Aggregate Demand and Supply Disturbances Revisited: An Application of the Blanchard-Quah Methodology

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1 Introduction and Literature Review

Understanding the driving forces behind macroeconomic fluctuations – whether they originate from the supply side of the economy (e.g., technological advancements, resource availability) or the demand side (e.g., changes in consumption, investment, government spending) – is a fundamental question in economics, critical for effective policy design. Vector Autoregression (VAR) models are commonly used to analyze these dynamics (Sims (1980)). However, standard VARs face an identification problem, as their correlated error terms do not directly represent structural shocks, necessitating theoretical restrictions to uncover these underlying disturbances. Various identification strategies exist, including short-run restrictions (e.g., Bernanke & Blinder (1992)), sign restrictions (e.g., Uhlig (2005)), and long-run restrictions. A seminal contribution in this area is Blanchard & Quah (1989)'s influential paper, "The Dynamic Effects of Aggregate Demand and Supply Disturbances." They proposed a long-run identification strategy based on the premise that aggregate demand shocks have only temporary effects on the level of output, while supply shocks can have permanent effects. Applying this to a bivariate VAR of US output growth and unemployment, they identified the dynamic impacts and relative importance of these shocks, finding supply shocks crucial for long-run output and demand shocks for short-run fluctuations. Their work has become a cornerstone in empirical macroeconomics. This paper replicates and extends Blanchard and Quah's (1989) structural VAR analysis. Adopting their core long-run identification strategy, I analyze aggregate demand and supply shocks in the United Kingdom economy using a dataset for nominal Gross Domestic Product (GDP) and the unemployment rate, covering the period 1971-2024. This focus on the UK and a distinct sample period represents a departure from the original study's analysis of US Gross National Product (GNP) over 1948-1987. Before the SVAR, a preliminary univariate analysis examines the time series properties of the UK variables, including correlograms and stationarity tests. Additionally, an AutoRegressive Distributed Lag (ARDL) model is estimated, with the first difference of the unemployment rate as the dependent variable and the first difference of GDP as the independent variable, to further investigate the relationship between GDP and unemployment. The primary empirical exercise involves estimating a bivariate VAR for UK GDP growth and unemployment, following Blanchard and Quah's simplest specification without deterministic dummies or breaks. Structural demand and supply shocks are identified by imposing the long-run restriction that demand shocks have no lasting impact on the level of GDP. The dynamic effects are analyzed via Impulse Response Functions (IRFs), and their relative importance is assessed through Forecast Error Variance Decomposition (FEVD).

2 The Model and Estimation Procedure

Empirical macroeconomic analysis often relies on modeling dynamic interdependencies among key economic variables. This study employs two distinct yet complementary time series models to analyze aggregate demand and supply disturbances and their relationship with unemployment: a single-equation AutoRegressive Distributed Lag (ARDL) model focused on the dynamics of the unemployment rate, and a Structural Vector Autoregression (SVAR) model, following the identification strategy of Blanchard and Quah (1989).

2.1 AutoRegressive Distributed Lag (ARDL) Model

In addition to the SVAR analysis, this study estimates a single-equation AutoRegressive Distributed Lag (ARDL) model to specifically examine the relationship between unemployment rate and GDP. While the SVAR provides a joint analysis of both variables and shocks, the ARDL offers a focused perspective on the dynamics of unemployment relative to output. The general form of the ARDL(p, q) model for the first difference of unemployment rate (Δu_t) as the dependent variable and the first difference of nominal GDP (ΔGDP_t) as the independent variable is:

$$\Delta u_t = c_0 + \sum_{i=1}^{p} \alpha_i \Delta u_{t-i} + \sum_{j=0}^{q} \beta_j \Delta \text{GDP}_{t-j} + \epsilon_t \tag{1}$$

2.2 The Reduced-Form Vector Autoregression (VAR) Model

The analysis begins with the estimation of a reduced-form Vector Autoregression (VAR) model. A VAR of order p, denoted VAR(p), models a vector of endogenous variables as a linear function of their own past values up to p lags and a vector of error terms. In this study, our VAR system comprises two key macroeconomic variables for the UK: the first difference of the logarithm of nominal GDP ($\Delta \ln \text{GDP}t$) and the unemployment rate (u_t). The choice of $\Delta \ln \text{GDP}_t$ reflects the assumption that the level of nominal GDP is integrated of order one, and thus its first difference is stationary. Similarly, the unemployment rate is treated as either stationary or weakly non-stationary in levels, consistently with the author's choice. The bivariate reduced-form VAR(p) model can be written as:

$$\begin{pmatrix} \Delta \ln \text{GDP}_t \\ u_t \end{pmatrix} = C + \sum_{i=1}^{p} A_i \begin{pmatrix} \Delta \ln \text{GDP}_{t-i} \\ u_{t-i} \end{pmatrix} + v_t \tag{2}$$

where C is a vector of constants, A_i are (2×2) matrices of coefficients for each lag i, and $v_t = (v_{1t}, v_{2t})'$ is a vector of contemporaneously correlated reduced-form error terms with a covariance matrix $E[v_t v_t'] = \Omega$. The estimation of the reduced-form VAR is typically performed using Ordinary Least Squares (OLS) applied to each equation separately, which is consistent and efficient. The lag length p is usually selected (among the ones that produce white noise residuals) based on information criteria such as the Akaike Information Criterion (AIC) or the Schwarz Criterion (SIC). Consistent with the simplest specification in Blanchard and Quah, my VAR model does not include deterministic dummy variables nor break terms.

2.3 Structural Vector Autoregression (SVAR) Model and Identification

The reduced-form errors (v_t) are generally correlated and do not represent economically meaningful structural shocks. To uncover the underlying structural disturbances, they imposed theoretical restrictions to identify the relationship between the reduced-form errors and a vector of structural shocks ($\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t})'$). The relationship between the two is typically represented by a linear transformation: $A_0 v_t = \varepsilon_t$, where A_0 is a (2x2) non-singular matrix capturing contemporaneous interactions among the variables. The structural shocks ε_t are assumed to be serially and mutually uncorrelated, with a normalized covariance matrix, often set to the identity matrix ($E[\varepsilon_t \varepsilon_t'] = I$). Substituting the structural relationship into the reduced-form VAR, we obtain the structural VAR (SVAR):

$$A_0 Y_t = A_0 C + \sum_{i=1}^{p} A_0 A_i Y_{t-i} + \varepsilon_t \tag{3}$$

The challenge in SVAR analysis is to identify the matrices A_0 and B from the estimated reduced-form covariance matrix Ω. Following Blanchard and Quah (1989), the identification of the structural demand and supply shocks is achieved by imposing a long-run restriction on the cumulative impulse responses. Let $\Psi(L) = (\sum_{i=0}^{\infty} A_i L^i)^{-1}$ be the lag polynomial of the reduced-form moving average representation, such that $Y_t = \Psi(L) A_0^{-1} \varepsilon_t$. The long-run impact of the reduced-form errors is given by $\Psi(1) = \sum_{i=0}^{\infty} \Psi_i$. The relationship between the long-run impact of reduced-form errors and structural shocks is given by:

$$\Psi(1) v_t = \Psi(1) A_0^{-1} \varepsilon_t \tag{4}$$

The matrix $\Psi(1)A_0^{-1}$ represents the matrix of long-run cumulative impulse responses of the variables (Y_t) to the structural shocks (ε_t). Let ε_{1t} represent the aggregate demand shock and ε_{2t} represent the aggregate supply shock. Blanchard and Quah's key identifying assumption is that aggregate demand shocks have no permanent effect on the level of output. Since their first variable is the first difference of log GDP, the long-run effect on the level of log GDP corresponds to the sum of the impulse responses of $\Delta \ln \text{GDP}t$. The cumulative impulse response of the level of output to the first structural shock (demand) being zero in the long run is imposed as the identifying restriction.

