Competing Technologies with Non-Linear Returns: An Extension of the Arthur Model

Paolo Sebastiani

July 27, 2025

Abstract

This paper extends W.B. Arthur's seminal model of competing technologies by replacing the assumption of linear increasing returns with a more realistic, non-linear learning function. Arthur's model demonstrates that small, random historical events can lock a market into a technology that is not necessarily superior. I investigate whether this conclusion holds when technologies exhibit learning-by-using dynamics that are subject to saturation, modeled here via a logistic growth function. Using Monte Carlo simulations, I explore three scenarios: an approximation of the original linear model, a symmetric non-linear competition, and an asymmetric "tortoise vs. hare" competition. My results reveal a richer set of market outcomes than predicted by the original model. I find that while path-dependent lock-in remains a powerful force, it is often incomplete due to heterogeneous preferences. Furthermore, non-linear returns create the possibility of a stable market-sharing equilibrium, an outcome impossible in the linear (increasing returns) case. Finally, I show that a sufficiently large difference in technological potential can overcome path dependence, leading to a predictable and efficient market outcome. These findings suggest that the inevitability of inefficient lock-in is sensitive to the specific form of the returns function, providing a more nuanced understanding of technological competition.

1 Introduction

The concept of path dependence, powerfully illustrated by W.B. Arthur's model of competing technologies, affirms that market outcomes can be determined by a sequence of small, seemingly inconsequential historical events. The model's core mechanism is a positive feedback loop: as a technology gains adopters, it improves, which in turn attracts more adopters. When these increasing returns to adoption are assumed to be linear—that is, they grow indefinitely at a constant rate for each new adopter—the model predicts a "winner-take-all" market. The process is non-ergodic; the final outcome is unpredictable and can lock the market into an inefficient technology, impervious to minor policy interventions.

While elegant in its simplicity, the assumption of linear returns is a significant abstraction. In reality, the benefits derived from learning-by-using are often subject to diminishing marginal gains and eventual saturation. A technology may improve rapidly in its early stages, but eventually, the opportunities for further enhancement become exhausted, and its performance level reaches a plateau.

This paper seeks to explore the ramifications of this more realistic assumption. I extend Arthur's model by incorporating heterogeneous and non-linear returns to adoption, specifically modeling them with a logistic S-shaped curve. My central research question is: How do learning dynamics subject to saturation affect the probability of technological lock-in, the potential for stable market sharing, and the overall efficiency of the adoption process? I hypothesize that introducing a learning plateau will fundamentally alter the path-dependent monopolies of the original model, revealing a richer and more complex landscape of possible market structures.

2 The Extended Model

I retain the core framework of Arthur's model: two unsponsored technologies, A and B, compete for adoption by a large, finite pool of agents. These agents are divided equally into two types, R and S, who possess different intrinsic preferences. R-agents have a natural preference for Technology A, while S-agents have a natural preference for Technology B. Agents make their adoption choice sequentially, and the order of arrival is random, representing the "small historical events" that drive path dependence.

The crucial modification lies in the payoff function. The total payoff an agent receives from adopting a technology is the sum of their intrinsic preference and the accumulated returns from learning-by-using. I model these learning returns not as a linear function, but as a logistic function of the number of previous adoptions (n_A and n_B).

The total payoff, Π, for an agent of a given type (R or S) choosing Technology A or B is defined as:

$$\Pi^A_{\text{type}}(n_A) = \text{IntrinsicPayoff}^A_{\text{type}} + \frac{L_A}{1 + e^{-k_A(n_A - n_{0A})}} \tag{1}$$

$$\Pi^B_{\text{type}}(n_B) = \text{IntrinsicPayoff}^B_{\text{type}} + \frac{L_B}{1 + e^{-k_B(n_B - n_{0B})}} \tag{2}$$

The parameters of the logistic function allow for a nuanced representation of the learning process:

- L **(Carrying Capacity):** This represents the maximum possible payoff gain from learning-by-using. It is the learning "plateau" or technological frontier for that technology.
- k **(Steepness):** This parameter controls the speed of the learning process. A higher k signifies a technology that improves more rapidly with each new adopter.
- n_0 **(Midpoint):** This is the number of adoptions at which the technology's rate of improvement is maximal—its "take-off" point.

This formulation decouples intrinsic preferences from learning dynamics, through additivity, and allows for the modeling of complex competitive scenarios, such as a technology with low initial appeal but massive long-term potential, or one that improves quickly but stagnates early.

3 Methodology

To investigate the dynamics of this extended model, I employ Monte Carlo simulations. This method allows us to observe the aggregate market outcome that emerges from the sequence of individual, micro-level choices across a large number of possible "histories."

