Basic Decision Analysis

B. Render, R.M. Stair and M.E. Hanna, Quantitative analysis for management, 9ed., 2006 Pearson

Learning Objectives

1. List the steps of the decision-making process.
2. Describe the types of decision-making environments.
3. Make decisions under uncertainty.
4. Use probability values to make decisions under risk.

Chapter Outline

Introduction

- Decision theory is an analytical and systematic way to tackle problems.
- A good decision is based on logic.

The Six Steps in Decision Theory

1. Clearly define the problem at hand.
2. List the possible alternatives.
3. Identify the possible outcomes.
4. List the payoff or profit of each combination of alternatives and outcomes.
5. Select one of the mathematical decision theory models.
6. Apply the model and make your decision.

John Thompson's Backyard Storage Sheds

Define problem	**To manufacture or market backyard storage sheds**
List alternatives	**1. Construct a large new plant** **2. A small plant** **3. No plant at all**
Identify outcomes	**The market could be favorable or unfavorable for storage sheds**
List payoffs	**List the payoff for each state of nature/decision alternative combination**
Select a model	**Decision tables and/or trees can be used to solve the problem**
Apply model and make decision	**Solutions can be obtained and a sensitivity analysis used to make a decision**

Decision Table for Thompson Lumber

Alternative	State of Nature		
	Favorable Market ($)	Unfavorable Market ($)	
Construct a large plant	200,000	-180,000	
Construct a small plant	100,000	-20,000	
Do nothing	0	0	

Types of Decision-Making Environments

- Type 1: Decision making under certainty.
  - Decision maker *knows with certainty* the consequences of every alternative or decision choice.
- Type 2: Decision making under risk.
  - The decision maker *does know* the probabilities of the various outcomes.
- Decision making under uncertainty.
  - The decision maker *does not know* the probabilities of the various outcomes.

Decision Making under Uncertainty

- Maximax
- Maximin
- Equally likely (Laplace)
- Criterion of realism
- Minimax

Decision Table for Thompson Lumber

- **Maximax: Optimistic Approach**
  - **Find the alternative that maximizes the maximum payoff.**

Alternative	State of Nature	
	Favorable Market ($)	Unfavorable Market ($)
Construct a large plant	200,000	-180,000
Construct a small plant	100,000	-20,000
Do nothing	0	0

Thompson Lumber: Maximax Solution

Alternative	State of Nature		Maximax
	Favorable Market ($)	Unfavorable Market ($)	
Construct a large plant	200,000	-180,000	
Construct a small plant	100,000	-20,000	
Do nothing	0	0	

Decision Table for Thompson Lumber

- **Maximin: Pessimistic Approach**
  - **Choose the alternative with maximum minimum output.**

Alternative	State of Nature	
	Favorable Market ($)	Unfavorable Market ($)
Construct a large plant	200,000	-180,000
Construct a small plant	100,000	-20,000
Do nothing	0	0

Thompson Lumber: Maximin Solution

Alternative	State of Nature		Maximin
	Favorable Market ($)	Unfavorable Market ($)	
Construct a large plant	200,000	-180,000	
Construct a small plant	100,000	-20,000	
Do nothing	0	0	

Thompson Lumber: Hurwicz

- **Criterion of Realism (Hurwicz)**
  - **Decision maker uses a weighted average based on optimism of the future.**

Alternative	State of Nature	
	Favorable Market ($)	Unfavorable Market ($)
Construct a large plant	200,000	-180,000
Construct a small plant	100,000	-20,000
Do nothing	0	0

Thompson Lumber: Hurwicz Solution

CR = α*(row max)+(1- α)*(row min)

Alternative	State of Nature		Criterion of Realism or Weighted Average (α = 0.8) ($)
	Favorable Market ($)	Unfavorable Market ($)	
Construct a large plant	200,000	-180,000	
Construct a small plant	100,000	-20,000	
Do nothing	0	0	

Decision Making under Uncertainty

- **Equally likely (Laplace)**
  - **Assume all states of nature to be equally likely, choose maximum Average.**

Alternative	State of Nature	
	Favorable Market ($)	Unfavorable Market ($)
Construct a large plant	200,000	-180,000
Construct a small plant	100,000	-20,000
Do nothing	0	0

Decision Making under Uncertainty

Alternative	State of Nature		Avg.
	Favorable Market ($)	Unfavorable Market ($)	
Construct a large plant	200,000	-180,000	
Construct a small plant	100,000	-20,000	
Do nothing	0	0	

