

Informed Intermediaries

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Abstract. I develop a theory of intermediation in a market in which agents meet bilaterally to trade assets and buyers have limited commitment to pay. Some agents observe the past trading history of traders in the market. These informed agents can secure trades by setting punishments to traders who have previously defaulted. Absent these punishments, no trade can be sustained. The punishment strategy affects prices in trades and also determines which trades are hindered due to the risk of default. Intermediation can be endogenously generated when punishment strategies are asymmetric and yield some agents either more effective opportunities to trade or the ability to extract more surplus in trades. I show that asymmetric equilibria typically yield higher value to informed agents, at the expense of value to uninformed ones, and are robust to the introduction of a cost of information.

1. INTRODUCTION

Often times, when people meet and bargain over a price at which to transact, they rely on the belief that, once they settle on a price, the seller will deliver the good and the buyer will pay the set price. For example, when someone is buying a house and reaches a price agreement with the current owner of the house, both parties sign a contract specifying this price as well as the conditions under which the house should be delivered. Both parties then rest assured that this contract will be followed, knowing that if not, there is a legal system that will punish whoever failed to honor the terms.

There are markets, however, in which agents cannot rely on an exogenous authority that guarantees that contracts are honored. For obvious reasons, in markets for stolen goods or corruption markets, agents who fail to honor their debts cannot be prosecuted through formal means. Even in markets that are not illegal, legal fees can be prohibitively high or certain contracts may be infeasible to write. In extreme cases, an effective state or justice system is not even in place.

In any of these mentioned scenarios, when people decide to transact, they trust that each party will honor their side of the bargain simply because they desire to maintain their reputation of being trustworthy. In their paper in 1990, Milgrom, North and Weingast write: “A good reputation can be an effective bond for honest behavior in a community of traders if members of the community know how others have behaved in the past – even if any particular pair of

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traders meets only infrequently”. In this paper, much in the spirit of the quote, I propose a model where agents trade bilaterally, contracts are not enforced by an outside authority, and a share of the traders are able to observe how others have behaved in the past.

In the absence of the outside authority, these informed agents who observe past history of trade emerge as a tacit “police” that secures trades. They have this power because they can choose to punish agents who default by not trading with them in the future. Anticipating this possibility of being excluded from a share of the market, agents can be incentivized to honor terms of trade. In my model, the particular strategy used by informed agents to punish defaulters can shape the terms of trade and even which trades actually take place in equilibrium. I show that there exist equilibria where a hierarchical structure emerges – some traders are able to transact at better prices or more often than others. In these equilibria, these central traders often intermediate trades between the peripheral agents.

This type of hierarchy is indeed observed in a variety of markets – some agents play the role of dealers, often trading assets not for their own use, but rather to profit from intermediating trades between other market participants. For example, Schneider (2005) interviews 50 “prolific burglars” and finds that their most common method for disposal of burgled or shoplifted goods is by selling them to fences, who then resell the goods to final consumers for a higher price. Della Porta and Vannucci (2016) also document the presence of brokers in corruption networks in Italy, as well as make a case for the importance of mafias as enforcers in the Italian market for corrupt exchange. There are also many papers documenting the hierarchical network structure of trade in different financial markets, such as Ashcraft and Duffie (2007), Bech and Atalay (2010) and Afonso and Lagos (2014) for the market for Federal Funds.

The model is a variation of the over the counter market in Duffie, Grleanu and Pedersen (DGP, 2005). A mass of agents meet bilaterally and continuously trade due to differences in their valuation for the asset, i.e., the flow utility they receive from holding it. In this context, an intermediation trade is one where an agent trades against their valuation, either by buying an asset when their valuation is low or selling one when their valuation is high. Unlike in DGP, buyers have no exogenous ability to commit within a meeting – sellers first transfer the asset and only then do buyers decide whether to make the agreed upon payment. If a buyer does not make the payment, the seller has no other recourse.

A share of the traders is informed and has access to a record of all past meetings and all other agents are uninformed and never observe this record. There is a continuum of agents, and so we know that no cooperation can be achieved without access to past history of play – the formal argument is made in Kandori (1992). Thus, without the presence of informed agents, the only equilibrium in the market is autarky. Informed agents can secure trade by refusing to trade with defaulters in the future. The strength of the punishment can be calibrated by only probabilistically excluding defaulters from future trade. A harsher punishment is then a higher probability of exclusion.

When a potential seller and a potential buyer meet, the seller makes a take it or leave it price offer to the buyer. When choosing what price offer to make, the seller is aware of what punishment the buyer is subject to in case of default, and therefore proposes that the buyer pay

the highest price to which they can commit given that punishment. If the buyer is punished very harshly for default, then the seller knows that they would choose not to default even if the price is high, and makes a high price offer. This price setting protocol highlights the role of the punishment in determining the terms of trade within meetings, but the value of defaulting could also impact the terms of trade if I had chosen a different bargaining protocol.

As a benchmark, I first show that if agents are patient enough and there are enough informed agents, an equilibrium exists in which no trades are hindered by the risk of default. This first equilibrium is also symmetric in that surplus is shared equally between buyers and sellers in all trades – a result of a punishing strategy that makes buyers exactly indifferent between paying a price equal to half of the surplus of the trade or defaulting. In this equilibrium, trades are such that assets efficiently flow from agents with low valuation to agents with high valuation, regardless of whether they are informed or uninformed. Aggregate welfare is maximized.

Next, I build an equilibrium with an asymmetric punishing strategy, where no punishment is assigned to uninformed buyers who default against uninformed sellers. Given this strategy, there can be no trade between two uninformed agents. Monitored opportunities of trade hence endogenously arrive faster to informed agents, who trade with both informed and uninformed agents, than to uninformed ones. In this equilibrium, since they are less well-connected, uninformed sellers are willing to sell assets to informed ones at a discount because it would take them longer to find a buyer on their own. Conversely, uninformed buyers are willing to buy assets from them at a higher price. This endogenous wedge in the speed of trade creates a motive for informed agents to effectively intermediate trades between uninformed agents. Deterring trade between uninformed agents comes at a loss of efficiency and thus aggregate welfare is not maximized in this equilibrium.

Finally, I design an equilibrium with an asymmetric punishing strategy in which informed buyers who default are punished less harshly than uninformed ones, and thus the former can only commit to paying lower prices than the latter. In this equilibrium, no trade is deterred by the risk of default, but the strength of the punishment still affects the prices at which agents trade. Due to the punishing structure, informed agents are endogenously able to extract higher rents in trades than uninformed ones. This wedge in agents' abilities to extract rents again generates motives for intermediation: informed agents are always willing to trade with uninformed ones because they know they can buy assets from them at a cheaper price and then sell them at a higher price. In this equilibrium, aggregate welfare is maximized despite the fact that informed agents trade faster and at better rates than uninformed ones.

Note that in the last two equilibria, the asymmetry is generated by the punishments in place. It is not the case that informed agents are inherently faster traders or better negotiators. Differences in trade speed and surplus sharing are endogenously generated when agents' information types are used as a coordinating device to assign asymmetric punishments for default.

At this point, it is clear that when agents are patient enough and there are enough informed agents, the model has multiple equilibria. I propose two ways of refining the equilibrium set. First, I look for equilibria that are best for informed agents – given that they are the ones coordinating on a punishing strategy, we can expect to observe their preferred equilibria. I show

that it is often profitable for informed agents not to secure trade between uninformed agents. When uninformed agents are unable to trade with each other, it becomes more important for them to trade with the informed agents, and thus the latter are able to extract higher value.

These equilibria where uninformed agents do not trade with each other are not welfare maximizing and provide high value to informed agents at the expense of value to uninformed agents. Another feature of this type of equilibrium is that it does not require that informed agents observe information about transactions between uninformed agents, and thus it is also robust to certain variations in the information environment in the model.

I also show that intermediation trades – where agents with high valuation sell assets or agents with low valuation buy assets – are a robust feature of equilibria where the value to informed agents is higher than the value to uninformed agents. Specifically, I show that if no trades are hindered due to risk of default, intermediation trades are observed in all equilibria where the value to informed agents is higher.

Another similar criterion asks what equilibria survive to information being costly. I adjust the environment by adding a stage before the market starts operating, where all agents start out uninformed, but can pay a fixed cost to acquire information. Of course, if agents anticipate that the value to informed agents is not higher than that to uninformed ones, they are not willing to pay said cost. I show that the symmetric equilibrium I built does not survive when information is costly, because it yields the same value to informed and uninformed agents. On the other hand, both asymmetric equilibria can be observed as an equilibrium of the two-stage game.

1.1. Related Literature. This paper is related to a recent and growing literature on intermediation in over-the-counter markets. Initial models feature exogenously given middlemen who facilitate trade (e.g., Duffie, Garleanu and Pedersen, 2005 and Lagos and Rocheteau, 2009). Many papers have since tackled the question of the endogenous emergence of some agents as intermediaries.¹ The papers that are closest to mine are Farboodi, Jarosch and Shimer (2017) and Farboodi, Jarosch and Menzio (2017). The former finds that agents with faster meeting rates, and hence more opportunities to trade, become intermediaries; while the latter shows that intermediation arises purely due to differences in bargaining power across agents.

I propose an alternative theory, which relies on agents' limited commitment to future payments. When trade is not secured by an outside source, the market structure with intermediaries can emerge as a way to reward agents who are informed about past history of trade and can thus punish defaulters. It is important to reward these informed agents, since with limited commitment their presence is what guarantees the existence of any non-autarky equilibrium. This new theory can be seen as a microfoundation for the two explanations I mentioned. The punishing

¹In Afonso and Lagos (2015), agents can accumulate assets and intermediation occurs because of heterogeneity in the marginal values of holding different asset positions. Hugonnier, Lester and Weil (2016) generate intermediation chains by modeling a market with a continuum of flow valuations. In Chang and Zhang (2016), agents have different trading needs in terms of the volatility of their taste for the asset – those with lower needs become intermediaries. Bethune, Sultanum and Trachter (2017) have agents who are better informed about others' types become intermediaries.

structures in my model can in fact be responsible for some agents endogenously having better trade opportunities or higher surplus extraction in trades.

Babus and Hu (2017) study a different environment with frictions that are very similar to those in my model and also show a link between intermediation and trade when agents cannot commit to payments. In their model, without intermediation, trade fails as the number of agents becomes large. When they introduce the possibility of intermediation, trade can be sustained regardless of the size of the market. The first main difference between their model and mine is that I only study the limit case where there is a continuum of agents. Secondly, here intermediation emerges dynamically, since agents choose to trade against their portfolio for future resale purposes. In their framework, this is not possible, as assets are not carried between periods. Finally, my paper highlights the effect of punishing strategies on terms of trade that are practiced in equilibrium.

My paper also relates to the sizable literature on limited commitment in search-theoretic models of liquidity. It is close to Carapella and Williamson (2015) and Bethume, Hu and Rocheteau (2018) in that both those papers consider asymmetric punishment strategies; and to Cavalcanti and Wallace (1999) in their environment with imperfect monitoring. Cavalcanti and Wallace assume that agents can be either monitored, having all their previous trades being recorded, or not monitored. Monitored agents are able to commit to future repayment since they can be punished for defaults. In my paper, agents are heterogeneous in their access to the public record rather than in their being recorded or not. Generally, the main difference between those models and mine is that the assets agents trade in my environment are long-lived and might be traded for speculative motives, rather than being a commodity that is consumed before the end of the period. This distinction is key for alinking the limited commitment friction to the presence of intermediaries.

