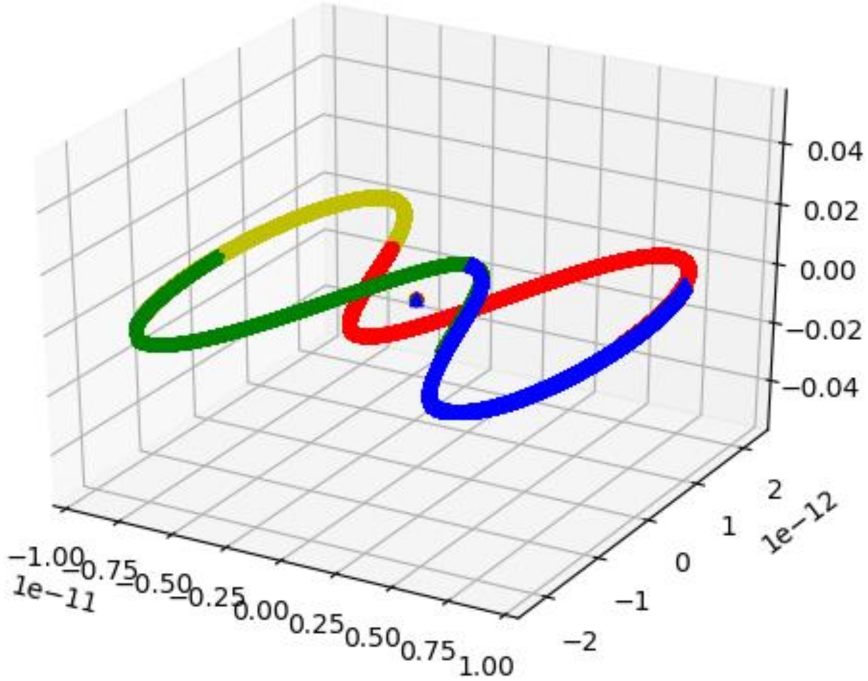
$$r = \frac{\frac{c}{k^2}}{1 + \frac{c^2q}{k}cos(\theta)} \tag{1.45}$$

