ABSTRACT

Optimal control techniques are nowadays used in human motion prediction and allow to anticipate the result of surgery and to improve the design and control of rehabilitation robots, prosthetic and orthotic devices. Considering orthotic devices, the human-orthosis behaviour can be predicted with the purpose of personalizing and optimizing the design of the orthosis to facilitate the patient adaptation process.

The main objective of this work is to apply optimal control techniques for predicting a squat exercise using a simple 2D biomechanical model and tracking a real movement. Firstly, a research of the state of the art of analysis and simulation of human motion using multibody dynamics techniques has been done. Moreover, the principal basis of formulation for an optimal control problem have been studied and applied to a squat motion prediction.

The musculoskeletal model used in this work is a two-dimensional lower limb model with 3 segments and 5 degrees of freedom which is actuated by five muscles. The motion capture equipment, which consists of an optical system with infrared, has been calibrated experimentally; and a squat exercise has been captured as an input for the optimal control problem.

The software used to solve the optimal control problem is GPOPS-II, a general-purpose MATLAB-based software for solving multiple-phase optimal control problems developed by the University of Florida. This software is a powerful tool for solving optimal control problems. An initial squat exercise has been predicted with GPOPS-II. Changing parameters has permitted to predict different motions, and finally, the captured motion has been used as an input to minimize the difference between the predicted motion and the real movement.

This work can be considered a first research study in optimal control formulations to predict human motion. In future works, these formulations will be combined with a more complex model of a patient wearing robotic orthoses in order to predict and anticipate the human-orthosis behaviour.

CONTENTS

1. INTRODUCTION

1.1. Motivation

Multibody dynamics techniques have been used in the last decades in the analysis and simulation of human motion to better understand how one specific motion or task is performed, to determine muscle forces, or to compare healthy motion versus pathological or impaired motion. Moreover, during the last years a growing interest in motion prediction has appeared to anticipate the result of surgery and to help in the design of prosthetic and orthotic devices.

In this line, a robotic orthosis prototype for patients with incomplete spinal cord injury has been developed in the Biomechanical Engineering Lab (BIOMEC) of the Department of Mechanical Engineering at the Barcelona School of Industrial Engineering (ETSEIB) and the Biomedical Engineering Research Centre (CREB), Universitat Politècnica de Catalunya (UPC). This orthosis has been developed in the frame of two national projects (DPI2009-13438-C03-03 and DPI2012-38331-C03-02), in collaboration with researchers from University of La Coruña (UDC) and University of Extremadura (UEX). A third coordinated UPC-UDC-UEX national project has been recently approved (DPI2015-65959-C3-2-R), which is entitled "Low-cost motor-FES hybrid orthosis for the gait of spinal cord injured subjects and simulation methods to support the design and adaptation". One of the objectives of the new project is to develope a computational tool that enables to virtually test different types and designs of active orthoses for gait assistance on a virtual patient-specific model of a disabled subject. The tools developed in this master's thesis, which are aimed at predicting motion, are aligned with this objective.

In previous projects, a 3D biomechanical model of human body has been developed for inverse and forward dynamic analyses of motion of healthy subjects, and the contact interface between the patient and the orthosis has been incorporated to the model. Moreover, different types of orthosis actuation have been studied so far by the BIOMEC group. At the present time, inverse and forward dynamic analyses of a known (or captured) motion have already been done. The next step is to predict movement with the purpose of personalizing and optimizing the design of the orthosis to facilitate the patient adaptation process. Optimal control techniques are nowadays used in human motion prediction and represent a suitable tool to face this new objective.

The present work, "Application of optimal control in the simulation of human motion", corresponds to the Master's Thesis of Biomedical Engineering. The main objective of this work is to acquire knowledge for the group in optimal control formulations to predict human motion using a simple 2D model. In future works, these formulations will be combined with a more complex model of a real patient wearing the designed robotic orthoses in order to anticipate the human-orthosis behaviour. This tool has a great interest to improve the design and control of rehabilitation robots. This work is also the start of a PhD thesis entitled "Dynamic simulation of walking using personalized neuromusculoskeletal models".

1.2. Objectives

The main objective of this work is to apply optimal control techniques for predicting a squat exercise using a simple biomechanical model.

In order to achieve this goal, some specific objectives are defined:

- Research the state of the art of analysis and simulation of human motion using multibody dynamics techniques.

- Study optimal control theory and how to formulate an optimal control problem for human motion prediction.

- Learn to use the optimal control software GPOPS-II, developed by the University of Florida.

- Develop a simple musculoskeletal model to predict a squat exercise.

- Calibrate experimentally an optical system with infrared cameras for motion capture.

- Capture a squat motion and use the experimental data as input for the optimal control problem.

1.3. Outline

The present master's thesis is structured as follows:

- In Chapter 2, the state of the art of multibody dynamics techniques applied to biomechanics of human motion is presented. Firstly, an overview of musculoskeletal modeling of the human body is illustrated with some examples. Then, the two main approaches for dynamic analysis, namely inverse dynamic analysis and forward dynamic analysis, are explained. Since the musculoskeletal system is an overactuated system, the two main methods for solving the muscle force redundancy problem are also presented. Finally, methods for simulation of the human motion are divided in three groups: inverse dynamics-based methods, forward dynamics-based methods, and predictive dynamics methods.

