

Informed Intermediaries – Online Appendix

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1. APPENDIX - PROOF OF PROPOSITION 2 (CONT.)

1.1. **Guesses.** Surplus sharing:

$$\beta(I, v_s, U, v_b) = \beta^I \geq \frac{1}{2} \quad \beta(U, v_s, I, v_b) = 1 - \beta^I \leq \frac{1}{2}$$

$$\beta(I, v_s, I, v_b) = \frac{1}{2}$$

Efficient trading iff an informed agent is involved:

$$\mathcal{I}(i_s, v_s, i_b, v_b) = 1 \Leftrightarrow (i_s, i_b) \neq (U, U) \text{ and } V_{v_s 0}^{i_s} - V_{v_s 1}^{i_s} + V_{v_b 1}^{i_b} - V_{v_b 0}^{i_b} > 0.$$

1.2. **Stationary Distribution.** Given the guesses for \mathcal{I} , the inflow equal to outflow equations for the stationary distribution become:

- (1) $\mu_{L1}^U (\eta + \lambda(\mu_{H0}^I + \mu_{L0}^I)) = \eta \mu_{L0}^U$
- (2) $\mu_{H0}^U (\eta + \lambda(\mu_{L1}^I + \mu_{H1}^I)) = \eta \mu_{H1}^U$
- (3) $\mu_{L1}^I (\eta + \lambda(\mu_{H0}^U + \mu_{H0}^I)) = (\eta + \lambda \mu_{L1}^U) \mu_{L0}^I$
- (4) $\mu_{H0}^I (\eta + \lambda(\mu_{L1}^U + \mu_{L1}^I)) = (\eta + \lambda \mu_{H0}^U) \mu_{H1}^I$

Combine (1) and (2), and using $\mu_{L1}^U + \mu_{L0}^U = \mu_{H0}^U + \mu_{H1}^U = \frac{1-\phi}{2}$ (since the half the uninformed agents have high valuation and half have low valuation), I get:

$$(5) \quad \mu_{L1}^U (2\eta + \lambda(\mu_{H0}^I + \mu_{L0}^I)) = \mu_{H0}^U (2\eta + \lambda(\mu_{L1}^I + \mu_{H1}^I)) = \frac{\eta(1-\phi)}{2}$$

Similarly use (3) and (4) and $\mu_{L1}^I + \mu_{L0}^I = \mu_{H0}^I + \mu_{H1}^I$ to get:

$$(6) \quad (\eta + \lambda \mu_{L1}^U) (\mu_{H0}^I + \mu_{L0}^I) = (\eta + \lambda \mu_{H0}^U) (\mu_{L1}^I + \mu_{L1}^I)$$

Use (5) and (6) to get $\mu_{L1}^U + \mu_{L1}^I + \mu_{H1}^I = \mu_{H0}^U + \mu_{H0}^I + \mu_{L0}^I$. Combining this with (5) yet again, conclude that $\mu_{L1}^U = \mu_{H0}^U \equiv \hat{\mu}^U$. This, along with (6), implies $(\mu_{H0}^I + \mu_{L0}^I) = (\mu_{L1}^I + \mu_{H1}^I)$;

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and hence $\mu_{H0}^I = \mu_{L1}^I \equiv \hat{\mu}^I$. Rewrite the inflow equals outflow conditions now as:

$$\hat{\mu}^U(\eta + \lambda(\phi/2)) = \eta \frac{(1 - \phi - 2\hat{\mu}^U)}{2} \quad \hat{\mu}^I(\eta + \lambda(\hat{\mu}^U + \hat{\mu}^I)) = \frac{\phi - 2\hat{\mu}^I}{2}(\eta + \lambda\hat{\mu}^U)$$

Solving these, I get:

$$\hat{\mu}^U = \frac{1 - \phi}{4 + \frac{\lambda}{\eta}} \quad \hat{\mu}^I = -\frac{\eta + \lambda\hat{\mu}^U}{\lambda} + \sqrt{\left(\frac{\eta + \lambda\hat{\mu}^U}{\lambda}\right)^2 + \frac{\phi\eta + \lambda\hat{\mu}^U}{2\lambda}}$$