The Keplero Problem

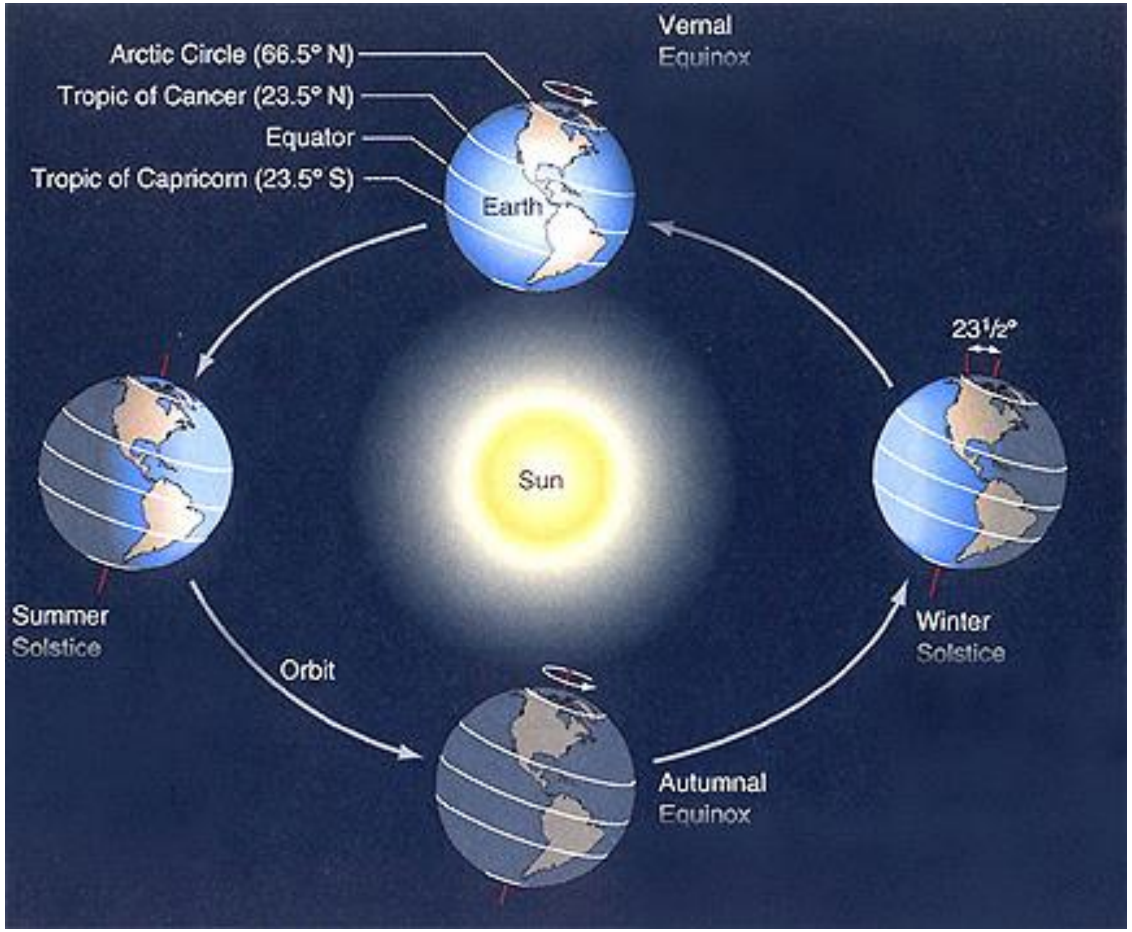


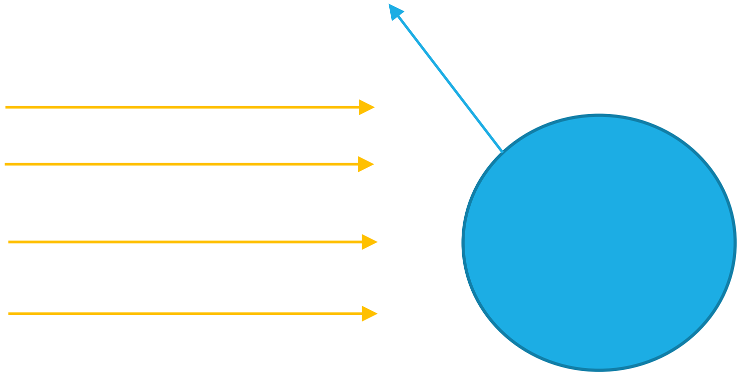
Three body solutions





The Seasons Calculation





The Seasons Calculation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2}$$

$$F(\sqrt{a^2 - b^2}; 0)$$

$$a = 0.999995; b = 0.999835$$

$$x = a\cos\theta(t)$$
$$y = b\sin\theta(t)$$

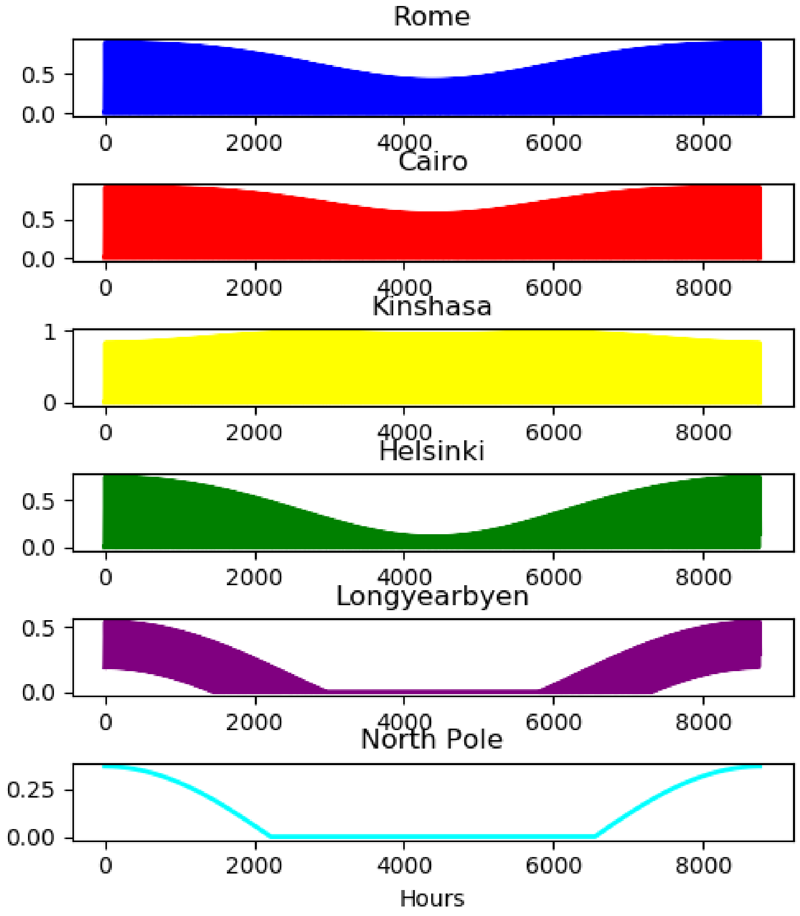
$$x_p = a\cos\theta(t) + R_t \sin\alpha\cos\beta(t)$$
$$y_p = b\sin\theta(t) + R_t \sin\alpha\sin\beta(t)$$
$$z_p = R_t \cos\alpha$$

$$x'_p = x_p \cos\Omega - z_p \sin\Omega$$
$$y'_p = y_p$$
$$z'_p = x_p \sin\Omega + z_p \cos\Omega$$

$$\frac{\phi(\theta)}{\sqrt{(a\cos(\theta t + \theta_0) + x_f)^2 + (b\cos(\theta t) + \theta_0)^2}} \begin{bmatrix} -a\cos(\theta t + \theta_0) + x_f \\ -b\sin(\theta t + \theta_0) \\ 0 \end{bmatrix} \bullet \begin{bmatrix} \cos(\beta)\sin(\alpha)\cos(\Omega) - \cos(\alpha)\sin(\Omega) \\ \sin(\beta)\sin(\alpha) \\ \cos(\beta)\sin(\alpha)\sin(\Omega) + \cos(\alpha)\cos(\Omega) \end{bmatrix}$$

