

# Information Acquisition and Disclosure by Advisors with Hidden Motives

Paula Onuchic<sup>†</sup> – March, 2021

**Abstract.** A sender acquires a signal about an object’s quality and commits to a rule to disclose its realizations to a receiver, who then chooses to buy the object or to keep an outside option of privately known value. Optimal disclosure rules typically conceal negative signal realizations when the object’s sale is very profitable to the sender and positive signal realizations when the sale is less profitable. Using such disclosure rules, the advisor is able to steer sales from lower- to higher-profitability objects. I show that, despite this strategic concealment of some signal realizations, the receiver may prefer being informed by a non-transparent sender, because the sender’s hidden motives produce an additional incentive to invest in acquiring a precise signal of the object’s quality. I use my model to evaluate policies commonly proposed in the context of financial advisors, such as mandatory disclosure of commissions and commission caps.

## 1. INTRODUCTION

Brokerage companies employ large teams of analysts to produce research on financial products for their clients. A typical report on an asset includes market forecasts, a detailed valuation model, and a recommendation to buy, sell, or hold. Though these reports provide valuable information to investors,<sup>1</sup> the interests of profit-seeking brokers may not align with maximizing their clients’ welfare. It is well documented that brokers conceal bad news about companies in which they have financial interests and that financial advisors recommend unsuitable products with high commissions.<sup>2</sup> There are many other contexts in which people consult advisors with hidden motives: followers watch Instagram influencers exalt products they are paid to review, studies on the effectiveness of drugs are sponsored by pharmaceutical companies, and schools selectively disclose grades of tuition-paying students to employers.

Why do such arrangements survive? Why do people seek information from sources they know to be conflicted? In this paper, I study an advisor’s decision to produce and share information

---

<sup>†</sup>New York University, p.onuchic@nyu.edu. I am grateful for guidance from Debraj Ray, Erik Madsen and Ariel Rubinstein. I am indebted to Joshua Weiss and Samuel Kapon for long discussions and helpful comments. I thank Margaret Meyer, David Pearce, Dilip Abreu, Heski Bar-Isaac, Dhruva Bhaskar, Sylvain Chassang, Navin Kartik, Laurent Mathevet, Luis Rayo, Mauricio Ribeiro, Nikhil Vellodi, Basil Williams and other audience members at NYU for their inputs. This research was supported by NSF Grant number SES-1629370 to Debraj Ray.

<sup>1</sup>According to Hung et al. (2008), a majority of Americans rely on professional advice from their brokers or other financial advisors when conducting stock market or mutual fund transactions.

<sup>2</sup>See, for example, Anagol, Cole and Sarkar (2017) and Eckardt and Rathke-Doppner (2010) about insurance brokers; Chalmers and Reuter (2020) on retirement plans; Inderst and Ottaviani (2012.2) on general financial advice. For a survey on quality disclosure and certification, see Dranove and Jin (2010).

with a receiver. I provide one possible answer to these questions, by showing that an advisor's hidden motives may provide an additional incentive to produce information.

There are two agents in the model: a sender (he) and a receiver (she). The receiver takes a binary action, interpreted as buying or not buying an object. The object has two relevant characteristics – its *quality* to the receiver and its *profitability* to the sender. In the financial advisor context, a high quality object is a good investment for the client, while a highly profitable object yields high commissions to the advisor. Prior to the realization of either the profitability or the quality, the sender takes two actions. He acquires a costly signal of the object's quality and commits to a disclosure rule. This rule assigns to each realization of the quality signal and each profitability a probability of disclosing the realization to the receiver.

The sender has *hidden motives*: the profitability of the object is not observed by the receiver. In particular, if the distribution of profitabilities is degenerate, then the object's profitability is known, and I say the sender has *transparent motives*.<sup>3</sup> The receiver is Bayesian and updates beliefs based on any information the sender reveals and on the sender's policy itself. She buys the object if her posterior about its quality, net of some exogenously given price, exceeds the value of a privately known outside option.

In Section 3, I take the sender's information acquisition choice as given and characterize the sender's *optimal disclosure rules*. When choosing what information to share with the receiver, the sender treads a fine path. He wants to disclose positive evidence of the object's quality, so as to incentivize the receiver to make a purchase. He also knows that he can hide any evidence that the object is of low quality. However, if he does so, the receiver becomes skeptical and reads non-disclosure itself as a negative sign, lowering the probability of sale.

Because disclosure can be conditioned on profitability, the sender solves this balancing act with a disclosure rule that hides some negative evidence when the object's sale is very profitable and some positive evidence when the sale of the object is less profitable. By concealing bad news, he increases the probability that a highly profitable object is sold. At the same time, by concealing good news, he decreases the probability that less profitable objects are sold, but improves the receiver's posterior upon non-disclosure. As such, the sender, who is biased in favor of more profitable objects, steers purchases from low to high profitability objects.

In Theorem 1, I show that optimal disclosure rules follow a simple *threshold structure*. Each signal realization about the object's quality is classified as either positive evidence, if above some endogenously determined threshold, or negative evidence, if below that same threshold. Moreover, for each piece of positive evidence, there is a profitability threshold such that the evidence is disclosed if the object's profitability is above the threshold, and concealed otherwise.

---

<sup>3</sup>The term “transparent motives” was coined by Lipnowski and Ravid (2020) to refer to a problem where the sender's preferences are state-independent. In my model, this is precisely the case when the distribution of profitabilities is degenerate.

Conversely, each piece of negative evidence is concealed if the object is sufficiently profitable, and revealed otherwise.

Theorem 2 further characterizes the sender's optimal thresholds, and Propositions 1 and 2 apply this characterization to the cases where the distribution of receiver outside options are linear, convex or concave. Using the optimal disclosure rules, I show the effect of the sender's hidden motives on his informativeness to the receiver depends on the shape of the receiver's distribution of outside options. Taking as given the acquired signal, a sender with hidden motives is less informative than a transparent one if the distribution of outside options is convex, and more informative than a transparent one if the distribution of outside options is concave.

In Section 4, I apply the optimal disclosure rule characterization and study the relation between the degree to which the sender's motives are hidden and his decision to invest in acquiring a costly signal of the object's quality. In Proposition 3, I show that, if the distribution of receiver outside options is linear, the sender's hidden motives are an additional incentive to acquire a quality signal. In particular, the stronger the sender's hidden motives, the more he invests in the signal precision. Therefore, in the linear case, the amount of information provided to the receiver, as well as the receiver's surplus, may be *increasing in the sender's hidden motives*.

For some intuition on this result, notice that the sender benefits from selectively disclosing information about the object's quality. But in order to manipulate it, he must have the information in the first place. The more precise the signal acquired by the sender, the more he is able to profitably steer the receiver. Moreover, the sender's gain from steering is higher for the stronger his hidden motives are, thereby increasing the incentive to acquire a precise signal.

This result has implications for commonly proposed policies aiming to mitigate advisors' biases. In the financial advisory context, regulators have proposed and implemented a variety of policies restricting the payment or requiring the disclosure of any commission payments to investors, as well as capping the size of commission payments.<sup>4</sup> In Section 5, I consider the effect of such regulations. I show that these policies may in fact reduce the receiver's surplus by curbing the sender's incentives to acquire a quality signal. Despite this negative effect on the surplus to the receiver, the policies may be welfare enhancing, as they prevent the sender from over-investing in signal acquisition.

**1.1. Related Literature.** My paper contributes to the large literature on information design, mainly stemming from Kamenica and Gentzkow (2011) and Rayo and Segal (2010).<sup>5</sup> Unlike most of the literature, which studies problems where a sender commits to an experiment design, I consider the problem of a sender who commits to a rule to disclose hard evidence.

<sup>4</sup>Since 2011, New York state law mandates that insurance agents disclose their general compensation scheme to clients. As a response to the Great Recession, the Dodd-Frank act also granted the SEC the ability to impose a fiduciary duty on broker-dealers, which is already required of financial advisors. Other countries, such as the UK and the Netherlands have altogether imposed bans on commission payments for some types of financial advisors.

<sup>5</sup>For a survey, see Kamenica (2019)

The main feature of the disclosure schemes optimally chosen by the sender is that they pool good realizations for low profitability objects with bad realizations for high profitability objects, and transfer value from the former to the latter by doing so. This feature is equally the highlight of the optimal schemes in Rayo and Segal (2010). My paper adds to that in three main ways. First, Rayo and Segal (2010) study an information design problem, while I am interested in characterizing disclosure of hard evidence. I characterize optimal disclosure for general distributions of receiver outside options, while Rayo and Segal (2010) focus on the linear case. Second, in my model, the sender endogenously acquires the underlying quality signal at a cost, while this signal is exogenous in their paper. Finally, while Rayo and Segal (2010) are mainly interested in characterizing the sender's optimal disclosure rule, my focus is in studying how the sender's hidden motives affects their decision to acquire and disclose information. The analogous comparative static is not studied in their framework.

Gentzkow and Kamenica (2017) consider the problem of a sender that acquires a costly signal and disclose some of it to the receiver. They show that the sender always fully discloses the acquired signal. A similar point is made in Pei (2015). In my paper, the fact that the sender has hidden motives implies that this statement no longer holds. Gentzkow and Kamenica (2014) show that, if transmitting a signal to the receiver is costly, and the cost function over signals satisfies certain requirements, then the optimal signal can be found with an adaptation of their earlier concavification arguments. In my paper, I consider a sender who only chooses whether to disclose hard evidence or not, and thus the concavification method does not apply.

A literature studying disclosure of hard evidence started with Grossman (1981), Milgrom (1981).<sup>6</sup> A common result in that literature is that when senders choose to voluntarily disclose information, *unraveling* takes place and equilibria feature full revelation. In my paper, the sender is able to commit to a disclosure rule prior to the realization of the signal and of the object's profitability, and the usual unravelling argument does not apply. In fact, full revelation is often not an optimal disclosure rule in my model.

Dye (1985) and Jung and Kwon (1988) study a variant of voluntary disclosure models where the sender is informed about the state with an exogenous probability, but uninformed senders cannot prove that they are uninformed. In these models, when the sender does not disclose a signal realization, the receiver is unsure if the sender strategically chose non-disclosure or if he is uninformed. The sender optimally uses "sanitization" disclosure rules that reveal only good realizations of the signal. Bad realizations get pooled with the uninformative signal. In my model, the sender has hidden motives and chooses disclosure rules that depend on the object's profitability. When the object has higher profitability, the optimal disclosure rule has a similar flavor to sanitization: only good outcomes are revealed. On the other hand, when profitability is lower, the sender does the opposite and reveals only bad outcomes.<sup>7</sup>

---

<sup>6</sup>For a survey, see Milgrom (2008)

<sup>7</sup>Shishkin (2019) and DeMarzo, Kremer, and Skrzypacz (2019) study information acquisition by the sender in a Dye (1985) framework.

Kartik, Lee and Suen (2017) and Che and Kartik (2009) also study environments with endogenous information acquisition and voluntary disclosure.<sup>8</sup> Kartik, Lee and Suen (2017) show that an advisee may prefer to solicit advice from just one biased expert even when others (of equal or opposite bias) are available, because, in the presence of more advisors, each individual expert free rides on the information acquired by the other experts. In Che and Kartik (2009), though sender and receiver share the same preferences, they hold different priors over the distribution of states. Che and Kartik (2009) show that the sender may have stronger incentives to invest in acquiring information when the “disagreement” between sender and receiver is larger.

In a series of papers in 2012, Inderst and Ottaviani (2012.1, 2012.2, 2012.3) propose models of brokers and financial advisors compensated through commissions. Competing sellers play a game of offering commissions to the advisor, knowing that he will steer business to the seller that offers highest compensation. In their model, the price of the asset is always equal to buyers’ expected value for it, which means that information is not valuable to the consumer, as their surplus is always equal to zero. In that environment, biased commissioned agents may achieve efficiency when providing buyers with less information and steering business to high commission firms who are also more cost efficient.

My model takes an alternative approach, taking the sender’s distribution of profitability as given and focusing on the value of the information provided to buyers. In my environment, information is always beneficial to the consumer and I show that hidden motives can improve surplus precisely by increasing the amount of information provided to the consumer.

The literature on commissioned financial advisors is scarce prior to the mentioned series and, after these papers, it has been mostly empirical. However, there is a large literature that studies the provision of information by Credit Rating Agencies that are financed by fees paid by issuers of financial products. Some key papers in this literature are Bolton, Freixas, and Shapiro (2012), Opp, Opp and Harris (2013), Bar-Isaac and Shapiro (2012) and Skreta and Veldkamp (2009). In this literature, the Credit Rating Agency receives payments equally from all issuers of financial products; while in my paper, the main concern is that the advisor might choose to benefit some products over other because they have different profitabilities.

## 2. ENVIRONMENT - INFORMATION DISCLOSURE

There are two players: a sender and a receiver. The receiver can buy an object of unobserved quality<sup>9</sup>  $x \in \mathcal{X} = [x_{min}, x_{max}]$ , and unobserved profitability  $w \in \mathcal{W} = [w_{min}, w_{max}]$ . Quality and profitability are drawn from a joint distribution known by both the sender and the receiver. The marginal distribution of profitabilities is  $W$ . The sender is said to have *transparent motives*

<sup>8</sup>Szalay (2005) and Ball and Gao (2020) study information acquisition by a biased agent in a delegation context.

<sup>9</sup>Quality  $x$  is the value of the product to the receiver, *net of its price*. In various applications, such as the highlighted one where the sender is a financial advisor who receives kickbacks for sales of financial products, products’ prices are pre-set and cannot be negotiated between the financial advisor and the buyer.

when  $W$  is a degenerate distribution, so that the object's profitability is known. He has *hidden motives* otherwise.

The receiver has an outside option  $y \in [y_{min}, y_{max}]$ , unknown to the sender, which is drawn from a continuous distribution  $Y$ .<sup>10</sup> If the receiver buys the object, she gets value  $x$  and the sender gets value  $w$ . If the receiver does not buy the object, then she gets her outside option  $y$  and the sender gets value 0.