Finally, I contribute to the literature linking intermediation and trade efficiency. Previous work has found intermediation to facilitate trade by minimizing transaction costs (Townsend, 1978), minimizing search frictions (Rubinstein and Wolinsky, 1987; Duffie, Garleanu and Pedersen, 2005), or reducing monitoring costs (Diamond, 1984). In terms of the monitoring motivation for intermediation, my paper is closely related to Diamond (1984) and the subsequent literature. In Diamond's model, intermediation increases efficiency by lowering the total cost of monitoring. In my model, intermediation can be welfare-improving by providing incentives for informed agents to join the market, who then monitor trades. However, as we see in the equilibrium with asymmetric trade opportunities, intermediation might be associated to preventing trade between uninformed agents.

2. MODEL

2.1. Environment. A measure one of heterogeneous agents trade in a market for a homogeneous and indivisible asset with supply fixed at $1/2$. Time is continuous and the horizon infinite. Future utility flows are discounted at exponential rate $r > 0$. There is also a numeraire good which can be produced by any agent at linear cost or consumed for linear payoff and it is used as medium of exchange.

At any time, each agent in the market either holds an asset or does not. This limited capacity means that agents cannot accumulate assets or *go short* by selling assets they do not yet have. Half of the agents have a high valuation and receive flow payoff δ_H when holding the asset; while the other half receive only a low flow payoff $\delta_L < \delta_H$ in the same situation. Asset holdings are subject to shocks – at a Poisson rate $\eta > 0$, an agent that is holding an asset loses it and, at that same rate, an agent that is not holding an asset receives one.

As usual, the difference in flow payoffs between high and low valuation agents, as well as the asset holding shocks, imply that agents have motive for continuous trading and re trading of the assets. In most of the literature stemming from Duffie, Garleanu and Pedersen (2005), this continuous trading motive is achieved through shocks in agents' valuations for the asset, rather than to the asset holdings as in my model. In the absence of limited commitment, these modeling choices are equivalent. However, here this equivalence is broken since, as I explain later, harsher punishments for default can be achieved when there are shocks to asset holdings.

2.2. Bilateral Trade. At a Poisson rate $\lambda > 0$, an agent meets another randomly selected agent. Upon meeting, an agent that holds an asset can make a take it or leave it price offer – in terms of the numeraire good – to an agent who does not already have an asset. If this potential buyer accepts the offer, they immediately receive the asset from the seller. Only after this transfer takes place, the buyer chooses to pay the agreed upon price or to default. The seller here is extending a very short-term credit to the buyer: the decision to repay or not is made within the meeting, before either buyer or seller have any other meetings.²

Naturally, the buyers decision to default or not depends on who learns about their default and how they react to that information. Of course, the seller who gets defaulted on knows about the default. However, there is no way in which this seller can use this information to punish the defaulter, since the population of agents is very large – a continuum – and the defaulting buyer will never meet this particular seller again. In turn, a seller, anticipating that a buyer will always default, chooses not to sell their asset. This means that, for trade to take place in this environment with limited commitment, information about past defaults has to spread beyond each meeting. This is formally argued in Kandori (1992).

2.3. Information and Punishment Structure. A measure ϕ of agents is *informed* and, upon meeting a potential trading partner, can observe all of their past trading history – the identity of past trading partners and whether they defaulted or not. The other $1 - \phi$ agents are *uninformed* and never observe said history. Agents' information types are independent of their valuation.

An agent's past trading history is a complex object, but it is only used by the informed agent to decide whether to trade with the agent they met or not. We can think of the set of trading histories plus the realization of a public randomization device as being partitioned in two – elements which induce informed agents to choose trade and elements that induce informed agents to choose no trade. I denote these elements *unflagged histories* and *flagged histories*,

²While here the buyer is the one who can choose to default, the model could seamlessly be flipped to the case where the buyer first pays and then the seller chooses whether to transfer the good or not.

respectively. I assume that all informed agents act symmetrically; there is anonymity, i.e., an informed agent who meets two agents with the same trading history chooses the same action in both meetings; and informed agents only use mixed strategies as the result of a public randomization device.

2.4. Grim Trigger Punishing Strategies. The set of punishment strategies available to informed agents is very large. However, throughout the paper, I restrict to a particular class of punishing strategies: grim trigger strategies. Under grim trigger strategies, when an unflagged buyer defaults, they become flagged with some probability and, once flagged, they are punished forever by being excluded from trade with informed agents and never return to the unflagged state.

If the probability of triggering the punishment after a default is higher, then the seller knows that they can charge the buyer a higher price and still expect them not to default, since the punishment is harsher. This means, as I more formally explain later, that there is a link between the strength of the punishment (probability of triggering the flagged state after default) and the price the buyer pays in a trade and consequently the sharing of the trade surplus between buyer and seller.

Of course, there are other, non-grim trigger, strategies which might also support the equilibria I find and there are parameter values for which other strategies might support equilibria which are not supported by the grim trigger class. However, it is worth noting that the harshest grim trigger strategy – the one in which the flagged state is triggered with probability 1 after default – is also the harshest punishment informed agents can inflict on defaulters across all the possible, even non-grim trigger, strategies.

2.5. Punishing Strategy and Price Determination. Let V_{va}^i be the value to the unflagged agent of information type $i \in \{I, U\}$, valuation type $v \in \{H, L\}$ and asset holding $a \in \{0, 1\}$. Accordingly, let \tilde{V}_{va}^i be the value of that same agent when he is in the flagged regime. There are 16 potential trades between unflagged agents and (i_s, v_s, i_b, v_b) denotes a trade between a seller of type $(i_s, v_s) \in \{I, U\} \times \{H, L\}$ and a buyer of type $(i_b, v_b) \in \{I, U\} \times \{H, L\}$. The surplus in this meeting is given by $V_{v_s 0}^{i_s} - V_{v_s 1}^{i_s} + V_{v_b 1}^{i_b} - V_{v_b 0}^{i_b}$, the value to the buyer of acquiring an asset minus the value to the seller of losing an asset.

Define the *punishing strategy* $\tau : (\{I, U\} \times \{H, L\})^2 \rightarrow [0, 1]$ where $\tau(i_s, v_s, i_b, v_b)$ is the probability that the buyer becomes flagged after defaulting on trade (i_s, v_s, i_b, v_b) . In meeting (i_s, v_s, i_b, v_b) , the buyer will choose to pay the seller if:

$$(1) \quad V_{v_b 1}^{i_b} - p(i_s, v_s, i_b, v_b) \geq (1 - \tau(i_s, v_s, i_b, v_b)) V_{v_b 1}^{i_b} + \tau(i_s, v_s, i_b, v_b) \tilde{V}_{v_b 1}^{i_b}$$

When set to equality, condition (1) determines the highest price the seller can charge while guaranteeing that the buyer will not default. Thus, the sellers' optimal take-it-or-leave-it price offer in meeting (i_s, v_s, i_b, v_b) is

$$(2) \quad p(i_s, v_s, i_b, v_b) = \tau(i_s, v_s, i_b, v_b)(V_{v_b 1}^{i_b} - \tilde{V}_{v_b 1}^{i_b})$$

If participating in the market is valuable, then $V_{v_b1}^{i_b}$ is larger than $\tilde{V}_{v_b1}^{i_b}$ and the seller can charge a positive price, knowing that the buyer will not default. Equation (2) shows that a harsher punishment to default (larger τ) raises the maximum price the seller can charge, by decreasing the buyer's deviation value. If there is no punishment to default ($\tau = 0$), the price is zero.

To fully describe the punishing strategy, I also need to assign punishments to informed agents who fail to punish when called upon to do so, i.e., informed agents who trade with flagged agents. It is enough to impose that informed agents who trade with flagged agents become flagged. If a flagged agent wants to buy from an informed agent, she has no incentive to repay, and hence the informed agent will not engage in this sale. If a flagged agent wants to sell to an informed agent, the informed agent will already get punished from engaging in this trade and will hence have no incentive to pay. Anticipating that, the flagged agent does not sell.

Sometimes it is convenient to talk about the share of the surplus that remains with the buyer and the share of the surplus that remains with the seller. Define $\beta : (\{I, U\} \times \{H, L\})^2 \rightarrow \mathbb{R}$, where $\beta(i_s, v_s, i_b, v_b)$ is the proportion of the surplus in such trade that remains with the seller in meeting (i_s, v_s, i_b, v_b) .

$$(3) \quad \beta(i_s, v_s, i_b, v_b) = \frac{V_{v_s0}^{i_s} - V_{v_s1}^{i_s} + p(i_s, v_s, i_b, v_b)}{V_{v_s0}^{i_s} - V_{v_s1}^{i_s} + V_{v_b1}^{i_b} - V_{v_b0}^{i_b}}$$

2.6. Value Functions. Given the surplus sharing rules defined above, we can write the unflagged and flagged agents' value functions. To that end, also let $\{\mu_{va}^i\}$ denote the stationary distribution of unflagged agents across valuations and asset holdings. Since I focus on equilibria with no default on path, I refrain from adding notation for the stationary measure of flagged agents, which must be zero.

These values are composed by the flow value received if holding an asset, the value due to asset holding shocks and flows from trade. Value functions for unflagged and flagged agents, respectively, are shown below.

$$(4) \quad rV_{v0}^i = \eta (V_{v1}^i - V_{v0}^i) + \lambda \sum_{v_s \in \{L, H\}} \mu_{v_s1}^I \mathcal{I}(I, v_s, i, v) (1 - \beta(I, v_s, i, v)) (V_{v1}^i - V_{v0}^i + V_{v_s0}^I - V_{v_s1}^I) \\ + \lambda \sum_{v_s \in \{L, H\}} \mu_{v_s1}^U \mathcal{I}(U, v_s, i, v) (1 - \beta(U, v_s, i, v)) (V_{v1}^i - V_{v0}^i + V_{v_s0}^U - V_{v_s1}^U)$$

$$(5) \quad rV_{v1}^i = \delta_v + \eta (V_{v0}^i - V_{v1}^i) + \lambda \sum_{v_b \in \{L, H\}} \mu_{v_b0}^I \mathcal{I}(i, v, I, v_b) \beta(i, v, I, v_b) (V_{v_b1}^I - V_{v_b0}^I + V_{v0}^i - V_{v1}^i) \\ + \lambda \sum_{v_b \in \{L, H\}} \mu_{v_b0}^U \mathcal{I}(i, v, U, v_b) \beta(i, v, U, v_b) (V_{v_b1}^U - V_{v_b0}^U + V_{v0}^i - V_{v1}^i)$$

$$(6) \quad r\tilde{V}_{v0}^i = \eta (\tilde{V}_{v1}^i - \tilde{V}_{v0}^i) + \lambda \sum_{v_s \in \{L, H\}} \mu_{v_s1}^U \mathcal{I}(U, v_s, i, v) (\tilde{V}_{v1}^i - \tilde{V}_{v0}^i)$$

$$(7) \quad r\tilde{V}_{v1}^i = \delta_v + \eta (\tilde{V}_{v0}^i - \tilde{V}_{v1}^i) + \lambda \sum_{v_b \in \{L, H\}} \mu_{v_b0}^U \mathcal{I}(i, v, U, v_b) \beta(i, v, U, v_b) (V_{v_b1}^U - V_{v_b0}^U + \tilde{V}_{v0}^i - \tilde{V}_{v1}^i)$$

The value experienced by agents in the flagged regime differs from these above in that flagged agents do not trade with informed agents, thus (6) and (7) do not account for value due to meetings with informed agents. Flagged agents can still trade with uninformed agents, who

are not able to observe their past defaults. Moreover, since the flagged agent is already in the grim trigger punishment regime and cannot be punished further, flagged buyers always default, which is also accounted for in (6). Since I work with equilibria with no default on path, these trades never take place. However, they still affect the value of the deviation.