3 Data Description

This study uses time series data for the United Kingdom economy to replicate and extend the analysis of aggregate demand and supply disturbances. I focus on two key macroeconomic indicators: nominal Gross Domestic Product (GDP) and the unemployment rate. Data on nominal GDP and unemployment rate for the United Kingdom were obtained from Office for National Statistics. The data are observed at a quarterly frequency, cover the sample period from 1971:Q1 to 2024:Q4, and are seasonally adjusted. Nominal GDP is measured in millions of pounds, while the unemployment rate is measured as a percentage of the civilian labour force aged 16 and over. The figures of these variables are reported in the appendix (1). Consistent with the methodology of Blanchard and Quah (1989) and standard practice in time series econometrics, nominal GDP is transformed into its natural logarithm ($\ln \text{GDP}_t$) to capture proportional changes. For the VAR analysis, I use the first difference of the log of nominal GDP ($\Delta \ln \text{GDP}_t$), which approximates the GDP growth rate. The unemployment rate (u_t) is used in first difference for the ARDL and in levels for the VAR.

3.1 Univariate Time Series Analysis

Before proceeding to the multivariate VAR and single-equation ARDL models, it is essential to examine the time series properties of the individual variables. Understanding whether the variables are stationary or non-stationary and their degree of persistence informs the appropriate modelling strategy. This univariate analysis involves inspecting the autocorrelation and partial autocorrelation functions through correlograms and conducting formal unit root tests. Note that the authors assume stationarity for both

$\Delta \ln \mathrm{GDP}_t$ and u_t. My univariate analysis, presented in the next sections, indicates that while $\Delta \ln \mathrm{GDP}_t$ appears stationary, the unemployment rate for the UK shows evidence of a unit root, suggesting it is I(1). Despite this finding, and to ensure our replication closely follows their methodology, I maintain the assumption that u_t is stationary within the VAR framework.

3.1.1 Correlograms

The figures (2) in the Appendix show the sample autocorrelation functions (ACF) and partial autocorrelation functions (PACF) for $\ln \mathrm{GDP}_t$ and u_t, and their first differences. The correlograms of $\ln \mathrm{GDP}_t$ and u_t in levels exhibit strong trends and persistent behaviour, consistent with non-stationary data. Taking the first difference of these series appears to remove this persistence, as seen in their respective figures. This initial visual assessment suggests that both variables are likely integrated of order one (I(1)).

3.1.2 Stationarity Tests

To formally assess the stationarity of the time series, I conduct unit root tests. Following common practice, I employ the Augmented Dickey-Fuller (ADF) test. The null hypothesis is that the series has a unit root (i.e., it is non-stationary). The alternative hypothesis is that the series is stationary. Figures (3) presents the results of the ADF test for both the levels and the first differences of our variables. While the ADF test with a constant and trend indicates stationarity for $\ln \mathrm{GDP}_t$, the visual evidence from the time series plot (Figure 1b) suggests a persistent, non-linear (concave) pattern not fully captured by the test's deterministic trend specification. Consistent with the prevalent treatment of GDP levels in the macroeconomic literature as I(1) and to adhere to the assumptions underlying the Blanchard and Quah methodology, I proceed assuming $\ln \mathrm{GDP}_t$ is integrated of order one. For the unemployment rate (u_t), the unit root tests provide strong evidence of non-stationarity, concluding that it is integrated of order one (I(1)). This finding diverges from Blanchard and Quah's assumption of unemployment stationarity. However, to maintain comparability with their core bivariate VAR framework, I assume u_t is stationary (I(0)) in my VAR analysis, highlighting this assumption's inconsistency with my univariate test results.

4 Empirical Results

This chapter presents the estimation results for the multivariate models used to analyze the dynamics of GDP and unemployment in the UK. I first discuss the estimated single-equation ARDL model. Then, I present the estimated reduced-form VAR model, followed by the results from the identified Structural VAR (SVAR). In the next section, the diagnostic of all these models will be presented.

4.1 Single Equation Model

To complement the structural VAR analysis and gain further insight into the short-run dynamics linking unemployment and GDP fluctuations, a single-equation AutoRegressive Distributed Lag (ARDL) model is estimated. This model focuses specifically on explaining the changes in the unemployment rate as a function of its own past changes and past changes in nominal GDP. The specified ARDL model takes the first difference of the unemployment rate (Δu_t) as the dependent variable and the first difference of nominal GDP (ΔGDPt) as the independent variable. Based on information criteria (such as the Akaike one and the Schwarz one), an ARDL(2,4) specification without a constant was selected, indicating two lags of the dependent variable and four lags of the independent variable. The estimated ARDL(2,4) model can be written as:

$$\Delta u_t = \sum_{i=1}^{2} \alpha_i \Delta u_{t-i} + \sum_{j=0}^{4} \beta_j \Delta \mathrm{GDP}_{t-j} + \epsilon_t \tag{5}$$

where α_i are the coefficients for the lagged changes in unemployment, β_j are the coefficients for the current and lagged changes in nominal GDP, and ϵ_t is the error term. This model was estimated using Ordinary Least Squares (OLS). Given evidence of heteroskedasticity in the model residuals, heteroskedasticity-consistent (White) standard errors were computed to ensure valid inference. Figure 4a in the Appendix presents the detailed estimation results for the ARDL(2,4) model, including coefficient estimates, standard errors, t-statistics, and p-values. The estimated coefficients from this ARDL(2,4) model, specified in first differences, capture the short-run dynamics between changes in unemployment and changes in nominal GDP. While the negative coefficient on the first 2 lags of $\Delta \mathrm{GDP}_t$ aligns with expectations of faster economic growth leading to a decrease in unemployment, the significant positive coefficients on the third and fourth lags of $\Delta \mathrm{GDP}_t$ suggest a more complex lagged relationship whose positive sign is not immediately intuitive from standard economic theory.

4.2 Reduced-form VAR Estimation

The foundation of our structural analysis is the estimation of a reduced-form Vector Autoregression (VAR) model. As outlined in Section 2.2, this bivariate VAR system comprises the first difference of the logarithm of GDP ($\Delta \ln \mathrm{GDP}_t$) and the unemployment rate (u_t). The choice of these variables and transformations aligns with the core specification of Blanchard and Quah, though applied to UK data. The reduced-form VAR(p) models each variable as a function of its own lagged values and the lagged values of the other variable in the system, plus a constant term. Preliminary analysis involving information criteria (such as AIC and BIC) suggested a relatively high optimal lag length. More importantly, diagnostic testing for autocorrelation in the residuals, detailed in Section 5, indicated that a VAR with 8 lags, as used by Blanchard and Quah, did not produce serially uncorrelated residuals for our UK data. To ensure that the VAR residuals are white noise, which is a crucial assumption for valid impulse response analysis and structural identification, I found it necessary to estimate a VAR(16). This higher lag length differs from the VAR(8) estimated by the authors for their US data, reflecting potentially different dynamic properties of the UK variables or the specific sample period analyzed. The estimated reduced-form VAR(16) model can be written as:

$$\begin{pmatrix} \Delta \ln \mathrm{GDP}_t \\ u_t \end{pmatrix} = C + \sum_{i=1}^{16} A_i \begin{pmatrix} \Delta \ln \mathrm{GDP}_{t-i} \\ u_{t-i} \end{pmatrix} + v_t \tag{6}$$

where C is a vector of constants, A_i are (2×2) matrices of estimated coefficients for each lag i from 1 to 16, and $v_t = (v_{1t}, v_{2t})'$ is the vector of estimated reduced-form errors. Consistent with the methodology, the VAR(16) model was estimated using Ordinary

Least Squares (OLS) equation by equation. Due to the extensive number of lags, the full estimation output is not presented, but it is available upon request from the author.