- Methods are detailed in Chapter 3. The musculoskeletal model is described first. Then, the laboratory equipment for the motion capture, as well as the signal processing is explained. A simple theoretical basis for optimal control is given, and the GPOPS-II software and optimal control formulations are presented. Last section includes a detailed explanation of the data analysis.

- Results are reported and discussed in Chapter 4. An initial prediction has been done to explore the model, and different parameters have been changed to study the influence of parameter changes on the predicted motion. At last, the captured motion has been used as an input for movement tracking.

- Finally, some conclusions are drawn to collect the work done and summarize the most important contributions of this work.

2. STATE OF THE ART

2.1. Multibody system dynamics

Simulation of the dynamics of human motion involves modelling the human neuromusculoskeletal system. It can be viewed as a servo-controlled multibody dynamic system, where bones are modelled as rigid bodies connected by joints, actuated by muscles and controlled by the central nervous system (CNS) [Fregly, 2009].

The skeleton is usually modelled as an open kinematic chain, where each rigid body represents a bone (or a group of bones) and contains the information of the physical characteristics of each segment (mass, length, tensor of inertia about the centre of mass, and position of the centre of mass in a local coordinate system), called body segment parameters (BSP). The joints that link each rigid body to the next one are usually considered ideal joints, and they restrict the relative movement between these rigid bodies [Ackermann and Schiehlen, 2006; Rodrigo *et al*., 2008].

Motion equations are obtained mainly with two approaches: Newton-Euler equations or Lagrange equations. The Newton-Euler equations express the sum of forces and sum of moments acting on a rigid body in relation to the body kinematics, and are applied consecutively from one rigid body to the next one. Examples of this formulation can be found in [Ackermann and Schiehlen, 2006]. The Lagrange equations allow to obtain the motion equations in a systematic way from the general expressions of the kinetic and potential energies and using the concept of generalized force. Examples of this second formulation can be found in [Silva and Ambrósio, 2002].

Depending on the purpose of the study models can be two- or three-dimensional, and can represent the whole body or only a part of it. In Figure 2.1. three different models are presented: a) a two-dimensional model [Millard et al., 2011]; b) a three-dimensional whole body model, where head, arms and trunk (HAT) are assembled in a single body [Anderson and Pandy, 2001a]; and c) a three-dimensional whole body model that includes arms and head, and an accurate representation of the spine [Xiang *et al*., 2007]

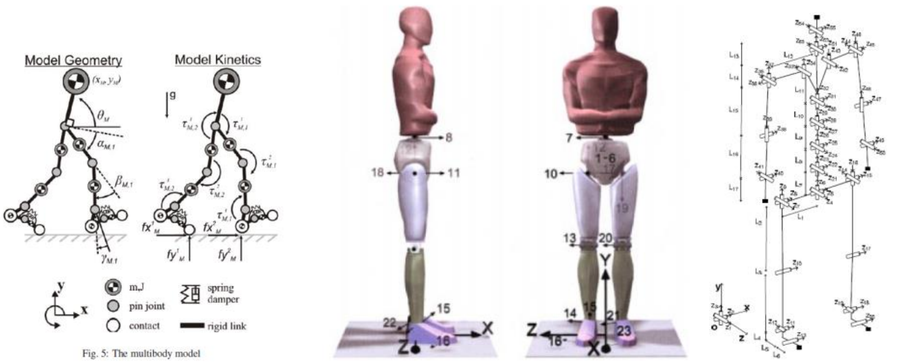

Figure 2. 1: Examples of biomechanical models from the literature review

Muscles and tendons are modelled as linear actuators attached to segments by insertion points. Commonly one joint is actuated by several muscles. Muscles are represented by massless actuators in the model; and their actual mass is considered to be rigidly attached to bones [Stelzer and von Stryck, 2005]. There are many different muscle models (see [Zajac, 1989] for extensive review), which differ on their internal parameters, but agree in the states that present (activation, fiber length, and velocity). The most used muscle-tendon model in the field of movement simulation is the Hill-type model, which describes the muscle-tendon actuator as an elastic element (tendon, SE) in series with an elastic element (connective tissue, PE) in parallel with the muscle fibers (contractile element, CE). The Hill-type model is shown in Figure 2.2. Muscles are controlled by neural excitations that produce muscle activations. Muscle activation and muscle-tendon contraction dynamics are described by differential equations [Ackermann and Schiehlen, 2006].