For each simulation run (5.000), a sequence of $N = 10.000$ agents is randomly generated, composed of equal numbers of R- and S-types. The agents choose sequentially. At each step, the choosing agent calculates the current payoffs for Technology A and B based on the number of existing adopters (n_A and n_B) and selects the technology with the higher payoff. This process is repeated until all N agents have made a choice.

To ensure statistical robustness, I perform a large number of independent runs for each scenario. The final market share of Technology A from each run is recorded, and the aggregated results are visualized in a histogram, which reveals the probability distribution of possible long-term market structures.

I investigate three distinct cases to systematically explore the impact of my model extension:

1. **Approximated Linear Returns.** To establish a baseline and validate my simulation framework, I approximate Arthur's original linear model. This is achieved by setting the learning potential (L) to a very high value and the steepness (k) to a very low value, making the initial segment of the logistic curve nearly linear.

2. **Symmetric Non-Linear Competition.** Here, both technologies are given identical non-linear learning parameters ($L_A = L_B, k_A = k_B$). This case isolates the effect of the learning plateau in a perfectly balanced competitive environment.

3. **Asymmetric "Tortoise vs. Hare" Competition.** This scenario models a competition between two fundamentally different technologies. Technology A ("the hare") is characterized by rapid initial improvement but a low ultimate potential (high k, low L). Technology B ("the tortoise") improves slowly but has a much higher long-term potential (low k, high L).

The specific parameters for each case are detailed in the Appendix.

4 Results

The simulation results reveal profoundly different market dynamics across the three cases.

Case 1: Approximated Linear Returns As expected, this simulation replicated the classic findings of Arthur's model. The resulting distribution of final market shares is sharply bimodal, with nearly all runs ending in a state of quasi-complete monopoly (see Appendix, Figure 1). Either Technology A or Technology B captures almost 100% of the market. The winner is determined entirely by early, random fluctuations in the sequence of agent types, confirming the powerful role of path dependence in a linear-returns world.

Case 2: Symmetric Non-Linear Competition The introduction of a symmetric learning plateau dramatically alters the outcome. The distribution of market shares becomes trimodal (Appendix, Figure 2). The two side-peaks represent path-dependent lock-in, but this lock-in is incomplete. They are centered around market shares of approximately 0.9 and 0.1 for Technology A, not 1.0 and 0.0. This occurs because once the winning technology's learning benefits saturate, its advantage is no longer overwhelming, and the minority agent type reverts to their intrinsically preferred, albeit less popular, technology.

Most strikingly, a third, small peak appears at exactly 0.5 market share. This represents a stable market-sharing equilibrium. This outcome arises in runs where the sequence of agents is sufficiently balanced in the early stages, allowing both technologies to reach their learning plateaus concurrently. Once learning benefits are exhausted for both, the competition is driven by intrinsic preferences, leading to a 50/50 market split. While this state is a possible outcome, the simulation shows it is infrequent, as the "path" to it is narrow and requires a specific balance of historical events. The system is still far more likely to "tip" onto the wider paths leading to near-monopoly.

Case 3: Asymmetric "Tortoise vs. Hare" Competition This scenario yields the most dramatic result. The distribution of final market shares is sharply unimodal, with a single peak indicating a near-total market capture by Technology B, the "tortoise" (Appendix, Figure 3). In almost every run, regardless of the initial random sequence of agents, Technology A finishes with a small niche market of around 10%, while Technology B dominates.

This outcome demonstrates that a sufficiently large difference in fundamental technological potential can override the forces of path dependence. While the "hare" (Technology A) may gain an early lead due to its rapid initial improvement, its growth stagnates at its low potential ceiling. The "tortoise" (Technology B), with its slow but persistent improvement towards a much higher ceiling, eventually surpasses the hare in payoff. Once this crossover point is reached, a landslide adoption for Technology B becomes inevitable. The market outcome is no longer path-dependent but is predictable and efficient, converging to the technologically superior option.

5 Discussion and Differences with the Original Model

The results from my extended model, while building on Arthur's framework, lead to a set of conclusions that are significantly more nuanced than those derived from the original linear model.

First, the concept of **lock-in becomes softer**. In the linear model, lock-in is absolute. In my non-linear model, even when path dependence leads to a dominant technology, the heterogeneity of preferences combined with a learning plateau ensures that the losing technology can retain a persistent niche market. This suggests that monopolies arising from network effects may be less total than previously thought, with space remaining for minority-preference products.

Second, the model reveals the possibility of a **stable market-sharing equilibrium**. This is a critical departure from the original model, where coexistence is impossible. The learning plateau acts as a dampening effect that can stop a runaway feedback loop, allowing the market to settle into a structure that reflects the underlying diversity of consumer tastes rather than a technological monopoly. However, my results also caution that the existence of such an equilibrium does not make it a likely outcome; the forces of path dependence still heavily favor tipping towards a single dominant player.