4.3 Structural VAR Estimation

Building on the estimated reduced-form VAR(16), I proceed to identify the fundamental macroeconomic disturbances driving fluctuations in UK GDP and unemployment using a Structural Vector Autoregression (SVAR) model. As detailed in Section 2.3, the relationship between the reduced-form errors and structural shocks is identified by imposing theoretical restrictions. Following the methodology of Blanchard and Quah, the key identifying restriction assumes that aggregate demand shocks have no permanent effect on the level of output.[1] The SVAR model was estimated using Maximum Likelihood estimation in EViews software, incorporating this long-run restriction. The estimation converged successfully, and the model is just-identified. The whole estimation output is presented in the figure 4b.

4.3.1 Structural Impulse Response Functions (IRFs)

The estimated SVAR allows us to analyze the dynamic impact of identified structural shocks on the UK economy through impulse response functions, presented in Figure 5a in the Appendix. These plots show the accumulated response of GDP (representing the level of log GDP) and the response of the unemployment rate to a one-standard-deviation aggregate demand shock ("Shock 1") and aggregate supply shock ("Shock 2"). The confidence intervals are computed using the bootstrap technique with 2000 iterations. The impulse responses exhibit both similarities and notable differences compared to the original findings of Blanchard and Quah. Firstly, the accumulated response of GDP to a demand shock does not show a significant impact, since the confidence interval is always at the zero level. This finding contrasts with the results of the authors, whose IRF typically shows a statistically significant hump-shaped positive response of output to a demand shock in the short to medium run. Consistently with their findings, instead, a supply shock is found to have a permanent positive effect on the level of GDP. For the unemployment rate, a demand shock leads to a permanent decrease in unemployment, while in their study it was only temporary. Lastly, a supply shocks has an initial negative effect on unemployment, but then it gets dissipated after 20 periods, broadly consistent with the findings of the authors.

4.3.2 Forecast Error Variance Decomposition (FEVD)

To assess the relative importance of the identified aggregate demand and supply shocks in explaining the fluctuations of GDP and the unemployment rate, we analyze the Forecast Error Variance Decomposition (FEVD). Figure 5b in the Appendix presents the percentage of the forecast error variance for each variable attributable to each structural shock at various forecast horizons, ranging from 1 to 40 quarters. For the forecast error variance of GDP, demand shocks account for less than 25% at a 1-quarter horizon, while supply shocks contribute for more than 75%. As the forecast horizon increases, the contribution of demand shocks slightly rises, but the supply shocks dominate the long-run variance of

[1] In EViews software, this restriction was imposed using the text syntax @LR1(@U1) = 0, where @LR1 denotes the first row of the long-run matrix and @U1 denotes the first structural shock.

GDP. This long-run dominance of supply shocks for output variance is consistent with the fundamental findings of Blanchard and Quah, although the specific proportions at shorter horizons differ (in their table 2C, the proportions are 5% for the demand shock and 95% for the supply shock). For the forecast error variance of the unemployment rate, demand shocks are found to play a more significant role, particularly in the very short run. At a 1-quarter horizon, demand shocks contribute more than 80% to unemployment variance, compared to less than 20% from supply shocks. As the horizon extends, the contributions may evolve. In the long run (40 quarters), demand shocks explain almost 70% of unemployment variance, while supply shocks account for less than 30%. Compared to Blanchard and Quah's results (Table 2C), my findings are almost inverse (they had a 40% from demand and 60% from supply shocks).

5 Evaluation of the Estimated Model and Hypothesis Testing

This chapter provides an evaluation of the statistical properties of the estimated ARDL, reduced-form VAR, and Structural VAR (SVAR) models. Assessing the adequacy of these models through diagnostic tests is essential for ensuring the validity of the empirical results and the reliability of the conclusions drawn. Additionally, I present the results of relevant hypothesis tests.

5.1 ARDL Model Diagnostics

For the estimated ARDL(2,4) model for the first difference of unemployment, it is also important to assess the properties of the residuals. As mentioned in Section 4.1, preliminary analysis indicated the presence of heteroskedasticity in the initial OLS estimation of the ARDL model. To address this, heteroskedasticity-consistent (White) standard errors were used for inference in the ARDL model, ensuring the validity of the t-statistics and p-values despite the presence of non-constant error variance. Regarding autocorrelation of the residuals, a visual inspection of the correlogram (Figure 6a) suggest its absence. The formal LM test (Figure 6b) confirms the absence of autocorrelation. Moreover, the Jarque-Bera test confirms that the residuals are normally distributed (Figure 6d). Finally, a last check on the residuals confirm that now they're homoskedastic (Figure 6c).

5.2 VAR Model Diagnostics

A crucial assumption for valid impulse response analysis and structural identification in a VAR model is that the reduced-form residuals are serially uncorrelated (i.e., they constitute white noise). A first look at their correlograms (Figure 7a) suggest no autocorrelation. The formal LM test for autocorrelation in the residuals confirms that the residuals are serially uncorrelated (Figure 7b). This supports the selection of a VAR(16), indicating that the chosen lag length is sufficient to capture the linear dynamic dependencies in the data and that the reduced-form residuals are indeed white noise. Lastly, since the application of the Wold's Theorem is required in the original work, I also checked that the system is stable, i.e. that every root lies inside the unit circle (Figure 7c).

5.3 SVAR Model Evaluation

The evaluation of the SVAR model centers on the plausibility of the imposed identification scheme and and the adequacy of the underlying reduced-form VAR model. As confirmed by the diagnostic tests in Section 5.2, the estimated VAR(16) provides a statistically sound basis with white noise residuals. As discussed in Section 4.3, the SVAR is just-identified by the long-run restriction that demand shocks have no permanent effect on the level of GDP, in addition to the standard assumption of uncorrelated structural shocks.

6 Conclusions and Further Extensions

This study replicated and extended the seminal work of Blanchard and Quah (1989), applying their structural VAR methodology to the UK economy with a focus on GDP and unemployment dynamics. Using a long-run restriction to identify demand and supply shocks, and complementing this with an ARDL model for unemployment, we gained insights into UK macroeconomic fluctuations. The SVAR analysis revealed findings both consistent with and diverging from the original US results. Supply shocks exhibited the expected permanent positive effect on GDP and temporary negative impact on unemployment, qualitatively aligning with Blanchard and Quah. However, the response of GDP and unemployment to demand shocks are qualitatively different from the original ones. The forecast error variance decomposition also supported the B-Q framework's finding of supply shocks dominating long-run GDP variance, but the precise variance proportions differed significantly from the original study across variables and horizons. These discrepancies likely stem from differences in data, sample period, the UK vs. US economic structure, and the higher-order VAR required for our analysis. This study contributes to the literature by demonstrating the applicability of a key SVAR methodology in a new country context and illustrating the sensitivity of results to data and sample specificities. Despite these insights, the study is limited by focusing solely on Blanchard and Quah's most basic model specification, which excluded deterministic breaks and dummy variables used to account for specific historical events. A significant avenue for future research is therefore to replicate and analyze the more complex models presented in their paper, which incorporated such deterministic terms, such as oil price shocks or COVID-19 crisis.