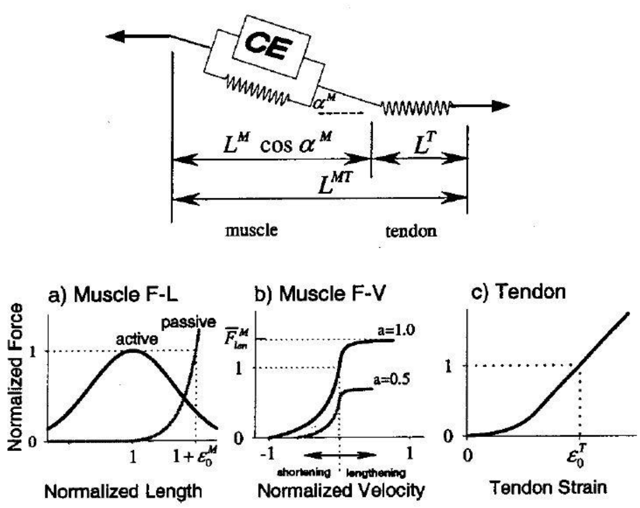

Figure 2. 2: Hill-type muscle model and force curves [Thelen, 2003]

2.2. Dynamic Analysis

Dynamic analysis of real movements can be carried out to better understand how one type of motion is performed [Stelzer and von Stryk, 2005; Anderson and Pandy, 2001b], to determine muscle forces [Zajac, 2002], to study how the central nervous system controls the motion [Neptune *et al.*, 1997], or to compare healthy motion versus pathological or impaired motion [Peasgood *et al*., 2005; Ackermann and Schiehlen, 2006].

Depending on the purpose of the study, inverse or forward dynamic analyses can be carried out. Roughly, it can be said that in inverse dynamic analysis (IDA), motion is known and forces are calculated, while in forward dynamic analysis (FDA), forces are known and motion is calculated. In this section, both approaches are used to analyse a known motion. In both approaches, it is necessary to capture data from a real movement and to process these data in order to filter noise and ensure kinematic and dynamic consistency [Reinbolt *et al.*, 2005; Thelen and Anderson, 2006]. The basic difference between the two approaches lies in whether or not the equations of motion are integrated [Xiang and Arora, 2010].

In this section only analyses at joint level will be considered. When muscles are taken into account, a redundancy problem appears that will be discussed in next section.

2.2.1. Inverse Dynamic Analysis

Inverse dynamic analysis is the most popular analysis technique used to analyse the motion of human musculoskeletal systems. It is used to calculate net moments that must be applied at the joints to produce the motion observed**.** Input data are usually acquired kinematic data, foot-ground contact forces (or ground reaction forces, GRF) and estimated body segment parameters (BSP). The use of this type of approaches is exemplified by the work of Silva and Ambrósio (2002).

This approach is mainly algebraic, because motion is known (*i.e.*, coordinates, velocities and accelerations), so by replacing their values for a time range in the motion equations, joint forces and torques can be computed.

Motion data is generally acquired with optical systems that consist on reflective markers located in anatomical positions and cameras to track their movement. Also, ground reaction forces are measured using force plates. However, marker trajectories cannot be directly used as inputs of the equations of motion, so a first step is to determine the model coordinates and its time derivatives (velocities and accelerations). This can be solved by means of an optimization problem, where the difference between captured marker trajectories and model marker trajectories is minimized. This process is usually referred as inverse kinematics.

The most analysed motion is gait. When studying gait, usually some assumptions are made in order to simplify the solution. Some works assume that gait is symmetric, and analyse only half of the gait cycle, or only one leg. There's a phase where both foot are in contact with ground, and it is not possible to distribute properly the reaction force at each foot. In [Pàmies-Vilà, 2012] a new strategy called corrected force plate sharing method (CFP) is proposed to solve this indeterminacy. The force plates measurements are not used directly as inputs of the analysis, they are used to solve the contact wrench sharing problem during the double support phase, where both feet contact the ground and kinematic measurements are insufficient to determine the individual wrench at each foot.

The IDA is highly dependent on the accurate collection and processing of body segmental kinematics [Anderson and Pandy, 2001b], and it lacks of the dynamic constistency of FDA, which follows the motion acquired by means of controllers. Small measurement errors in the given kinematic trajectories may lead to large errors in the calculated forces [Pàmies-Vilà *et al.*, 2012].

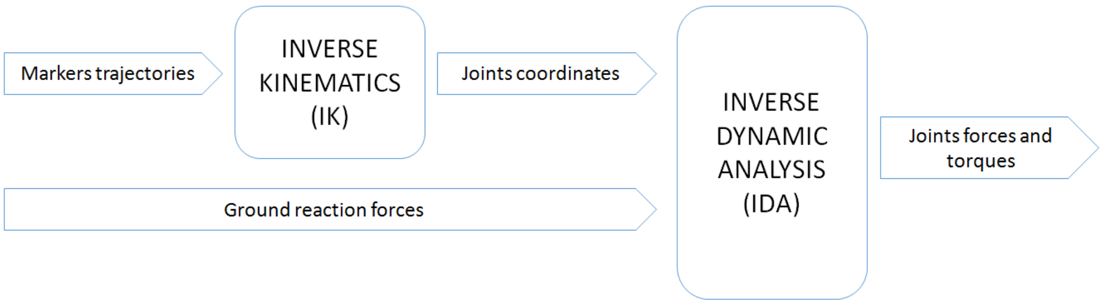

Figure 2. 3: Diagram block of inverse kinematics and inverse dynamics

2.2.2. Forward Dynamics Analysis

Forward dynamic analysis (FDA) determines how a mechanical system will move when certain forces are applied. The input data are the muscle forces or the resultant joint torques (together with BSP), and motion is determined by means of integrating the differential equations of motion. The work by Pandy and Anderson (2001b) exemplifies the use of forward dynamic simulation in biomechanical studies of the human motion. When input data are muscle excitations, the simulation is called muscle-driven simulation. An example can be found in [Zajac 2002], where jumping, pedalling and walking are analysed using this method.