2.7. Equilibrium Trades. A trade will be mutually beneficial if both seller and buyer retain a positive surplus. In equilibrium, only mutually beneficial trades take place. Additionally, all strictly beneficial trades must take place.

$$(8) \quad \mathcal{I}(i_s, v_s, i_b, v_b) = 1 \Rightarrow \beta(i_s, v_s, i_b, v_b) \in [0, 1] \text{ and } V_{v_s 0}^{i_s} - V_{v_s 1}^{i_s} + V_{v_b 1}^{i_b} - V_{v_b 0}^{i_b} \geq 0$$

$$(9) \quad \beta(i_s, v_s, i_b, v_b) \in (0, 1) \text{ and } V_{v_s 0}^{i_s} - V_{v_s 1}^{i_s} + V_{v_b 1}^{i_b} - V_{v_b 0}^{i_b} > 0 \Rightarrow \mathcal{I}(i_s, v_s, i_b, v_b) = 1$$

2.8. Stationary Distribution. The distribution of agents across types must satisfy the adding up constraints given below.

$$(10) \quad \sum_{v \in \{H, L\}} \sum_{a \in \{0, 1\}} \mu_{va}^I = \phi$$

$$(11) \quad \sum_{v \in \{H, L\}} \sum_{a \in \{0, 1\}} \mu_{va}^U = 1 - \phi$$

$$(12) \quad \sum_{i \in \{I, U\}} \sum_{v \in \{H, L\}} \mu_{v1}^i = \frac{1}{2}$$

Finally, in any stationary equilibrium, the stationary distribution must be so that the inflow into each state is equal to the outflow. For any $v \in H, L$ and $i \in \{I, U\}$, the following must hold:

$$(13) \quad \mu_{v1}^i \left[\eta + \sum_{v_b \in \{H, L\}} \sum_{i_b \in \{I, U\}} \mu_{v_b 0}^{i_b} \mathcal{I}(i, v, i_b, v_b) \right] = \mu_{v0}^i \left[\eta + \sum_{v_s \in \{H, L\}} \sum_{i_s \in \{I, U\}} \mu_{v_s 1}^{i_s} \mathcal{I}(i_s, v_s, i, v) \right]$$

2.9. Equilibrium Definition. A stationary equilibrium with no default is a set of unflagged value functions $\{V_{va}^i\}$, flagged value functions $\{\tilde{V}_{va}^i\}$, trade indicator \mathcal{I} , seller surplus shares β and punishment strategy τ and stationary distribution $\{\mu_{va}^i\}$ such that (3)-(13) are satisfied.

3. SYMMETRIC AND ASYMMETRIC EQUILIBRIA

As argued, the presence of informed agents is necessary for any trade to happen. On the other hand, if the share of informed agents is high enough and agents are patient enough, many different equilibria with trade can be supported, with different structures of trade and different values to informed and uninformed agents.

In this section, I propose two ways in which informed agents can design punishment strategies so as to guarantee themselves higher value in the market and show that using either of those channels leads to a market structure with intermediation. While the equilibria proposed here

are not exhaustive, I argue in the next section that they assign higher value to informed agents than other equilibria and hence should be expected if either acquiring information is costly or informed agents can coordinate on their preferred equilibrium.

3.1. Symmetric Equilibrium. To build intuition and notation, I first show an equilibrium, with no intermediation, in which the limited commitment friction is completely overcome – no positive surplus potential trades are hindered by the threat of default.

As we saw in equations (2) and (3), there is a mapping between the punishing strategy τ and the surplus sharing rule β . Throughout this paper, instead of starting with a punishing strategy and finding the equilibrium that is supported by this strategy, I take a reverse approach and first conjecture a surplus sharing rule β and then look for a punishing strategy τ which supports this rule. For this equilibrium, I conjecture symmetric surplus sharing: in all trades, sellers and buyers share the surplus equally.

Proposition 1. *There exists $R \in (0, 1) \times [0, 1]$, a neighborhood of $(0, 1)$, in which an equilibrium exists where:*

(i) *All positive surplus trades take place:*

$$V_{v_s 0}^{i_s} - V_{v_s 1}^{i_s} + V_{v_b 1}^{i_b} - V_{v_b 0}^{i_b} > 0 \Rightarrow \mathcal{I}(i_s, v_s, i_b, v_b) = 1$$

(ii) *Surplus is shared equally in all trades:*

$$\mathcal{I}(i_s, v_s, i_b, v_b) = 1 \Rightarrow \beta(i_s, v_s, i_b, v_b) = \frac{1}{2}$$

The proof of the proposition is in the appendix. We can fully characterize the values, prices and trades in this equilibrium. First of all, the symmetry and the fact that limited commitment doesn't have a bite imply that informed and uninformed agents trade under the same terms. Naturally, in this case, the positive surplus trades are the portfolio balancing trades – those between sellers with low valuation and buyers with high valuation, irrespective of their information types. Formally, $\mathcal{I}(i_s, v_s, i_b, v_b) = 1 \Leftrightarrow v_s = L$ and $v_b = H$.

With this in place, (10)-(13) give us the stationary distribution. The distribution is symmetric in that the measure of uninformed (informed, respectively) agents with high valuation but without an asset is equal to that of uninformed (informed) agents with low valuation and holding an asset. That is, $\mu^U := \mu_{H0}^U = \mu_{L1}^U$ and $\mu^I := \mu_{H0}^I = \mu_{L1}^I$. These are the measures of agents with unbalanced portfolios, which in the context of this equilibrium are the ones trading.

Now define $S_v^i := V_{v1}^i - V_{v0}^i$, the value of holding an asset to agent of information type i and valuation type v . In this equilibrium, the symmetry of the trading pattern implies that $S_v^I = S_v^U =: S_v$ for $v \in \{H, L\}$. These values, in this equilibrium, are given by

$$S_H = \left[\frac{1}{2(r + 2\eta)} \right] (\delta_H + \delta_L) + \alpha(\delta_H - \delta_L) \quad S_L = \left[\frac{1}{2(r + 2\eta)} \right] (\delta_H + \delta_L) - \alpha(\delta_H - \delta_L)$$

where $\alpha = \frac{1}{2(r + 2\eta + \lambda(\mu^U + \mu^I))}$.

The value of holding an asset is higher to an agent with high valuation than to an agent with low valuation ($S_H > S_L$), since the high valuation agent enjoys a higher flow payoff today from holding it. This confirms that portfolio balancing trades indeed are the ones with positive surplus.

The constant α is an effective discount rate which determines the wedge between S_H and S_L . If α is higher, this difference is bigger - the value of holding an asset in the high valuation state is higher and in the low valuation state is lower. Note that α is higher whenever agents get less opportunities to trade, either due to more frictions (lower λ) or less potential trading partners (less agents with unbalanced portfolios, $\mu^U + \mu^I$). For instance, when an agent with high valuation gets few opportunities to trade, it is important for them to hold onto an asset. If on the other hand opportunities to trade were abundant, they would not be so hurt from losing an asset, as they could easily buy another one.

A measure of ex-ante efficiency is the proportion of unbalanced agents in the economy in the stationary distribution, i.e. $\mu^I + \mu^U$. If there are no unbalanced agents, it means all the assets are successfully allocated to high valuation agents and the overall flow payoff generated is maximal. In this equilibrium, constrained efficiency is met, in that the measure of unbalanced agents is the lowest possible, subject to the meeting technology and the asset holding shocks. The total measure of unbalanced agents in this equilibrium is given by

$$(\mu^U + \mu^I) = \left[-\frac{\eta}{\lambda} + \sqrt{\left(\frac{\eta}{\lambda}\right)^2 + \frac{1}{2} \frac{\eta}{\lambda}} \right]$$

Note that, if search frictions vanish, i.e. $\lambda \rightarrow +\infty$, the proportion of unbalanced agents goes to 0. On the other hand, when η is much bigger than λ , $\mu^U + \mu^I$ converges to $1/4$. This is equivalent to the asset being randomly assigned across agents in the market, that is, the trading is not enough to make the agents any more balanced than if they were not trading at all.

The price at which agents trade the asset in the portfolio balancing trades in this equilibrium is

$$p = \frac{\delta_H + \delta_L}{2(r + 2\eta)}$$

When agents are considering the decision to pay the set price or default, they pay if the price of the asset is lower than the punishment value. To support this equilibrium, I argue that, as agents become very patient, the punishment value gets unboundedly high, while the price is bounded. Notice, however, that if $\eta = 0$, this price goes to ∞ as $r \rightarrow 0$. This implies that, if there were no asset holding shocks, the punishment strategy I propose would not be able to support this equilibrium. What asset holding shocks do is that they make the value of purchasing an asset bounded, no matter how patient an agent is, since they anticipate that eventually they will lose the asset they just acquired.

3.2. Equilibrium with Asymmetric Trade Opportunities. I now build an equilibrium in which informed agents do not punish uninformed agents who default on each other. When that is the case, uninformed agents cannot trade amongst themselves due to the risk of default. As

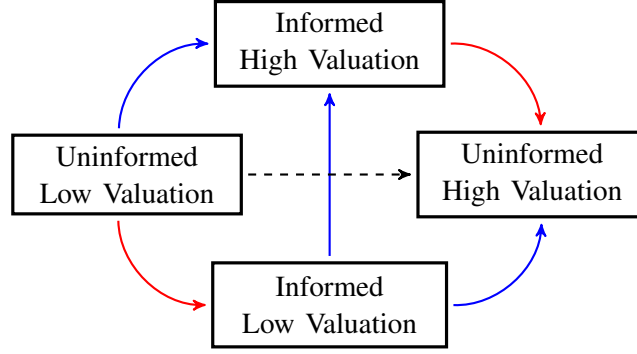


FIGURE 1. Trade pattern supported with asymmetric trade opportunities. Blue arrows indicate portfolio balancing trades. Red arrows indicate intermediation trades. Dashed arrow indicates a trade that does not occur in this pattern due to risk of default.

before, I build an equilibrium in which surplus is shared evenly between buyer and seller in the trades that do take place.