The sender has access to a signal about the object's quality. The signal is a mapping between the object's quality and a distribution of messages. Given the shared prior, the signal induces a distribution of posterior means about the object's quality. Formally, take a signal:  $\pi : \mathcal{X} \rightarrow \Delta\mathcal{M}$ , where  $\mathcal{M}$  is a rich enough set of possible messages. Using  $\pi$  and the players' common prior, each message  $m \in \mathcal{M}$  can be mapped into the expected quality of the object given that message – the posterior mean induced by the message – using Bayesian updating.

This procedure maps signal  $\pi$ , a distribution over messages, into a distribution over posterior means. Since the receiver is risk-neutral, her choice between buying the object or not depends only on her expectation of the object's quality, and thus the induced distribution of posterior means is a sufficient description of a signal. Let  $S_w(\cdot)$  be the distribution of posterior means induced by the signal when the object's profitability is  $w$ . This distribution varies with the object's profitability because quality and profitability may not be independently distributed.

At an ex-ante stage, the sender commits to a *rule to disclose* signal realizations to the receiver. Each of the signal realizations can be either revealed to the receiver or not, and this choice can depend on the profitability of the object. A *disclosure rule* is a measurable mapping from profitabilities and signal realizations into probabilities of disclosure:  $d : \mathcal{W} \times \mathcal{X} \rightarrow [0, 1]$ . A combination of signal and disclosure rule induces a *disclosed signal*, which is observed by the receiver.

The receiver chooses to buy the object or not after observing the realization of the disclosed signal. She buys the object if its posterior expected quality is higher than her outside option. Therefore, if a realization of the disclosed signal induces a posterior expected quality of  $\hat{x}$  on the receiver, then the sender expects that the object will be purchased with probability  $Y(\hat{x})$ , the probability that the outside option is lower than  $\hat{x}$ . The distribution of the receiver's outside option,  $Y$ , can therefore be seen as the demand function faced by the sender. It maps the expected quality of the object (from the receiver's perspective) into probabilities of purchase. Throughout the paper, I refer to the distribution of outside options and to the demand function interchangeably. The solution concept is Perfect Bayesian Equilibrium.

Figure 1 summarizes the timing of play.

<sup>10</sup>This setup can be interpreted either as the sender disclosing to a population of receivers with a distribution of outside options or literally to a receiver with unknown outside option. With some notational hurdles, Theorem 1 can be extended to allow for distributions  $Y$  with mass points, encompassing the case where the outside option is known (where the distribution is degenerate).



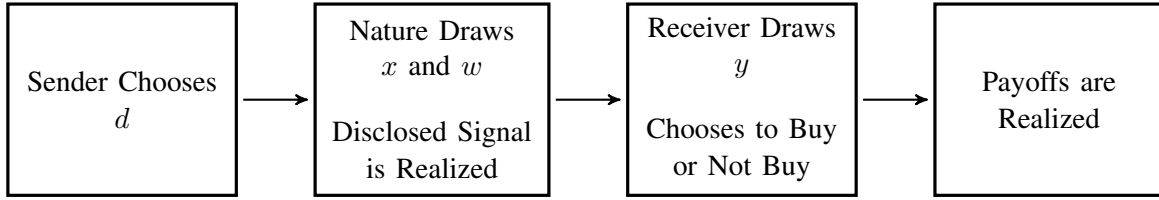


FIGURE 1. Timing of play.

**2.1. Informativeness and Receiver's Surplus.** When a signal realization  $x$  is disclosed, the receiver's posterior mean after observing it is  $x$  itself. On the other hand, when it is not disclosed, the receiver's posterior mean is given by<sup>11</sup>

$$(1) \quad x^{ND} = \frac{\int_{\mathcal{W}} \int_{\mathcal{X}} [x(1 - d(w, x))] dS_w(x) dW(w)}{\int_{\mathcal{W}} \int_{\mathcal{X}} [1 - d(w, x)] dS_w(x) dW(w)}$$

which is the expected quality over all the signal realizations that are not disclosed.

Given a disclosure rule  $d$ , we can use  $S_w(\cdot)$  for each profitability  $w$  and (1) to generate the distribution of posterior means observed by the receiver, denoted  $R(\cdot; d)$ .<sup>12</sup> A disclosed signal produced by  $d$  is *more informative than* one produced by  $d'$  if  $R(\cdot; d)$  is more informative than  $R(\cdot; d')$  in the Blackwell order.<sup>13</sup> The next observation is a known implication of this ordering.

**Observation 1.** *Receiver surplus is increasing in the informativeness of the disclosed signal.*

**2.2. An Interpretation.** The model describes a game between two players, where a sender informs a receiver about the quality of an object and also sells the object to this receiver. This setup where an advisor's interests are not aligned with those of the advisee is ubiquitous, but I highlight the case of financial advisors and brokers of financial products.

When you open a brokerage account at Morgan Stanley, you get access to research reports put together by their Equity Research team. In addition to publicly available data about companies

<sup>11</sup>If  $\int_{\mathcal{W}} \int_{\mathcal{X}} [1 - d(w, x)] dS_w(x) dW(w) = 0$ , we let  $x^{ND} = 0$ . This is a harmless assumption, since if not disclosing is used only for a measure zero of signals, then the event of a signal not being disclosed does not enter the sender's value.

<sup>12</sup>If  $x < x^{ND}$ ,

$$R(x; d) = \int_{\mathcal{W}} \int_{[x_{min}, x)} dS_w(\hat{x}) dW(w)$$

If  $x \geq x^{ND}$ ,

$$R(x; \theta, d) = \int_{\mathcal{W}} \int_{[x_{min}, x)} dS_w(\hat{x}) dW(w) + \int_{\mathcal{W}} \int_{\mathcal{X}} (1 - d(w_L, \hat{x})) dS_w(\hat{x}) dW(w)$$

<sup>13</sup>Equivalently, if  $R(\cdot; d)$  is a mean preserving spread of  $R(\cdot; d')$ .

and industries, a report on a particular product includes forecasts, valuations and recommendations to buy, sell or keep the asset in your portfolio. Upon seeing the provided research, an investor compares the perceived value of the product to an outside option, which could depend on their current appetite for investment, desire to reallocate their current portfolio, or even independent information she may have sourced about the financial product at hand.

As in the model, the incentives of the advisor/broker and those of the investor may not always be aligned. For instance, some of the products available in the brokerage system are proprietary products, which are investments that are issued or managed by Morgan Stanley. Upon selling one of these assets, the broker receives extra compensation. Another source of conflict is that third parties commonly pay the broker for marketing and selling their products, which may make the sale of some products more desirable than others. These considerations can fuel the broker's desire to produce advice that steers investors to the more profitable products.

A feature of the model is that the sender produces information about a product and chooses only whether to share the outcome of the receiver or not. Similarly, when the research team at Morgan Stanley creates a report on a product, they cannot outright lie about their forecasts or valuation models, but they do have discretion in choosing whether and how to disclose the outcome of their research, as well as in picking which models to disclose.

The information that is *not* revealed by a report is as important as information that is provided. To the receiver, seeing that the outcome of research is not displayed is in itself an important signal. In the advisor/broker example, when an investor sees that the research team chose to not make a buy or sell recommendation and to display only very short-term forecasts for the performance of a stock or not much information about the expected trends of a particular industry targeted by a mutual fund, she creates a conjecture about the value of the product.

Suppose, for instance, that each time the advisor chooses not to disclose information it later comes out that the performance of the product is bad. In equilibrium, a buyer should become skeptical and understand the absence of news as bad news. In the other direction, now in the language of the model, if the sender commits to often concealing high realizations of the signal, the receiver interprets non-disclosure as a signal of the object's high quality. Importantly, when the sender in the model chooses a disclosure rule, they understand that it will have effects on their *reputation* – specifically, on the interpretation of non disclosed signals. When the Equity Research team chooses to disclose bad forecasts on a product they wish to sell, they do so eyeing the fact that they are building the reputation of Morgan Stanley's research reports.

### 3. OPTIMAL DISCLOSURE RULE

The expected value to the sender who chooses disclosure rule  $d$  is

$$(2) \quad \Pi(d) = \mathbb{E}[wP(w; d)]$$



where  $P(w; d) = \int_{\mathcal{X}} Y(x) d(w, x) dS_w(x) + \int_{\mathcal{X}} Y(x^{ND}) (1 - d(w, x)) dS_w(x)$  is the expected probability of sale when the object has profitability  $w$ . The expectation in (2) is taken with respect to the marginal distribution of profitabilities. The sender is biased towards higher profitability objects, since the expected probability of their sale is weighted more highly than that of lower profitability ones.

One useful rewriting of the sender's value is

$$(3) \quad \Pi(d) = \mathbb{E}(w) \mathbb{E}[P(w; d)] + \text{Cov}[w, P(w; d)]$$

In choosing the disclosure rule  $d$ , the sender's objective is twofold: first, the sender wishes to maximize the overall expected probability of sale, which is multiplied by the average profitability in the first term; second, the sender seeks to maximize the covariance between the object's profitability and its probability of sale.

Theorem 1 brings a first characterization of disclosure rules that maximize the sender's value. Optimal disclosure rules exhibit a threshold structure. Each signal realization about the object's quality is classified as either positive evidence, if  $x$  is larger than  $\bar{x}$ , the quality threshold, or negative evidence, if  $x < \bar{x}$ . Additionally, for each piece of positive evidence, there is a profitability threshold such that the evidence is disclosed if the object's profitability is above the threshold, and concealed otherwise. Conversely, each piece of negative evidence is concealed if the object is sufficiently profitable, and revealed otherwise.

Figure 2 depicts the threshold structure of optimal disclosure rules. Note that, in the picture, the profitability threshold is a continuous function, but that is not guaranteed by Theorem 1.

**Theorem 1.** *An optimal disclosure rule  $d^*$  exists and has a threshold structure:*

*There is a **quality threshold**  $\bar{x} \in \mathcal{X}$  and a **profitability threshold**  $\bar{w} : \mathcal{X} \rightarrow \mathcal{W}$  such that  $d^*$  almost everywhere satisfies<sup>14</sup>*

$$d^*(w, x) = 1 \text{ when } x > \bar{x} \text{ and } w > \bar{w}(x), \text{ or } x < \bar{x} \text{ and } w < \bar{w}(x);$$

$$d^*(w, x) = 0 \text{ when } x > \bar{x} \text{ and } w < \bar{w}(x), \text{ or } x < \bar{x} \text{ and } w > \bar{w}(x).$$

*Moreover, in the optimal disclosure rule, the quality threshold satisfies  $\bar{x} = x^{ND}$ .*

To prove Theorem 1, I show that any disclosure rule that does not satisfy the threshold structure described in the theorem can be improved upon. Start with such a disclosure rule  $d$ , and let  $x^{ND}$  be the expected quality given that a realization is not disclosed that is implied by  $d$  – as defined in equation (1).

Now define an alternative disclosure rule,  $\hat{d}$ , that discloses each signal realization  $x$  with the same probability as  $d$ , but has a threshold structure. That is, for each  $x \in [x_{min}, x_{max}]$  there is

---

<sup>14</sup>Almost everywhere with respect to the joint distribution of profitabilities  $w$  and signal realizations  $x$ .

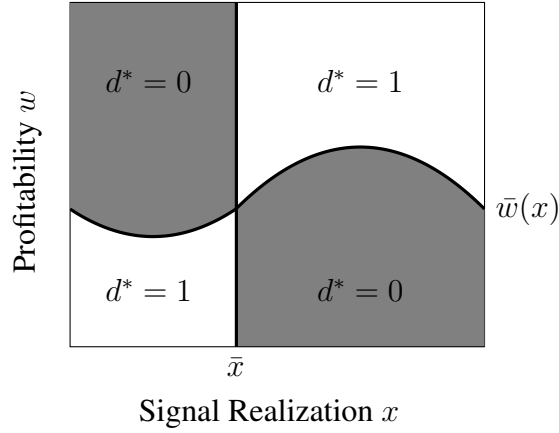


FIGURE 2. A disclosure rule with a threshold structure: quality threshold  $\bar{x}$  and profitability threshold  $\bar{w}(x)$ . The sender discloses when both the signal realization and profitability are sufficiently large or when both are sufficiently small.

some  $\hat{w}(x)$  such that, if  $x \leq x^{ND}$ :

$$\hat{d}(w, x) = \begin{cases} 1, & \text{if } w < \hat{w}(x) \\ \in [0, 1], & \text{if } w = \hat{w}(x) \\ 0, & \text{if } w > \hat{w}(x) \end{cases}$$

If  $x > x^{ND}$ :

$$\hat{d}(w, x) = \begin{cases} 0, & \text{if } w < \hat{w}(x) \\ \in [0, 1], & \text{if } w = \hat{w}(x) \\ 1, & \text{if } w > \hat{w}(x) \end{cases}$$

And for any signal realization  $x$ :

$$(4) \quad \int_{\mathcal{W}} \hat{d}(w, x) dW_x(w) = \int_{\mathcal{W}} d(w, x) dW_x(w)$$

where  $W_x$  is the profitability distribution conditional on a signal realization  $x$ . Because  $d$  and  $\hat{d}$  disclose each realization with the same probability, they also induce the same  $x^{ND}$ .<sup>15</sup>

In Appendix A, I show two facts comparing the two disclosure rules: 1.  $d$  and  $\hat{d}$  produce the same overall probability of sale; and 2.  $\hat{d}$  induces a larger covariance between sales and

<sup>15</sup>To see this, note that the definition of  $x^{ND}$  in (1) is equivalent to

$$x^{ND} = \frac{\int_{\mathcal{X}} x \int_{\mathcal{W}} (1 - d(w, x)) dW_x(w) dS(x)}{\int_{\mathcal{X}} \int_{\mathcal{W}} (1 - d(w, x)) dW_x(w) dS(x)}$$

where  $W_x$  is the profitability distribution conditional on a signal realization  $x$  and  $S$  is the marginal distribution of signal realizations.