When forward dynamic simulation is carried out, both the analysis of a given motion and the calculation and optimization of a free motion are possible [Stelzer and von Stryk, 2006]. So the forces that are known in FDA can be determined by a previous IDA and then some aspects that cannot be studied in an IDA, like contact forces between a subject and orthoses, can be studied using the forward dynamics approach [Ilzarbe *et al.*, 2014].

One advantage that presents this approach is that measurement errors may be compensated by controllers that force to follow the captured motion [Thelen, 2003; Ren *et al.*, 2007], by controllers that compute a reference trajectory [Millard et al., 2011], or by including an optimization problem that minimizes differences between the real motion and the simulated motion. The quality of the FDA solution is closely related to the reliability of the model, to the control system used and to the definition that is used to measure how human-like a particular gait is [Pàmies-Vilà, 2012].

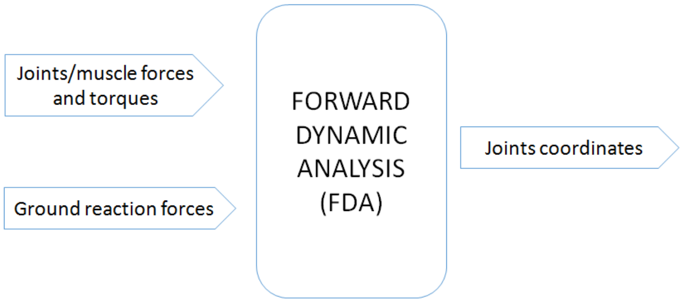

Figure 2. 4: Diagram block of forward dynamics

2.3. Muscle force redundancy problem

Muscle forces are difficult to be measured precisely and with non-invasive methods. For this reason, analysis and simulation of human motion are useful tools to determine muscle forces. However, once forces and toques are known for each joint, there are multiple solutions of muscle forces that yield the known joint torque. This is because the neuromusculoskeletal model is an over-actuated system, since each joint is actuated by different muscles. So, it can be exactly known the forces and torques at the joint level from the biomechanical model dynamics, but there are infinite solutions for the forces of each muscle. This is called the muscle force redundancy problem (or muscle force-sharing problem).

This problem is usually solved using optimization procedures based in physiological criteria [Ackermann and Schiehlen, 2006; Anderson and Pandy, 2001b], where a cost function is minimized. Several cost functions have been published in different studies. For example, Anderson and Pandy (2001a) propose that the cost function for gait is the metabolical energy cost per unit of distance travelled and Prilutsky and Zatsiorsky (2002) use a measure of the muscle fatigue and the sense of perceived effort as different cost functions. Ackermann and van den Bogert (2010) compare the results given by eight different effort or fatigue-like cost functions and conclude that fatigue minimization may be one of the primary optimality principles governing human gait. In [Berret *et al.*, 2011], results demonstrate that the recorded movements were closely linked to the combination of two complementary functions related to mechanical energy expenditure and joint-level smoothness. These and other results might suggest that movements are not the result of the minimization of single but rather of composite cost functions [Stokes *et al.*, 2001].

There are mainly two approaches to solve this muscle redundancy problem: in static optimization, an optimization problem is solved in each time step; whereas in dynamic optimization, the objective funcion is computed along the whole movement. In what follows, these two approaches are explained in more detail.

2.3.1. Static Optimization

Static Optimization (SO) approaches solve the muscle force distribution problem for each time step, without considering the muscle activation and contraction dynamics [Anderson and Pandy, 2001b]. In this approach, the goal is to find muscle forces as optimization variables such that an instantaneous cost function is minimized.

The main advantage of SO is its low computational cost. However, there are some drawbacks that may be considered. For example, it can lead to nonphysiological results (e.g., inability to predict contraction among antagonist muscles [Zajac and Neptune, 2002]) as it does not consider muscle activation-contraction dynamics. New approaches based on SO that attempt to solve these problems are Computed Muscle Control [Thelen *et al*., 2003, 2006]), and Modified Static Optimization [Ackerman, 2007], that explicitly accounts for delays in muscle force production resulting from activation and contraction dynamics while using a general static optimization framework to resolve muscle redundancy. The advantage to use these methods instead of Dynamic Optimization is that they present low computional cost.

Another disadvantage that presents SO is that it does not allow to define a time-integral cost function, such as a metabolical energy, because of its time-independent nature (*i.e.,* the cost function is minimized at each time step). Another approach, presented in [Ackermann and Schiehlen, 2006], called Extended Inverse Dynamics, solves this problem and permits the use of time-integral cost functions as total metabolic cost.

2.3.2. Dynamic Optimization

Dynamic optimization (DO) approaches solve the muscle force distribution problem for all the simulation time range and take muscle-tendon dynamics into account. An example of DO to solve the muscle force redundancy problem can be found in [Umberger *et al.*, 2006; Menegaldo *et al.*, 2006].