1.3. Unflagged Values. Again taking into account the guesses for \mathcal{I} and β , unflagged values are given by the system below

$$rV_{H0}^I = \eta(V_{H1}^I - V_{H0}^I) + \lambda\hat{\mu}^I \frac{V_{H1}^I - V_{H0}^I + V_{L0}^I - V_{L1}^I}{2} + \lambda\hat{\mu}^U \beta^I (V_{H1}^I - V_{H0}^I + V_{L0}^U - V_{L1}^U)$$

$$rV_{H1}^I = \delta_H + \eta(V_{H0}^I - V_{H1}^I) + \lambda\hat{\mu}^U \beta^I (V_{H0}^I - V_{H1}^I + V_{H1}^U - V_{H0}^U)$$

$$rV_{L1}^I = \delta_L + \eta(V_{L0}^I - V_{L1}^I) + \lambda\hat{\mu}^I \frac{V_{L0}^I - V_{L1}^I + V_{H1}^I - V_{H0}^I}{2} + \lambda\hat{\mu}^U \beta^I (V_{L0}^I - V_{L1}^I + V_{H1}^U - V_{H0}^U)$$

$$rV_{L0}^I = \eta(V_{L1}^I - V_{L0}^I) + \lambda\hat{\mu}^U \beta^I (V_{L1}^I - V_{L0}^I + V_{L0}^U - V_{L1}^U)$$

$$rV_{H0}^U = \eta(V_{H1}^U - V_{H0}^U) + \lambda\hat{\mu}^I(1 - \beta^I) (V_{H1}^U - V_{H0}^U + V_{L0}^I - V_{L1}^I) \\ + \lambda\hat{\mu}^I(1 - \beta^I) (V_{H1}^U - V_{H0}^U + V_{H0}^I - V_{H1}^I)$$

$$rV_{H1}^U = \delta_H + \eta(V_{H0}^U - V_{H1}^U)$$

$$rV_{L1}^U = \delta_L + \eta(V_{L0}^U - V_{L1}^U) + \lambda\hat{\mu}^I(1 - \beta^I) (V_{L0}^U - V_{L1}^U + V_{H1}^I - V_{H0}^I) \\ + \lambda\hat{\mu}^I(1 - \beta^I) (V_{L0}^U - V_{L1}^U + V_{L1}^I - V_{L0}^I)$$

$$rV_{L0}^U = \eta(V_{L1}^U - V_{L0}^U)$$

In terms of the values of holding an asset, this system becomes:

$$\begin{aligned}
rS_H^I &= \delta_H - 2\eta S_H^I + \lambda \hat{\mu}^U \beta^I (S_H^U - S_H^I) + \lambda \hat{\mu}^I \frac{(S_L^I - S_H^I)}{2} + \lambda \hat{\mu}^U \beta^I (S_L^U - S_H^I) \\
rS_L^I &= \delta_L - 2\eta S_L^I + \lambda \hat{\mu}^U \beta^I (S_L^U - S_L^I) + \lambda \hat{\mu}^I \frac{(S_H^I - S_L^I)}{2} + \lambda \hat{\mu}^U \beta^I (S_H^U - S_L^I) \\
rS_H^U &= \delta_H - 2\eta S_H^U + \lambda \hat{\mu}^I (1 - \beta^I) (S_L^I - S_H^U) + \lambda \frac{\phi - 2\hat{\mu}^I}{2} (1 - \beta^I) (S_H^I - S_H^U) \\
rS_L^U &= \delta_L - 2\eta S_L^U + \lambda \hat{\mu}^I (1 - \beta^I) (S_H^I - S_L^U) + \lambda \frac{\phi - 2\hat{\mu}^I}{2} (1 - \beta^I) (S_L^I - S_L^U)
\end{aligned}$$