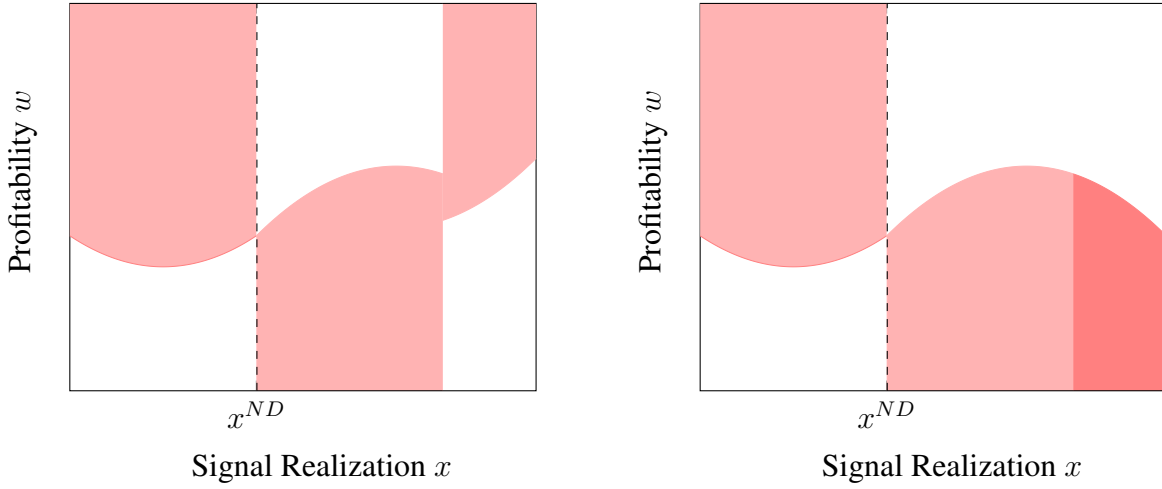


FIGURE 3. The disclosure rule depicted on the right panel is an improvement over the disclosure rule in the left panel: it produces the same overall probability of sale, but a larger covariance between sales and profitability.

profitability than  $d$ . These, along with the sender's value as stated in (3) imply that  $\hat{d}$  produces a higher sender value than  $d$ , proving that the optimal disclosure rule must have a threshold structure. In Appendix A, I also prove that an optimal disclosure rule exists.

Figure 3 shows an example of a disclosure rule  $d$  that does not have a threshold structure and an improvement  $\hat{d}$ . By shifting all the disclosure probability to low profitability objects when  $x < x^{ND}$  and to high profitability objects when  $x > x^{ND}$ , the sender maintains the distribution of posterior means over quality that is induced on the receiver. At the same time, this change increases the probability that very profitable objects are sold, and decreases that probability for less profitable objects. This change is beneficial to the sender.

Theorem 1 argues that optimal disclosure rules feature threshold structures. Theorem 2 below provides further characterization of the profitability threshold in the optimal disclosure rule. Define  $w^{ND}$  to be the average object profitability given that a signal realization is not disclosed – this object can be computed analogously to  $x^{ND}$  in (1).

**Theorem 2.** *If  $Y$  is differentiable, the profitability threshold in an optimal disclosure rule satisfies, for  $x \neq x^{ND}$ :*

$$(5) \quad \bar{w}(x) = w^{ND} \left[ \frac{Y'(x^{ND})(x^{ND} - x)}{Y(x^{ND}) - Y(x)} \right]$$

Moreover, there exists a disclosure rule with  $\bar{w}(x^{ND}) = w^{ND}$ .

The Proof of Theorem 2 is in Appendix A. I differentiate the sender's value with respect to the probability of disclosure of each signal realization  $x$  and profitability  $w$ , and show that the

value to the sender can be marginally improved whenever the disclosure rule does not satisfy a threshold structure with the profitability threshold as described in the Theorem.

When the joint distribution of profitability and signal realizations has no mass points, Theorems 1 and 2 achieve a significant simplification of the sender's problem. All the disclosure rules with the described threshold structure are fully defined by two numbers:  $x^{ND}$  and  $w^{ND}$ .<sup>16</sup> As such, the dimensionality of the sender's problem is reduced to two.

Also note that the distribution of profitability and signal realizations only affects the shape of the optimal thresholds through the sender's choice of  $x^{ND}$  and  $w^{ND}$ . From equation (5), we can see that some features of the optimal profitability threshold are defined by  $Y$ , the distribution of outside options of the receiver. These features, further described in the Propositions below, do not depend on the correlation between the object's quality and profitability.

**3.1. Linear Demand.** With more information about the distribution of the receiver's outside option (the demand function faced by the sender), we can apply Theorem's 1 and 2 and further characterize the sender's optimal disclosure rule.

First, take the distribution of receiver outside options to be uniform, so that  $Y$  is a linear function. Observation 2 below states that the expected probability of sale is constant and equal to the underlying expected quality of the object. This means that, in choosing a disclosure policy, there is no scope for the sender to increase or decrease the overall probability that the receiver buys the object. Rather, a sender with hidden motives solely distributes this constant value between the high and low realizations of the object's profitability.

**Observation 2.** *Let  $Y$  be linear. Then, for any disclosure rule  $d$ ,  $\mathbb{E}[P(w; d)] = \mathbb{E}(x)$ .*

A straightforward implication of this Observation is that a sender with transparent motives is indifferent between all disclosure rules.<sup>17</sup> Moreover, applying Theorem 2 to the linear demand case, we find that the profitability threshold is constant across all signal realizations.

**Proposition 1.** *Let  $Y$  be linear.*

- (1) *A sender with transparent motives is indifferent between all disclosure rules;*
- (2) *To a sender with hidden motives, an optimal disclosure rule has a threshold structure, with  $\bar{x} = x^{ND}$  and  $\bar{w}(x) = w^{ND}$ , for all  $x \in \mathcal{X}$ .*

Figure 4 displays an optimal disclosure rule  $d^*$ . In Appendix B, Figure 8 shows the posterior mean induced on the receiver by  $d^*$  as a function of profitability and the signal realization.

Given linear demand, the objective of the sender is simply to maximize the covariance between the sale probability and the object's profitability. This maximal covariance is achieved by

<sup>16</sup>To be more detailed, the threshold structure proposed in the Theorems does not fully describe the disclosure rules, because it does not define them exactly at the thresholds (note that all inequalities are strict in Theorem 1).

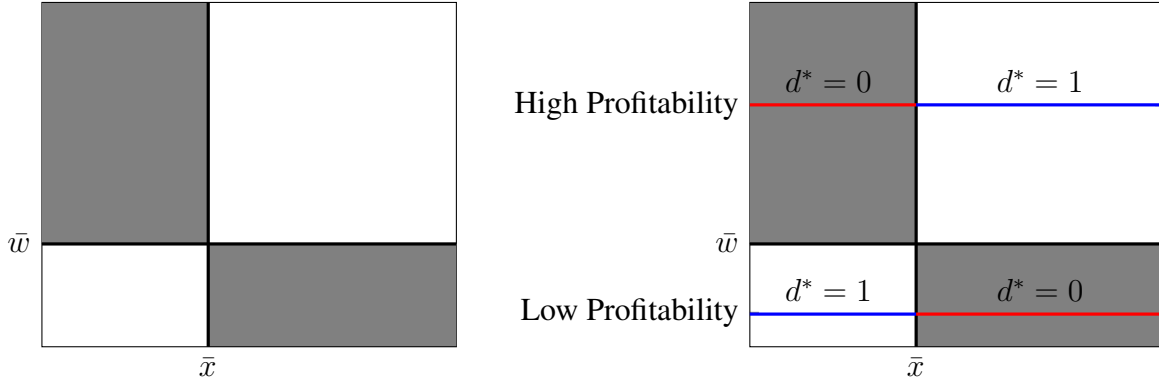


FIGURE 4. Optimal Disclosure Rule when  $Y$  is **linear**. The gray shaded areas represent signal realizations that are optimally concealed by the sender, and the white areas are optimally disclosed. On the right panel, two disclosure regimes are highlighted: for High Profitability objects ( $w > \bar{w}$ ), low realizations are concealed; for Low Profitability objects ( $w < \bar{w}$ ), high realizations are concealed.

assigning objects to two classes, based on their profitability: high profitability, with  $w > w^{ND}$ , and low profitability, with  $w < w^{ND}$ .

When the sender uses the optimal disclosure rule, the receiver's posterior mean for low profitability objects is *at best* equal to  $x^{ND}$ , which is the posterior mean induced when the signal realizations above  $x^{ND}$  are not disclosed. Conversely, the receiver's induced posterior mean for high profitability is *at least* equal to  $x^{ND}$ , which is induced when signal realizations below  $x^{ND}$  are not disclosed. By strategically using the option of non-disclosure, the sender is able to *steer* sales from low to high profitability objects.

Proposition 1 also shows that there is a discontinuity between the policy chosen by the sender with transparent motives and that chosen by any sender with motives that are even “slightly” hidden. The former is indifferent between all policies and, in particular, disclosing all signals is an optimal disclosure policy. However, any sender with hidden motives uses a threshold structure with two profitability classes.

**3.2. Nonlinear Demand.** We saw that, when the distribution of receiver outside options is linear, the sender can use strategic disclosure to distribute probability of sale between realizations of the object's profitability. However, regardless of the disclosed signal chosen by the sender, the overall probability of sale is a constant. With a nonlinear demand, distributing probability of sale across profitabilities comes at the expense of the total probability that the

This is not an issue if the joint distribution of profitability and signal realizations has no mass points, since in that case the sender's value is not affected by the disclosure rule at the thresholds.

<sup>17</sup>When the sender has transparent motives,  $W$  is a degenerate distribution, and so  $\text{Cov}[w, P(w; d)] = 0$ . Along with Observation 2, this means that  $\Pi(d) = \mathbb{E}(w)\mathbb{E}(x)$ , for any disclosure rule  $d$ .

object is sold. If the demand is not linear, the informativeness of the disclosed signal impacts the overall probability of sale.

**Observation 3.** *Let disclosure rule  $d$  be strictly more informative than disclosure rule  $d'$ .*

- (1) *If  $Y$  is strictly convex, then  $d$  yields a higher probability of sale than  $d'$ ;*
- (2) *If  $Y$  is strictly concave, then  $d$  yields a lower probability of sale than  $d'$ .*

If the sender has transparent motives and faces a convex demand function, he is incentivized to produce and disclose information about the object's quality. On the other hand, if facing a concave demand, his incentives are to conceal any information from the receiver.

Let  $\bar{d}$  be the disclosure rule that reveals all signal realizations. That is,  $\bar{d}(x, w) = 1$  for all  $x \in [0, 1]$  and  $w \in [\underline{w}, \bar{w}]$ . Conversely, let  $\underline{d}$  be the disclosure rule that conceals all signal realizations –  $\underline{d}(x, w) = 0$  for all  $x$  and  $w$ .

**Proposition 2.** *If  $Y$  is strictly convex:*

- (1) *To a sender with transparent motives,  $\bar{d}$  is an optimal disclosure rule<sup>18</sup>;*
- (2) *To a sender with hidden motives, an optimal disclosure rule has a threshold structure with  $\bar{x} = x^{ND}$ , and  $\bar{w}(x)$  a strictly decreasing function with  $\bar{w}(x^{ND}) = w^{ND}$ .*

*If  $Y$  is strictly concave:*

- (1) *To a sender with transparent motives,  $\underline{d}$  is an optimal disclosure rule;*
- (2) *To a sender with hidden motives, an optimal disclosure rule has a threshold structure with  $\bar{x} = x^{ND}$ , and  $\bar{w}(x)$  a strictly increasing function with  $\bar{w}(x^{ND}) = w^{ND}$ .*

When  $Y$  is strictly convex, a transparent sender maximizes the overall probability of sale by disclosing every signal realization. On the other hand, a sender with hidden motives chooses to strategically hide some of the signal realizations in order to improve the covariance between sales and profitability.

An optimal disclosure rule is represented in Figure 5. There is some threshold signal realization  $\bar{x}$  such that an interval of quality signal realizations right below  $\bar{x}$  are optimally concealed when profitability is high. Conversely, an interval of signal realizations right above  $\bar{x}$  is optimally concealed when profitability is low. These intervals are larger for more extreme profitability values. A striking difference when comparing to disclosure rules when  $Y$  is linear is that some very bad signal realizations may be optimally disclosed even when the object

<sup>18</sup>There is a trivial multiplicity of optimal disclosure rules. If there is only one signal realization that is not disclosed, then, to the receiver, this is equivalent to all signal realizations being disclosed. Despite  $\bar{d}$  not being the only solution, all optimal disclosure rules induce the same receiver's distribution of posterior means, equal to  $R(\cdot; \bar{d}) = S(\cdot)$ .

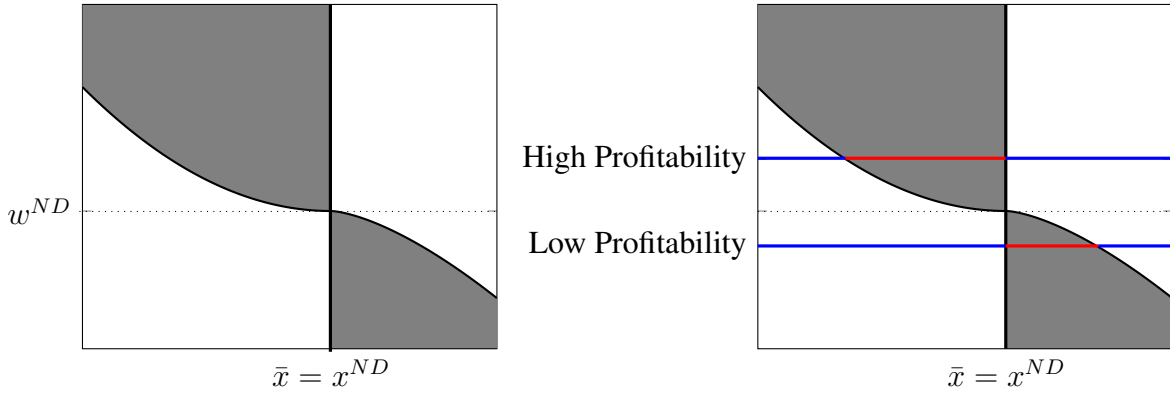


FIGURE 5. Optimal Disclosure Rule when  $Y$  is **convex**. The gray shaded areas represent signal realizations that are optimally concealed by the sender, and the white areas are optimally disclosed.

