Problem set 1

I.1) The model assumptions are:

- Labor and government are absent
- $Y_t = Z_t {K^\alpha}_t$ (production function is a Cobb-Douglas with capital-augmenting technology)
- There is uncertainty and agents are rational and forward-looking
- The market is perfectly competitive
- $Y_t = C_t + I_t$
- Capital K depreciates at a rate δ
- The utility function of agents is a C.R.R.A. (Constant Relative Risk Aversion)
- Infinitely-lived agents
- The degree of impatience of agents is measured by the parameter β

The corresponding optimization problem is:

$$V(K_{t-1}, Z_t) = max_{C_t}\{u(C_t) + \beta E_t[V_t(K_t, Z_{t+1})] | Z_t\}$$

$$s.t. \begin{cases} Y_t = C_t + I_t \\ K_{t+1} = K_t(1-\delta) + I_t \\ Y_t = Z_t {K^\alpha}_t \\ Z_t = (1-\rho_z) + \rho_z Z_{t-1} + \varepsilon_{z,t} \end{cases}$$

I.2) As we indicated in the constraints above, the production function Y_t is:

$$Y_t = Z_t {K^\alpha}_t$$

Therefore the first two FOCs can be expressed as:

$$C_t^{-\sigma} = \beta E_t(C_{t+1}^{-\sigma}[Y_{t+1}' + (1-\delta)])$$

$$C_t + K_{t+1} = Y_t + (1-\delta)K_t$$

II) *Please see the attached .mod file*

III.1) The steady state values are:

STEADY-STATE RESULTS:

C	1.97219
Y	2.50809
K	21.4361
Z	1

<u>III.2)</u> We can derive the policy functions from the following table (rounding at 2° decimal):

POLICY AND TRANSITION FUNCTIONS

	C	Y	K	Z
Constant	1.972191	2.508093	21.436072	1.000000
K(-1)	0.032524	0.034314	0.977577	0
Z(-1)	0.424665	2.321610	1.832618	0.900000
epsilon_z	0.471850	2.579567	2.036243	1.000000

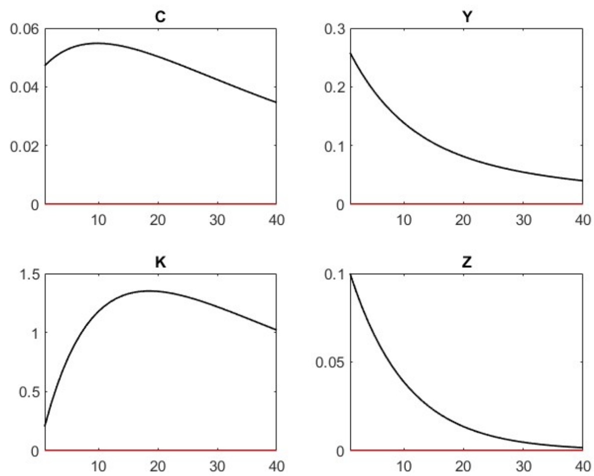
$$C_t - 1{,}97 = 0{,}03(K_{t-1} - 21{,}44) + 0{,}42(Z_{t-1} - 1) + 0{,}47\varepsilon_t$$

$$Y_t - 2{,}51 = 0{,}03(K_{t-1} - 21{,}44) + 2{,}32(Z_{t-1} - 1) + 2{,}58\varepsilon_t$$

$$K_t - 21{,}44 = 0{,}98(K_{t-1} - 21{,}44) + 1.83(Z_{t-1} - 1) + 2{,}04\varepsilon_t$$

$$Z_t - 1 = 0{,}9(Z_{t-1} - 1) + \varepsilon_t$$

<u>III.3)</u> As we can see from the following plots, all the endogenous variables respond positively to a 1% technology shock, but then they tend to converge over time to their initial steady-state values:



<u>III.4)</u> From the following tables we can conclude that:

THEORETICAL MOMENTS

VARIABLE	MEAN	STD. DEV.	VARIANCE
C	1.9722	0.3432	0.1178
Y	2.5081	0.7635	0.5829
K	21.4361	8.7737	76.9782
Z	1.0000	0.2294	0.0526