Thompson Lumber; Minimax Regret

- **Minimax Regret:**
  - **Choose the alternative that minimizes the maximum opportunity loss .**

Alternative	State of Nature	
	Favorable Market ($)	Unfavorable Market ($)
Construct a large plant	200,000	-180,000
Construct a small plant	100,000	-20,000
Do nothing	0	0

Thompson Lumber: Opportunity Loss Table

Alternative	State of Nature	
	Favorable Market ($)	Unfavorable Market ($)
Construct a large plant		
Construct a small plant		
Do nothing		

Thompson Lumber: Minimax Regret Solution

Alternative	State of Nature		Maximum Opportunity Loss
	Favorable Market ($)	Unfavorable Market ($)	
Construct a large plant	0	180,000	
Construct a small plant	100,000	20,000	
Do nothing	200,000	0	

In-Class Example 1

- Let's practice what we've learned. Use the decision table below to compute

(1) Maximax (2) Maximin (3) Minimax regret

Alternative	State of Nature		
	Good Market ($)	Average Market ($)	Poor Market ($)
Construct a large plant	75,000	25,000	-40,000
Construct a small plant	100,000	35,000	-60,000
Do nothing	0	0	0

In-Class Example 1: Maximax

Alternative	State of Nature			Maximax
	Good Market ($)	Average Market ($)	Poor Market ($)	
Construct a large plant	75,000	25,000	-40,000	
Construct a small plant	100,000	35,000	-60,000	
Do nothing	0	0	0	

In-Class Example 1: Maximin

Alternative	State of Nature			Maximin
	Good Market ($)	Average Market ($)	Poor Market ($)	
Construct a large plant	75,000	25,000	-40,000	
Construct a small plant	100,000	35,000	-60,000	
Do nothing	0	0	0	

In-Class Example 1: Minimax Regret Opportunity Loss Table

Alternative	State of Nature			Maximum Opp. Loss
	Good Market ($)	Average Market ($)	Poor Market ($)	
Construct a large plant	25,000	10,000	40,000	
Construct a small plant	0	0	60,000	
Do nothing	100,000	35,000	0	

Decision Making under Risk

Expected Monetary Value:

$$\text{EMV(Alternative)} = \sum_{j=1}^{n} \text{Payoff } S_j * P(S_j)$$

where n = number of states of nature.

In other words:

EMV(Alternative n) = Payoff $_1$ * P(S_1) + Payoff $_2$ * P(S_2) + … + Payoff $_n$ * P(S_n)

Thompson Lumber: EMV

Alternative	State of Nature		EMV
	Favorable Market ($)	Unfavorable Market ($)	
Construct a large plant	200,000	-180,000	
Construct a small plant	100,000	-20,000	
Do nothing	0	0	
Probabilities	0.50	0.50	

Thompson Lumber: EMV Solution

Alternative	State of Nature		EMV
	Favorable Market ($)	Unfavorable Market ($)	
Construct a large plant	200,000	-180,000	
Construct a small plant	100,000	-20,000	
Do nothing	0	0	

Expected Value of Perfect Information (*EVPI*)

- *EVPI* places an upper bound on what one would pay for additional information.
- *EVPI* is the expected value with perfect information (EV|PI) minus the maximum EMV.

Expected Value with Perfect Information ($EV|PI$)

$$EV \mid PI = \sum_{j=1}^{n} (\text{Best payoff for state of nature}) * P(S_j)$$

n = number of states of nature.

In other words

$EV|PI$ = Best Payoff of S_1 * $P(S_1)$ + Best Payoff of S_2 * $P(S_2)$ +… + Best Payoff of S_n * $P(S_n)$

Expected Value with Perfect Information ($EV|PI$)

Alternative	State of Nature		EMV
	Favorable Market ($)	Unfavorable Market ($)	
Construct a large plant	200,000	-180,000	10,000
Construct a small plant	100,000	-20,000	40,000
Do nothing	0	0	0
Perfect Information			EV|PI =

Expected Value of Perfect Information

$EVPI = EV|PI$ - maximum EMV

Expected value with perfect information

Expected value with no additional information

Thompson Lumber: EVPI Solution

$EVPI$ = expected value with perfect information - max(EMV)

= $100,000 - $40,000

= $60,000

In-Class Example 2

Let's practice what we've learned. Using the table below compute EMV, EV|PI, and EVPI.