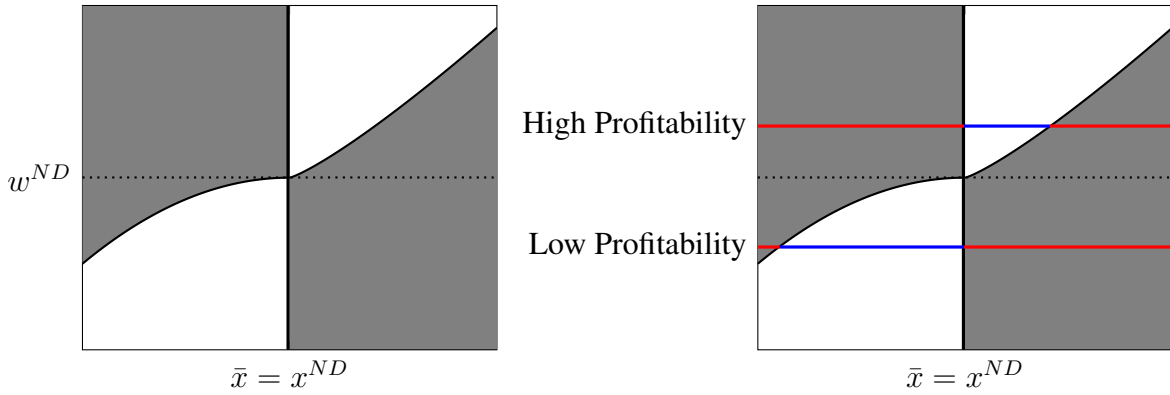


FIGURE 6. Optimal Disclosure Rule when  $Y$  is **concave**. The gray shaded areas represent signal realizations that are optimally concealed by the sender, and the white areas are optimally disclosed.

is highly profitable; and, conversely, some very good signal realizations are disclosed to the receiver even when the object has low profitability.

For each profitability, the posterior mean induced on the receiver by the optimal disclosure rule is increasing in the signal realization. Despite the fact that some signal realizations are not disclosed, higher signal realizations always map into higher probabilities that the object is sold. This feature is depicted in Figure 9 in Appendix B.

Consider the opposite case, where  $Y$  is strictly concave. Observation 3 tells us that more informative disclosed signals yield lower total sale probability. Therefore, a transparent sender optimally conceals all signal realizations. A sender with hidden motives, again, has to balance two objectives: maximizing total sales, and distributing them from lower profitability objects



to higher profitability ones. To that end, it is profitable to strategically reveal some outcomes of the quality signal, rather than fully concealing all signal realizations.

The optimal disclosure rule reveals signal realizations that fall in an interval right above  $x^{ND}$ , when the object has higher profitability, and an interval right below  $x^{ND}$  when the object has lower profitability, as shown in Figure 6. Unlike in the convex case, the posterior mean induced on the receiver may not be increasing (see Figure 10 in Appendix B). This means that, for a given profitability, higher signal realizations may map into lower sale probabilities.

The next result is a corollary of Proposition 2 which compares transparent senders and senders with hidden motives in terms of their informativeness.

**Corollary 1.** *A sender with hidden motives: i. generates less probability of sale than a transparent sender; ii. is less informative than a transparent sender, if  $Y$  is strictly convex; and iii. is more informative than a transparent sender, if  $Y$  is strictly concave.*

In Appendix C, I compute the optimal disclosure rule in an example where  $Y$  is convex, and show that informativeness strictly decreases as the sender's motives become "more hidden".

#### 4. HIDDEN MOTIVES AND SIGNAL ACQUISITION

In the baseline model of Section 3, the signal about the object's quality that is observed by the sender is taken as given, and I characterize the sender's optimal policy to disclose that signal realizations to the receiver. In this section, I study the sender's decision to acquire a costly signal about the object's quality, as well as how this decision is impacted by the extent to which the sender's motives are hidden.

**4.1. Signal Acquisition Technology.** Prior to either the profitability or quality being drawn, the sender commits to two actions: he *acquires a costly signal* of the object's quality and chooses a rule to disclose that signal (as before).

Signals are indexed by their precision  $\theta$ , and are acquired at a cost  $c(\theta)$ , where  $c$  is non-decreasing, with  $c(0) = 0$  and  $c(\theta) > 0$  for all  $\theta > 0$ .<sup>19</sup> In this section, I assume that quality and profitability are independently distributed, and let  $S_w(\cdot; \theta) = S(\cdot; \theta)$  be the distribution of posterior means induced by a signal of precision  $\theta$  for every profitability  $w \in \mathcal{W}$ .

**ASSUMPTIONS:** (i)  $\theta' \geq \theta$  implies  $S(\cdot; \theta')$  is a mean-preserving spread of  $S(\cdot; \theta)$ ;

(ii)  $S(\cdot; 0)$  is the degenerate distribution at  $\mathbb{E}(x)$ ;

<sup>19</sup>In much of the literature on costly information acquisition or disclosure, signals are not indexed by a one dimensional "precision" parameter. Rather, it is usual to assume that there is a cost function defined over the space of signals, and such that more informative signals are associated to higher costs. Here, I choose to index signals by a scalar precision mostly for ease of exposition, but the result can be adapted to a model where the cost is defined over the space of signals.

Assumption (i) defines a more precise signal: more precise signals are Blackwell more informative than less precise ones. Assumption (ii) states that the signal with precision  $\theta = 0$  is the perfectly uninformative signal.

**4.2. An Order on the Sender's Hidden Motives.** A sender's motives are more hidden when his distribution of profitabilities is more spread. In particular, I focus on linear spreads of the profitability distribution, as defined below. On the one end, if the sender is transparent, his profitability distribution is degenerate. At the other end, his motives grow more opaque when the profitabilities become more diffuse, while keeping the average profitability fixed.

**Definition 3.** A distribution  $H'$  is a linear mean preserving spread of a distribution  $H$  if  $\mathbb{E}_{H'}(z) = \mathbb{E}_H(z)$  and one of the following holds:

- (a)  $H$  is the degenerate distribution at  $\mathbb{E}_H(z)$ ;
- (b) There exist  $\alpha \geq 1$  and  $\beta \in \mathbb{R}$  such that, for every  $q \in [0, 1]$ ,<sup>20</sup>

$$H'^{-1}(q) = \alpha H^{-1}(q) + \beta$$

A linear mean preserving spread of a distribution is essentially a renormalization of that distribution that makes it more spread out. For example, increasing the variance of a normal distribution leads to a linear mean preserving spread of the original distribution. Likewise, increasing the support of a uniform distribution symmetrically around the mean also leads to a mean preserving spread of the original distribution. More generally, the defining feature of a linear mean preserving spread is that the distance between every two quantiles of the distribution increases by a factor  $\alpha \geq 1$ .

**Definition 4.** A sender with profitability distribution  $W'$  has **more hidden motives** than one with profitability distribution  $W$  if  $W'$  is a linear mean preserving spread of  $W$ .

**4.3. Linear Demand.** The proposition below states that hidden motives provide incentives for the sender to acquire a precise signal. In fact, when the sender's motives are transparent, the sender is not willing to invest at all in signal acquisition.

**Proposition 3.** Let  $Y$  be linear. Senders with more hidden motives acquire weakly more precise signals. In particular, a sender with transparent motives acquires  $\theta = 0$ , a perfectly uninformative signal.

The full proof is in the Appendix. Although the proposition is stated weakly, there are mild assumptions (described in footnote 21) under which information acquisition strictly increases as the sender's motives become more hidden.

<sup>20</sup>If  $H$  and  $H'$  are continuous and strictly increasing, their inverses are well defined. If not, then let  $H^{-1}(q) = \inf\{x : H(x) \geq q\}$  and  $H'^{-1}$  accordingly.

The main observation is that an increase in the signal precision increases the spread between the sale probability to high and low profitability objects. When the sender has access to more precise information, then his ability to reveal to the receiver that a high profitability object is “very good”, or that a low profitability object is “very bad”, is greater. Recall that by linearity of the demand function, the sender cannot affect the overall probability of sale by acquiring more information. In the linear setting, then, the *only* use of information acquisition is to create a transfer of sale probability from from low to high profitability objects. But the extent to which the sender’s motives are hidden determine the value he gets from steering – when motives are more hidden, steering is more profitable. It is natural, then, that senders with more hidden motives have stronger incentives to invest in acquiring precise signals.

A bit more formally, we can write the value of acquiring a signal of precision  $\theta$  as

$$(6) \quad \max_d \Pi(\theta, d; W) = \mathbb{E}(w)\mathbb{E}(x) + \pi(\theta, W) - c(\theta)$$

$$\text{where } \pi(\theta, F) = \max_d \{\text{Cov}(w, P(w; \theta, d); W)\}$$

First notice that, since for a transparent sender,  $\text{Cov}(w, P(w; \theta, d)) = 0$  for all  $d$ , then it must be that there is no benefit from acquiring a more precise signal and he chooses  $\theta = 0$ .

In the Appendix, I show two things. First, that  $\pi(\theta, W)$  is increasing in  $\theta$ .<sup>21</sup> Second, I show that, if  $W'$  is a linear mean preserving spread of  $W$ , with factor  $\alpha > 1$ , then  $\pi(\theta, W') = \alpha\pi(\theta, W)$ . Putting these two facts together, I find that the marginal benefit of acquiring precision is larger for senders with more hidden motives.

Proposition 3 has an almost immediate implication that senders with more hidden motives are no less informative than more transparent senders.

**Proposition 4.** *Informativeness of the optimal disclosed signal does not decrease as the sender’s motives become more hidden. In particular, a sender with hidden motives produces a weakly more informative disclosed signal than a sender with transparent motives.*

Notice that, since the informativeness order is not complete, a not less informative signal is not the same as a weakly more informative signal. One can produce examples where, as motives become more hidden, the sender acquires more information, discloses a no less informative signal, and yet, the receiver’s surplus shrinks. However, Proposition 4 does guarantee that

<sup>21</sup>I show in the Appendix that we can write

$$\pi(\theta, W) = \int_{[\hat{w}, \bar{w}]} (w - \mathbb{E}(w)) dW(w) \left[ \int_{[0, \hat{x}]} S(x; \theta) dx + \int_{[\hat{x}, 1]} (1 - S(x; \theta)) dx \right]$$

where  $\hat{x}$  and  $\hat{w}$  are as given in Proposition 1. Both  $\int_{[0, \hat{x}]} S(x; \theta) dx$  and  $\int_{[\hat{x}, 1]} (1 - S(x; \theta)) dx$  weakly increase in  $\theta$ , by the mean preserving spread property. I say that, for  $\theta' > \theta$ ,  $S(\cdot; \theta')$  provides more information than  $S(\cdot; \theta)$  across  $\hat{x}$  if either  $\int_{[0, \hat{x}]} S(x; \theta) dx$  or  $\int_{[\hat{x}, 1]} (1 - S(x; \theta)) dx$  strictly increase with  $\theta$ . In that case, Proposition 3 can be stated strictly.

the receiver is weakly better off when the sender has hidden motives, as opposed to fully transparent motives.

**4.4. An Example.** Suppose  $W$  is such that, with probability  $1/2$ , the object is of high profitability ( $w = w_H$ ) and, with probability  $1/2$ , it has low profitability ( $w = w_L$ , where  $w_H \geq w_L$ ). In that case, we can pin down the threshold quality in the optimal disclosure rule, as described in Proposition 1.

**Observation 4.** *The threshold  $\bar{x}$ , described in Proposition 1, is equal to  $\mathbb{E}(x)$ , the underlying average quality of the object.*

In this case, the optimal disclosure rule does not depend on the precision acquired by the sender, since, for all  $\theta$ , the expected value of  $x$  under  $S(\cdot; \theta)$  is always  $\mathbb{E}(x)$ . The disclosure rule also does not vary with the degree to which the sender's motives are hidden (the difference between  $w_H$  and  $w_L$ ). In fact, the sender always discloses *above average* realizations when profitability is high and *below average* realizations when profitability is low. In this case, we can show that the receiver's surplus is weakly increasing in the sender's hidden motives.

**Observation 5.** *Informativeness is increasing in the sender's hidden motives.*

Now let's further assume that the true underlying distribution of the object's quality is uniform over  $[0, 1]$  and the available quality signals, indexed by  $\theta \in [0, 1]$ , are such that they reveal the object's true quality with probability  $\theta$  and are perfectly uninformative with probability  $1 - \theta$ .

If the true quality is  $x$  and the signal perfectly reveals the quality, then the sender understands the quality to be exactly  $x$ . If, on the other hand, the signal is uninformative, then the sender gains no insight into the object's quality, so his quality estimate is equal to the average quality,  $\mathbb{E}(x) = 1/2$ . Hence, for a given  $\theta$ , the quality signal is defined by:

$$S(x; \theta) = \begin{cases} \theta x, & \text{if } x < 1/2 \\ \theta x + (1 - \theta), & \text{if } x \geq 1/2 \end{cases}$$

With probability  $1 - \theta$ , the signal is uninformative, which induces a mass point at  $1/2$ . Otherwise,  $S$  follows the uniform distribution, which is the true underlying distribution of the object's quality. Applying the optimal disclosure rule, we find the distribution of receiver posterior means when the acquired precision is  $\theta$ .

$$R(x; \theta, d^*) = \begin{cases} \frac{\theta}{2}x, & \text{if } x < 1/2 \\ \frac{\theta}{2}x + \left(1 - \frac{\theta}{2}\right), & \text{if } x \geq 1/2 \end{cases}$$

This distribution is pictured in Figure 7, for two different values of precision  $\theta$ . Note that when the sender acquires a higher precision, the amount of information provided to the receiver is larger. If  $\theta' > \theta$ , then  $R(\cdot; \theta', d^*)$  is a mean preserving spread of  $R(\cdot; \theta, d^*)$ . In the picture, we can see that the mass point on  $1/2$  gets smaller and there is more weight on the tails of  $R$ .