Proposition 2. *There exists $R \in (0, 1) \times [0, 1]$, a neighborhood of $(0, 1)$, in which an equilibrium exists where:*

(i) *Positive surplus trades takes place if and only if it involves an informed agent:*

$$\mathcal{I}(i_s, v_s, i_b, v_b) = 1 \Leftrightarrow (i_s, i_b) \neq (U, U) \text{ and } V_{v_s 0}^{i_s} - V_{v_s 1}^{i_s} + V_{v_b 1}^{i_b} - V_{v_b 0}^{i_b} > 0$$

(ii) *Surplus is shared equally in all trades:*

$$\mathcal{I}(i_s, v_s, i_b, v_b) = 1 \Rightarrow \beta(i_s, v_s, i_b, v_b) = \frac{1}{2}$$

Again, we can fully characterize values, prices and trades in this equilibrium. Portfolio balancing trades involving at least one informed agent take place: $\mathcal{I}(i_s, L, i_b, H) = 1$ if $(i_s, i_b) \neq (U, U)$. A different type of trade also takes place, when an uninformed seller with low valuation meets an informed buyer with low valuation or when an informed seller with high valuation meets an uninformed buyer with high valuation, the trade goes through as well. Formally, $\mathcal{I}(I, H, U, H) = \mathcal{I}(U, L, I, L) = 1$. These are *intermediation trades*, where informed agents are trading against their valuation – either selling their assets when they have a high valuation or buying assets when they have a low valuation. This trading pattern is represented in Figure 1.

In the left panel of Figure 2, I show an example of the region R where the proposed equilibrium exists. In the right panel, the value of r is fixed and the punishing strategy τ is displayed as a function of ϕ . In black are the probabilities of becoming flagged when uninformed buyers default – there are two different lines, referring to the two different trades where uninformed agents are buyers. Accordingly, the gray lines correspond to the probability of flagging informed buyers that default in each of the three trades where informed agents are buyers.

The symmetry in the stationary distribution holds as in the previous equilibrium, $\mu^U := \mu_{H0}^U = \mu_{L1}^U$ and $\mu^I := \mu_{H0}^I = \mu_{L1}^I$. Unlike before, the value of holding an asset now depends on the agent's information type as well as on their valuation type:

$$S_H^i = \left[\frac{1}{2(r+2\eta)} \right] (\delta_H + \delta_L) + \alpha^i (\delta_H - \delta_L) \quad S_L^i = \left[\frac{1}{2(r+2\eta)} \right] (\delta_H + \delta_L) - \alpha^i (\delta_H - \delta_L)$$

where $\alpha^I = \frac{1}{2(r+2\eta+\lambda(\mu^U+\mu^I))}$ and $\alpha^U = \frac{r+2\eta+\lambda\mu^U+\frac{\lambda\phi}{4}}{2(r+2\eta+\frac{\lambda\phi}{4})(r+2\eta+\lambda(\mu^U+\mu^I))}$.

Informed agents get effective opportunities to trade at a higher rate than uninformed ones, since informed agents trade with both informed and uninformed ones, while trades between two uninformed agents are hindered by the risk of default. This difference in effective opportunities to trade leads to $\alpha^U > \alpha^I$. This in turn implies that the values of holding an asset are ordered $S_H^U > S_H^I > S_L^I > S_L^U$. Importantly, note that the difference in opportunities to trade is not inherent to the agents, but endogenously generated by the punishing strategy in place.

This wedge between informed and uninformed values leads to intermediation trades also having positive surplus. For instance, $S_H^U > S_H^I$ means that the value to the uninformed of having an asset when in the high valuation state is higher than that of the informed agent. The informed agent knows that, if they forego the asset, they might soon engage in a portfolio balancing trade and recover the asset. As for the uninformed agent, they can only do so less often. This implies that it is more important for the uninformed to be balanced.

There is also a spread in the prices at which informed agents buy assets from uninformed agents and the price at which informed agents sell assets to uninformed agents. Informed agents are able to buy cheaper and sell for a more expensive value, both in portfolio balancing trades and intermediation trades. When informed agents balance their portfolios with each other, they trade at the same price as in the symmetric equilibrium.

Unlike in the symmetric equilibrium, ex-ante efficiency is not achieved in this equilibrium with asymmetric trade opportunities. The fact that portfolio balancing trades between uninformed agents are hindered implies that, in the stationary distribution, the proportion of unbalanced agents is higher³ and hence the total flow payoff to the economy is not as large as it could be. Note that, even though intermediation trades do not move assets from low valuation agents to high valuation agents, they do increase efficiency. Since informed agents trade at a faster effective rate than uninformed ones, it is preferable in terms of efficiency to have an unbalanced informed agent than an unbalanced uninformed agent, and intermediation trades guarantee that assets move in that direction.

3.3. Equilibria with Asymmetric Surplus Sharing. In equation (3), we saw the link between the punishing strategy τ and the surplus sharing rule in equilibrium. While in both of the equilibria we saw before, we built punishing strategies that guaranteed that surplus was

³This is shown in the appendix.

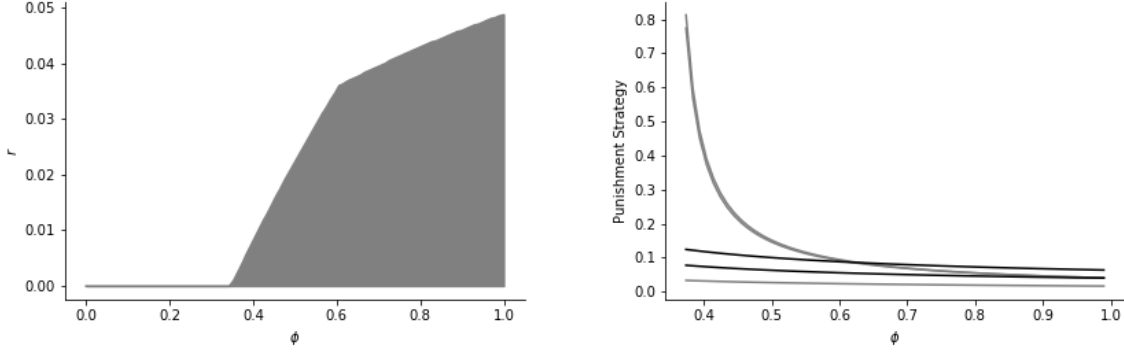


FIGURE 2. In the left panel, the shaded gray area represents the parameter region where an equilibrium with asymmetric trade opportunities exists. Other parameters are fixed at $\lambda = 2$, $\eta = .2$, $\delta_H = 1$ and $\delta_L = 0$. In the right panel, I fix the discount factor $r = .003$ and show, for each ϕ , the punishment strategy τ . The τ values in black correspond to punishment in trades in which uninformed agents are buyers subject to the punishment and the τ values in gray to punishments to informed buyers.

shared equally between buyer and seller, I am now interested in using punishing strategies to design equilibria with asymmetric surplus sharing.

In particular, I build an equilibrium where when informed and uninformed agents meet, the informed agent keeps more than half of the surplus. Such a surplus split is supported by punishing uninformed defaulters very harshly, with a high probability of exclusion from the market, and informed defaulters lightly, with a smaller probability of triggering exclusion from the market. This implies that uninformed buyers can be induced to pay a higher price than informed buyers.

Proposition 3. *For each $\beta^I \in (.5, 1)$, there exists $R \in (0, 1) \times [0, 1]$, a neighborhood of $(0, 1)$, in which an equilibrium exists where:*

(i) *All trades with positive surplus take place:*

$$V_{v_s 0}^{i_s} - V_{v_s 1}^{i_s} + V_{v_b 1}^{i_b} - V_{v_b 0}^{i_b} > 0 \Rightarrow \mathcal{I}(i_s, v_s, i_b, v_b) = 1$$

(ii) *Informed agents keep a higher share of surplus in trades:*

$$\beta(I, v_s, U, v_b) = \beta^I > \frac{1}{2} \quad \beta(U, v_s, I, v_b) = 1 - \beta^I < \frac{1}{2}$$

$$\beta(I, v_s, I, v_b) = \frac{1}{2} \quad \beta(U, v_s, U, v_b) = \frac{1}{2}$$

In Figure 4, I display parameter regions where the proposed equilibrium holds. In the top-left panel, I fix $\beta^I = .7$ and plot out the region R in the (ϕ, r) space. In the bottom-left panel, I fix $\phi = .7$ and show that, the higher the β^I , the more patient need to be agents in order for the equilibrium to be sustained.

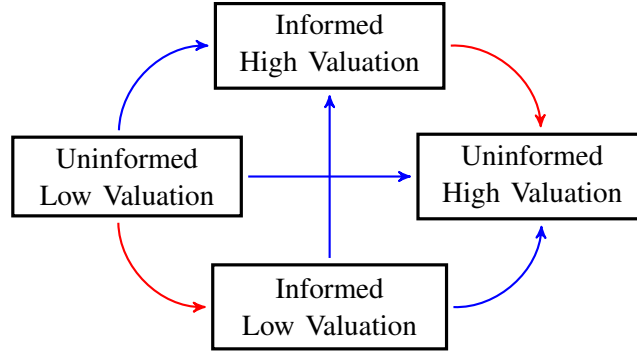


FIGURE 3. Trade pattern supported with asymmetric surplus sharing. Blue arrows indicate portfolio balancing trades. Red arrows indicate intermediation trades.

In the top and bottom right panels of Figure 4, I fix the value of r and display the punishing strategy τ as a function of ϕ and β^I . Again, in black are the probabilities of becoming flagged when uninformed buyers default and the gray lines correspond to the probability of flagging informed buyers that default in each of the three trades where informed agents are buyers. Notice that uninformed agents are always punished more harshly for defaults – the black lines are always above the gray ones. This leads uninformed agents to being charged higher prices in trades and thus retaining a lower share of surplus.

The value of holding an asset in this equilibrium is, for $i \in \{U, I\}$:

$$S_H^i = \left[\frac{1}{2(r+2\eta)} \right] (\delta_H + \delta_L) + \frac{\tilde{\alpha}^i}{2} (\delta_H - \delta_L) \quad S_L^i = \left[\frac{1}{2(r+2\eta)} \right] (\delta_H + \delta_L) - \frac{\tilde{\alpha}^i}{2} (\delta_H - \delta_L)$$

where $\tilde{\alpha}^I = \frac{1}{(r+2\eta+\lambda(2\mu^U\beta^I+\mu^I))}$ and $\tilde{\alpha}^U = \frac{r+2\eta+\lambda\beta^I(2\mu^U+\mu^I)+\lambda(1-\beta^I)(\phi/2-\mu^I)}{(r+2\eta+\lambda(\mu^U+\phi/2(1-\beta^I)))(r+2\eta+\lambda(2\mu^U\beta^I+\mu^I))}$. It is easy to check that, for $\beta^I > .5$, $\tilde{\alpha}^U > \tilde{\alpha}^I$.

These values are thus ordered $S_H^U > S_H^I > S_L^I > S_L^U$, and the positive surplus trades are all portfolio balancing trades, as well as the intermediation trades. All of these take place in equilibrium. This trading pattern is depicted in Figure 3.

Informed agents buy assets even in the low valuation state because they know they are able to buy at a lower price than the one they expect to sell for in a future encounter. Conversely, they sell assets even in the high valuation state, since they are able to secure a higher price than the one they expect to buy a new asset for in a future trade.