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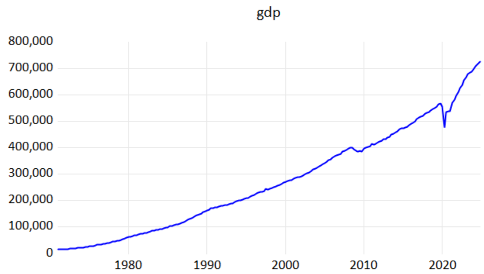
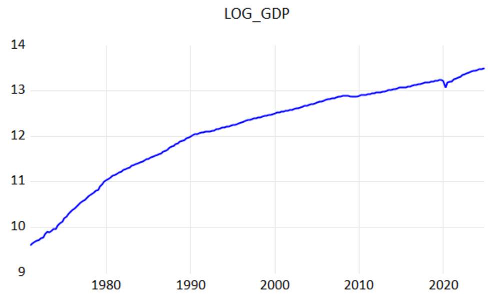
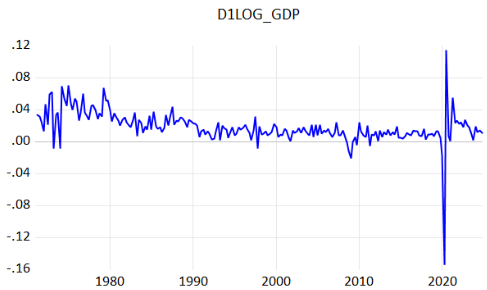
A Appendix



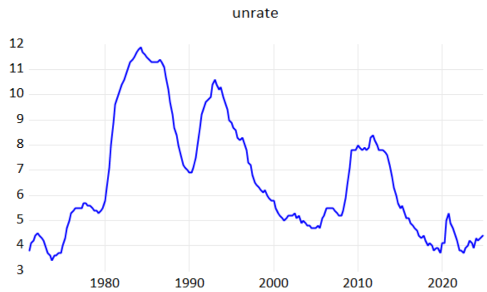
(a) GDP in levels



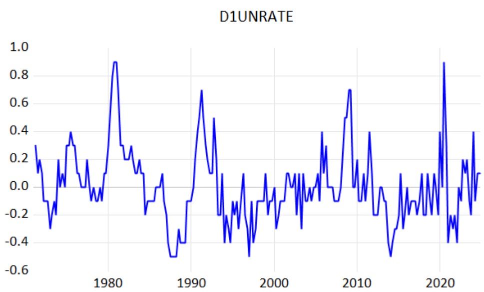
(b) Logarithm of GDP



(c) First difference of logarithm of GDP

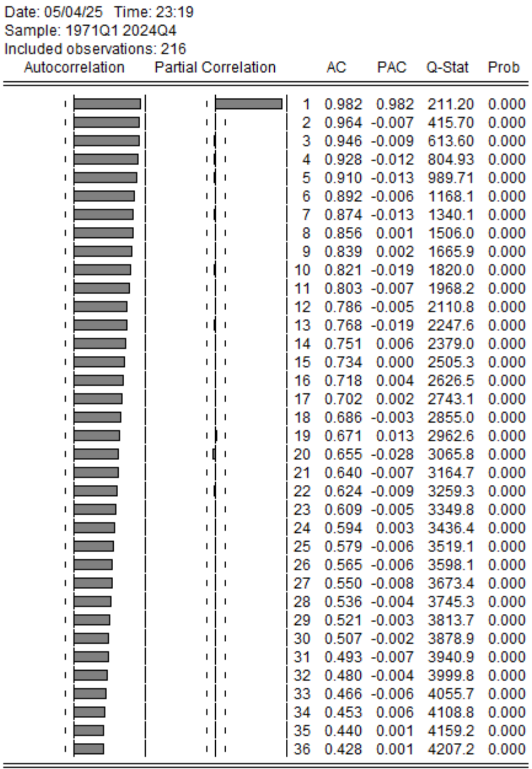


(d) Unemployment rate



(e) First difference of unemployment rate

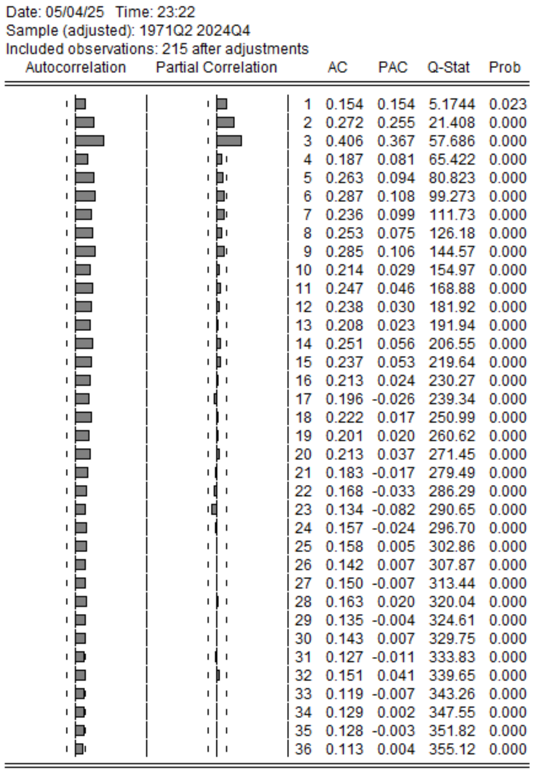
Figure 1: Plots of the variables



Date: 05/04/25 Time: 23:19
Sample: 1971Q1 2024Q4
Included observations: 216

Lag	AC	PAC	Q-Stat	Prob
1	0.982	0.982	211.20	0.000
2	0.964	-0.007	415.70	0.000
3	0.946	-0.009	613.60	0.000
4	0.928	-0.012	804.93	0.000
5	0.910	-0.013	989.71	0.000
6	0.892	-0.006	1168.1	0.000
7	0.874	-0.013	1340.1	0.000
8	0.856	0.001	1506.0	0.000
9	0.839	0.002	1665.9	0.000
10	0.821	-0.019	1820.0	0.000
11	0.803	-0.007	1968.2	0.000
12	0.786	-0.005	2110.8	0.000
13	0.768	-0.019	2247.6	0.000
14	0.751	0.006	2379.0	0.000
15	0.734	0.000	2505.3	0.000
16	0.718	0.004	2626.5	0.000
17	0.702	0.002	2743.1	0.000
18	0.686	-0.003	2855.0	0.000
19	0.671	0.013	2962.6	0.000
20	0.655	-0.028	3065.8	0.000
21	0.640	-0.007	3164.7	0.000
22	0.624	-0.009	3259.3	0.000
23	0.609	-0.005	3349.8	0.000
24	0.594	0.003	3436.4	0.000
25	0.579	-0.006	3519.1	0.000
26	0.565	-0.006	3598.1	0.000
27	0.550	-0.008	3673.4	0.000
28	0.536	-0.004	3745.3	0.000
29	0.521	-0.003	3813.7	0.000
30	0.507	-0.002	3878.9	0.000
31	0.493	-0.007	3940.9	0.000
32	0.480	-0.004	3999.8	0.000
33	0.466	-0.006	4055.7	0.000
34	0.453	0.006	4108.8	0.000
35	0.440	0.001	4159.2	0.000
36	0.428	0.001	4207.2	0.000