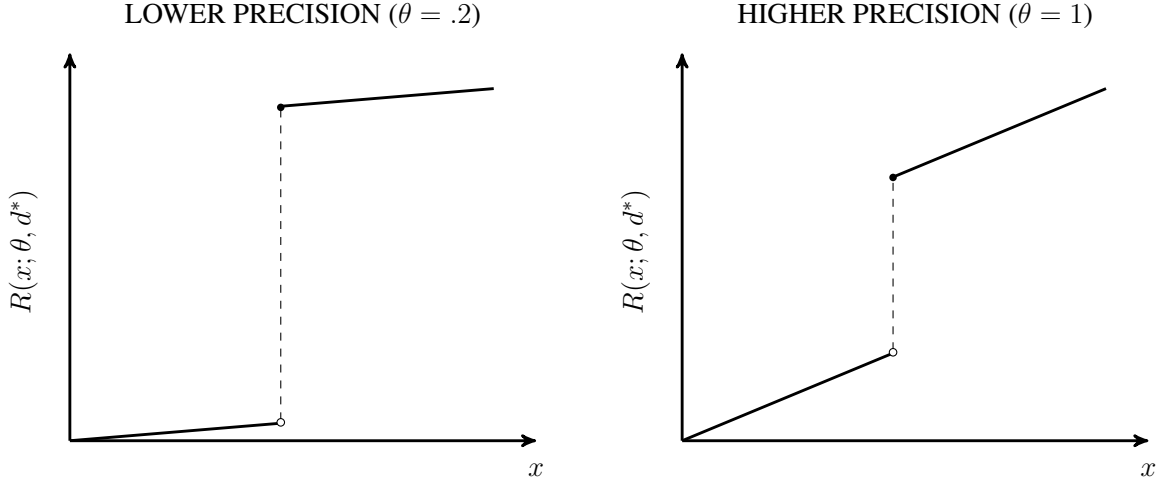


FIGURE 7. Example 4.4. I display the receiver's distribution of posterior means when the sender uses the optimal disclosure rule  $d^*$  and acquires signal precision  $\theta$ .

At one extreme, if  $\theta = 0$ , the sender is perfectly uninformative and the receiver's distribution is degenerate at  $1/2$ . At the other end, when  $\theta = 1$ , the sender is *not* perfectly informative. Even though the sender perfectly observes the object's quality, he optimally hides “half” of the realizations from the receiver. Half of the time, the receiver is not informed of the object's quality, which leads to a mass point of probability  $1/2$  at  $\bar{x} = 1/2$ .

Substituting the quality signal  $S$  from this example into (6), we find that the sender's profit from acquiring precision  $\theta$  is

$$\Pi(\theta, d^*) = \frac{\tilde{w}}{2} + \Delta \frac{\theta}{4} - c(\theta)$$

where  $\tilde{w} \equiv (w_H + w_L)/2$  is the average profitability, and  $\Delta \equiv (w_H - w_L)/2$  is the degree to which the sender's motives are hidden. Assuming that  $c$  is differentiable and strictly convex, the sender's optimal precision choice is an increasing function of the sender's opaqueness  $\Delta$ :

$$\theta^* = \min \left\{ c'^{-1} \left( \frac{\Delta}{4} \right), 1 \right\}$$

**4.5. Nonlinear Demand.** In the case of a linear receiver type distribution, the sender's hidden motives act as an incentive to acquire more precise quality signals. This result holds for any increasing precision cost function.

When  $Y$  is not linear, the precision acquired by the sender is not necessarily increasing in the sender's hidden motives for such a general class of cost function. However, Proposition 5 below shows that this still holds when acquiring a signal is a discrete choice — the sender acquires a signal of some precision at a positive fixed cost or no signal at all at zero cost.

**Proposition 5.** *Suppose precision  $\hat{\theta}$  can be acquired at a fixed cost  $k > 0$ .<sup>22</sup> Senders with more hidden motives acquire weakly more precise signals. Moreover, a sender that acquires precision  $\hat{\theta}$  is strictly more informative than one that does not.*

If the sender does not acquire the signal, then regardless of the disclosure policy, the distribution of posterior means induced on the receiver is the degenerate distribution at  $\mathbb{E}(x)$ . The value to the sender in that case is  $wY(\mathbb{E}(x))$ , which does not depend on the degree of the sender's hidden motives. On the other hand, the value to the sender conditional on acquiring the signal is higher for senders with more hidden motives. When the sender acquires the informative signal, then he has the ability to choose a disclosure rule that benefits high profitability objects at the expense of low profitability ones, and the benefit from doing so is higher when motives are more hidden.

## 5. REGULATING ADVISORS

In this section, I focus on the case where  $W$  has binary support. With probability  $1/2$ , the object is of high profitability ( $w = w_H$ ) and, with probability  $1/2$ , it has low profitability ( $w = w_L$ , where  $w_H \geq w_L$ ).

**5.1. Mandatory Disclosure of Commissions.** In the context of financial advisors, a commonly proposed mechanism that aims at protecting clients of is to mandate they be informed of commissions paid by product providers to their advisors/brokers.

In the model, this would mean that the receiver would not only observe the disclosed/not disclosed signal realization, but also the profitability of the object. If the signal realization is disclosed, then knowing the profitability of the object has no effect. However, when the sender does not disclose the signal realization, his extra information affects the receiver's posterior. Remember that, when the receiver is not informed about the profitability, upon observing that the signal is not disclosed, her posterior mean is

$$x^{ND} = \frac{\int_{\mathcal{W}} \int_{\mathcal{X}} [x(1 - d(w, x))] dS_w(x) dW(w)}{\int_{\mathcal{W}} \int_{\mathcal{X}} [1 - d(w, x)] dS_w(x) dW(w)}$$

Now if she is informed about the profitability, then upon observing that the signal was not disclosed and that the profitability is  $w$ , her posterior mean is

$$x_w^{ND} = \frac{\int_{\mathcal{X}} [1 - d(w, x)] x dS(x; \theta)}{\int_{\mathcal{X}} [1 - d(w, x)] dS(x; \theta)}$$

<sup>22</sup>There is some  $\hat{\theta} \geq 0$  such that  $c(0) = 0$ ,  $c(\theta) = k$  for all  $\theta \in (0, \hat{\theta}]$  and  $c(\theta) = \infty$  for all  $\theta > \hat{\theta}$ . The revenue to the sender is weakly increasing in  $\theta$ , so that the sender never chooses  $\theta \in (0, \hat{\theta})$ .

And the value to the sender that chooses the disclosed signal  $(\theta, d)$  is

$$\Pi(\theta, d) = w_H P(w_H; \theta, d) + w_L P(w_L; \theta, d) - c(\theta)$$

$$\text{where } P(w; \theta, d) = \int_{\mathcal{X}} Y(x) d(w, x) dS(x; \theta) + \int_{\mathcal{X}} Y(x_w^{ND}) (1 - d(w, x)) dS(x; \theta)$$

Notice that in this case the disclosure rule for low profitability objects does not affect the expected sale probability to high profitability objects and vice-versa. This implies that any optimal disclosure rule must yield the same expected sale probability to both high and low profitability objects. That is, if  $d^*$  is an optimal disclosure rule, then  $P(w_H; \theta, d^*) = P(w_L; \theta, d^*)$ . And in this case, the value to the sender is

$$\Pi(\theta, d) = (w_H + w_L) P(w_H; \theta, d) - c(\theta) = (w_H + w_L) P(w_L; \theta, d) - c(\theta)$$

which is exactly the objective function faced by a sender with transparent motives with average profitability  $(w_H + w_L)/2$ . Hence in order to evaluate the effectiveness of the policy, we need to compare the disclosed signal chosen by the sender with hidden motives to that of the transparent sender with the same average profitability. In the linear case, Observation 5 shows that the optimally chosen disclosed signal is more informative when the sender's motives are more hidden. This implies that the receiver's surplus decreases as a result of the policy.

**Proposition 6.** *If  $Y$  is linear, mandating commission disclosure **decreases receiver's surplus**.*

*There exists  $\bar{\Delta} \in \mathbb{R}_+ \cup \{+\infty\}$  such that, if  $\Delta \equiv (w_H - w_L) > \bar{\Delta}$ , mandating commission disclosure **improves welfare**.*

Welfare takes into account the surplus to the receiver (positively) and the cost of acquiring information (negatively). By defining welfare in this way, I am taking the view that all other values are simply transfers between agents in the market. Proposition 6 states that, if the sender's hidden motives are too strong, he over-invests in acquiring signal precision. This may happen because the sender's value from investing in the signal is not aligned to the welfare value of the signal. Given the linear  $Y$ , to the sender, the optimal signal acquisition is:

$$(7) \quad \theta^* = \arg \max \{ \mathbb{E}(w) \mathbb{E}(x) + \Delta \mathbb{E}[|x - \mathbb{E}(x)|] - c(\theta) \}$$

Taking into account that the sender will optimally disclose signal realizations, the welfare without mandating disclosure is given by  $\frac{1}{2} (\bar{x}^2 + \mathbb{E}(x^2 | \theta^*)) - c(\theta^*)$ . With mandatory disclosure, the sender acquires no information and welfare is equal to  $\bar{x}^2$ . If  $c(\theta^*) > \mathbb{E}(x^2 | \theta^*) - \bar{x}^2$ , mandatory disclosure is welfare-improving.

For nonlinear distributions of receiver outside options, we can evaluate the mandatory disclosure policy in the case of a fixed cost of signal acquisition.

**Proposition 7.** *Suppose there is a fixed cost to acquiring an informative signal. The surplus to the receiver weakly decreases with the policy if the transparent sender does not acquire the signal. If  $Y$  is convex, the converse also holds.*



If the transparent sender does not acquire the signal, then the receiver is weakly better off without the policy, in which case he is informed by a sender with hidden motives who acquires and discloses weakly more information (as per Proposition 5). If  $Y$  is convex and the transparent sender acquires the signal, then he also fully discloses that signal (Proposition 3), and so the receiver is weakly better off with the policy.

**5.2. Commission Caps.** A second commonly proposed policy is to cap the level of commissions that financial advisors are allowed to accept from product providers. In my model, a (perhaps coarse) way of imposing a commission cap is to set the high profitability  $w_H$  to be equal to the cap,  $w_{CAP} \in (w_L, w_H)$ . By doing this, the difference between high and low profitability becomes smaller, and so the sender's motives become less hidden.

The effect of commission caps is similar to that of mandatory disclosure. While mandatory disclosure makes the sender act as if he had transparent motives, commission caps can partially reduce the extent of the sender's hidden motives. To evaluate the effect on welfare, we can define the welfare-maximizing signal acquisition, taking into account the sender's optimal disclosure rule. It is given by

$$(8) \quad \theta^{**} = \arg \max \left[ \frac{1}{2} (\bar{x}^2 + \mathbb{E}(x^2|\theta)) - c(\theta) \right]$$

**Proposition 8.** *Let  $Y$  be linear.*

*Imposing a commission cap  $w_{CAP}$  decreases the surplus to the receiver.*

*If  $\theta^{**}$ , as defined in (8), is smaller than  $\theta^*$ , as defined in (7), then there is a commission cap  $w_{CAP} \in [w_L, w_H)$  that improves welfare.*

## 6. CONCLUSION

The paper studies an environment where a sender acquires and discloses information about an object's quality. A receiver observes the information disclosed by the sender and chooses whether to buy the object or take an outside option. The sender's motives are hidden: his profitability from the object's sale is not observed by the receiver. A sender with more hidden motives has stronger incentives to push the sale of some objects over others.

My analysis shows that the sender's hidden motives affect the amount of information he provides to the receiver through two channels. First, a more biased sender can be *more informative* because he has stronger incentives to acquire a precise signal of the object's quality. Secondly, his hidden motives determine his Optimal Disclosure Rule. In Section 3, I show that Optimal Disclosure Rules are characterized by a threshold structure. Typically, the sender discloses good signal realizations when the object is sufficiently profitable. Conversely, he discloses bad signal realizations when the object's profitability is sufficiently low.

These results inform the evaluation of instituting policies that are commonly proposed in the context of financial advisors, such as mandatory disclosure of commissions or commission caps. I find that instituting such a policies can backfire and reduce the surplus to the receiver by stripping the sender of his incentive to invest in acquiring precise quality signals.

## 7. BIBLIOGRAPHY

- Anagol, S., S. Cole, and S. Sarkar. (2017) “Understanding the Advice of Commissions-Motivated Agents: Evidence from the Indian Life Insurance Market.” *Review of Economics and Statistics*, **99**, 1-15.
- Ball, I. and X. Gao. (2020) “Benefitting from Bias.” *working paper*.
- Bar-Isaac, H., and J. Shapiro. (2013) “Ratings Quality over the Business Cycle.” *Journal of Financial Economics*, **108**, 62-78.
- Bolton, P., X. Freixas, and J. Shapiro. (2012) “The Credit Ratings Game.” *The Journal of Finance*, **67**, 85-111. NBR 6023
- Chalmers, J. and J. Reuter. (2020) “Is Conflicted Investment Advice Better than no Advice?.” *Journal of Financial Economics*, forthcoming.
- Chang, B., and M. Szydlowski. (2020) “The Market for Conflicted Advice.” *The Journal of Finance*, **75**, 867-903.
- Che, Y.K., and N. Kartik (2009) “Opinions as Incentives.” *Journal of Political Economy*, **117**, 815-860.
- DeMarzo, P. M., I. Kremer and A. Skrzypacz (2019) “Test Design and Minimum Standards.” *American Economic Review*, **109**: 2173-2207.
- Di Pei, H. (2015) “Communication with Endogenous Information Acquisition.” *Journal of Economic Theory*, **160**, 132-149.
- Dranove, D. and G. Jin. (2010) “Quality Disclosure and Certification: Theory and Practice.” *Journal of Economic Literature*, **48**: 935-63.
- Dye, R. A. (1985) “Disclosure of Nonproprietary Information.” *Journal of Accounting Research*, **23**: 123-145.
- Eckardt, M., and S. Rathke-Doppner. (2010) “The Quality of Insurance Intermediary Services – Empirical Evidence for Germany.” *Journal of Risk and Insurance*, **77**, 667-701.
- Gentzkow, M., and E. Kamenica. (2014) “Costly Persuasion.” *American Economic Review: Papers & Proceedings*, **104**, 457-62.