One clear advantage is that the cost function can be calculated over the motion period [Shariff, 2013], so the goal of the motor task can be included in the formulation of the problem, or differences of measured and calculated motion may be included in the optimality criterion by an additional term consisting, e.g., on the integral of the square of the deviation [Stelzer and von Stryk, 2006].

Since it is inherently a forward dynamics method, the problem may be formulated independent of experimental data. Ground reaction forces as well as feet positions are calculated, so the alignment of foot position with the measured ground reaction force is no longer a problem. And finally, the simulation is not limited by the number of measured gait cycles (usually one or two), since gait patterns can be extrapolated [Wojtyra, 2000].

The main disadvantage is that dynamic optimization is computationally very expensive, due to the integration of the equations of human motion, and this is why previous solutions for walking have been greatly simplified [Anderson and Pandy 2001a]. The work by Anderson and Pandy (2001b) has been for more than ten years the best published gait simulation using a full three-dimensional musculoskeletal model. They compared two solutions based on static optimization (first, muscles treated as ideal force generators; second, muscles constrained by their force/length/velocity properties) and a solution based on dynamic optimization (considering activation dynamics). Their main conclusion was that static and dynamic optimization solutions for gait are practically equivalent. Thus, for normal gait, the use of dynamic optimization rather than static optimization is currently not justified. However, scenarios in which the use of dynamic optimization is justified were also suggested.

2.4. Simulation

A growing interest in motion prediction has appeared during the last years, e.g., to anticipate the result of surgery, to help in the design of prosthetic/orthotic devices, or to study human motion dynamics performing various tasks [Fregly *et al*., 2007].

Simulation of human motion is used to predict movements without the need of knowing them in advance. Motions can be studied and analysed without the limitation of reproducing real movements. Another application is to predict how will walk one subject when some characteristic is modified, e.g., when a surgical intervention modifies some biomechanical characteristics, a prosthesis is implanted or an orthosis is worn. For example, Ackermann and Schiehlen (2006) study different mechanical disturbances in gait, with the goal of designing orthotic and prosthetic devices.

The most accepted approach to predict how a human will move is to state an optimization problem (so as to find the most likely motion) according to some cost functions such as minimizing weighted normalized torques, minimum metabolic energy cost, weighted joint accelerations, minimum muscle activation or minimum energy consumption [Ackermann and Schiehlen, 2006; Anderson and Pandy 2001a].

Methods used for the simulation of human movement are mainly the analysis methods presented previously: inverse dynamics, forward dynamics, and a new approach that unifies both, predictive dynamics. More details and examples for each approach are given in the following sections.

2.4.1. Inverse Dynamics-Based Methods

In inverse dynamics-based methods, the design variables are the joint coordinates that define the motion. That is, joint coordinates are obtained solving an optimization problem were a cost function is minimized and some constrains are satisfied. Forces are then directly calculated from equations of motion by replacing the obtained joint coordinates, so numerical integration is avoided. For this reason, some authors have taken profit of this computational efficiency and have used IDA for predictive dynamics.

Ren *et al.* (2007) have used this approach to predict a complete gait cycle using a two-dimensional model. In Figure 2.5 three local minima and the optimal solution are reproduced. Multi-objective optimization is also possible in these methods. An exemple can be found in [Xiang *et al*., 2004], which have used this approach to predict upper body posture in a 3D upper body. One clear advantage of prediction is that different conditions can be studied, as for example in the work of Xiang *et al* (2007), where normal walking with a shoulder backpack of varying loads is predicted.

Menegaldo *et al.* (2006) proposes an hybrid methodology which increases the numerical robustness, called Invese Dynamics Optimal Control. It uses the inverse dynamics of the multi-body system, but incorporates optimal control algorithms and muscle dynamics to the solution of the force-sharing optimization problem. It provides very similar muscle force patterns when compared to the forward dynamics solution, but the computational cost is much smaller.

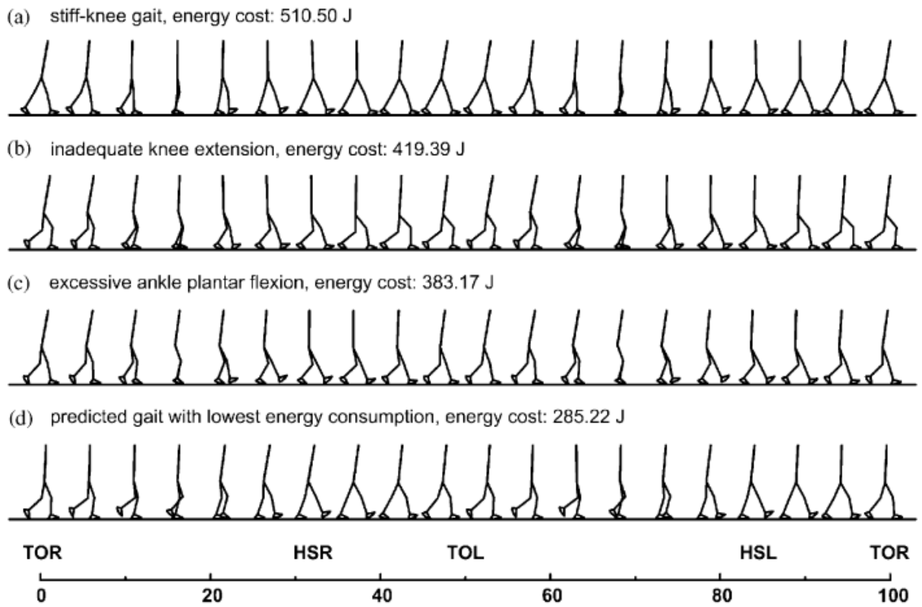

Figure 2. 5: Local minima (a-c) and optimal solution for a complete gait cycle prediction [Ren et al., 2007]