Add up the first two and the last two to get:

$$\begin{aligned}
(r + 2\eta)(S_H^I + S_L^I) &= \delta_H + \delta_L + 2\lambda \hat{\mu}^U \beta^I (S_H^U + S_L^U) - 2\lambda \hat{\mu}^U \beta^I (S_H^I + S_L^I) \\
(r + 2\eta)(S_H^U + S_L^U) &= \delta_H + \delta_L + \frac{\lambda \phi}{2} (1 - \beta^I) (S_H^I + S_L^I) - \frac{\lambda \phi}{2} (1 - \beta^I) (S_H^U + S_L^U)
\end{aligned}$$

These imply $(S_H^I + S_L^I) = (S_H^U + S_L^U) = \frac{\delta_H + \delta_L}{r + 2\eta}$. Now from the original system, subtract the second equation from the first and the fourth from the third to find:

$$\begin{aligned}
(r + 2\eta)(S_H^I - S_L^I) &= \delta_H - \delta_L - \lambda(2\hat{\mu}^U \beta^I + \hat{\mu}^I)(S_H^I - S_L^I) \\
(r + 2\eta)(S_H^U - S_L^U) &= \delta_H - \delta_L - \frac{\lambda \phi}{2} (1 - \beta^I) (S_H^U - S_L^U) + \left(\frac{\lambda \phi}{2} - \hat{\mu}^I \right) (1 - \beta^I) (S_H^I - S_L^I)
\end{aligned}$$

Rearrange these to get the following expressions:

$$\begin{aligned}
(S_H^I - S_L^I) &= \hat{\alpha}^I (\delta_H - \delta_L) \\
(S_H^U - S_L^U) &= \hat{\alpha}^U (\delta_H - \delta_L) \\
\text{where } \hat{\alpha}^I &= \frac{1}{2(r + 2\eta + \lambda(2\hat{\mu}^U \beta^I + \hat{\mu}^I))} \\
\text{and } \hat{\alpha}^U &= \hat{\alpha}^U = \left[\frac{r + 2\eta + \lambda \hat{\mu}^U + \frac{\lambda \phi}{4}}{r + 2\eta + \frac{\lambda \phi}{4}} \right] \hat{\alpha}^I
\end{aligned}$$

Which finally implies:

$$\begin{aligned}
S_H^i &= \left[\frac{1}{2(r + 2\eta)} \right] (\delta_H + \delta_L) + \frac{\hat{\alpha}^i}{2} (\delta_H - \delta_L) \\
S_L^i &= \left[\frac{1}{2(r + 2\eta)} \right] (\delta_H + \delta_L) - \frac{\hat{\alpha}^i}{2} (\delta_H - \delta_L)
\end{aligned}$$

1.4. Flagged Values. I solve for D_{H1}^I , D_{L1}^I and D_{H1}^U . The system defining $\{D_{va}^I\}$ is:

$$rD_{H1}^I = (\eta + \lambda\hat{\mu}^U)(D_{H0}^I - D_{H1}^I)$$

$$rD_{L1}^I = (\eta + \lambda\hat{\mu}^U)(D_{L0}^I - D_{L1}^I) + \lambda\hat{\mu}^I \frac{S_H^I - S_L^I}{2}$$

$$rD_{L0}^I = (\eta + \lambda\hat{\mu}^U)(D_{L1}^I - D_{L0}^I) - \lambda\hat{\mu}^U (\beta^I S_L^U + (1 - \beta^I)S_L^I)$$

$$rD_{H0}^I = (\eta + \lambda\hat{\mu}^U)(D_{H1}^I - D_{H0}^I) + \lambda\hat{\mu}^I \frac{S_H^I - S_L^I}{2} - \lambda\hat{\mu}^U (\beta^I S_H^I + (1 - \beta^I)S_L^U)$$

Combining the first with the fourth and the second with the third:

$$r(D_{H1}^I - D_{H0}^I) = -2(\eta + \lambda\hat{\mu}^U)(D_{H1}^I - D_{H0}^I) - \lambda\hat{\mu}^I \frac{S_H^I - S_L^I}{2} + \lambda\hat{\mu}^U (\beta^I S_H^I + (1 - \beta^I)S_L^U)$$

$$r(D_{L1}^I - D_{L0}^I) = -2(\eta + \lambda\hat{\mu}^U)(D_{L1}^I - D_{L0}^I) + \lambda\hat{\mu}^I \frac{S_H^I - S_L^I}{2} + \lambda\hat{\mu}^U (\beta^I S_L^U + (1 - \beta^I)S_L^I)$$