- Gentzkow, M., and E. Kamenica. (2017) "Disclosure of Endogenous Information." *Economic Theory Bulletin*, **5**, 47-56.
- Grossman, S. (1981) "The Informational Role of Warranties and Private Disclosure about Product Quality." *Journal of Law & Economics*, **24**: 461-484.
- Hung, A., N. Clancy, J. Dominitz, E. Talley, C. Berrebi, and F. Suvankulov. (2008) "Investor and Industry Perspectives on Investment Advisers and Broker-Dealers." *RAND Institute for Civil Justice Technical Report*.
- Jung, W.-O. and Y. K. Kwon (1988) "Disclosure When the Market Is Unsure of Information Endowment of Managers." *Journal of Accounting Research*, **26**: 146-153.
- Inderst, R., and M. Ottaviani. (2012) "Competition through Commissions and Kickbacks." *American Economic Review*, **102**, 780-809.
- Inderst, R., and M. Ottaviani. (2012) "Financial Advice". *Journal of Economic Literature*, **50**, 494-512.
- Inderst, R., and M. Ottaviani. (2012) "How (not) to Pay for Advice: A Framework for Consumer Financial Protection." *Journal of Financial Economics*, **105**, 393-411.
- Kamenica, E. (2019) "Bayesian Persuasion and Information Design." *Annual Review of Economics*, **11**.
- Kamenica, E., and M. Gentzkow. (2011) "Bayesian Persuasion." *American Economic Review*, **101**, 2590-2615.
- Kartik, N., F. X. Lee, and W. Suen. (2017) "Investment in Concealable Information by Biased Experts." *The RAND Journal of Economics*, **48**: 24-43.
- Lipnowski, E., D. Ravid, and D. Shishkin. (2019) "Persuasion via Weak Institutions." *working paper*.
- Lipnowski, E. and D. Ravid. (2020) "Cheap Talk with Transparent Motives." *Econometrica*, **88**: 1631-1660.
- Mathevet, L., D. Pearce, and E. Stacchetti. (2019) "Reputation and Information Design." *working paper*.
- Milgrom, P. (1981) "Good News and Bad News: Representation Theorems and Applications." *The Bell Journal of Economics*, **12**: 380-391.
- Milgrom, P. (2008) "What the Seller Won't Tell You: Persuasion and Disclosure in Markets." *Journal of Economic Perspectives*, **22**: 115-131.
- Min, D. (2020) "Bayesian Persuasion under Partial Commitment." *working paper*.

Opp, C., M. Opp, and M. Harris. (2013) “Rating Agencies in the Face of Regulation.” *Journal of Financial Economics*, **108**, 46-61.

Rappoport, D. (2020) “Evidence and Skepticism in Verifiable Disclosure Games.” *working paper*.

Rayo, L., and I. Segal (2010) “Optimal Information Disclosure.” *Journal of Political Economy*, **118**, 949–987.

Shishkin, D. (2020) “Evidence Acquisition and Voluntary Disclosure.” *working paper*.

Skreta, V., and L. Veldkamp. (2009) “Ratings Shopping and Asset Complexity: A Theory of Ratings Inflation.” *Journal of Monetary Economics*, **56**, 678-695.

Szalay, D. (2005) “The Economics of Clear Advice and Extreme Options.” *The Review of Economic Studies*, **72**: 1173-1198.

## 8. APPENDIX A - PROOFS

**8.1. Proof of Observation 1.** The surplus to the receiver that has a posterior mean of  $x$  and outside option  $y$  is the maximum of  $(x - y)$  and 0. To find the ex-ante expected surplus to the receiver when the sender picks  $(\theta, d)$ , we integrate with respect to the distribution of receiver outside options, as well as the distribution of posterior means the receiver faces:

$$\text{Receiver Surplus} = \int_{\mathcal{X}} \int_{[y_{\min}, y_{\max}]} \max\{x - y, 0\} dY(y) dR(x; d) = \int_{\mathcal{X}} \int_{[y_{\min}, x]} Y(y) dy dR(x; d)$$

Since  $Y$  is a cdf, and thus nondecreasing, then  $\int_{[y_{\min}, x]} Y(y) dy$  is a weakly convex function of  $x$ . Using this and the definition of the informativeness order, we find that the receiver benefits from facing more informative disclosed signals. ■

**8.2. Proof of Theorem 1. Step 1.** Verifying that  $d$  and  $\hat{d}$  produce the same overall probability of sale:

$$\begin{aligned} \mathbb{E}[P(w; \hat{d})] - \mathbb{E}[P(w; d)] &= \int_{\mathcal{W}} \int_{\mathcal{X}} [Y(x) - Y(x^{ND})] [\hat{d}(w, x) - d(w, x)] dS_w(x) dW(w) \\ &= \int_{\mathcal{X}} [Y(x) - Y(x^{ND})] \int_{\mathcal{W}} [\hat{d}(w, x) - d(w, x)] dW_x(w) dS(x) = 0 \end{aligned}$$

where  $W_x$  is the profitability distribution conditional on a signal realization  $x$  and  $S$  is the marginal distribution of signal realizations. The first equality uses the definition of  $P(w, d)$  and the third is due to  $d$  and  $\hat{d}$  disclosing each realization with the same probability, as in (4).

**Step 2.** Showing that  $\hat{d}$  induces a larger covariance between sales and profitability than  $d$ :

$$\begin{aligned} \text{Cov}[w, P(w; \hat{d})] - \text{Cov}[w, P(w; d)] &= \mathbb{E} \left[ \left( P(w; \hat{d}) - P(w; d) \right) (w - \mathbb{E}(w)) \right] \\ &= \int_{\mathcal{W}} \int_{\mathcal{X}} [Y(x) - Y(x^{ND})] [\hat{d}(w, x) - d(w, x)] [w - \mathbb{E}(w)] dS_w(x) dW(w) \\ (9) \quad &= \int_{\mathcal{X}} [Y(x) - Y(x^{ND})] \int_{\mathcal{W}} [\hat{d}(w, x) - d(w, x)] [w - \mathbb{E}(w)] dW_x(w) dS(x) \end{aligned}$$

By the definition of  $\hat{d}$ , for  $x < x^{ND}$ ,  $\hat{d}(w, x) - d(w, x) \geq 0$  when  $w < \hat{w}(x)$  and  $\hat{d}(w, x) - d(w, x) \leq 0$  when  $w > \hat{w}(x)$  – but, as given by (4), the expected difference  $\hat{d}(w, x) - d(w, x)$  is 0. This, along with the fact that  $w - \mathbb{E}(w)$  is increasing in  $w$ , implies that

$$\int_{\mathcal{W}} [\hat{d}(w, x) - d(w, x)] [w - \mathbb{E}(w)] dW_x(w) \leq 0$$

when  $x \leq x^{ND}$ . Analogously, we can show that  $\int_{\mathcal{W}} [\hat{d}(w, x) - d(w, x)] [w - \mathbb{E}(w)] dW_x(w) \geq 0$  when  $x > x^{ND}$ . Moreover, these inequalities are strict for a positive measure of signal realizations. These observations, along with the fact that  $Y$  is strictly increasing, deliver that the expression in (9) is strictly positive. ■

**8.3. Proof of Theorem 2.** Using (2), for  $w \in \mathcal{W}$  and  $x \in \mathcal{X}$ , we can take a derivative of the sender's value with respect to  $d(w, x)$ , to get

$$(10) \quad \frac{\partial \Pi}{\partial d(w, x)} = w (Y(x) - Y(x^{ND})) dS_w(x) dW(w) + \left( \int_{\mathcal{W}} \int_{\mathcal{X}} \hat{w} [1 - d(\hat{w}, \hat{x})] dS_{\hat{w}}(\hat{x}) dW(\hat{w}) \right) Y'(x^{ND}) \frac{\partial x^{ND}}{\partial d(w, x)}$$

Now from (1), we get

$$\frac{\partial x^{ND}}{\partial d(w, x)} = \frac{\int_{\mathcal{W}} \int_{\mathcal{X}} (\hat{x} - x) (1 - d(\hat{w}, \hat{x})) dS_{\hat{w}}(\hat{x}) dW(\hat{w})}{\left( \int_{\mathcal{W}} \int_{\mathcal{X}} (1 - d(\hat{w}, \hat{x})) dS_{\hat{w}}(\hat{x}) dW(\hat{w}) \right)^2} dS_w(x) dW(w)$$

Substituting this into the previous equation, we get

$$(11) \quad \frac{\partial \Pi}{\partial d(w, x)} = [w (Y(x) - Y(x^{ND})) - w^{ND} Y'(x^{ND}) (x - x^{ND})] dS_w(x) dW(w)$$

where  $w^{ND}$  is the average object profitability given that a signal realization is not disclosed.

It is easy to check that, if  $x < x^{ND}$ ,

$$\frac{\partial \Pi}{\partial d(w, x)} \begin{cases} > 0, \text{ if } w < w^{ND} \left[ \frac{Y'(x^{ND})(x^{ND} - x)}{Y(x^{ND}) - Y(x)} \right] \\ < 0, \text{ if } w > w^{ND} \left[ \frac{Y'(x^{ND})(x^{ND} - x)}{Y(x^{ND}) - Y(x)} \right] \end{cases}$$

Conversely, if  $x > x^{ND}$ ,

$$\frac{\partial \Pi}{\partial d(w, x)} \begin{cases} > 0, \text{ if } w > w^{ND} \left[ \frac{Y'(x^{ND})(x^{ND} - x)}{Y(x^{ND}) - Y(x)} \right] \\ < 0, \text{ if } w < w^{ND} \left[ \frac{Y'(x^{ND})(x^{ND} - x)}{Y(x^{ND}) - Y(x)} \right] \end{cases}$$

Now suppose there is a positive measure of signal realizations where either  $d(w, x) \neq 0$  when (11) is negative or  $d(w, x) \neq 1$  when (11) is positive. Then  $d$  cannot be optimal. So any optimal disclosure rule must have a threshold structure where the profitability threshold satisfies

$$\bar{w}(x) = w^{ND} \left[ \frac{Y'(x^{ND})(x^{ND} - x)}{Y(x^{ND}) - Y(x)} \right]$$

for all  $x \neq x^{ND}$ . ■

**8.4. Proof of Theorem 1 (continued): On Existence of  $d^*$ .** If the joint distribution of profitability and signal realizations has finite support, with cardinality  $N$ , it is simple to argue that  $d^*$  must exist. Just note that effectively the sender picks  $N$  numbers in  $[0, 1]$ :

$$\{d(w_1, x_1), d(w_2, x_2), \dots, d(w_N, x_N)\}$$

where  $(w_i, x_i)$  for  $i \in \{1, \dots, N\}$  are the profitability and signal realization in each point in the support of the distribution. Since  $[0, 1]^N$  is compact and the objective is continuous, then a maximizer exists.

Now suppose instead that the distribution is continuous and strictly increasing.<sup>23</sup> Let  $\Pi^* \equiv \sup_d \Pi(d)$ . Also let  $F^N$  be a discretized version of the joint distribution of profitability and signal realizations, with with  $N$  “bins”: it has a mass  $1/N$  at each  $(x_n, w_n)$  where

$$\begin{aligned} x_n &= \mathbb{E} \left[ x \mid \frac{n-1}{N} \leq S_w(x; \theta) W(w) < \frac{n}{N} \text{ for some } w \right] \\ w_n &= \mathbb{E} \left[ w \mid \frac{n-1}{N} \leq S_w(x; \theta) W(w) < \frac{n}{N} \text{ for some } x \right] \end{aligned}$$

for  $n \in \{1, \dots, N\}$ . We know that, for any  $N \in \mathbb{N}_+$ , there exists a disclosure rule that solves the sender’s *discretized* problem.

**Fact 1.** For any  $\xi > 0$ , there exists some  $N \in \mathbb{N}_+$  such that if  $d_N$  is a solution to the sender’s discretized problem, then  $\Pi^* - \Pi(d_N) < \xi$ .

Furthermore, define a class of *threshold* disclosure rules  $\mathcal{D}$  where for some  $\bar{x}, \bar{w} \in \mathcal{X} \times \mathcal{W}$ ,  $d(w, x) = 1$  if

$$(\bar{x} - x) \frac{w}{\bar{w}} > (\bar{x} - x) \Gamma(x, \bar{x})$$

And  $d(w, x) = 0$  if

$$(\bar{x} - x) \frac{w}{\bar{w}} < (\bar{x} - x) \Gamma(x, \bar{x})$$

where  $\Gamma(x, \bar{x}) = \frac{Y'(\bar{x})(\bar{x}-x)}{Y(\bar{x})-Y(x)}$  if  $x \neq \bar{x}$  and  $\Gamma(x, \bar{x}) = 1$  if  $x = \bar{x}$

Let  $\mathcal{D}$  also include any disclosure rules that differ from that in at most a measure 0 set – where this measure is computed with respect to the original distribution of profitability and signal realizations, not the discretized version. Using the part of Theorem 1 proved in section 8.2, Theorem 2, and the fact that a solution to the discretized problem exists, we have that

**Fact 2.** For each  $N$ , there is a solution  $d_N$  to the sender’s problem that belongs to  $\mathcal{D}$ .

Now take some  $d \notin \mathcal{D}$ . Again by Theorems 1 and 2, we know that  $\Pi(d) < \Pi^*$ . But then, by Fact 1 and 2, it must be that there is a  $\hat{d} \in \mathcal{D}$  such that  $\Pi(d) \leq \Pi(\hat{d})$ .

<sup>23</sup>This same argument applies if it has mass points or flat regions, but the notation becomes more cumbersome.



So it must be that, if a disclosure rule is a solution to a constrained sender problem where the constraint is  $d \in \mathcal{D}$ , then it must also be a solution to the unconstrained problem. But, in fact, it is easy to show that a solution to the constrained problem must exist: just note that  $(\bar{x}, \bar{w}) \in \mathcal{X} \times \mathcal{W}$  is a compact set, and that the sender's objective is continuous.