Alternative	State of Nature		
	Good Market ($)	Average Market ($)	Poor Market ($)
Construct a large plant	75,000	25,000	-40,000
Construct a small plant	100,000	35,000	-60,000
Do nothing	0	0	0
Probability	0.25	0.50	0.25

In-Class Example 2: EMV and EV|PI Solution

Alternative	State of Nature			EMV
	Good Market ($)	Average Market ($)	Poor Market ($)	
Construct a large plant	75,000	25,000	-40,000	
Construct a small plant	100,000	35,000	-60,000	
Do nothing	0	0	0	
Probability	0.25	0.50	0.25	

In-Class Example 2: EVPI Solution

***EVPI* = expected value with perfect information - max(*EMV*)**

= $ *0.25 + *0.50 + *0.25 -27,500

= $ - 27,500

= $

Expected Opportunity Loss (EOL)

- *EOL* is the cost of not picking the best solution.
  EOL = Expected Regret

Thompson Lumber: Payoff Table

Alternative	State of Nature	
	Favorable Market ($)	Unfavorable Market ($)
Construct a large plant	200,000	-180,000
Construct a small plant	100,000	-20,000
Do nothing	0	0
Probabilities	0.50	0.50

Thompson Lumber: EOL The Opportunity Loss Table

Alternative	State of Nature	
	Favorable Market ($)	Unfavorable Market ($)
Construct a large plant	200,000 – 200,000	0- (-180,000)
Construct a small plant	200,000 - 100,000	0 – (-20,000)
Do nothing	200,000 - 0	0-0
Probabilities	0.50	0.50

Thompson Lumber: Opportunity Loss Table

Alternative	State of Nature	
	Favorable Market ($)	Unfavorable Market ($)
Construct a large plant	0	180,000
Construct a small plant	100,000	20,000
Do nothing	200,000	0
Probabilities	0.50	0.50

Thompson Lumber: EOL Solution

Alternative		EOL
Large Plant		
Small Plant		
Do Nothing		

Summary

- **Minimum EOL has the same conclusion as maximum EMV.**
- **EVPI has the same value as EOL.**

Thompson Lumber: Sensitivity Analysis

Let P = probability of favorable market

EMV(Large Plant):

= \$200,000P + (-\$180,000)(1-P)

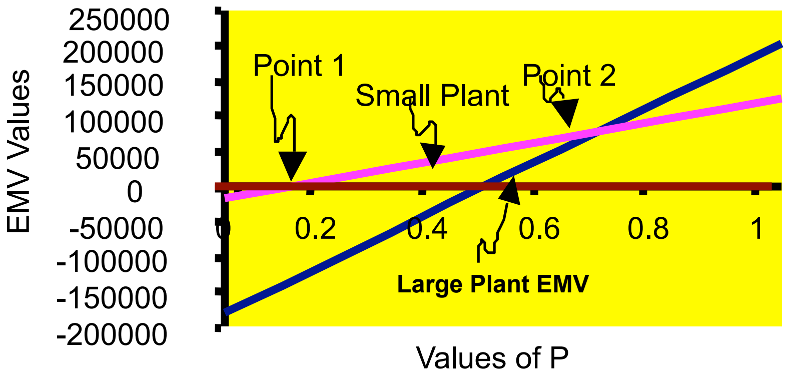
EMV(Small Plant):

= \$100,000P + (-\$20,000)(1-P)

EMV(Do Nothing):

= \$0P + 0(1-P)

Thompson Lumber: Sensitivity Analysis *(continued)*



Sensitivity Analysis

- **Point 1: EMV (do nothing) = EMV (small plant)**
  - **P=20000/120000 = 0.167**
- **Point 2: EMV (small plant) = EMV (large plant)**
  - **P=160000/260000 = 0.615**

Best alternative	Range of P values
Do nothing	<0.167
Construct a small plant	0.167-0.615
Construct a large plant	>0.615

Example

- **A manufacturer must decide whether to expand his plant capacity now or wait at least another year. His advisors tell him that if he expands now and economic conditions remain good, there will be a profit of $164,000 during the next year; if he expands now and there is a recession, there will be a loss of $40,000; if he waits at least another year and economic conditions remain good, there will be a profit of $80,000; and if he waits at least another year and there is a recession, there will be a small profit of $8,000. What should the manufacturer decide to do if he wants to minimize the expected loss during the next year and he feels that the odds are 2 to 1 that there will be a recession?**

Example

- Solution:

  All these "payoffs" can be represented as in the following table, where the entries are the losses that correspond to the various possibilities and, hence, gains are represented by negative numbers:

	Expand now	**Delay expansion**
remain good	**-164,000**	**-80,000**
recession	**40,000**	**-8,000**

We are working with losses here rather than profits to make this example fit the general scheme. Since the prob. that economic conditions will remain good and recession are 1/3 and 2/3 respectively.