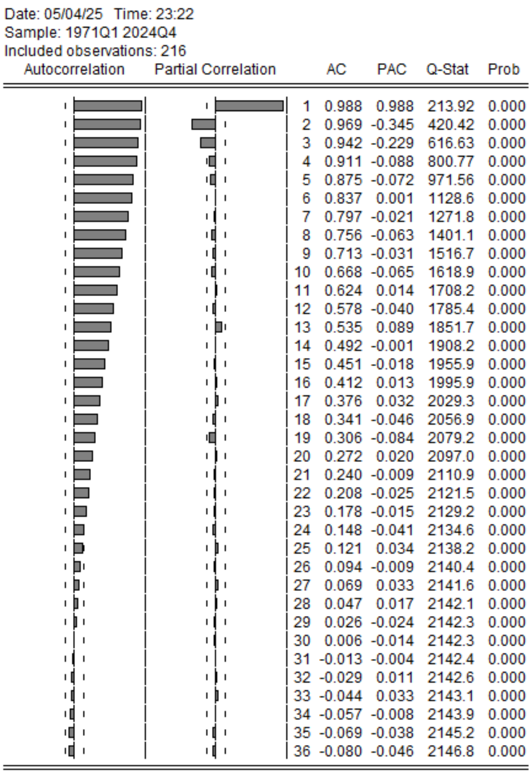
(a) Correlogram of $\ln \mathrm{GDP}_t$



Date: 05/04/25 Time: 23:22
Sample (adjusted): 1971Q2 2024Q4
Included observations: 215 after adjustments

Lag	AC	PAC	Q-Stat	Prob
1	0.154	0.154	5.1744	0.023
2	0.272	0.255	21.408	0.000
3	0.406	0.367	57.686	0.000
4	0.187	0.081	65.422	0.000
5	0.263	0.094	80.823	0.000
6	0.287	0.108	99.273	0.000
7	0.236	0.099	111.73	0.000
8	0.253	0.075	126.18	0.000
9	0.285	0.106	144.57	0.000
10	0.214	0.029	154.97	0.000
11	0.247	0.046	168.88	0.000
12	0.238	0.030	181.92	0.000
13	0.208	0.023	191.94	0.000
14	0.251	0.056	206.55	0.000
15	0.237	0.053	219.64	0.000
16	0.213	0.024	230.27	0.000
17	0.196	-0.026	239.34	0.000
18	0.222	0.017	250.99	0.000
19	0.201	0.020	260.62	0.000
20	0.213	0.037	271.45	0.000
21	0.183	-0.017	279.49	0.000
22	0.168	-0.033	286.29	0.000
23	0.134	-0.082	290.65	0.000
24	0.157	-0.024	296.70	0.000
25	0.158	0.005	302.86	0.000
26	0.142	0.007	307.87	0.000
27	0.150	-0.007	313.44	0.000
28	0.163	0.020	320.04	0.000
29	0.135	-0.004	324.61	0.000
30	0.143	0.007	329.75	0.000
31	0.127	-0.011	333.83	0.000
32	0.151	0.041	339.65	0.000
33	0.119	-0.007	343.26	0.000
34	0.129	0.002	347.55	0.000
35	0.128	-0.003	351.82	0.000
36	0.113	0.004	355.12	0.000

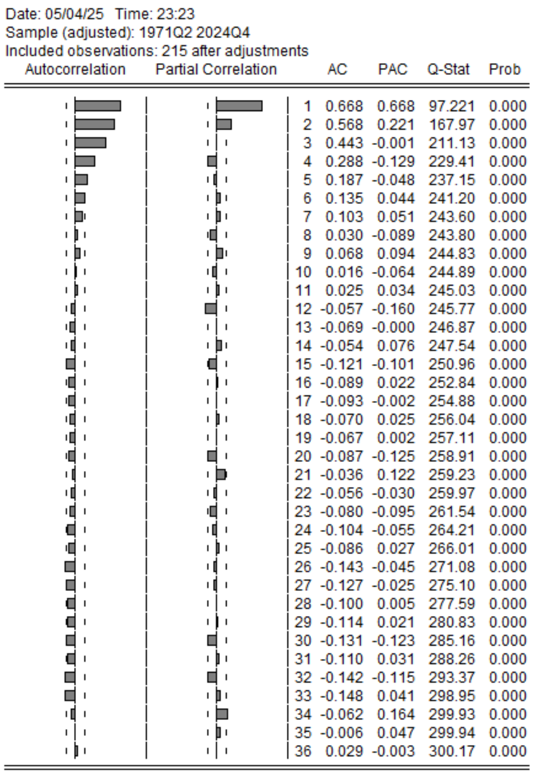
(b) Correlogram of $\Delta \ln \mathrm{GDP}_t$



Date: 05/04/25 Time: 23:22
Sample: 1971Q1 2024Q4
Included observations: 216

Lag	AC	PAC	Q-Stat	Prob
1	0.988	0.988	213.92	0.000
2	0.969	-0.345	420.42	0.000
3	0.942	-0.229	616.63	0.000
4	0.911	-0.088	800.77	0.000
5	0.875	-0.072	971.56	0.000
6	0.837	0.001	1128.6	0.000
7	0.797	-0.021	1271.8	0.000
8	0.756	-0.063	1401.1	0.000
9	0.713	-0.031	1516.7	0.000
10	0.668	-0.065	1618.9	0.000
11	0.624	0.014	1708.2	0.000
12	0.578	-0.040	1785.4	0.000
13	0.535	0.089	1851.7	0.000
14	0.492	-0.001	1908.2	0.000
15	0.451	-0.018	1955.9	0.000
16	0.412	0.013	1995.9	0.000
17	0.376	0.032	2029.3	0.000
18	0.341	-0.046	2056.9	0.000
19	0.306	-0.084	2079.2	0.000
20	0.272	0.020	2097.0	0.000
21	0.240	-0.009	2110.9	0.000
22	0.208	-0.025	2121.5	0.000
23	0.178	-0.015	2129.2	0.000
24	0.148	-0.041	2134.6	0.000
25	0.121	0.034	2138.2	0.000
26	0.094	-0.009	2140.4	0.000
27	0.069	0.033	2141.6	0.000
28	0.047	0.017	2142.1	0.000
29	0.026	-0.024	2142.3	0.000
30	0.006	-0.014	2142.3	0.000
31	-0.013	-0.004	2142.4	0.000
32	-0.029	0.011	2142.6	0.000
33	-0.044	0.033	2143.1	0.000
34	-0.057	-0.008	2143.9	0.000
35	-0.069	-0.038	2145.2	0.000
36	-0.080	-0.046	2146.8	0.000

(c) Correlogram of u_t



Date: 05/04/25 Time: 23:23
Sample (adjusted): 1971Q2 2024Q4
Included observations: 215 after adjustments