**8.5. Proof of Observation 2.** For a given  $d$ , we have:

$$\mathbb{E}[P(w; d)] = \int_{\mathcal{X}} Y(x) dR(x; d) = \int_{\mathcal{X}} x dR(x; \theta, d) = \mathbb{E}(x)$$

where the last equality is due to  $R(\cdot; d)$  being a mean preserving contraction of the underlying distribution of the object's quality. ■

**8.6. Proof of Observation 3.** For a given  $d$ ,

$$\mathbb{E}[P(w; d)] = \int_{\mathcal{X}} Y(x) dR(x; \theta, d)$$

If  $d$  is more informative than  $d'$ , then  $R(\cdot; d)$  is a mean preserving spread of  $R(\cdot; d')$ . This immediately implies that, if  $Y$  is strictly convex,  $\mathbb{E}[P(w; d)] > \mathbb{E}[P(w; d')]$ . And, if  $Y$  is strictly concave,  $\mathbb{E}[P(w; d)] < \mathbb{E}[P(w; d')]$ . ■

**8.7. Proof of Proposition 3.** I use three steps in this proof.

**Step 1.** Showing that  $\pi(\theta, W)$  is weakly increasing in  $\theta$ . Using Proposition 1, we can write  $\pi(\theta, W)$ , defined in (6), as:

$$\begin{aligned} \pi(\theta, W) = & \int_{(-\infty, \bar{w}]} (w - \mathbb{E}(w)) dW(w) \left[ \int_{(-\infty, \bar{x}]} (x - \mathbb{E}(x)) dS(x; \theta) + \int_{(\bar{x}, \infty)} (\bar{x} - \mathbb{E}(x)) dS(x; \theta) \right] \\ & + \int_{(\bar{w}, \infty)} (w - \mathbb{E}(w)) dW(w) \left[ \int_{(-\infty, \bar{x}]} (\bar{x} - \mathbb{E}(x)) dS(x; \theta) + \int_{(\bar{x}, \infty)} (x - \mathbb{E}(x)) dS(x; \theta) \right] \end{aligned}$$

where  $\bar{x}$  and  $\bar{w}$  are as given in Proposition 1. Using integration by parts:

$$\begin{aligned} \pi(\theta, W) = & \int_{(-\infty, \bar{w}]} (w - \mathbb{E}(w)) dW(w) \left[ (\bar{x} - \mathbb{E}(x)) - \int_{(-\infty, \bar{x}]} S(x; \theta) dx \right] \\ & + \int_{[\bar{w}, \infty)} (w - \mathbb{E}(w)) dW(w) \left[ (\bar{x} - \mathbb{E}(x)) + \int_{[\bar{x}, \infty)} (1 - S(x; \theta)) dx \right] \end{aligned}$$

Finally, noticing that  $\int_{(-\infty, \bar{w}]} (w - \mathbb{E}(w)) dW(w) = - \int_{[\bar{w}, \infty)} (w - \mathbb{E}(w)) dW(w)$ , we get:

$$(12) \quad \pi(\theta, W) = \int_{[\bar{w}, \infty)} (w - \mathbb{E}(w)) dW(w) \left[ \int_{(-\infty, \bar{x}]} S(x; \theta) dx + \int_{[\bar{x}, +\infty)} (1 - S(x; \theta)) dx \right]$$

Both  $\int_{(-\infty, \bar{x}]} S(x; \theta) dx$  and  $\int_{[\bar{x}, +\infty)} (1 - S(x; \theta)) dx$  weakly increase in  $\theta$ , by the mean preserving spread property. This, along with (12), implies that  $\pi$  is increasing in  $\theta$ .

**Step 2.** Showing that if  $\hat{W}$  is a linear mean preserving spread of  $W$ , with factor  $\alpha > 1$ , then  $\pi(\theta, \hat{W}) = \alpha\pi(\theta, W)$ .

Take any  $d$ , and let  $\hat{d}$  be such that, for all  $q \in [0, 1]$ ,  $\hat{d}(\hat{W}^{-1}(q), x) = d(W^{-1}(q), x)$ . This implies that  $P(\hat{W}^{-1}(q); \theta, \hat{d}) = P(W^{-1}(q); \theta, d)$ .

Moreover, since  $\hat{W}$  is a linear mean preserving spread of  $W$  with factor  $\alpha$ , we have that  $\hat{W}^{-1}(q) - \mathbb{E}(w) = \alpha(W^{-1}(q) - \mathbb{E}(w))$ . Putting all this together, we get that:

$$\text{Cov}[w, P(w; \theta, d); W] = \alpha \text{Cov}[w, P(w; \theta, \hat{d}); \hat{W}]$$

Now suppose  $d^*$  maximizes  $\text{Cov}[w, P(w; \theta, d); W]$ . Then it must be that  $\hat{d}^*$  such that, for all  $q \in [0, 1]$ ,  $\hat{d}^*(\hat{W}^{-1}(q), x) = d^*(W^{-1}(q), x)$ , maximizes  $\text{Cov}[w, P(w; \theta, \hat{d}); \hat{W}]$ . And so, it must be that:

$$\pi(\theta; \hat{W}) = \text{Cov}[w, P(w; \theta, \hat{d}^*); \hat{W}] = \alpha \text{Cov}[w, P(w; \theta, d^*); W] = \alpha\pi(\theta, W)$$

### Step 3.

Let  $\Pi^*(\theta; W) \equiv \max_d \Pi(\theta, d; W)$ . And let  $\hat{W}$  be a linear mean preserving spread of  $F$ , with factor  $\alpha$ . Step 1 and 2 imply that, for any  $\theta' > \theta$ , if  $\Pi^*(\theta'; W) \geq \Pi^*(\theta; W)$ , then  $\Pi^*(\theta'; \hat{W}) \geq \Pi^*(\theta; \hat{W})$  as well. And so it must be that a sender with profitability distribution  $\hat{W}$  acquires weakly higher precision than a sender with profitability distribution  $W$ . ■

**8.8. Proof of Proposition 4.** From Proposition 3, we know that a transparent sender acquires  $\theta = 0$ . Thus, he provides the receiver with a perfectly uninformative signal. And we can conclude that a sender with hidden motives is weakly more informative than a transparent sender.

Also from Proposition 3, we know that if there are two senders, such that sender 1 has more hidden motives than sender 2, then if  $\theta_i$  is a signal optimally acquired by sender  $i \in \{1, 2\}$ ,  $\theta_1 \geq \theta_2$ . Moreover, from Proposition 1, we know that the optimal disclosure rule has a threshold structure. Let  $(\bar{x}_i, \bar{w}_i)$  be the optimal thresholds for sender  $i$ .

Then the distribution of posterior means optimally induced on the receiver by sender  $i$  is as follows. If  $x < \bar{x}_i$ ,

$$R_i(x) = W(\bar{w}_i)S(x; \theta_i)$$

If  $x \geq \bar{x}_i$ ,

$$R_i(x) = W(\hat{w}_i) + (1 - W(\bar{w}_i))S(x; \theta_i)$$

Now suppose  $\bar{w}_1 > \bar{w}_2$ . This, along with the fact that  $\theta_1 \geq \theta_2$  implies that, for all  $0 < q < \min\{W(\bar{w}_1)S(\bar{x}_1; \theta_1); W(\bar{w}_2)S(\bar{x}_2; \theta_2)\}$ :

$$\mathbb{E}_{R_1}(x | R_1(x) \leq q) < \mathbb{E}_{R_2}(x | R_2(x) \leq q)$$

which implies that  $R_2$  is *not* a mean preserving spread of  $R_1$  (and so sender 1 is no less informative than sender 2).

Now suppose otherwise that  $\bar{w}_1 < \bar{w}_2$ . Then for all  $\max\{W(\bar{w}_1)S(\bar{x}_1; \theta_1); W(\bar{w}_2)S(\bar{x}_2; \theta_2)\} < q < 1$ :

$$\mathbb{E}_{R_1}(x|R_1(x) \geq q) > \mathbb{E}_{R_2}(x|R_2(x) \geq q)$$

which again implies that  $R_2$  is *not* a mean preserving spread of  $R_1$ .

Finally, suppose that  $\bar{w}_1 = \bar{w}_2 \equiv \bar{w}$ . Then, by optimality, it must be that either  $R_1 = R_2$  or:

$$\mathbb{E}_{R_1}(x|R_1(x) > W(\bar{w})) - \mathbb{E}_{R_1}(x|R_1(x) < W(\bar{w})) > \mathbb{E}_{R_2}(x|R_2(x) > W(\bar{w})) - \mathbb{E}_{R_2}(x|R_2(x) < W(\bar{w}))$$

which implies that either  $\mathbb{E}_{R_1}(x|R_1(x) > W(\bar{w})) > \mathbb{E}_{R_2}(x|R_2(x) > W(\bar{w}))$  or  $\mathbb{E}_{R_1}(x|R_1(x) < W(\bar{w})) < \mathbb{E}_{R_2}(x|R_2(x) < W(\bar{w}))$ . Therefore,  $R_2$  is *not* a mean preserving spread of  $R_1$ . ■

**8.9. Proof of Observation 4.** Suppose  $w_L < w^{ND} < w_H$ . Let's verify that  $d^*(w_H, x) = 0$  if  $x < x^{ND}$ ,  $d^*(w_H, x) = 1$  if  $x > x^{ND}$ ,  $d^*(w_L, x) = 1$  if  $x < x^{ND}$  and  $d^*(w_L, x) = 0$  if  $x > x^{ND}$  implies that  $x^{ND} = \mathbb{E}(x)$ . Suppose first that  $\mathbb{E}(x)$  is not a mass point of  $S(\cdot; \theta)$ . Then using  $d$  and (1), we get

$$x^{ND} = \frac{\int_{x^{ND}}^{\infty} x dS(x; \theta) + \int_{-\infty}^{x^{ND}} x dS(x; \theta)}{\int_{x^{ND}}^{\infty} dS(x; \theta) + \int_{-\infty}^{x^{ND}} dS(x; \theta)} = \int_{-\infty}^{\infty} x dS(x; \theta) = \mathbb{E}(x)$$

If otherwise  $\mathbb{E}(x)$  is a mass point of  $S(\cdot; \theta)$ , then set  $s(\bar{x}; \theta) \equiv S(x; \theta) - S^\uparrow(x; \theta)$  and find

$$x^{ND} = \frac{\int_{-\infty}^{\infty} x dS(x; \theta) - \mathbb{E}(x)s(\mathbb{E}(x); \theta) \left( \frac{1}{2}d(w_H, \mathbb{E}(x)) + \frac{1}{2}d(w_L, \mathbb{E}(x)) \right)}{1 - s(\mathbb{E}(x); \theta) \left( \frac{1}{2}d(w_H, \mathbb{E}(x)) + \frac{1}{2}d(w_L, \mathbb{E}(x)) \right)} = \mathbb{E}(x)$$

Now I want to show that there are no optimal disclosure rules where  $w_L = w^{ND}$  or  $w_H = w^{ND}$ . To that end, let's check that  $P(w_H; \theta, d^*) > P(w_L; \theta, d^*)$ .

$$\begin{aligned} P(w_H; \theta, d^*) &= \int_{-\infty}^{\mathbb{E}(x)} \mathbb{E}(x) dS(x; \theta) + \int_{\mathbb{E}(x)}^{+\infty} x dS(x; \theta) \\ &> \int_{-\infty}^{\mathbb{E}(x)} x dS(x; \theta) + \int_{\mathbb{E}(x)}^{\infty} \mathbb{E}(x) dS(x; \theta) = P(w_L; \theta, d^*) \end{aligned}$$

This implies that the value to the sender when using  $d^*$  satisfies

$$\begin{aligned} \Pi(\theta, d^*) &= \frac{w_H + w_L}{2} \mathbb{E}(x) + \frac{w_H - w_L}{2} (P(w_H; \theta, d^*) - P(w_L; \theta, d^*)) - c(\theta) \\ &> \frac{w_H + w_L}{2} \mathbb{E}(x) - c(\theta) \end{aligned}$$

Now suppose by contradiction that  $w_L = w^{ND} < w_H$ , which means that  $d(w_H, x) = 0$  almost everywhere. Then we can use (1) and (2) to see that  $P(\theta, d; w_H) = P(\theta, d; w_L) = \mathbb{E}(x)$ , and

thus  $\Pi(\theta, d) = \frac{w_H + w_L}{2} \mathbb{E}(x) - c(\theta)$  which is strictly lower than the value to the sender under  $d^*$ . Hence, it cannot be that  $w_L = w^{ND}$  in any optimal disclosure rule. The same argument can be made to show that  $w_H = w^{ND}$  cannot hold under an optimal disclosure rule. ■

**8.10. Proof of Proposition 5.** Let  $\hat{W}$  be a linear mean preserving spread of  $W$ . I want to show that, if a sender with profitability distribution  $W$  acquires the signal at cost  $k$ , then a sender with profitability  $\hat{W}$  must also acquire the signal at cost  $k$ .

First note that, since  $\hat{W}$  is a mean preserving spread of  $W$ , then for any disclosure rule  $d$ , there exists another disclosure rule  $\hat{d}$  such that  $\Pi(\hat{\theta}, \hat{d}; \hat{W}) = \Pi(\hat{\theta}, d; W)$ . This in turn implies that  $\Pi^*(\hat{\theta}; \hat{W}) \geq \Pi^*(\hat{\theta}; W)$ .

Second, the value from not acquiring the signal is the same under  $W$  and  $\hat{W}$ :  $\Pi^*(\theta = 0; \hat{W}) = \Pi^*(\theta = 0; W) = \mathbb{E}(w)Y(\mathbb{E}(x))$ .