Example

The manufacturer's expected loss is

if he expands his plant capacity now, and

if he waits at least another year.

Since an expected profit (negative expected loss) of $ is preferable to an expected profit (negative expected loss) of $, it follows that the manufacturer should delay expanding the capacity of his plant.

Example (con't)

- With reference to previous example, suppose that the manufacturer has no idea about the odds that there will be a recession. What should he decide to do if he is a confirmed pessimist?

  Solution
- Being the kind of person who always expects the worst to happen, he might argue that if he expands his plant capacity now he could lose $40,000, if he delays expansion there would be a profit of at least $8,000 and, hence, that be will minimize the maximum loss if he waits at least another year.

- The criterion used in this example is called the **minmax criterion**, and it is only one of many different criteria that can be used in this kind of situation.

Decision Making with Uncertainty: Using the Decision Trees

- *Decision trees* are useful tools when a sequence of decisions must be made.
- All information included in a payoff table is also included in a decision tree.

Five Steps to Decision Tree Analysis

1. Define the problem.
2. Structure or draw the decision tree.
3. Assign probabilities to the states of nature.
4. Estimate payoffs for each possible combination of alternatives and states of nature.
5. Solve the problem by computing expected monetary values (EMVs) at each state of nature node.

Structure of Decision Trees

A graphical representation where:

A decision node (indicated by a square) from which one of several alternatives may be chosen.

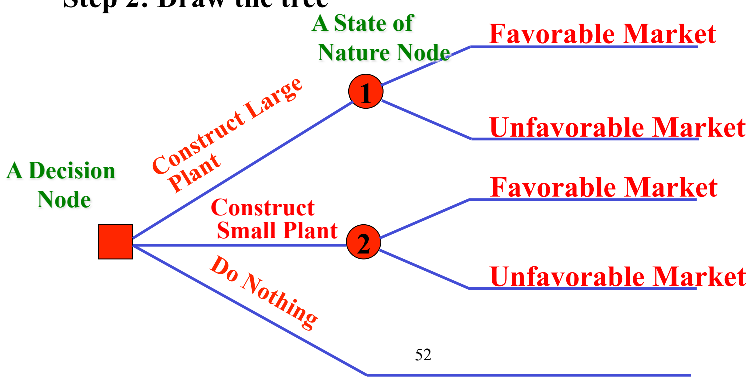
A state-of-nature node (indicated by a circle) out of which one state of nature will occur.

Thompson's Decision Tree

Step 1: Define the problem

Lets re-look at John Thompson's decision regarding storage sheds. This simple problem can be depicted using a decision tree.