$$0.94 < \phi(\theta) <= 1$$

The Seasons Calculation



Rome: 41.8933203

Cairo: 31.2357257

Kinshasa: -4.3217055

Helsinki: 60.16749

Longyearbyen: 78.22

North Pole: 90

The Einstein Equations

$$c^2 d\tau^2 = dx^2 + dy^2 + dz^2$$

$$c^2 d\tau^2 = c^2 dt^2$$

$$d\tau = dt$$

$$c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

$$d\tau = dt\sqrt{1 - \frac{v^2}{c^2}}$$

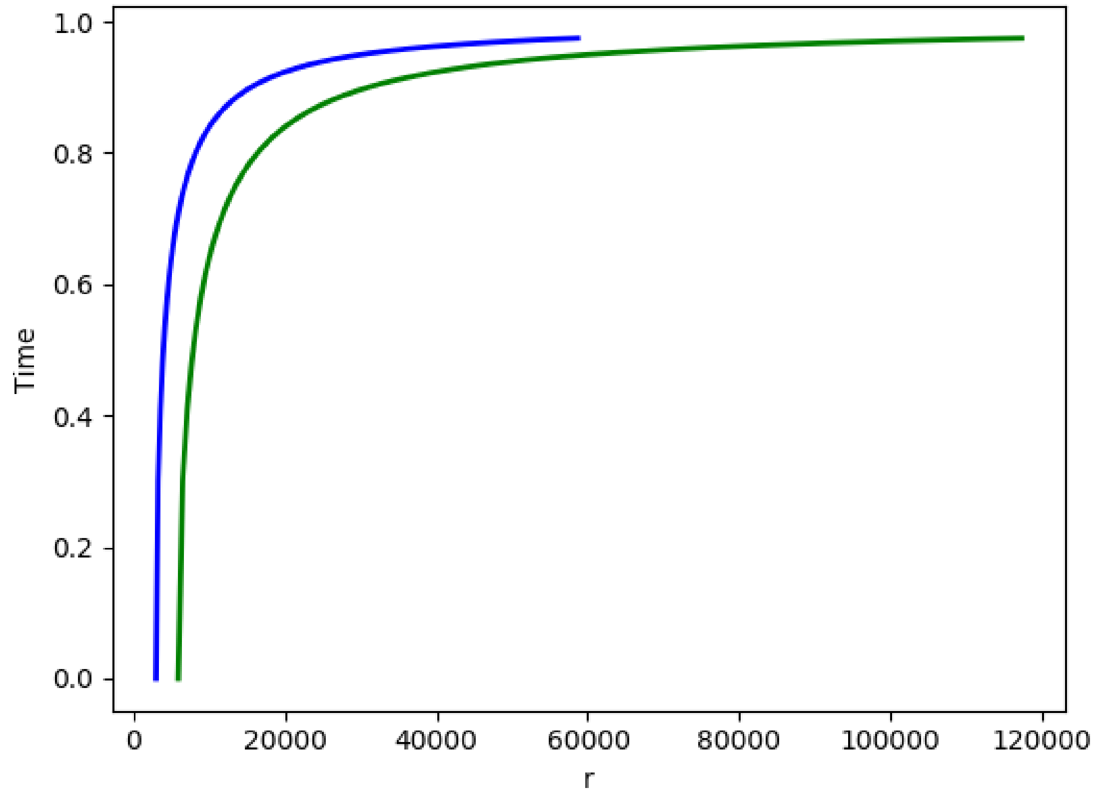
$$R_{\nu\eta} - \frac{1}{2}Rg_{\nu\eta} = \frac{8\pi G}{c^4}T_{\nu\eta}$$

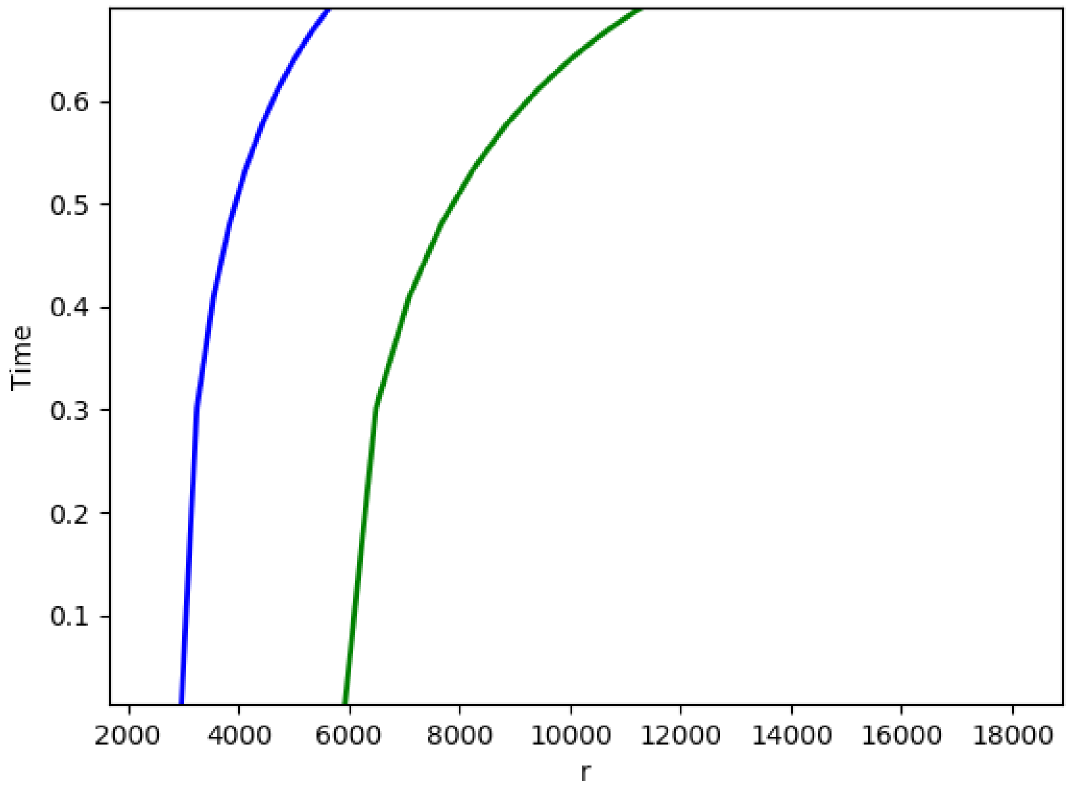
$$ds^2 = (1 - \frac{2GM}{c^2 r})c^2 dt^2 - (1 - \frac{2GM}{c^2 r})^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2$$