So it must be that, if  $\Pi^*(\hat{\theta}; W) - k \geq \Pi^*(\theta = 0; W)$ , then  $\Pi^*(\hat{\theta}; \hat{W}) - k \geq \Pi^*(\theta = 0; \hat{W})$ . ■

## 9. APPENDIX B - OTHER FIGURES

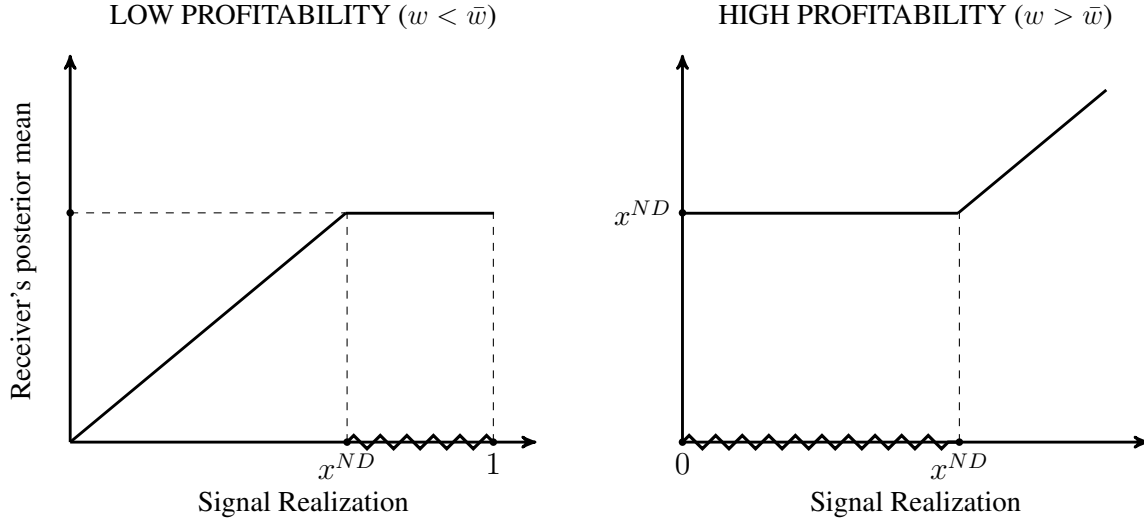


FIGURE 8. Posterior means induced on the receiver by the optimal disclosure rule when  $Y$  is **linear**. In both charts, I plot the posterior mean induced on the receiver as a function of the signal realization – it equals  $x$  when  $x$  is a disclosed signal realization and  $x^{ND}$  when  $x$  is not disclosed. The snaked intervals represent the signal realizations that are not disclosed by the sender under  $d^*$ .

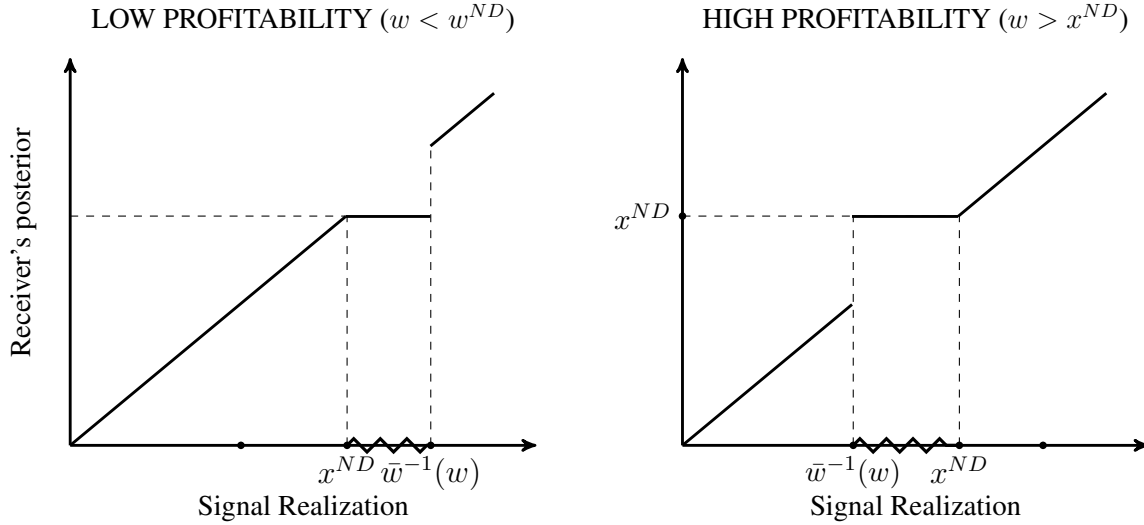


FIGURE 9. Posterior means induced on the receiver by the optimal disclosure rule when  $Y$  is **convex**. The snaked intervals represent non-disclosed signal realizations.

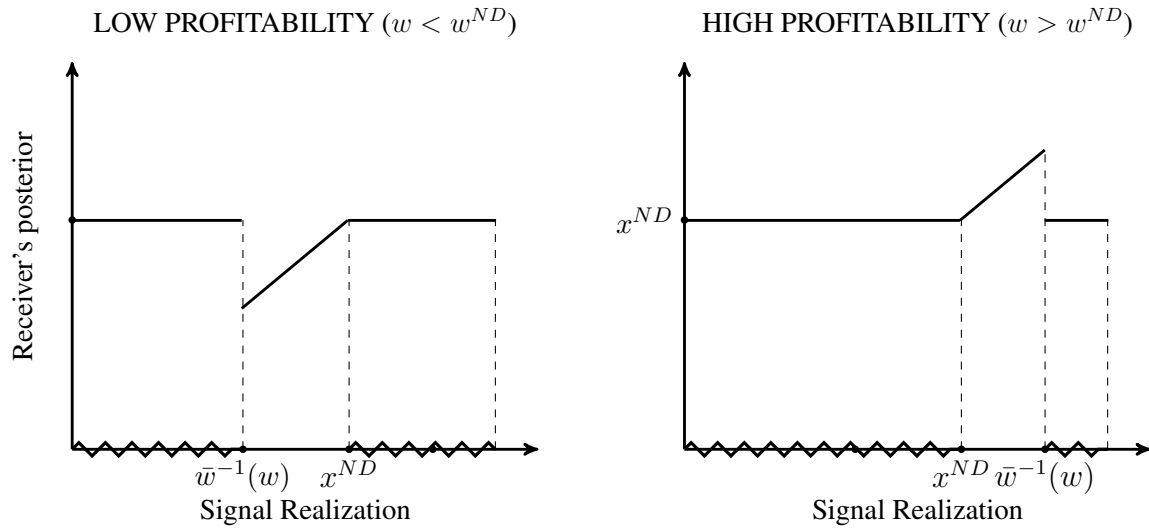


FIGURE 10. Posterior means induced on the receiver by the optimal disclosure rule when  $Y$  is **concave**. The snaked intervals represent non-disclosed signal realizations.

## 10. APPENDIX C - AN EXAMPLE FOR SECTION 3.2

**10.1. An Example.** Suppose quality and profitability are independent.  $W$  is such that, with probability  $1/2$ , the object is of high profitability ( $w = w_H$ ) and, with probability  $1/2$ , it has low profitability ( $w = w_L$ , where  $w_H \geq w_L$ ). Moreover, let the true underlying quality distribution be uniform over  $[0, 1]$  and the signal is such that sender perfectly observes the object's quality. Finally, let the distribution of receiver outside options be given by  $Y(y) = y^2$ , so we are looking at the strictly convex demand case.

From Proposition 2, we know that the optimal disclosure rule has a threshold structure. Another way to describe the threshold structure is: there are  $X(w_H)$ ,  $x^{ND}$  and  $X(w_L)$  such that signal realizations in  $[X(w_H), x^{ND}]$  are not disclosed when the object's profitability is high and signal realizations in  $[x^{ND}, X(w_L)]$  are not disclosed when the object's profitability is low. All other realizations are revealed. In the proof below, I show that the optimal thresholds are:

$$(13) \quad X(w_H) = \frac{\tilde{w} + \Delta/2}{\tilde{w} + 3\Delta/2}, \quad X(w_L) = 1 \quad \text{and} \quad x^{ND} = \frac{X(w_L) + X(w_H)}{2}$$

where  $\tilde{w} \equiv (w_H + w_L)/2$  is the average profitability, and  $\Delta \equiv (w_H - w_L)/2$ . The optimal disclosure rule based on (13) is pictured in Figure 11. In the Figure, I take  $\tilde{w} = 1$  and vary  $\Delta$  between 0 (so that  $w_L = w_H = 1$ ) and 2 (so that  $w_L = 0$  and  $w_H = 2$ ).

As the difference between high and low profitability ( $\Delta$ ) increases, the non-disclosure region when the object is of low profitability increases, as  $X(w_L)$  does not change and  $x^{ND}$  decreases. The non-disclosure region when the object has high profitability also increases in area as the sender's bias increases, because the difference between  $x^{ND}$  and  $X(w_H)$  increases with  $\Delta$ .

We can interpret an increase in  $\Delta$  as the sender's motives becoming more opaque. In that sense, as the sender's motives become more opaque, the disclosed signal becomes strictly less informative, since the overall area of non-disclosure, given by  $[X(w_H), X(w_L)]$  increases.

**10.2. Proof.** Since  $x^{ND}$  is the average quality amongst non-disclosed signal realizations, then, given the uniform distribution, we must have  $x^{ND} = \frac{X(w_H) + X(w_L)}{2}$ . Moreover, again because of the uniform distribution, we must have  $w^{ND} = \tilde{w} = \frac{w_H + w_L}{2}$ .

A candidate solution must satisfy the following three conditions:

- I. Either  $X(w_H) = 0$  and  $\frac{\partial \Pi}{\partial d(w_H, 0)} < 0$  (corner solution) or  $\frac{\partial \Pi}{\partial d(w_H, X(w_H))} = 0$ .
- II. Either  $X(w_L) = 1$  and  $\frac{\partial \Pi}{\partial d(w_L, 1)} < 0$  (corner solution) or  $\frac{\partial \Pi}{\partial d(w_L, X(w_L))} = 0$ .
- III.  $\frac{X(w_H) + X(w_L)}{2} = x^{ND}$ .



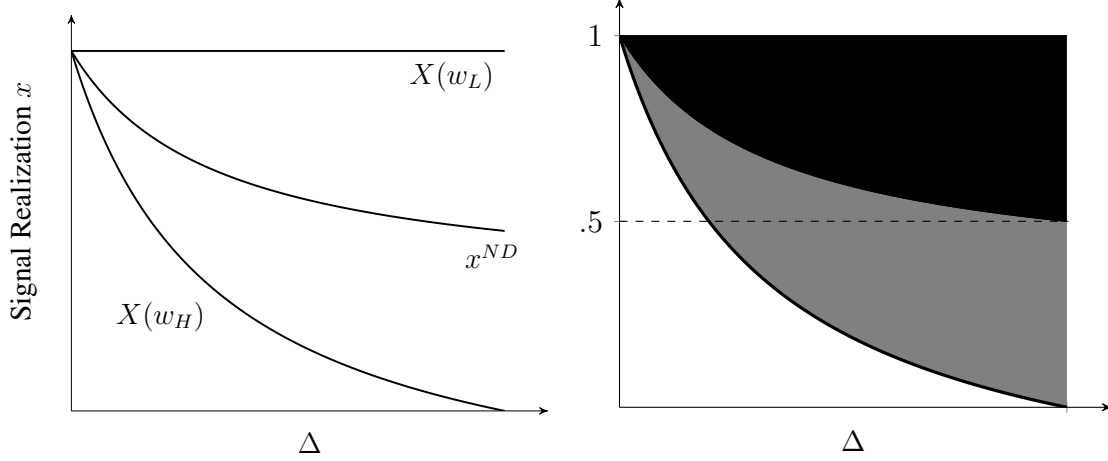


FIGURE 11. Optimal disclosure rules in the example, as a function of  $\Delta$ , the difference between high and low profitability. In the left panel,  $X(w_H)$ ,  $x^{ND}$  and  $X(w_L)$  are as given in (13), which determine the optimal non-disclosure regions. In the right panel, the area in black is the non-disclosure region when the object's profitability is low; and the grey area is the non-disclosure region when the object's profitability is high.

Using (11), we find that

$$\begin{aligned}
 \frac{\partial \Pi}{\partial d(w_H, X(w_H))} &\leq 0 \Leftrightarrow w_H \left[ X(w_H)^2 - x^{ND2} \right] - 2\tilde{w}x^{ND} [X(w_H) - x^{ND}] \leq 0 \\
 &\Leftrightarrow (\tilde{w} + \Delta/2) [X(w_H) + x^{ND}] - 2\tilde{w}x^{ND} \geq 0 \Leftrightarrow \Delta x^{ND} - (\tilde{w} + \Delta/2) (x^{ND} - X(w_H)) \geq 0 \\
 (14) \qquad \qquad \qquad &\Leftrightarrow x^{ND} - X(w_H) \leq \frac{\Delta}{\tilde{w} + \Delta/2} x^{ND}
 \end{aligned}$$

Again using (11), we have

$$\begin{aligned}
 \frac{\partial \Pi}{\partial d(w_L, X(w_L))} &\leq 0 \Leftrightarrow w_L \left[ X(w_L)^2 - x^{ND2} \right] - 2\tilde{w}x^{ND} [X(w_L) - x^{ND}] \leq 0 \\
 &\Leftrightarrow (\tilde{w} - \Delta/2) [X(w_L) + x^{ND}] - 2\tilde{w}x^{ND} \leq 0 \Leftrightarrow -\Delta x^{ND} + (\tilde{w} - \Delta/2) (X(w_L) - x^{ND}) \leq 0 \\
 (15) \qquad \qquad \qquad &\Leftrightarrow X(w_L) - x^{ND} \leq \frac{\Delta}{\tilde{w} - \Delta/2} x^{ND}
 \end{aligned}$$

From (14) and (15), we see that if both candidate  $X(w_H)$  and  $X(w_L)$  are interior, then the distance between  $X(w_H)$  and  $x^{ND}$  is strictly smaller than the distance between  $X(w_L)$  and  $x^{ND}$ . But this contradicts condition **III**.

So it must be that either  $X(w_H) = 0$  or  $X(w_L) = 1$ . If  $X(w_H) = 0$ , then  $\frac{\partial \Pi}{\partial d(w_H, 0)} < 0$  does not hold, because  $\Delta/2 \leq \tilde{w}$  (since  $w_L \geq 0$ ). So we must have  $X(w_L) = 1$ .

Plugging  $X(w_L) = 1$  and condition **III** into (14) and setting it to equality (so that  $X(w_H)$  is interior), we have

$$X(w_H) = \frac{\tilde{w} - \Delta/2}{\tilde{w} + 3\Delta/2} \Rightarrow x^{ND} = \frac{\tilde{w} + \Delta/2}{\tilde{w} + 3\Delta/2}$$

Plugging this into (15), we can confirm that  $\frac{\partial \Pi}{\partial d(w_L, 1)} < 0$  is satisfied. ■