It is important to note that this surplus acquiring ability is not inherent to the agents, but rather an endogenous object in this equilibrium sustained by the punishment strategy in place. For instance, we could conversely build an equilibrium with $\beta^I \in (0, 1/2)$, so that the uninformed agents are the ones extracting more surplus and playing the intermediating rule. How could that be possible? Simply informed agents would need to coordinate on a high punishment for informed defaulters and a low punishment for uninformed defaulters. Since each informed

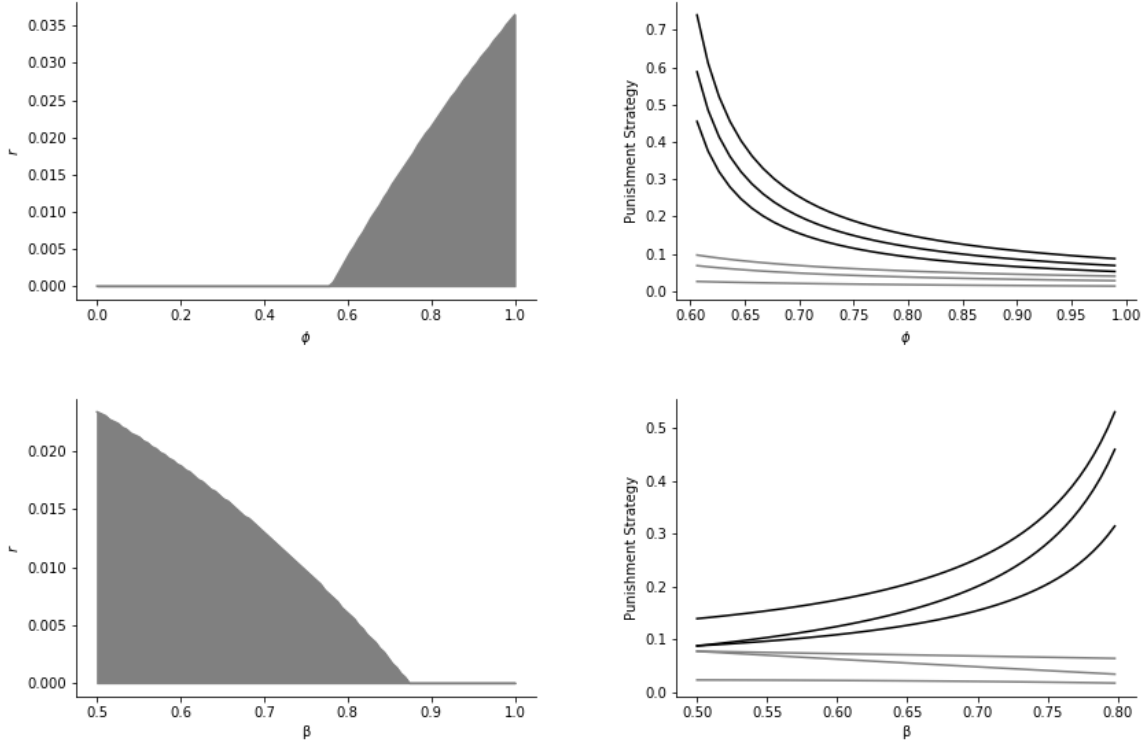


FIGURE 4. In the left panels, the shaded gray area represents the parameter region where an equilibrium with asymmetric surplus sharing exists. Other parameters are fixed at $\lambda = 2$, $\eta = .2$, $\delta_H = 1$, $\delta_L = 0$. For the top-left panel, $\beta^I = .7$ and for the bottom-left, $\phi = .7$. In the top-right panel, I fix the discount factor $r = .003$ and $\beta^I = .7$ and show, for each ϕ , the punishment strategy τ . The τ values in black correspond to punishment in trades in which uninformed agents are buyers subject to the punishment and the τ values in gray to punishments to informed buyers. Accordingly, the bottom-right panel displays τ as a function of β^I when $r = .003$ and $\phi = .7$.

agent is small, she would not be able to unilaterally deviate from this norm. Of course, such equilibrium is not intuitive – why would informed agents coordinate on giving uninformed agents a higher share of the surplus? In the next section, I discuss refinements which would rule out that equilibrium.

Despite the limited commitment friction, all positive surplus trades take place and ex-ante efficiency is achieved as before. In fact, the proportion of unbalanced agents, $\mu^I + \mu^U$, is the same as in the symmetric equilibrium.

In the limit when $\beta^I \rightarrow \frac{1}{2}$, we have $S_H^U = S_H^I > S_L^I = S_L^U$, in which case agents are indifferent between performing the intermediation trades or not. In this case, the equilibrium trades and prices are the same as in the symmetric equilibrium shown in section 3.1.

3.4. Other Equilibria. The complete equilibrium set in this model is obviously not exhausted by the equilibria introduced here. For one, hybrid equilibria in which both the asymmetric trade opportunities and asymmetric surplus sharing channel are activated can be easily designed.

We can also flip the proposed punishment structures so that the roles of the informed and uninformed agents are reversed. As mentioned before, this is possible because even if all informed agents are coordinating on a social norm that is beneficial to uninformed agents, each individual informed agent cannot profitably deviate from that norm.

We saw that equilibrium trading is determined by the ordering of the objects $(S_H^I, S_H^U, S_L^I, S_L^U)$, as well as if when agents meet the trades are secured by the punishment strategy or not. Since agents with high valuation have a higher flow value from holding the asset, it is the case that in any equilibrium $S_H^i > S_L^j$ for $i, j \in \{I, U\}$. While the whole set of equilibria is not exhausted by the ones shown above, they do illustrate that in essence, intermediation trades happen whenever $S_H^I \neq S_H^U$ or $S_L^I \neq S_L^U$ and that these differences can be generated by two different channels.

4. WHICH EQUILIBRIUM SHOULD WE EXPECT?

The equilibria shown in the last section help us understand the channels that generate motive for intermediation and how those can be determined by the punishment in place. In this section, I propose two ways to refine the equilibrium set. First, I look for equilibria that are best for informed agents. This is a natural requirement in this model, since informed agents are the ones with the ability to punish. After that, I propose a different refinement, but with the same flavor, asking which equilibria survive if information is costly to acquire.

4.1. Maximizing Value to Informed Agents. Define the objects V^I and V^U , the value of being informed and uninformed, respectively. Half the agents have high valuation and half have low valuation, regardless of their information type, and this is reflected in the weighting in V^I and V^U .

These values are also defined considering an agent that enters the market without holding an asset. Despite the fact that these are values for agents who initially do not have an asset, it already accounts for the value of holding the asset, since these agents know that over time they will be hit by asset holding shocks, and may also trade assets. Having no asset is simply the starting point. The results in this section equally hold if we look at the opposite case, where the value accounts for an agent that enters the market holding an asset.

$$(14) \quad V^I := \frac{V_{H0}^I + V_{L0}^I}{2}$$

$$(15) \quad V^U := \frac{V_{H0}^U + V_{L0}^U}{2}$$

Any equilibrium with trade is supported by informed agents coordinating on a punishing strategy. It is then natural to expect that informed agents would coordinate on strategies that maximize their own value in the market. In this section, I look for properties of equilibria that maximize the value to informed agents for each given set of parameters.

Observation 1. *The equilibrium which maximizes the value to informed agents has $V^I \geq V^U$.*

If any equilibrium with $V^U > V^I$ exists, then all the labels in that equilibrium can be flipped so that the roles of informed and uninformed agents are reversed. This generates an equilibrium which improves on the original value to informed agents, and thus the original equilibrium did not maximize the value to informed agents. The next result compares the value to informed agents in two of the equilibria that were built in the previous section.

Proposition 4. *The value to informed agents is higher in the equilibrium with asymmetric trade opportunities than in the symmetric equilibrium.*

Proof. In the symmetric equilibrium, we have:

$$\begin{aligned}
 rV_{H0} &= \eta \left[\frac{\delta_H + \delta_L}{2(r + 2\eta)} + \frac{\delta_H - \delta_L}{2(r + 2\eta + \lambda(\mu^U + \mu^I))} \right] + \frac{\lambda(\mu^U + \mu^I)(\delta_H - \delta_L)}{2(r + 2\eta + \lambda(\mu^U + \mu^I))} \\
 rV_{L0} &= \eta \left[\frac{\delta_H + \delta_L}{2(r + 2\eta)} - \frac{\delta_H - \delta_L}{2(r + 2\eta + \lambda(\mu^U + \mu^I))} \right] \\
 (16) \quad \Rightarrow V^I &= \frac{\eta(\delta_H + \delta_L)}{2r(r + 2\eta)} + \frac{\lambda(\mu^U + \mu^I)(\delta_H - \delta_L)}{4r(r + 2\eta + \lambda(\mu^U + \mu^I))}
 \end{aligned}$$

where μ^I and μ^U are given by the stationary distribution in this equilibrium, as defined before.

In the equilibrium with asymmetric trade opportunities, we have:

$$\begin{aligned}
 rV_{H0}^I &= \eta S_H^I + \frac{\lambda}{2} \hat{\mu}^I (S_H^I - S_L^I) + \frac{\lambda}{2} \hat{\mu}^U (S_H^I - S_L^U) \\
 &> \eta S_H^I + \frac{\lambda}{2} \hat{\mu}^I (S_H^I - S_L^I) + \frac{\lambda}{2} \hat{\mu}^U (S_H^I - S_L^I) \\
 &= \eta \left[\frac{\delta_H + \delta_L}{2(r + 2\eta)} + \frac{\delta_H - \delta_L}{2(r + 2\eta + \lambda(\hat{\mu}^U + \hat{\mu}^I))} \right] + \frac{\lambda(\hat{\mu}^U + \hat{\mu}^I)(\delta_H - \delta_L)}{2(r + 2\eta + \lambda(\hat{\mu}^U + \hat{\mu}^I))} \\
 rV_{L0} &= \eta S_L^I + \frac{\lambda}{2} \hat{\mu}^U (S_L^I - S_L^U) > \eta S_L^I = \eta \left[\frac{\delta_H + \delta_L}{2(r + 2\eta)} - \frac{\delta_H - \delta_L}{2(r + 2\eta + \lambda(\hat{\mu}^U + \hat{\mu}^I))} \right] \\
 (17) \quad \Rightarrow V^I &> \frac{\delta_L}{2r} + \frac{\eta(\delta_H + \delta_L)}{2r(r + 2\eta)} + \frac{\lambda(\hat{\mu}^U + \hat{\mu}^I)(\delta_H - \delta_L)}{4r(r + 2\eta + \lambda(\hat{\mu}^U + \hat{\mu}^I))}
 \end{aligned}$$

where $\hat{\mu}^I$ and $\hat{\mu}^U$ are given by the stationary distribution in this equilibrium, as defined before.

We previously showed that $\hat{\mu}^U + \hat{\mu}^I > \mu^U + \mu^I$; and this implies that the value in (17) is greater than the value in (16), which proves the statement. ■

The difference between the symmetric equilibrium and the equilibrium with asymmetric trade opportunities is simply that in the latter, uninformed agents cannot trade with each other. In essence, when trade between uninformed agents is shut down, uninformed agents become more desperate to trade with informed agents, which increases the surplus in meetings between uninformed and informed agents. This increased surplus means that informed agents can now capture a higher value from these trades. Additionally, informed agents now also capture surplus in intermediation trades, which did not take place in the symmetric equilibrium.