Solve to find:

$$(D_{H1}^I - D_{H0}^I) = -\frac{\lambda\hat{\mu}^I}{(r + 2\eta + 2\lambda\hat{\mu}^U)} \frac{S_H^I - S_L^I}{2} + \frac{\lambda\hat{\mu}^U}{(r + 2\eta + 2\lambda\hat{\mu}^U)} (\beta^I S_H^I + (1 - \beta^I)S_L^U)$$

$$(D_{L1}^I - D_{L0}^I) = \frac{\lambda\hat{\mu}^I}{(r + 2\eta + 2\lambda\hat{\mu}^U)} \frac{S_H^I - S_L^I}{2} + \frac{\lambda\hat{\mu}^U}{(r + 2\eta + 2\lambda\hat{\mu}^U)} (\beta^I S_L^U + (1 - \beta^I)S_L^I)$$

Plug this back into the original system to get:

$$(7) \quad D_{H1}^I = \frac{\eta + \lambda\hat{\mu}^U}{r(r + 2\eta + 2\lambda\hat{\mu}^U)} \left[\lambda\hat{\mu}^I \frac{S_H^I - S_L^I}{2} - \lambda\hat{\mu}^U (\beta^I S_H^I + (1 - \beta^I)S_L^U) \right]$$

$$(8) \quad D_{L1}^I = \frac{r + \eta + \lambda\hat{\mu}^U}{r(r + 2\eta + 2\lambda\hat{\mu}^U)} \lambda\hat{\mu}^I \frac{S_H^I - S_L^I}{2} - \frac{\lambda\hat{\mu}^U (\eta + \lambda\hat{\mu}^U)}{r(r + 2\eta + 2\lambda\hat{\mu}^U)} (\beta^I S_L^U + (1 - \beta^I)S_L^I)$$

The system defining $\{D_{va}^U\}$ is:

$$\begin{aligned}
rD_{H1}^U &= \eta(D_{H0}^U - D_{H1}^U) \\
rD_{H0}^U &= \eta(D_{H1}^U - D_{H0}^U) + \lambda\hat{\mu}^I(1 - \beta^I)(S_H^U - S_L^I) + \lambda(\phi/2 - \hat{\mu}^I)(1 - \beta^I)(S_H^U - S_H^I) \\
rD_{L1}^U &= \eta(D_{L0}^U - D_{L1}^U) + \lambda\hat{\mu}^I(1 - \beta^I)(S_H^I - S_L^U) + \lambda(\phi/2 - \hat{\mu}^I)(1 - \beta^I)(S_L^I - S_L^U) \\
rD_{L0}^U &= \eta(D_{L1}^U - D_{L0}^U)
\end{aligned}$$

Subtract the second from the first to get:

$$\begin{aligned}
r(D_{H1}^U - D_{H0}^U) &= -2\eta(D_{H1}^U - D_{H0}^U) - \lambda\hat{\mu}^I(1 - \beta^I)(S_H^U - S_L^I) - \lambda(\phi/2 - \hat{\mu}^I)(1 - \beta^I)(S_H^U - S_H^I) \\
\Rightarrow (D_{H1}^U - D_{H0}^U) &= -\frac{\lambda\hat{\mu}^I(1 - \beta^I)}{r + 2\eta}(S_H^U - S_L^I) - \frac{\lambda(\phi/2 - \hat{\mu}^I)(1 - \beta^I)}{r + 2\eta}(S_H^U - S_H^I)
\end{aligned}$$

Finally plug back into the original system to get:

$$(9) \quad D_{H1}^U = \frac{\eta\lambda\hat{\mu}^I(1 - \beta^I)}{r(r + 2\eta)}(S_H^U - S_L^I) + \frac{\eta\lambda(\phi/2 - \hat{\mu}^I)(1 - \beta^I)}{r(r + 2\eta)}(S_H^U - S_H^I)$$