Lag	AC	PAC	Q-Stat	Prob
1	0.668	0.668	97.221	0.000
2	0.568	0.221	167.97	0.000
3	0.443	-0.001	211.13	0.000
4	0.288	-0.129	229.41	0.000
5	0.187	-0.048	237.15	0.000
6	0.135	0.044	241.20	0.000
7	0.103	0.051	243.60	0.000
8	0.030	-0.089	243.80	0.000
9	0.068	0.094	244.83	0.000
10	0.016	-0.064	244.89	0.000
11	0.025	0.034	245.03	0.000
12	-0.057	-0.160	245.77	0.000
13	-0.069	-0.000	246.87	0.000
14	-0.054	0.076	247.54	0.000
15	-0.121	-0.101	250.96	0.000
16	-0.089	0.022	252.84	0.000
17	-0.093	-0.002	254.88	0.000
18	-0.070	0.025	256.04	0.000
19	-0.067	0.002	257.11	0.000
20	-0.087	-0.125	258.91	0.000
21	-0.036	0.122	259.23	0.000
22	-0.056	-0.030	259.97	0.000
23	-0.080	-0.095	261.54	0.000
24	-0.104	-0.055	264.21	0.000
25	-0.086	0.027	266.01	0.000
26	-0.143	-0.045	271.08	0.000
27	-0.127	-0.025	275.10	0.000
28	-0.100	0.005	277.59	0.000
29	-0.114	0.021	280.83	0.000
30	-0.131	-0.123	285.16	0.000
31	-0.110	0.031	288.26	0.000
32	-0.142	-0.115	293.37	0.000
33	-0.148	0.041	298.95	0.000
34	-0.062	0.164	299.93	0.000
35	-0.006	0.047	299.94	0.000
36	0.029	-0.003	300.17	0.000

(d) Correlogram of Δu_t

Figure 2: Correlograms of the series

Null Hypothesis: LOG_GDP has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 1 (Automatic - based on SIC, maxlag=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-4.288127	0.0040
Test critical values:	1% level	-4.001516	
	5% level	-3.430963	
	10% level	-3.139114	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LOG_GDP)
Method: Least Squares
Date: 05/06/25 Time: 16:41
Sample (adjusted): 1971Q3 2024Q4
Included observations: 214 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LOG_GDP(-1)	-0.017623	0.004110	-4.288127	0.0000
D(LOG_GDP(-1))	-0.121022	0.067453	-1.794159	0.0742
C	0.221665	0.043381	5.109680	0.0000
@TREND("1971Q1")	0.000113	6.66E-05	1.689426	0.0926

R-squared	0.270098	Mean dependent var	0.017968
Adjusted R-squared	0.259671	S.D. dependent var	0.020345
S.E. of regression	0.017505	Akaike info criterion	-5.234137
Sum squared resid	0.064350	Schwarz criterion	-5.171222
Log likelihood	564.0527	Hannan-Quinn criter.	-5.208714
F-statistic	25.90327	Durbin-Watson stat	1.997667
Prob(F-statistic)	0.000000		

(a) Unit root test on $\ln \mathrm{GDP}_t$

Null Hypothesis: D1LOG_GDP has a unit root
Exogenous: Constant
Lag Length: 2 (Automatic - based on SIC, maxlag=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-4.416395	0.0004
Test critical values:	1% level	-3.461178	
	5% level	-2.874997	
	10% level	-2.574019	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(D1LOG_GDP)
Method: Least Squares
Date: 05/07/25 Time: 16:06
Sample (adjusted): 1972Q1 2024Q4
Included observations: 212 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D1LOG_GDP(-1)	-0.400295	0.090638	-4.416395	0.0000
D(D1LOG_GDP(-1))	-0.581385	0.084347	-6.892740	0.0000
D(D1LOG_GDP(-2))	-0.368937	0.064323	-5.735688	0.0000
C	0.007028	0.002060	3.411798	0.0008

R-squared	0.535236	Mean dependent var	-6.69E-05
Adjusted R-squared	0.528533	S.D. dependent var	0.026593
S.E. of regression	0.018260	Akaike info criterion	-5.149520
Sum squared resid	0.069353	Schwarz criterion	-5.086188
Log likelihood	549.8491	Hannan-Quinn criter.	-5.123922
F-statistic	79.84628	Durbin-Watson stat	2.056601
Prob(F-statistic)	0.000000		

(b) Unit root test on $\Delta \ln \mathrm{GDP}_t$

Null Hypothesis: UNRATE has a unit root
Exogenous: Constant
Lag Length: 2 (Automatic - based on SIC, maxlag=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.341439	0.1600
Test critical values:	1% level	-3.461030	
	5% level	-2.874932	
	10% level	-2.573985	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(UNRATE)
Method: Least Squares
Date: 05/07/25 Time: 15:59
Sample (adjusted): 1971Q4 2024Q4
Included observations: 213 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
UNRATE(-1)	-0.013154	0.005618	-2.341439	0.0202
D(UNRATE(-1))	0.515950	0.066618	7.744906	0.0000
D(UNRATE(-2))	0.238814	0.066870	3.571330	0.0004
C	0.087994	0.039831	2.209190	0.0282

R-squared	0.489348	Mean dependent var	0.000939
Adjusted R-squared	0.482019	S.D. dependent var	0.268467
S.E. of regression	0.193218	Akaike info criterion	-0.431392
Sum squared resid	7.802659	Schwarz criterion	-0.368269
Log likelihood	49.94322	Hannan-Quinn criter.	-0.405882
F-statistic	66.76035	Durbin-Watson stat	2.009346
Prob(F-statistic)	0.000000		

(c) Unit root test on u_t

Null Hypothesis: D(UNRATE) has a unit root
Exogenous: None
Lag Length: 1 (Automatic - based on SIC, maxlag=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-4.727993	0.0000
Test critical values:	1% level	-2.575864	
	5% level	-1.942324	
	10% level	-1.615707	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(UNRATE,2)
Method: Least Squares
Date: 05/04/25 Time: 23:18
Sample (adjusted): 1971Q4 2024Q4
Included observations: 213 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(UNRATE(-1))	-0.257541	0.054472	-4.727993	0.0000
D(UNRATE(-1),2)	-0.220073	0.066933	-3.287955	0.0012

R-squared	0.205622	Mean dependent var	2.19E-17
Adjusted R-squared	0.201857	S.D. dependent var	0.218053
S.E. of regression	0.194806	Akaike info criterion	-0.424278
Sum squared resid	8.007333	Schwarz criterion	-0.392717
Log likelihood	47.18560	Hannan-Quinn criter.	-0.411523
Durbin-Watson stat	1.996244		

(d) Unit root test on Δu_t

Figure 3: Unit root (Augmented Dickey-Fuller) tests

Dependent Variable: D1UNRATE
Method: Least Squares (Gauss-Newton / Marquardt steps)
Date: 05/05/25 Time: 18:04
Sample (adjusted): 1972Q2 2024Q4
Included observations: 211 after adjustments
Huber-White-Hinkley (HC1) heteroskedasticity consistent standard errors and covariance
D1UNRATE = C(1)*D1UNRATE(-1) + C(2)*D1UNRATE(-2) + C(3)*D1GDP(-1) + C(4)*D1GDP(-2) + C(5)*D1GDP(-3) + C(6)*D1GDP(-4)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.524757	0.075567	6.944303	0.0000
C(2)	0.249468	0.071797	3.474635	0.0006
C(3)	-9.10E-06	1.11E-06	-8.190011	0.0000
C(4)	-2.76E-06	1.46E-06	-1.889217	0.0603
C(5)	6.75E-06	1.09E-06	6.167951	0.0000
C(6)	5.28E-06	1.19E-06	4.429629	0.0000

R-squared	0.579362	Mean dependent var	-0.000474
Adjusted R-squared	0.569103	S.D. dependent var	0.269302
S.E. of regression	0.176777	Akaike info criterion	-0.599828
Sum squared resid	6.406292	Schwarz criterion	-0.504515
Log likelihood	69.28189	Hannan-Quinn criter.	-0.561301
Durbin-Watson stat	2.031213		