MATRIX OF CORRELATIONS

Variables	C	Y	K	Z
C	1.0000	0.8789	0.9702	0.6470
Y	0.8789	1.0000	0.7372	0.9323
K	0.9702	0.7372	1.0000	0.4430
Z	0.6470	0.9323	0.4430	1.0000

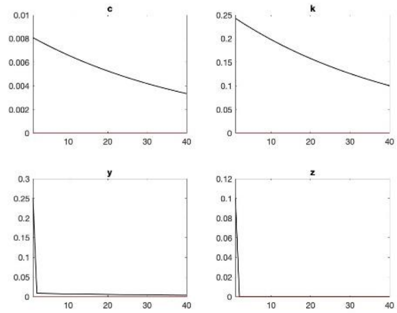
COEFFICIENTS OF AUTOCORRELATION

Order	1	2	3	4	5
C	0.9904	0.9797	0.9681	0.9558	0.9427
Y	0.9411	0.8871	0.8377	0.7923	0.7506
K	0.9988	0.9955	0.9904	0.9837	0.9755
Z	0.9000	0.8100	0.7290	0.6561	0.5905

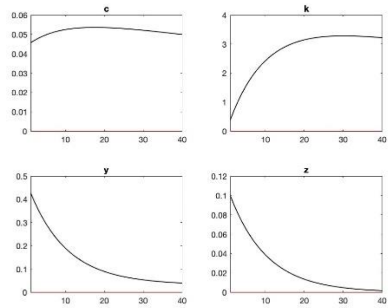
- The first theoretical moments, i.e. the means, are equal to their respective steady-state values
- The second theoretical moments, i.e. the variances, are low for C, Y and Z, and much higher for K
- C and K are highly correlated (since the correlation index is 0.97, very close to 1)
- C and K are highly autocorrelated (since the values are around 0.9), this means that they are both very influenced by their previous value

III.5) Now let's see what happens when we change the values of the parameters:

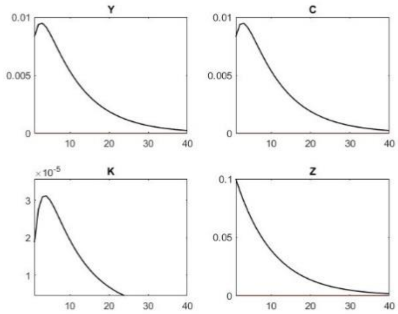
- $\rho = 0$ → The shock is not persistent, therefore both Y and Z return to their steady-state values right after the first-period peak



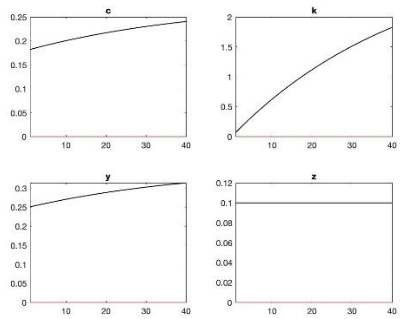
- $\delta = 0$ → In this case, K does not depreciate, therefore after a technological shock it will reach a new steady-state value



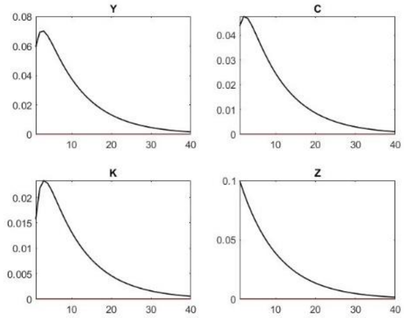
- $\beta = 0.01$ → Agents are very "impatient", therefore a shock causes the fact that their consumption peaks immediately and then the convergence (towards the initial steady-state value) is very quick



- $\rho = 1$ → The shock is fully persistent, and therefore the endogenous variables will converge to a new steady-state value which will be higher than the previous one



- $\delta = 1$ → With a full depreciation of capital, all the variables will peak right after the shock and then converge very quickly towards the initial steady-state value



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