Third, and perhaps most importantly, my extension identifies a clear boundary condition for the power of path dependence. The "tortoise vs. hare" scenario demonstrates that **path dependence is not absolute**. When the underlying differences in the long-term potential of competing technologies are sufficiently large, the market can "see through" the noise of historical chance and select the superior option. This implies that while small events can determine the winner in a competition between relatively similar technologies, they are less likely to lock the market into a grossly inferior one. The system can, under these conditions, become ergodic and predictable once more.

In essence, moving from a linear to a non-linear returns function transforms the model's conclusion from "history is everything" to "history is important, but fundamentals can be more so." It replaces a world of inevitable, absolute monopolies with one of incomplete lock-in, possible coexistence, and the ultimate triumph of superior potential.

6 Conclusion

This paper has extended W.B. Arthur's model of competing technologies to incorporate non-linear, saturating returns to adoption. Through numerical simulation, I have shown that this modification, while simple in its premise, yields a significantly richer and more realistic set of outcomes. The harsh, winner-takes-all world of the linear model is replaced by a landscape where incomplete lock-in is the norm, stable market-sharing is a possibility, and fundamental technological superiority can ultimately override the randomness of historical events.

The key contribution of this work is to demonstrate that the powerful conclusions of path dependence are highly sensitive to the underlying assumptions about the nature of increasing returns. By showing that market forces can, under plausible conditions, avoid inefficient lock-in and select for genuinely superior technologies, my findings offer a more optimistic and nuanced perspective on the evolution of technological markets. Future work could extend this model further to include more than two agents, more than two technologies, and factors such as strategic pricing by technology sponsors or the arrival of new, disruptive technologies that reset the competitive landscape.

A Simulation Parameters, Code, and Results

A.1 Code Availability

The Python code used for the simulations presented in this paper is available as an interactive Google Colab notebook. The notebook allows for the replication of the results and experimentation with different parameters. It can be accessed at the following link:

View Simulation Code on Google Colab

A.2 Case 1: Approximated Linear Returns

Table 1: Parameters for Case 1 Simulation.

Parameter	Technology A	Technology B
Intrinsic Payoff (R-Agent)	1.2	1.0
Intrinsic Payoff (S-Agent)	1.0	1.2
L (Max Learning)	10.0	10.0
k (Steepness)	0.01	0.01
n_0 (Midpoint)	500	500

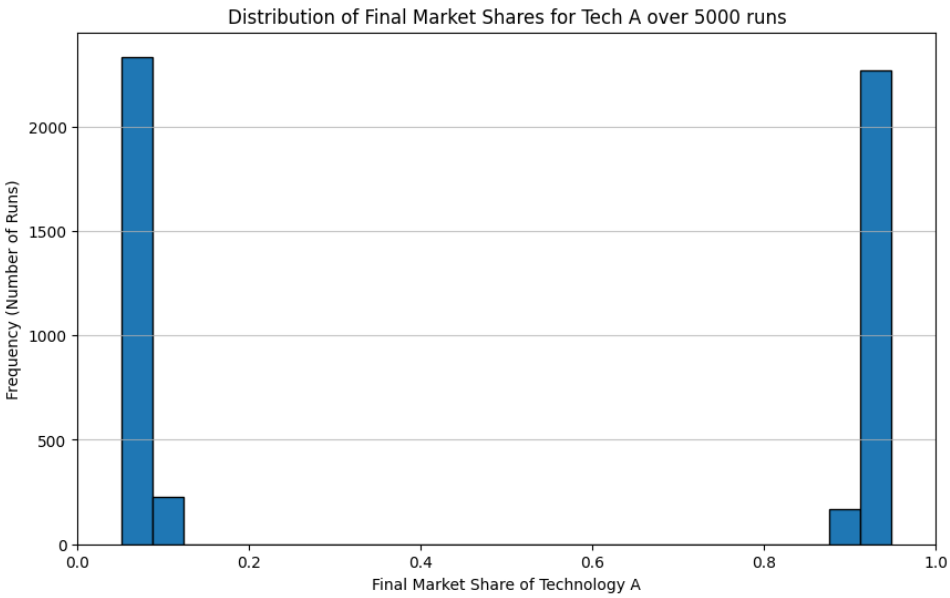

Figure 1: Distribution of final market shares for Technology A under approximated linear returns. The outcome is a quasi-complete monopoly for either A or B.

A.3 Case 2: Symmetric Non-Linear Competition

Table 2: Parameters for Case 2 Simulation.