(a) ARDL(2,4) estimate with "White" standard errors

Structural VAR Estimates
Date: 05/06/25 Time: 18:46
Sample (adjusted): 1975Q2 2024Q4
Included observations: 199 after adjustments
Estimation method: Maximum likelihood via Newton-Raphson (analytic derivatives)
Convergence achieved after 26 iterations
Structural VAR is just-identified

Model: e = Phi*Fu where E[uu']=I
F =

0	C(2)
C(1)	C(3)

including the restriction(s)
@LR1(@U1) = 0

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	9.646607	0.483541	19.94993	0.0000
C(2)	-0.067168	0.003367	-19.94993	0.0000
C(3)	-5.652140	0.740197	-7.635997	0.0000

Log likelihood 599.6140

Estimated S matrix:

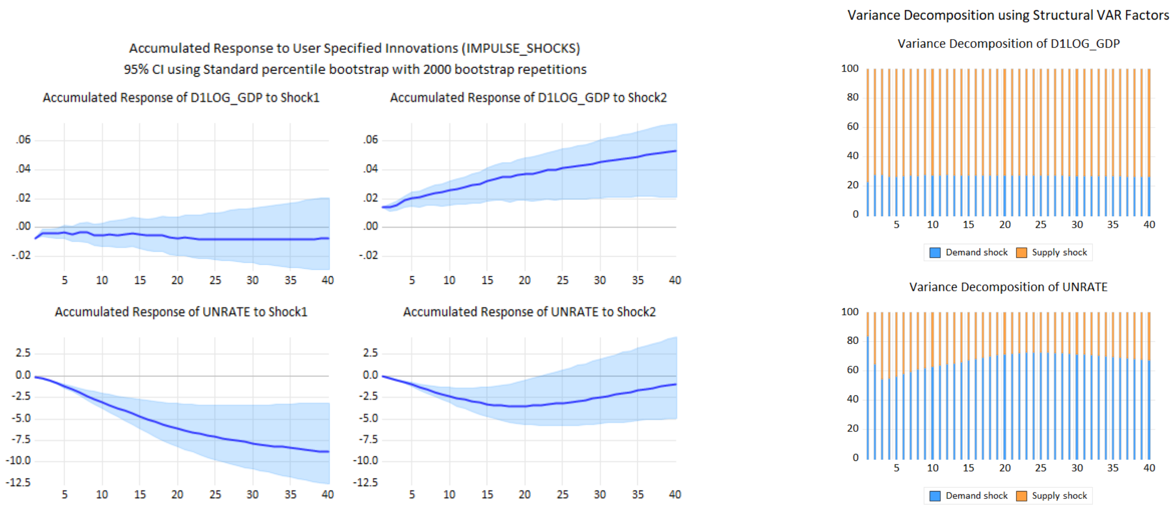
0.008232	-0.015188
0.152671	0.067808

Estimated F matrix:

0.000000	-0.067168
9.646607	-5.652140

(b) Structural VAR estimate

Figure 4: Estimation of the models



(a) Structural IRFs with 95% confidence interval

(b) FEVD

Figure 5: Impulse Response Functions and Forecast Error Variance Decompositions

Date: 05/05/25 Time: 18:04
Sample (adjusted): 1972Q2 2024Q4
Q-statistic probabilities adjusted for 2 dynamic regressors

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.020	-0.020	0.0820	0.775
		2	0.033	0.033	0.3220	0.851
		3	0.110	0.111	2.9139	0.405
		4	-0.052	-0.049	3.4960	0.478
		5	-0.117	-0.128	6.4656	0.264
		6	0.056	0.044	7.1547	0.307
		7	0.055	0.082	7.8196	0.349
		8	-0.192	-0.177	15.994	0.042
		9	0.107	0.076	18.549	0.029
		10	-0.014	-0.014	18.591	0.046
		11	0.006	0.056	18.600	0.069
		12	-0.043	-0.076	19.025	0.088
		13	0.001	-0.039	19.025	0.122
		14	0.013	0.052	19.061	0.163
		15	-0.153	-0.137	24.422	0.058
		16	-0.010	-0.058	24.444	0.080
		17	-0.063	-0.038	25.357	0.087
		18	0.009	0.036	25.378	0.115
		19	0.041	0.069	25.769	0.137
		20	-0.022	-0.094	25.882	0.170
		21	0.051	0.058	26.498	0.188
		22	0.025	0.051	26.641	0.225
		23	0.021	-0.017	26.747	0.267
		24	-0.054	-0.057	27.458	0.284
		25	0.063	0.028	28.409	0.289
		26	-0.076	-0.023	29.824	0.275
		27	-0.034	-0.033	30.113	0.309
		28	0.007	-0.056	30.125	0.357
		29	-0.030	0.017	30.353	0.397
		30	-0.085	-0.095	32.127	0.362
		31	0.018	0.001	32.209	0.407
		32	-0.066	-0.120	33.302	0.404
		33	-0.165	-0.130	40.206	0.181
		34	0.055	0.059	40.969	0.191
		35	0.020	0.023	41.066	0.222
		36	-0.106	-0.100	43.944	0.170

*Probabilities may not be valid for this equation specification.

(a) Correlogram of the residuals

Breusch-Godfrey Serial Correlation LM Test:
Null hypothesis: No serial correlation at up to 4 lags

F-statistic	1.877085	Prob. F(4,201)	0.1158
Obs*R-squared	7.598065	Prob. Chi-Square(4)	0.1075

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 05/05/25 Time: 18:05
Sample: 1972Q2 2024Q4
Included observations: 211
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.680177	0.432635	1.572172	0.1175
C(2)	-0.664878	0.367355	-1.809906	0.0718
C(3)	-5.85E-08	1.59E-06	-0.036809	0.9707
C(4)	6.14E-06	3.80E-06	1.617006	0.1074
C(5)	-1.22E-06	1.59E-06	-0.766706	0.4442
C(6)	-5.09E-06	3.23E-06	-1.577398	0.1163
RESID(-1)	-0.698874	0.435153	-1.606043	0.1098
RESID(-2)	0.341645	0.172057	1.985649	0.0484
RESID(-3)	0.106846	0.095463	1.119233	0.2644
RESID(-4)	0.015147	0.089082	0.170039	0.8652

R-squared	0.036005	Mean dependent var	0.000380
Adjusted R-squared	-0.007159	S.D. dependent var	0.174660
S.E. of regression	0.175284	Akaike info criterion	-0.598588
Sum squared resid	6.175603	Schwarz criterion	-0.439732
Log likelihood	73.15101	Hannan-Quinn criter.	-0.534375
Durbin-Watson stat	1.993944		

(b) Autocorrelation test on the residuals

Heteroskedasticity Test: White
Null hypothesis: Homoskedasticity

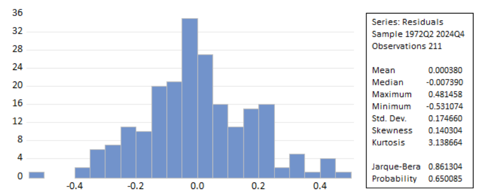
F-statistic	1.335729	Prob. F(6,204)	0.2427
Obs*R-squared	7.976031	Prob. Chi-Square(6)	0.2399
Scaled explained SS	8.055455	Prob. Chi-Square(6)	0.2341