Since the symmetric equilibrium maximizes aggregate welfare, it delivers the highest value to informed agents amongst the equilibria that yield equal value to all agents. Hence, if the symmetric equilibrium is dominated by another equilibrium, we know that the informed agents' favorite equilibrium must yield strictly lower value to uninformed agents than informed agents.

The feature that blocking trade between uninformed agents increases the value to informed ones extends further than just the comparison between the two equilibria in proposition 4. I show in the next result that the proposed equilibria with asymmetric surplus sharing, where uninformed agents do trade with each other, can also be improved upon from the perspective of informed agents by shutting down trade between uninformed agents.

Proposition 5. *Let there be an equilibrium with asymmetric surplus sharing, indexed by β^I as before. Then if ϕ is large enough and r low enough, there exists another equilibrium, in which uninformed agents do not trade amongst themselves, that yields higher value to informed agents.*

Proof. Proof in the appendix. ■

Another relevant feature of equilibria in which trade amongst uninformed agents does not take place is that the information about trades between uninformed agents is never used. This means that they are also robust to variations in the information technology: if informed agents only had access to information about trades involving at least one informed agent, these equilibria would still be supported. On the other hand, equilibria in which uninformed agents trade with each other require informed agents to have access to all the information available.

4.2. Costly Information Acquisition. So far, the measure of informed agents in the market was taken as given. However, this measure itself might be a result of agents deciding to invest in acquiring information about transactions in the market or not. Suppose there is a stage prior to the market stage in which all agents start out uninformed. In this pre-play stage, agents can pay a cost $k > 0$ to become informed.

When considering whether to make this investments, agents weigh the cost k against the benefit to becoming informed, i.e., the difference between the value to informed and uninformed agents in the market stage. Well, this difference depends on what equilibrium agents expect to be played in the market interaction. Of course, if they expect that in the market stage, informed

agents have lower or equal value to that of uninformed agents, they choose to not become informed. If all agents choose to not become informed, then the only equilibrium that can be supported in the market stage is autarky.

First, let's define an equilibrium of the two-stage game.

Definition 1. A stationary equilibrium of the two-stage game with no default is a probability of becoming informed $\phi \in [0, 1]$, a benefit of becoming informed, B , a set of unflagged value functions $\{V_{va}^i\}$, flagged value functions $\{\tilde{V}_{va}^i\}$, trade indicator \mathcal{I} , seller surplus shares β and punishment strategy τ and stationary distribution $\{\mu_{va}^i\}$ such that:

- (i) Given ϕ , $\{\{V_{va}^i\}, \{\tilde{V}_{va}^i\}, \mathcal{I}, \beta, \tau, \{\mu_{va}^i\}\}$ is a symmetric stationary equilibrium of the second stage game;
- (ii) $B = V^I - V^U$ as given by (14) and (15);
- (iii) Given B , $\phi > 0 \Rightarrow B \geq k$ and $\phi < 1 \Rightarrow B \leq k$.

Knowing that no trade can be sustained in the market stage without the presence of informed agents, the following observation is immediate.

Observation 2. Any non-autarky equilibrium of the two-stage game must have $V^I > V^U$.

This observation already tells us that adding costly acquisition of information rules out the symmetric equilibrium, since it assigns the same value to informed and uninformed agents. On the other hand, both the equilibrium with asymmetric trade opportunities and the equilibrium with asymmetric surplus sharing might be supported as an equilibrium of the two-stage game, as the benefit to becoming informed is no longer 0. In the equilibrium with asymmetric trade opportunities, using (14) and (15) and the stationary distribution found before, I find that

$$\begin{aligned}
 V^I &= \frac{\delta_H}{2} + \frac{\eta}{2}(S_L^I - S_H^I) + \frac{\lambda\mu^U}{4}(S_H^U - S_H^I + S_L^I - S_L^U) - \frac{1}{2}S_H^I \\
 V^U &= \frac{\delta_H}{2} + \frac{\eta}{2}(S_L^U - S_H^U) - \frac{1}{2}S_H^U \\
 \Rightarrow V^I - V^U &= \left(\frac{\eta}{2} + \frac{\lambda\mu^U}{4}\right)(S_L^I - S_L^U + S_H^U - S_H^I) + \frac{1}{2}(S_H^U - S_H^I) \\
 (18) \quad &= \left(1 + \eta + \frac{\lambda\mu^U}{4}\right)(\alpha^U - \alpha^I)(\delta_H - \delta_L)
 \end{aligned}$$

The value of being informed in this equilibrium, as expressed in (18) is positive for any $\phi < 1$. As $\phi \rightarrow 1$, we have $\alpha^U \rightarrow \alpha^I$ and the benefit goes to 0. This positive benefit almost immediately implies the existence of an equilibrium of the two-stage game where the equilibrium with asymmetric trade opportunities is played in the second stage. Analogous steps could be

followed to show that the equilibrium with asymmetric surplus sharing also assigns a higher value to informed agents than uninformed ones.

Proposition 6. *There is no equilibrium of the two-stage game in which the symmetric equilibrium is played in the market stage. On the other hand, if r and k are low enough, there exist equilibria of the two-stage game where either the equilibrium with asymmetric trade opportunities or the equilibrium with asymmetric surplus sharing are played in the second stage.*

In the equilibria I have constructed, we find $V^I > V^U$ to be associated with the presence of intermediation trades. However, it is not the case that intermediation is a necessary feature of any equilibrium in which informed agents have higher value. One counterexample is an equilibrium in which informed agents extract higher surplus, but are prevented from performing intermediation trades because the intermediation trades are not monitored.

However, when no trades are hindered by risk of default, all equilibria with $V^I > V^U$ nest some intermediation trades, i.e., some trade where agents with high valuation sell their asset or agents with low valuation buy assets. This result tells us that, when information is costly, intermediation is a robust feature of equilibria.

Proposition 7. *If an equilibrium has $V^I > V^U$ and is so that all trades with strictly positive surplus take place, then at least one type of intermediation trade takes place in it.*

Proof. Proof in the appendix. ■

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6. APPENDIX

6.1. Symmetric Equilibrium. The building of this equilibrium is a big guess and verify.

Guesses.

Symmetric surplus sharing: $\beta(i_s, v_s, i_b, v_b) = \frac{1}{2}$ if $\mathcal{I}(i_s, v_s, i_b, v_b) = 1$.

Efficient trading: $\mathcal{I}(i_s, v_s, i_b, v_b) = 1 \Leftrightarrow v_s = L$ and $v_b = H$.

Stationary Distribution. Given the guesses for \mathcal{I} , the inflow equal to outflow equations for the stationary distribution become, for $i \in \{I, U\}$:

$$\begin{aligned}\mu_{L1}^i (\eta + \lambda(\mu_{H0}^U + \mu_{H0}^I)) &= \eta \mu_{L0}^i \\ \mu_{H0}^i (\eta + \lambda(\mu_{L1}^U + \mu_{L1}^I)) &= \eta \mu_{H1}^i\end{aligned}$$

Combining these with the fact that half of the agents have high valuation and half have low valuation (uncorrelated with information types), i.e., $\mu_{v0}^I + \mu_{v1}^I = \frac{\phi}{2}$ and $\mu_{v0}^U + \mu_{v1}^U = \frac{1-\phi}{2}$ for $v \in \{H, L\}$, I get:

$$(19) \quad \mu_{L1}^I (2\eta + \lambda(\mu_{H0}^U + \mu_{H0}^I)) = \mu_{H0}^I (2\eta + \lambda(\mu_{L1}^U + \mu_{L1}^I)) = \frac{\eta\phi}{2}$$

$$(20) \quad \mu_{L1}^U (2\eta + \lambda(\mu_{H0}^U + \mu_{H0}^I)) = \mu_{H0}^U (2\eta + \lambda(\mu_{L1}^U + \mu_{L1}^I)) = \frac{\eta(1-\phi)}{2}$$

Adding up equations (19) and (20):

$$(21) \quad (\mu_{L1}^I + \mu_{L1}^U) (2\eta + \lambda(\mu_{H0}^U + \mu_{H0}^I)) = (\mu_{H0}^I + \mu_{H0}^U) (2\eta + \lambda(\mu_{L1}^U + \mu_{L1}^I)) = \frac{\eta}{2}$$

$$(22) \quad \Rightarrow (\mu_{L1}^I + \mu_{L1}^U) = (\mu_{H0}^I + \mu_{H0}^U)$$

Now plug (22) back into (21) to get:

$$\begin{aligned}(23) \quad & \lambda(\mu_{L1}^I + \mu_{L1}^U)^2 + 2\eta(\mu_{L1}^I + \mu_{L1}^U) - \frac{\eta}{2} = 0 \\ & \Rightarrow (\mu_{L1}^I + \mu_{L1}^U) = (\mu_{H0}^I + \mu_{H0}^U) = -\frac{\eta}{\lambda} + \sqrt{\left(\frac{\eta}{\lambda}\right)^2 + \frac{1}{2}\frac{\eta}{\lambda}}\end{aligned}$$

Plug (23) back into (19) and (20) to get $\mu_{H0}^U = \mu_{L1}^U =: \mu^U$ and $\mu_{H0}^I = \mu_{L1}^I =: \mu^I$, and finally:

$$\begin{aligned}\mu^I &= \phi \left[-\frac{\eta}{\lambda} + \sqrt{\left(\frac{\eta}{\lambda}\right)^2 + \frac{1}{2}\frac{\eta}{\lambda}} \right] \\ \mu^U &= (1-\phi) \left[-\frac{\eta}{\lambda} + \sqrt{\left(\frac{\eta}{\lambda}\right)^2 + \frac{1}{2}\frac{\eta}{\lambda}} \right]\end{aligned}$$

Unflagged Values. Taking into account the guesses for \mathcal{I} and β , and also the result for the stationary distribution, unflagged values are the same for informed and uninformed agents and

are defined below, where the index for the information type was removed.

$$\begin{aligned} rV_{H0} &= \eta(V_{H1} - V_{H0}) + \lambda(\mu^I + \mu^U) \left[\frac{V_{H1} - V_{H0} + V_{L0} - V_{L1}}{2} \right] \\ rV_{H1} &= \delta_H + \eta(V_{H0} - V_{H1}) \\ rV_{L1} &= \delta_L + \eta(V_{L0} - V_{L1}) + \lambda(\mu^I + \mu^U) \left[\frac{V_{H1} - V_{H0} + V_{L0} - V_{L1}}{2} \right] \\ rV_{L0} &= \eta(V_{L1} - V_{L0}) \end{aligned}$$

From these, I find:

$$\begin{cases} (r + 2\eta)S_H = \delta_1 - \frac{\lambda(\mu^U + \mu^I)}{2}(S_H - S_L) \\ (r + 2\eta)S_L = \delta_0 + \frac{\lambda(\mu^U + \mu^I)}{2}(S_H - S_L) \end{cases} \Rightarrow S_H - S_L = \frac{\delta_1 - \delta_0}{r + 2\eta + \lambda(\mu^U + \mu^I)}$$

This can now be rearranged to find the values of S_H and S_L in the main text.

This confirms that the conjectured \mathcal{I} and β indeed satisfy (8) and (9), since trades have positive surplus if and only if $v_s = L$ and $v_b = H$.