2.4.2. Forward Dynamics-Based Methods

In forward dynamics-based methods, the design variables for the optimization problem are forces and torques, or muscle excitations. The motion is obtained from the integration of the system differential equations. So this approach has a higher computational cost.

The best published solution of gait using this approach and a full 3D musculoskeletal model can be found in [Anderson and Pandy, 2001b].

With this approach, motions of implants and prosthesis can be predicted. Piazza and Delp (2001) predict the motions of knee implants during a step-up activity. Patterns of muscle activity, initial joint angles and velocities, and kinematics of the hip and ankle were measured experimentally and used as inputs to the simulation. Measurements of real moviments are normally used to stabilize the dynamical system, for example using a closed-loop control that follows a gait pattern, as it is explained in [Wojtyra, 2000].

2.4.3. Predictive Dynamics Methods

In predictive methods, both forces and motion are design variables. So the number of variables increases, but this approach presents the advantages from inverse dynamics- and forward dynamics-based methods. Equations of motion are introduced by means of constraints. The problem then can be formulated as an Optimal Control problem, and it presents the advantages of predictive methods avoiding the integration of the equations of motion, so its computational cost is relatively low.

Ackermann and van den Bogert (2010) state an optimal control problem for a family of different cost functions for gait prediction, and investigate the effects of different performance criteria on predicted gait patterns. Stelzer and von Stryk (2006) have solved a time optimal kicking motion for a two-dimensional lower limb model.

A particular case is hybrid predictive dynamics, which minimizes the difference between a real movement and the simulated one, and more or less convergence can be required [Xiang *et al.*, 2012].

3. METHODS

3.1. Biomechanical model

The model used in this project is a two-dimensional lower limb model with 3 segments and 5 degrees of freedom (for its unconstrained motion). It is a simplified planar model of a person performing a squatting motion (see Figure 3.1). This model was proposed by Nathan Sauder, Anil Rao and B.J. Fregly, from the University of Florida [Fregly *et al.*, 2015]. The model is controlled by five fictitious muscles that span the knee and ankle joints, and contains four reference frames (the absolute or inertial frame, and the segment local reference frames).

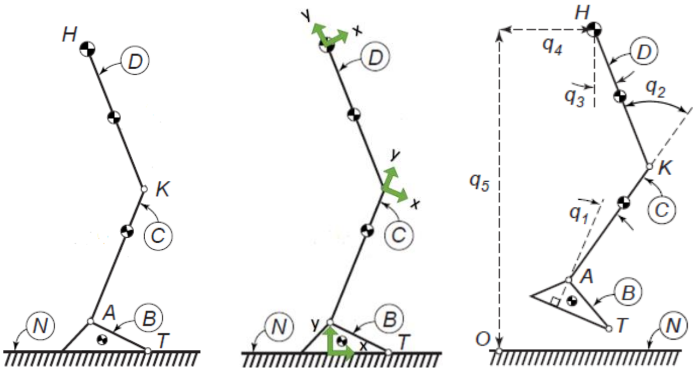

Figure 3. 1: Reference frames, points and coordinates

3.1.1. Skeletal model

The skeletal model contains three segments, which represent both right and left legs: thigh (contains local reference frame D), shank (contains local reference frame C) and foot (contains local reference frame B). The upper body is modelled as a single point mass at the hip H. Points A and K represent the ankle and knee joint locations, respectively, and point T is the application point for the ground reaction forces. There's a fourth reference frame which is the absolute one (reference frame N).

The anthropometric parameters can be seen in Table 2.1. The moment of inertia of the segments is expressed with respect to their centre of mass (COM), and its location is given by ρ, in the local frame for each body.

Segment	Formed by	Length (L)	COM location (ρ)	Mass (m)	Principal Moment of Inertia (I)
		[m]	[m]	[kg]	[kg·m^2]
H	Head, arms and trunk	-	-	51.220	-
D	Thigh (left and right)	0.400	0.200	15.150	0.126
C	Shank (left and right)	0.435	0.200	7.500	0.065
B	Foot (left and right)	0.150	0.100	2.200	0.008

Table 3. 1: Values of anthropometric parameters

3.1.2. Generalized coordinates

Generalized coordinate q_1 represents the ankle angle relative to reference frame B, q_2 represents the knee angle relative to reference frame C, q_3 is the orientation of the thigh segment in reference frame N, and q_4 and q_5 define the position (x and y, respectively) of the hip (point H) in reference frame N (see Figure 3.1).