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 05/05/25 Time: 18:05
Sample: 1972Q2 2024Q4
Included observations: 211
Huber-White-Hinkley (HC1) heteroskedasticity consistent standard errors
and covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.027044	0.003660	7.389973	0.0000
(D1UNRATE(-1))^2	0.076266	0.040552	1.880693	0.0614
(D1UNRATE(-2))^2	-0.017434	0.037444	-0.465595	0.6420
(D1GDP(-1))^2	-3.96E-12	1.27E-12	-3.119763	0.0021
(D1GDP(-2))^2	-5.63E-12	5.20E-12	-1.083492	0.2799
(D1GDP(-3))^2	4.26E-13	5.71E-12	0.074546	0.9406
(D1GDP(-4))^2	-3.81E-12	3.13E-12	-1.215487	0.2256

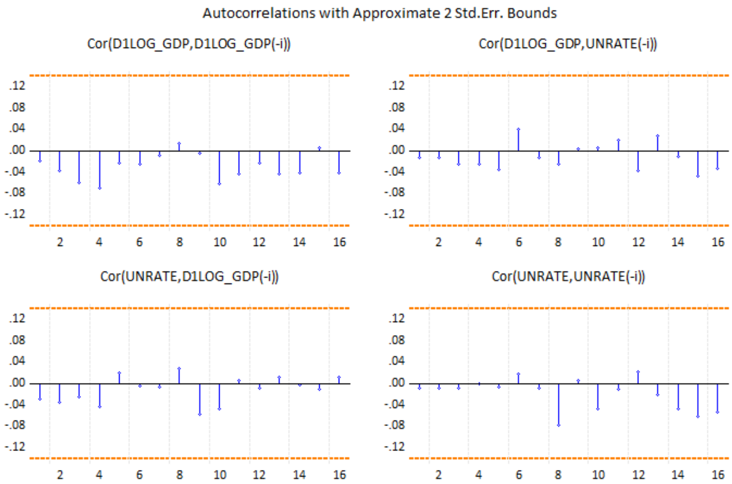
R-squared	0.037801	Mean dependent var	0.030362
Adjusted R-squared	0.009501	S.D. dependent var	0.044520
S.E. of regression	0.044308	Akaike info criterion	-3.362709
Sum squared resid	0.400485	Schwarz criterion	-3.251510
Log likelihood	361.7658	Hannan-Quinn criter.	-3.317760
F-statistic	1.335729	Durbin-Watson stat	1.841652
Prob(F-statistic)	0.242716		

(c) Residuals Heteroskedasticity test



(d) Residuals Normality test

Figure 6: ARDL Diagnostics



(a) Correlogram of the residuals

VAR Residual Serial Correlation LM Tests
Date: 05/06/25 Time: 18:49
Sample: 1971Q1 2024Q4
Included observations: 199

Null hypothesis: No serial correlation at lag h

Lag	LRE* stat	df	Prob.	Rao F-stat	df	Prob.
1	4.630457	4	0.3274	1.162284	(4, 326.0)	0.3274
2	6.603527	4	0.1584	1.662564	(4, 326.0)	0.1584
3	9.433460	4	0.0511	2.385395	(4, 326.0)	0.0511
4	12.61798	4	0.0133	3.206310	(4, 326.0)	0.0133
5	3.959601	4	0.4115	0.992872	(4, 326.0)	0.4115
6	3.193950	4	0.5259	0.799946	(4, 326.0)	0.5259
7	0.415578	4	0.9812	0.103643	(4, 326.0)	0.9812
8	7.997376	4	0.0917	2.017804	(4, 326.0)	0.0917
9	3.661778	4	0.4537	0.917774	(4, 326.0)	0.4537
10	8.730962	4	0.0682	2.205376	(4, 326.0)	0.0682
11	2.303517	4	0.6801	0.576145	(4, 326.0)	0.6801
12	2.520334	4	0.6410	0.630584	(4, 326.0)	0.6410
13	2.861684	4	0.5812	0.716363	(4, 326.0)	0.5812
14	2.749845	4	0.6005	0.688249	(4, 326.0)	0.6005
15	1.857246	4	0.7620	0.464209	(4, 326.0)	0.7620
16	2.291996	4	0.6822	0.573253	(4, 326.0)	0.6822
17	2.995309	4	0.5586	0.749967	(4, 326.0)	0.5586

Null hypothesis: No serial correlation at lags 1 to h

Lag	LRE* stat	df	Prob.	Rao F-stat	df	Prob.
1	4.630457	4	0.3274	1.162284	(4, 326.0)	0.3274
2	7.903539	8	0.4429	0.990822	(8, 322.0)	0.4430
3	13.76425	12	0.3160	1.153672	(12, 318.0)	0.3161
4	18.17202	16	0.3139	1.143033	(16, 314.0)	0.3141
5	20.83206	20	0.4071	1.045998	(20, 310.0)	0.4075
6	23.99682	24	0.4618	1.002643	(24, 306.0)	0.4624
7	26.10121	28	0.5675	0.931786	(28, 302.0)	0.5684
8	32.80250	32	0.4275	1.028927	(32, 298.0)	0.4288
9	35.34383	36	0.4996	0.982901	(36, 294.0)	0.5014
10	40.10317	40	0.4657	1.004720	(40, 290.0)	0.4681
11	47.11255	44	0.3464	1.078106	(44, 286.0)	0.3495
12	48.61848	48	0.4479	1.015274	(48, 282.0)	0.4519
13	54.87893	52	0.3660	1.061518	(52, 278.0)	0.3709
14	57.42429	56	0.4222	1.028435	(56, 274.0)	0.4282
15	59.15644	60	0.5065	0.984495	(60, 270.0)	0.5137
16	60.33743	64	0.6068	0.936253	(64, 266.0)	0.6146
17	70.82106	68	0.3837	1.045457	(68, 262.0)	0.3939

*Edgeworth expansion corrected likelihood ratio statistic.

(b) Autocorrelation test on the residuals

Roots of Characteristic Polynomial
Endogenous variables: D1LOG_GDP UNRATE
Exogenous variables: C
Lag specification: 1 16
Date: 05/06/25 Time: 18:50

Root	Modulus
0.972755 - 0.027769i	0.973151
0.972755 + 0.027769i	0.973151
-0.853360 + 0.344025i	0.920096
-0.853360 - 0.344025i	0.920096
0.905791 - 0.154175i	0.918818
0.905791 + 0.154175i	0.918818
0.826284 - 0.352394i	0.898291
0.826284 + 0.352394i	0.898291
-0.185372 + 0.878716i	0.898056
-0.185372 - 0.878716i	0.898056
0.574512 - 0.687674i	0.896080
0.574512 + 0.687674i	0.896080
-0.397773 + 0.796367i	0.890182
-0.397773 - 0.796367i	0.890182
0.225354 + 0.861060i	0.890061
0.225354 - 0.861060i	0.890061
0.754018 + 0.471664i	0.889387
0.754018 - 0.471664i	0.889387
0.272156 + 0.841013i	0.883952
0.272156 - 0.841013i	0.883952
-0.558949 - 0.680256i	0.880438
-0.558949 + 0.680256i	0.880438
-0.798445 + 0.351485i	0.872385
-0.798445 - 0.351485i	0.872385
-0.842021 - 0.033672i	0.842694
-0.842021 + 0.033672i	0.842694
-0.545572 - 0.612406i	0.820176
-0.545572 + 0.612406i	0.820176
-0.159812 - 0.772955i	0.789303
-0.159812 + 0.772955i	0.789303
0.510494 + 0.543383i	0.745566
0.510494 - 0.543383i	0.745566

No root lies outside the unit circle.
VAR satisfies the stability condition.

(c) Stability Condition

Figure 7: Reduced-form VAR Diagnostics