Punishing Strategy. To confirm the existence of this equilibrium, we now only need to build a punishing strategy to support it.

For the trades for which $\mathcal{I}(i_s, v_s, i_b, v_b) = 0$, which have non-positive surplus, as found above, we can set any punishment level. For instance, let $\tau(i_s, v_s, i_b, v_b) = 0$ if $\mathcal{I}(i_s, v_s, i_b, v_b) = 0$. All that is left to show is that there exists a punishing strategy τ under which $\beta(i_s, L, i_b, H) = \frac{1}{2}$ satisfies (3). I need to find τ such that:

$$\frac{1}{2} = \frac{V_{L0}^{i_s} - V_{L1}^{i_s} + \tau(i_s, L, i_b, H) (V_{H1}^{i_b} - \tilde{V}_{H1}^{i_b})}{V_{L0}^{i_s} - V_{L1}^{i_s} + V_{H1}^{i_b} - V_{H0}^{i_b}}$$

Since in the symmetric equilibrium values do not depend on agents' information types, I set $\tau(i_s, L, i_b, H) = \hat{\tau}$ for all $i_s, i_b \in \{U, I\}$ and look for $\hat{\tau}$ such that:

$$\frac{1}{2} = \frac{V_{L0} - V_{L1} + \hat{\tau} (V_{H1} - \tilde{V}_{H1})}{V_{L0} - V_{L1} + V_{H1} - V_{H0}} \Leftrightarrow \hat{\tau} (V_{H1} - \tilde{V}_{H1}) = \frac{S_H + S_L}{2}$$

It is easier to work with the object $D_{va} = V_{va} - \tilde{V}_{va}$, hence I look for $\hat{\tau}$ such that $D_{H1} = \frac{S_H + S_L}{2\hat{\tau}}$. The system that defines values $\{D_{va}\}$ is:

$$(24) \quad rD_{H0} = (\eta + \lambda\mu^U)(D_{H1} - D_{H0}) - \lambda\mu^U \frac{S_H + S_L}{2} + \lambda\mu^I \frac{S_H - S_L}{2}$$

$$(25) \quad rD_{H1} = \eta(D_{H0} - D_{H1})$$

$$(26) \quad rD_{L1} = (\eta + \lambda\mu^U)(D_{L0} - D_{L1}) + \lambda\mu^I \frac{S_H - S_L}{2}$$

$$(27) \quad rD_{L0} = \eta(D_{L1} - D_{L0})$$

From (24) and (25), I get:

$$(28) \quad (D_{H1} - D_{H0}) = \frac{\lambda\mu^U}{r + 2\eta + \lambda\mu^U} \left(\frac{S_H + S_L}{2} \right) - \frac{\lambda\mu^I}{r + 2\eta + \lambda\mu^U} \left(\frac{S_H - S_L}{2} \right)$$

Plug (28) back into (25) to get:

$$D_{H1} = \frac{\eta\lambda\mu^I}{r(r + 2\eta + \lambda\mu^U)} \left(\frac{S_H - S_L}{2} \right) - \frac{\eta\lambda\mu^U}{r(r + 2\eta + \lambda\mu^U)} \left(\frac{S_H + S_L}{2} \right)$$

If $D_{H1}(0) \geq \frac{S_H + S_L}{2\hat{\tau}}$, I can conclude that there exists $\hat{\tau} \in (0, 1]$ such that $D_{H1} = \frac{S_H + S_L}{2\hat{\tau}}$, as desired. Plugging in values for S_H and S_L and the expression for D_{H1} , this condition is satisfied if:

$$(29) \quad \frac{\eta\lambda\mu^I}{r + 2\eta + \lambda\mu^U} \frac{\delta_H - \delta_L}{2(r + 2\eta + \lambda(\mu^I + \mu^U))} \geq \left[r + \frac{\eta\lambda\mu^U}{r + 2\eta + \lambda\mu^U} \right] \frac{\delta_H + \delta_L}{2(r + 2\eta)}$$

At $\phi = 1$ (hence $\mu^I > 0$ and $\mu^U = 0$) and $r = 0$, this condition is satisfied. By continuity, I know that there is a neighborhood of $(\phi, r) = (1, 0)$ in which (29) holds. In that neighborhood, the punishment strategy $\tau(i_s, v_s, i_b, v_b) = \hat{\tau}$ for $(v_s, v_b) = (L, H)$ and $\tau(i_s, v_s, i_b, v_b) = 0$ otherwise supports the original guess for β and a symmetric equilibrium exists.

Also note that, if $\phi \rightarrow 0$, then the left hand side of (29) is 0 while the right hand side is positive and hence, for any value of r , (29) cannot hold. This means that there must be a strictly positive mass of informed agents in order for a symmetric equilibrium to exist.

6.2. Equilibrium with Asymmetric Trade Opportunities. Once again, we build this equilibrium with a big guess and verify. The steps to solving for the equilibrium objects in this case are similar to the ones in the Symmetric Equilibrium. They can be found in the Online Appendix. The exception is the punishing strategy that supports the equilibrium, which I fully report here.

Guesses.

Symmetric surplus sharing: $\beta(i_s, v_s, i_b, v_b) = \frac{1}{2}$ if $\mathcal{I}(i_s, v_s, i_b, v_b) = 1$.

Efficient trading iff an informed agent is involved:

$$\mathcal{I}(i_s, v_s, i_b, v_b) = 1 \Leftrightarrow (i_s, i_b) \neq (U, U) \text{ and } V_{v_s0}^{i_s} - V_{v_s1}^{i_s} + V_{v_b1}^{i_b} - V_{v_b0}^{i_b} > 0.$$

Stationary Distribution.

$$\mu_{L1}^U = \mu_{H0}^U = \mu^U = \frac{1-\phi}{4+\frac{\lambda}{\eta}} \quad \mu_{H0}^I = \mu_{L1}^I = \mu^I = -\frac{\eta+\lambda\mu^U}{\lambda} + \sqrt{\left(\frac{\eta+\lambda\mu^U}{\lambda}\right)^2 + \frac{\phi}{2} \frac{\eta+\lambda\mu^U}{\lambda}}$$

Unflagged Values.

$$S_H^i = \left[\frac{1}{2(r+2\eta)} \right] (\delta_H + \delta_L) + \frac{\alpha^i}{2} (\delta_H - \delta_L) \quad S_L^i = \left[\frac{1}{2(r+2\eta)} \right] (\delta_H + \delta_L) - \frac{\alpha^i}{2} (\delta_H - \delta_L)$$

where $\alpha^I = \frac{1}{(r+2\eta+\lambda(\mu^U+\mu^I))}$ and $\alpha^U = \frac{r+2\eta+\lambda\mu^U+\frac{\lambda\phi}{4}}{(r+2\eta+\frac{\lambda\phi}{4})(r+2\eta+\lambda(\mu^U+\mu^I))}$.

This confirms that the conjectured \mathcal{I} and β indeed satisfy (8) and (9).

Punishing Strategy. To conclude that an Intermediation Equilibrium exists, all that is left to show is that there exists a punishing strategy τ under which the conjectured β satisfies (3). Once again, for the trades for which $\mathcal{I}(i_s, v_s, i_b, v_b) = 0$, this is trivial, since we already checked that they have non-positive surplus, in which case to fully specify the punishing strategy, set no punishment, i.e., let $\tau(i_s, v_s, i_b, v_b) = 0$ when $\mathcal{I}(i_s, v_s, i_b, v_b) = 0$.

Analogously to the case of the symmetric equilibrium, define $D_{va}^i = V_{va}^i - \tilde{V}_{va}^i$. The other conditions which need to be satisfied by τ in order to support the conjectured β are:

$$(30) \quad \begin{aligned} \tau(I, L, I, H)D_{H1}^I &= \frac{S_H^I + S_L^I}{2} & \tau(U, L, I, L)D_{L1}^I &= \frac{S_L^U + S_L^I}{2} & \tau(U, L, I, H)D_{H1}^I &= \frac{S_L^U + S_H^I}{2} \\ \tau(I, H, U, H)D_{H1}^U &= \frac{S_H^U + S_H^I}{2} & \tau(I, L, U, H)D_{H1}^U &= \frac{S_H^U + S_L^I}{2} \end{aligned}$$

First, I solve for the values D_{H1}^I , D_{L1}^I and D_{H1}^U . The system defining $\{D_{va}^I\}$ is:

$$\begin{aligned} rD_{H1}^I &= (\eta + \lambda\mu^U)(D_{H0}^I - D_{H1}^I) \\ rD_{L1}^I &= (\eta + \lambda\mu^U)(D_{L0}^I - D_{L1}^I) + \lambda\mu^I \frac{S_H^I - S_L^I}{2} \\ rD_{L0}^I &= (\eta + \lambda\mu^U)(D_{L1}^I - D_{L0}^I) - \lambda\mu^U \frac{S_L^U + S_L^I}{2} \\ rD_{H0}^I &= (\eta + \lambda\mu^U)(D_{H1}^I - D_{H0}^I) + \lambda\mu^I \frac{S_H^I - S_L^I}{2} - \lambda\mu^U \frac{S_H^I + S_L^U}{2} \end{aligned}$$

Combining the first with the fourth and the second with the third:

$$\begin{aligned} r(D_{H1}^I - D_{H0}^I) &= -2(\eta + \lambda\mu^U)(D_{H1}^I - D_{H0}^I) - \lambda\mu^I \frac{S_H^I - S_L^I}{2} + \lambda\mu^U \frac{S_H^I + S_L^U}{2} \\ r(D_{L1}^I - D_{L0}^I) &= -2(\eta + \lambda\mu^U)(D_{L1}^I - D_{L0}^I) + \lambda\mu^I \frac{S_H^I - S_L^I}{2} + \lambda\mu^U \frac{S_L^U + S_L^I}{2} \end{aligned}$$

Solve to find:

$$\begin{aligned} (D_{H1}^I - D_{H0}^I) &= -\frac{\lambda\mu^I}{(r + 2\eta + 2\lambda\mu^U)} \frac{S_H^I - S_L^I}{2} + \frac{\lambda\mu^U}{(r + 2\eta + 2\lambda\mu^U)} \frac{S_H^I + S_L^U}{2} \\ (D_{L1}^I - D_{L0}^I) &= \frac{\lambda\mu^I}{(r + 2\eta + 2\lambda\mu^U)} \frac{S_H^I - S_L^I}{2} + \frac{\lambda\mu^U}{(r + 2\eta + 2\lambda\mu^U)} \frac{S_L^U + S_L^I}{2} \end{aligned}$$

Plug this back into the original system to get:

$$(31) \quad D_{H1}^I = \frac{\eta + \lambda\mu^U}{r(r + 2\eta + 2\lambda\mu^U)} (\lambda\mu^I \frac{S_H^I - S_L^I}{2} - \lambda\mu^U \frac{S_H^I + S_L^U}{2})$$

$$(32) \quad D_{L1}^I = \frac{r + \eta + \lambda\mu^U}{r(r + 2\eta + 2\lambda\mu^U)} \lambda\mu^I \frac{S_H^I - S_L^I}{2} - \frac{\eta + \lambda\mu^U}{r(r + 2\eta + 2\lambda\mu^U)} \lambda\mu^U \frac{S_L^U + S_L^I}{2}$$