As the simulated movement is a squat exercice, the foot is constrained to the floor, so q_3, q_4 and q_5 can be calculated from q_1 and q_2 (Eqs. 3.1-3.3):

$$q_3 = q_2 - q_1 \tag{3.1}$$

$$q_4 = LC * \sin(q_1) + LD * \sin(q_1 - q_2) - LB \tag{3.2}$$

$$q_5 = HB + LC * \cos(q_1) + LD * \cos(q_1 - q_2) \tag{3.3}$$

3.1.3. Muscles

The model is controlled by five fictitious muscles that emulate the action of the agonistic and antagonistic knee and ankle muscles in the human body. Five muscles and groups of muscles from the knee and ankle (soleus, tibialis anterior, biceps femoris shorthead, vastus and gastrocnemius) have been simplified in four uniarticular muscles (two for each joint) and a fifth biarticular. The origin is the center of mass of the segment where the muscle is attached, and the insertion point is a point at a certain distance from the joint where the muscle actuates (see Figure 3.2).

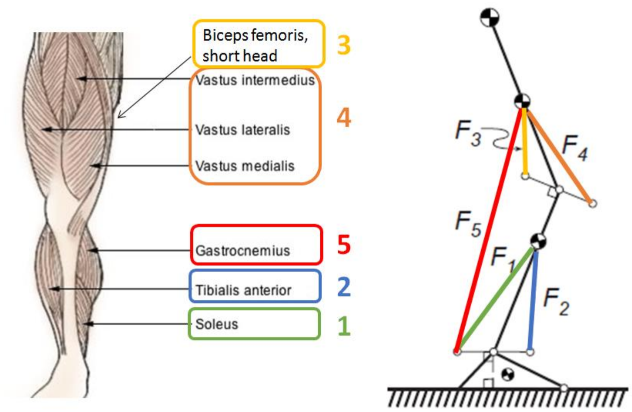

Figure 3. 2: Real lower limb muscles (left) and model muscles (right).

Muscles are modelled as Hill-type muscles with force-length-velocity properties and possess activations dynamics (see [Fregly *et al.*, 2015] for a detailed information). In Table 3.2. the values for muscle parameters are shown.

Parameter	Description [Units]	Value M1	Value M2	Value M3	Value M4	Value M5
F_0^M	Maximum isometric force [N]	18000	3000	8700	18900	7700
l_0^M	Muscle optimum length [m]	0.1287	0.1400	0.1699	0.1890	0.3536
l_s^T	Tendon slack length [m]	0.1118	0.0866	0.0701	0.1197	0.2840
α_0	Pennation angle [rad]	0.4939	0.1676	0.2025	0.5166	0.1728
v_{max}^M	Maximum muscle velocity [m/s]	1.2865	1.4004	1.6993	1.8899	3.5364

Table 3. 2: Values for muscle parameters

3.1.4. Marker protocol

The marker protocol consists on 3 markers located in the lower limb joints (hip, knee and ankle) and two markers on the foot (see Figure 3.3). Only the right leg has been considered, because we assume that both legs perform the same motion pattern. It has been adapted from [Pàmies-Vilà *et al.,* 2015], where also a 2D movement was analysed.

The locations for the five markers are the following:

M_1: Metatarsal head V
M_2: Lateral femoral epicondyle
M_3: Calcaneus
M_4: Lateral malleolus
M_5: Femoral greater trochanter

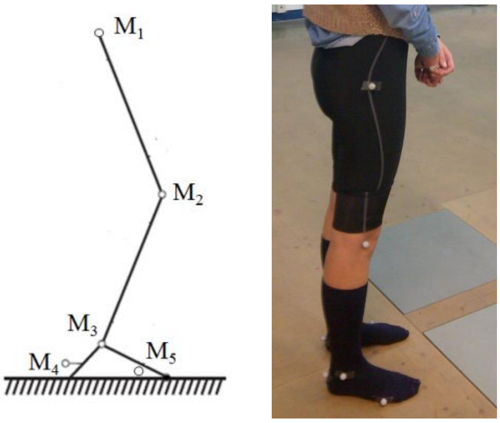

Figure 3. 3: Model markers (left) and real markers (right)

3.2. Experimental data

The biomechanical model presented in the previous section is used together with the measurements from the UPC Biomechanics Lab, placed in the Department of Mechanical Engineering of the Barcelona School of Industrial Engineering (ETSEIB). This laboratory includes an optical marker-based motion capture system.

The captured motion is a squatting movement. Squat is a full body exercise in strength training and fitness, that consists on lower body by bending knees, as illustrated in Figure 3.4. Squats can be performed to varying depths and weights are often used.

Data have been collected for a 18 year old girl, mass 50 kg and height 1.65 m.

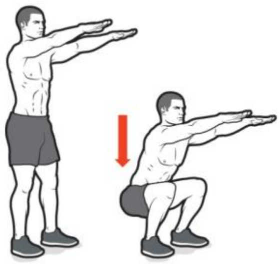

Figure 3. 4: Squat exercice

3.2.1. Motion capture equipment

Motion capture is the process of tracking marker positions along time with the aim of obtaining the motion of a computational model. In this project optical sensors have been used. These markers are located in anatomical positions which are matched in the computational model.