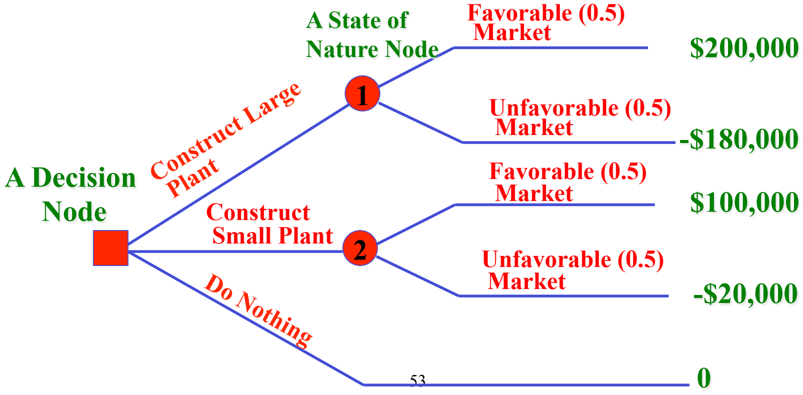
Step 2: Draw the tree



Thompson's Decision Tree

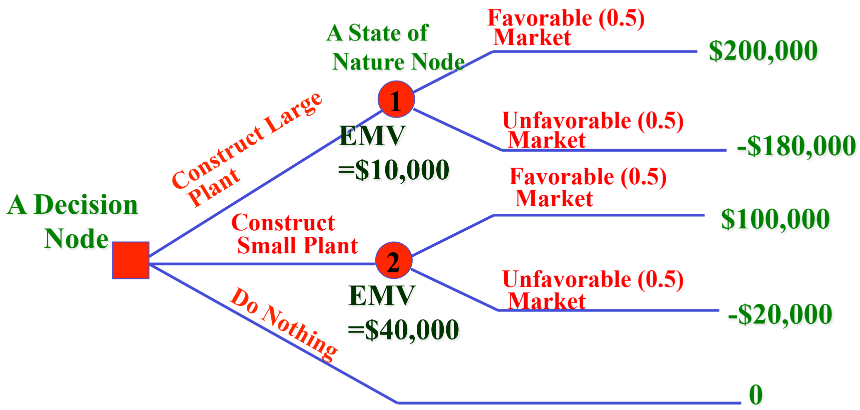
Step 3: Assign probabilities to the states of nature.

Step 4: Estimate payoffs.



Thompson's Decision Tree

Step 5: Compute EMVs and make decision.



Thompson's Decision: A More Complex Problem

- John has the opportunity of obtaining a market survey that will give additional information about the nature, i.e., percent change of a favorable market.
- Historical data show market surveys accurately predict favorable markets 78 % of the time.

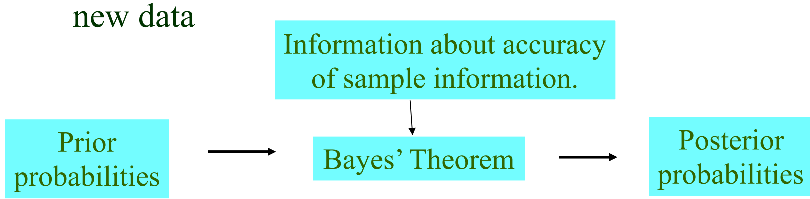
  P(Fav. Mkt | Fav. Survey Results) = .78
- Likewise, if the market survey predicts an unfavorable market, there is a 73 % chance of its occurring.

  P(Unfav. Mkt | Unfav. Survey Results) = .73

Now that we have redefined the problem (Step 1), let's use this additional data to redraw the decision tree (Step 2).

Estimating Probability Values with Bayes' Theorem

- Management experience or intuition
- History
- Existing data
- Need to be able to *revise* probabilities based upon new data



Bayesian Analysis

The probabilities of a favorable/unfavorable state of nature can be obtained by analyzing the Market Survey Reliability in Predicting Actual States of Nature.

Market Survey Reliability in Predicting Actual States of Nature		
	Actual States of Nature	
Result of Survey	Favorable Market (FM)	Unfavorable Market (UM)
Positive (favorable market)	P(survey pos.|FM) =0.7	P(survey pos.|UM) =0.2
Negative (unfavorable market)	P(survey neg.|FM) =0.3	P(survey neg.|UM) =0.8

Bayes' theorem

$$P(FM \mid \text{survey positive})$$

$$= \frac{P(\text{survey positive} \mid FM) \cdot P(FM)}{P(\text{survey positive} \mid FM) \cdot P(FM) + P(\text{survey positive} \mid UM) \cdot P(UM)}$$

$$= \frac{(0.70)(0.50)}{(0.70)(0.50) + (0.20)(0.50)} = \frac{0.35}{0.45} = 0.78$$

$$P(UM \mid \text{survey positive})$$

$$= \frac{P(\text{survey positive} \mid UM) \cdot P(UM)}{P(\text{survey positive} \mid UM) \cdot P(UM) + P(\text{survey positive} \mid FM) \cdot P(FM)}$$

$$= \frac{(0.20)(0.50)}{(0.20)(0.50) + (0.70)(0.50)} = \frac{0.10}{0.45} = 0.22$$

Bayes' theorem

$$P(FM \mid \text{survey negative})$$

$$= \frac{P(\text{survey negative} \mid FM) \cdot P(FM)}{P(\text{survey negative} \mid FM) \cdot P(FM) + P(\text{survey negative} \mid UM) \cdot P(UM)}$$

$$= \frac{(0.30)(0.50)}{(0.30)(0.50) + (0.80)(0.50)} = \frac{0.15}{0.55} = 0.27$$

$$P(UM \mid \text{survey negative})$$

$$= \frac{P(\text{survey negative} \mid UM) \cdot P(UM)}{P(\text{survey negative} \mid UM) \cdot P(UM) + P(\text{survey negative} \mid FM) \cdot P(FM)}$$

$$= \frac{(0.80)(0.50)}{(0.80)(0.50) + (0.30)(0.50)} = \frac{0.40}{0.55} = 0.73$$