Parameter	Technology A	Technology B
Intrinsic Payoff (R-Agent)	1.2	1.0
Intrinsic Payoff (S-Agent)	1.0	1.2
L (Max Learning)	3.0	3.0
k (Steepness)	0.02	0.02
n_0 (Midpoint)	500	500

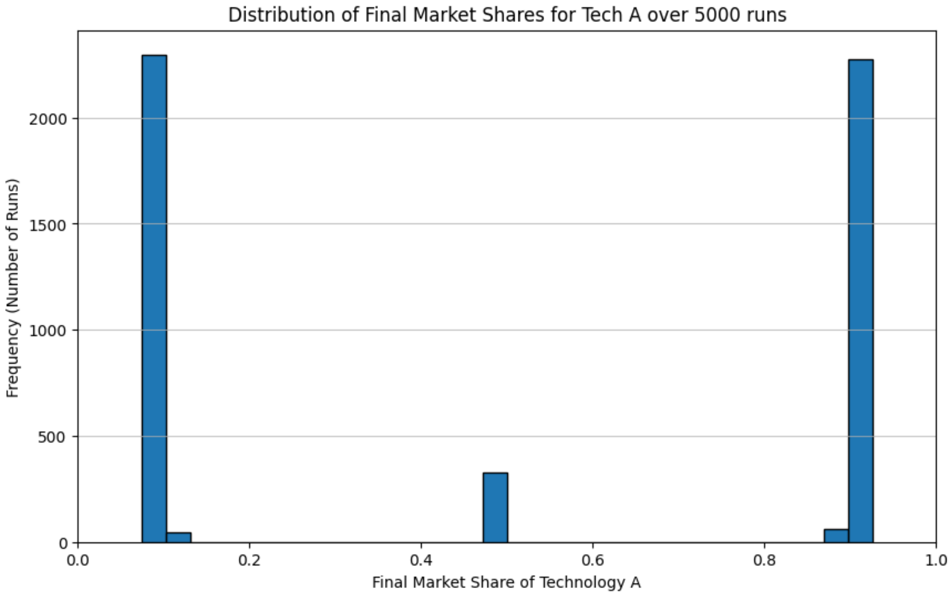

Figure 2: Distribution of final market shares for Technology A under symmetric non-linear returns. The outcome is trimodal: incomplete lock-in or stable market sharing.

A.4 Case 3: Asymmetric "Tortoise vs. Hare" Competition

Table 3: Parameters for Case 3 Simulation.

Parameter	Technology A (Hare)	Technology B (Tortoise)
Intrinsic Payoff (R-Agent)	1.2	1.0
Intrinsic Payoff (S-Agent)	1.0	1.2
L (Max Learning)	1.5	4.0
k (Steepness)	0.05	0.01
n_0 (Midpoint)	500	500

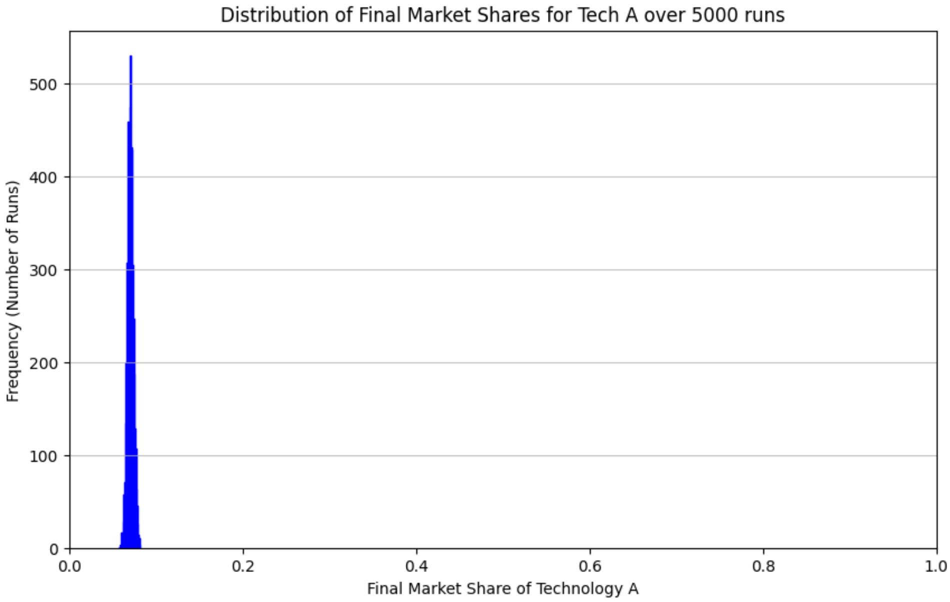

Figure 3: Distribution of final market shares for Technology A in the "tortoise vs. hare" scenario. The outcome is a predictable win for the superior technology (B).