The system defining $\{D_{va}^U\}$ is:

$$\begin{aligned} rD_{H1}^U &= \eta(D_{H0}^U - D_{H1}^U) \\ rD_{H0}^U &= \eta(D_{H1}^U - D_{H0}^U) + \lambda\mu^I \frac{S_H^U - S_L^I}{2} + \lambda(\phi/2 - \mu^I) \frac{S_H^U - S_H^I}{2} \\ rD_{L1}^U &= \eta(D_{L0}^U - D_{L1}^U) + \lambda\mu^I \frac{S_H^I - S_L^U}{2} + \lambda(\phi/2 - \mu^I) \frac{S_L^I - S_L^U}{2} \\ rD_{L0}^U &= \eta(D_{L1}^U - D_{L0}^U) \end{aligned}$$

Subtract the second from the first to get:

$$\begin{aligned} r(D_{H1}^U - D_{H0}^U) &= -2\eta(D_{H1}^U - D_{H0}^U) - \lambda\mu^I \frac{S_H^U - S_L^I}{2} - \lambda(\phi/2 - \mu^I) \frac{S_H^U - S_H^I}{2} \\ \Rightarrow (D_{H1}^U - D_{H0}^U) &= -\frac{\lambda\mu^I}{r + 2\eta} \frac{S_H^U - S_L^I}{2} - \frac{\lambda(\phi/2 - \mu^I)}{r + 2\eta} \frac{S_H^U - S_H^I}{2} \end{aligned}$$

Finally plug back into the original system to get:

$$(33) \quad D_{H1}^U = \frac{\eta\lambda\mu^I}{r(r + 2\eta)} \frac{S_H^U - S_L^I}{2} + \frac{\eta\lambda(\phi/2 - \mu^I)}{r(r + 2\eta)} \frac{S_H^U - S_H^I}{2}$$

If $D_{H1}^I(0) \geq \frac{S_H^I + S_L^I}{2}$, $D_{L1}^I(0) \geq \frac{S_L^U + S_L^I}{2}$ and $D_{H1}^U(0) \geq \frac{S_H^U + S_H^I}{2}$, I can conclude that there exist $\tau(I, L, I, H)$, $\tau(U, L, I, L)$, $\tau(U, L, I, H)$, $\tau(I, H, U, H)$, and $\tau(I, L, U, H)$ such that (30) are satisfied, as desired.

It is easy to check, analogously to in the symmetric equilibrium, that these conditions hold as $\phi \rightarrow 1$ (i.e., $\mu^I > 0$, $\mu^U \rightarrow 0$) and $r \rightarrow 0$. Hence, I conclude that, in a neighborhood of $(\phi, r) = (1, 0)$, there exists a punishing strategy τ which supports the conjectured β .

Showing that there are more unbalanced agents than in the symmetric equilibrium.

In the symmetric equilibrium, we had:

$$\begin{aligned}
 \mu^U(\eta + \lambda(\mu^U + \mu^I)) &= \left(\frac{1-\phi}{2} - \mu^U\right) \eta \\
 \mu^I(\eta + \lambda(\mu^U + \mu^I)) &= \left(\frac{\phi}{2} - \mu^I\right) \eta \\
 (34) \quad &\Rightarrow 2\eta(\mu^U + \mu^I)^2 + 4\eta(\mu^U + \mu^I) = \eta
 \end{aligned}$$

In the equilibrium with asymmetric trade opportunities:

$$\begin{aligned}
 \mu^U(\eta + \lambda\mu^I + \lambda(\phi/2 - \mu^I)) &= \left(\frac{1-\phi}{2} - \mu^U\right) \eta \\
 \mu^I(\eta + \lambda(\mu^U + \mu^I)) &= \left(\frac{\phi}{2} - \mu^I\right) (\eta + \lambda\mu^U) \\
 (35) \quad &\Rightarrow 2\eta(\mu^U + \mu^I)^2 + 4\eta(\mu^U + \mu^I) = \eta + \lambda(\mu^U)^2
 \end{aligned}$$

Now notice that $\mu^U + \mu^I$ must be higher in the equilibrium with asymmetric trade opportunities as given by (35) then in the symmetric equilibrium as given by (34).

6.3. Equilibria with Asymmetric Surplus Sharing. Once more, we build this equilibrium with a big guess and verify. The steps to solving for the equilibrium objects in this case are similar to the ones in the other equilibria. They can be found in the Online Appendix.

Guesses.

Asymmetric surplus sharing:

$$\begin{aligned}
 \beta(I, v_s, U, v_b) &= \beta^I > \frac{1}{2} & \beta(U, v_s, I, v_b) &= 1 - \beta^I < \frac{1}{2} \\
 \beta(I, v_s, I, v_b) &= \frac{1}{2} & \beta(U, v_s, U, v_b) &= \frac{1}{2}
 \end{aligned}$$

Efficient trading:

$$\mathcal{I}(i_s, v_s, i_b, v_b) = 1 \Leftrightarrow V_{v_s 0}^{i_s} - V_{v_s 1}^{i_s} + V_{v_b 1}^{i_b} - V_{v_b 0}^{i_b} > 0.$$

Stationary Distribution.

$$(\mu^U + \mu^I) = -\frac{\eta}{\lambda} + \sqrt{\left(\frac{\eta}{\lambda}\right)^2 + \frac{1}{2} \frac{\eta}{\lambda}}$$

Unflagged Values.

$$S_H^i = \left[\frac{1}{2(r+2\eta)} \right] (\delta_H + \delta_L) + \frac{\tilde{\alpha}^i}{2} (\delta_H - \delta_L) \quad S_L^i = \left[\frac{1}{2(r+2\eta)} \right] (\delta_H + \delta_L) - \frac{\tilde{\alpha}^i}{2} (\delta_H - \delta_L)$$

where $\tilde{\alpha}^I = \frac{1}{(r+2\eta+\lambda(2\mu^U\beta^I+\mu^I))}$ and $\tilde{\alpha}^U = \frac{r+2\eta+\lambda\beta^I(2\mu^U+\mu^I)+\lambda(1-\beta^I)(\phi/2-\mu^I)}{(r+2\eta+\lambda(\mu^U+\phi/2(1-\beta^I)))(r+2\eta+\lambda(2\mu^U\beta^I+\mu^I))}$.

6.4. Proof of Proposition 5. Suppose an equilibrium with asymmetric surplus sharing exists in which informed agents keep β^I share of the surplus in meetings with uninformed agents. Let's build a new equilibrium, with the same surplus sharing rule, but where uninformed agents do not trade with each other.

Guesses.

Surplus sharing:

$$\begin{cases} \beta(I, v_s, U, v_b) = \beta^I \text{ if } \mathcal{I}(I, v_s, U, v_b) = 1 \\ \beta(U, v_s, I, v_b) = 1 - \beta^I \text{ if } \mathcal{I}(U, v_s, I, v_b) = 1 \\ \beta(i, v_s, i, v_b) = \frac{1}{2} \text{ if } \mathcal{I}(i, v_s, i, v_b) = 1, \text{ for } i \in \{U, I\} \end{cases}$$

Equilibrium trades:

$$\begin{cases} \mathcal{I}(i_s, L, i_b, H) = 1 \text{ if } (i_s, i_b) \neq (U, U) \\ \mathcal{I}(I, H, U, H) = \mathcal{I}(U, L, I, L) = 1 \\ \mathcal{I}(i_s, v_s, i_b, v_b) = 0 \text{ otherwise} \end{cases}$$

With these guesses, use the same steps as in the other equilibria to find that:

$$\mu^U = \frac{1-\phi}{4+\frac{\lambda}{\eta}} \quad \mu^I = -\frac{\eta+\lambda\mu^U}{\lambda} + \sqrt{\left(\frac{\eta+\lambda\mu^U}{\lambda}\right)^2 + \frac{\phi}{2} \frac{\eta+\lambda\mu^U}{\lambda}}$$

$$S_H^i = \left[\frac{1}{2(r+2\eta)} \right] (\delta_H + \delta_L) + \tilde{\alpha}^i (\delta_H - \delta_L) \quad S_L^i = \left[\frac{1}{2(r+2\eta)} \right] (\delta_H + \delta_L) - \tilde{\alpha}^i (\delta_H - \delta_L)$$

where $\tilde{\alpha}^I = \frac{1}{2(r+2\eta+\lambda(2\mu^U\beta^I+\mu^I))}$ and $\tilde{\alpha}^U = \frac{r+2\eta+\lambda\beta^I(2\mu^U+\mu^I)+\lambda(1-\beta^I)(\frac{\phi}{2}-\mu^I)}{2(r+2\eta+\lambda\frac{\phi}{2}(1-\beta^I))(r+2\eta+\lambda(2\mu^U\beta^I+\mu^I))}$.

Again, since $\beta^I \in [1/2, 1)$, these are ordered $S_H^U \geq S_H^I > S_L^I \geq S_L^U$, strict if $\beta^I > 1/2$. Also, $\mu^U > \tilde{\mu}^U$ and $\mu^I > \tilde{\mu}^I$, where $\tilde{\mu}^U$ and $\tilde{\mu}^I$ are the ones given in the stationary distribution of the original equilibrium with asymmetric surplus sharing.

This implies that $\tilde{\alpha}^U > \alpha^U$ and $\tilde{\alpha}^I < \alpha^I$, where α^U and α^I are the ones given in the original asymmetric surplus sharing equilibrium. Now using the same steps as in the proof of proposition 4, we can check that the value to informed agents is higher in this new equilibrium where uninformed agents do not trade with each other.

As before, we can check that if ϕ is high enough and r low enough, a punishing strategy supporting this new equilibrium exists. ■

6.5. Proof of Proposition 7. Assume that all positive surplus trades take place: $V_{v_s 0}^{i_s} - V_{v_s 1}^{i_s} + V_{v_b 1}^{i_b} - V_{v_b 0}^{i_b} > 0 \Rightarrow \mathcal{I}(i_s, v_s, i_b, v_b) = 1$. Also assume that no intermediation trades take place, i.e., agents with balanced portfolios do not trade: $\mathcal{I}(i_s, v_s, i_b, v_b) = 0$ if $v_s = H$ or $v_b = L$. Then I show that we must have $V_I - V_U = 0$.

If agents with balanced portfolios do not trade, then for $i \in \{U, I\}$:

$$\begin{aligned} V_{L0}^i &= \eta S_L^i \\ V_{H1}^i &= \delta_H - \eta S_H^i \end{aligned}$$

In that case, we have:

$$\begin{aligned} V^I &= \frac{1}{2}(V_{H1}^I + V_{L0}^I) - \frac{1}{2}S_H^I = \frac{\delta_H}{2} - \frac{\eta}{2}(S_H^I - S_L^I) - \frac{1}{2}S_H^I \\ V^U &= \frac{1}{2}(V_{H1}^U + V_{L0}^U) - \frac{1}{2}S_H^U = \frac{\delta_H}{2} - \frac{\eta}{2}(S_H^U - S_L^U) - \frac{1}{2}S_H^U \end{aligned}$$

For intermediation trades not to have positive surplus, it must be that $S_H^I = S_H^U$ and $S_L^I = S_L^U$, and thus $V^I = V^U$. ■