$$\frac{dt_2}{dt_1} = \sqrt{\frac{g_{00}(x_1)}{g_{00}(x_2)}}$$

The Schwarzschild Dilatation Time

Sun and twice Sun mass





The Friedmann Model

$$\vec{x(t)} = \vec{x(t_0)}\frac{R(t)}{R(t_0)}$$

$$\rho_0 = \frac{3}{8\pi G}(\frac{k}{R_0^2} + H_0^2)$$

$$p_0 = -\frac{1}{8}[\frac{k}{R_0^2} + H_0^2(1 - 2q_0)]$$

$$\frac{\rho_g}{\rho_c} = 0.028$$

Matter Dominated Era

$$\dot{R}^2 + k + = \frac{8\pi G}{3}R^2$$

$$\frac{\rho}{\rho_0} = (\frac{R}{R_0})^{-3}$$

$$\frac{k}{R_0^2} = (2q_0 - 1)H_0^2$$

$$\frac{8\pi G\rho_0}{3} = 2q_0 H_0^2$$

$$(\frac{\dot{R}}{R_0})^2 = H_0^2[1 - 2q_0 + 2q_0(\frac{R_0}{R})]$$

$$t = \frac{1}{H_0}\int_0^{\frac{R}{R_0}}[1 - 2q_0 + \frac{2q_0}{x}]^{-1/2}dx$$

The Friedmann Model

$$\frac{H^2}{H_0^2} = \Omega_{0,R} r^{-4} + \Omega_{0,M} r^{-3} + \Omega_{0,k} r^{-2} + \Omega_{0,\lambda}$$

$$H = \frac{\dot{a}}{a}$$

$$H^2 = H_0^2(\Omega_{0,R} r^{-4} + \Omega_{0,M} r^{-3} + \Omega 0, k r^{-2} + \Omega_{0,\lambda})$$

$$H = H_0\sqrt{(\Omega_{0,R} r^{-4} + \Omega_{0,M} r^{-3} + \Omega 0, k r^{-2} + \Omega_{0,\lambda})}$$

$$\frac{da}{dt} = H_0\sqrt{(\Omega_{0,R} r^{-4} + \Omega_{0,M} r^{-3} + \Omega 0, k r^{-2} + \Omega_{0,\lambda})}$$

$$t = \int_0^a \frac{da'}{H_0\sqrt{(\Omega_{0,R} r^{-4} + \Omega_{0,M} r^{-3} + \Omega 0, k r^{-2} + \Omega_{0,\lambda})}}$$

$$a = \frac{R(t)}{R_0}$$

The Friedmann Model