Bayesian Analysis *(continued):* Favorable Survey

Probability Revisions Given a Favorable Survey				
	Conditional Probability			
State of Nature	P(Survey positive|State of Nature)	Prior Probability	Joint Probability	Posterior Probability
FM	0.70	* 0.50	0.35	$\frac{0.35}{0.45} = 0.78$
UM	0.20	* 0.50	0.10	$\frac{0.10}{0.45} = 0.22$
			0.45	1.00

Bayesian Analysis *(continued)*: Unfavorable Survey

Probability Revisions Given an Unfavorable Survey				
	Conditional Probability			
State of Nature	P(Survey negative|State of Nature)	Prior Probability	Joint Probability	Posterior Probability
FM	0.30	* 0.50	0.15	$\frac{0.15}{0.55} = 0.27$
UM	0.80	* 0.50	0.40	$\frac{0.40}{0.55} = 0.73$
			0.55	1.00

Thompson's Decision Tree

Thompson's Decision Tree

Step 3: Assign the new probabilities to the states of nature.

Step 4: Estimate the payoffs.

Step 5: Compute the EMVs and make decision.

John Thompson Dilemma

John Thompson wants to determine the monetary worth of the survey. He is also interested in how sensitive his decision is to changes in the market survey results. What should he do?

- Expected Value of Sample Information
- Sensitivity Analysis

Expected Value of Sample Information

- The survey cost $10,000.
- The expected value of $49,200 (when the survey is used) is based on payoffs after the $10,000 cost was subtracted.
- The expected value *with* sample information (EV with SI) := expected value of using the survey assuming no cost to gather it. Thus, in this example
- EV with SI = $49,200 + $10,000 = $59,200.
- Without SI, the best EV is $40,000.
- Thus, the expected value would increase by $19,200 if the survey was available free.

Expected Value of Sample Information

$$EVSI = \left\{ \begin{array}{l} \text{EV } \textit{with} \text{ SI,} \\ \text{assuming no} \\ \text{cost to gather} \\ \text{it} \end{array} \right\} - \left\{ \begin{array}{l} \text{EV of best} \\ \text{decision} \\ \textit{without} \text{ SI} \end{array} \right\}$$

EVSI for Thompson Lumber = $59,200 - $40,000
= $19,200

Thompson could pay up to $19,200 and come out ahead.

Sensitivity Analysis

How sensitive are the decisions to changes in the probabilities?

e.g. If the probability of a favorable survey were less than the current value, would the survey still be selected? How low would this cause a change in the decision?

Let p = probability of a favorable survey

Calculations for Thompson Lumber Sensitivity Analysis

$$\text{EMV(node 1)} = (\$106{,}400)\,p + (1 - p)(\$2{,}400)$$
$$= \$104{,}000\,p + 2{,}400$$

Equating the *EMV* with the survey to the *EMV* without the survey, we have

$$\$104{,}000\,p + \$2{,}400 = \$40{,}000$$
$$\$104{,}000\,p = \$37{,}600$$

or

$$p = \frac{\$37{,}600}{\$104{,}000} = 0.36$$

In-Class Problem 3

Leo can purchase a house for $200,000 or land in a for $50,000. There is a 60% chance the economy will grow and a 40% chance it will not. If it grows, the house will appreciate in value by 15% ($30,000 profit). If it does not grow, the profit is only $10,000. If Leo purchases the land he will hold it for 1 year to assess the economic growth. If the economy grew during the first year, there is an 80% chance it will continue to grow. If it did not grow during the first year, there is a 30% chance it will grow in the next 4 years. After a year, if the economy grew, Leo will decide either to build and sell a house or simply sell the land. It will cost Leo $75,000 to build a house that will sell for a profit of $55,000 if the economy grows, or $15,000 if it does not grow. Leo can sell the land for a profit of $15,000. If, after a year, the economy does not grow, Leo will either develop the land, which will cost $75,000, or sell the land for a profit of $5,000. If he develops the land and the economy begins to grow, he will make $45,000. If he develops the land and the economy does not grow, he will make $5,000.

In-Class Problem 3: Solution