The motion data has been collected using fifteen infrared cameras (Natural Point, OptiTrack^{TM} FLEX:V100R2 sampling at 100 Hz). Cameras have been positioned in order to achieve more accuracy [Bosch, 2013]. Cameras have been calibrated experimentally before the motion capture.

Markers are small spheres covered by a reflector tissue, and reflect the infrared (IR) light emitted by the 26-LED ring surrounding the cameras. Figure 3.5 shows a passive reflective marker and a camera. This reflected light is captured discretely by the optical cameras system, where each camera positions each marker in a plane perpendicular to its optical axis. With this information from different cameras, the system is able to locate the points (where the markers are) in the three-dimensional space.

Collected data is transmitted to a PC by wires (USB 2.0) and is processed using a software, named Motive, which provides the 3D trajectories of the eleven markers attached to the human body. Each camera has 2 digit numeric LEDs as a status indicator, to be sure that are connected and to identify each camera in Motive.

Figure 3. 5: Passive marker and camera used in the motion capture

3.2.2. Measurements and signal processing

Two measurements have been performed: a static capture and a squat movement, with ten cycles. Markers are identified manually in Motive and exported to Matlab.

The first step is to filter data to eliminate electrical interferences and errors from the capture system. This low-level noise is amplified when numerical differentiation is used to calculate their corresponding velocities and accelerations. The signals have been filtered with a second order Butterworth filter with cutt-of frequency between 10 Hz [Silva and Ambrósio, 2002].

For the captured motion, the sagittal plane is where much of the movement takes places. So only values for markers position in this plane have been considered. To ensure kinematic consistency, kinematic errors due to markers movement and skin motion can be removed by modifying the markers trajectories in order to fulfill the constraint equations associated to the biomechanical model. Segment lengths have been computed from static capture, and new markers positions are calculated from the angles of the motion capture.

3.3. Optimal Control

Optimal control consists on finding a control law for a given system such that a certain optimality criterion is achieved. It is also sometimes referred as trajectory optimization. Optimal control problems are used in different applications in engineering (orbit transfers in aerospace engineering), economics (optimal investment of production strategies), and medicine (radiotherapy) [Rodrigues *et al*., 2014]. Because optimal control applications have increased in complexity in recent years, over the past two decades the subject of optimal control has transitioned from theory to computation [Patterson and Rao, 2014].

3.3.1. Optimal Control Problem Statement

The problem of optimal control can be stated in the following general form: Determine state $y(t)$ and control $u(t)$ that minimize cost functional[1] J (Eq. 3.4) subject to some constraints (Eqs. 3.5-3.10) [Betts, 2010].

$$J = \Phi\big(y(t_0), t_0, y(t_f), t_f\big) + \int_{t_0}^{t_f} \mathcal{L}[y(t), u(t), t]dt \tag{3.4}$$

In optimal control problems, variables are separated into two classes, namely the state (or phase) variables and the control variables. Note that by definition, a "state variable" is a differentiated variable that appears in the differential equation that describes the system dynamics (Eq. 3.2). In contrast, a "control variable" is an algebraic variable. Although this terminology is convenient for most purposes, there is often some ambiguity present when constructing a mathematical model of a physical system. For example, in multibody dynamics, it is common to model some physical states by algebraic variables.

Constraints can be:

- State equations (differential or dynamic constraints):

$$\dot{y} = f(y(t), u(t), t) \tag{3.5}$$

- Boundary conditions:

$$\phi\big(y(t_0), t_0, y(t_f), t_f, p\big) = 0 \tag{3.6}$$

  where t_0 and t_f are the initial and final time, respectively, and p are parameters independent from time.

[1] A functional is a "function of functions": states and controls are functions of time, and the functional J to be minimized is a function of states, control and time.

- Simple bounds on states and control variables:

$$y_L \leq y(t) \leq y_U \tag{3.7}$$
$$u_L \leq u(t) \leq u_U \tag{3.8}$$

  An equality constraint can be imposed if the upper (U) and lower (L) bounds are equal for some specific variables and/or phases.

- Algebraic path constraints (can be equality or inequality equations):

$$0 = g[y(t), u(t), t] \tag{3.9}$$
$$0 \leq g[y(t), u(t), t] \tag{3.10}$$

Some constraints may be viewed as "continuous" since they must be satisfied over the entire interval $t_0 \leq t \leq t_f$, whereas other constraints may be viewed as "discrete" since they are imposed at a specific time. Collectively, those functions evaluated during the phase can be referred to as the vector of continuous functions. Similarly, functions evaluated at specific points, such as the boundary conditions, are referred to as point functions.

An optimal control problem can be formulated as a collection of *N* phases. For many applications, the independent variable *t* is time, and the phases are sequential, but neither of these assumptions is required. Many complex problem descriptions require different dynamics and/or constraints within each phase.

The cost function may depend on quantities computed in each of the *N* phases. Furthermore, the cost function includes contributions evaluated at the phase boundaries (point functions) and over the phase (continue functions).

3.3.2. Direct Collocation Methods

Two main groups of methods for solving optimal control problems are indirect and direct methods. Betts (2010) presents some practical difficulties with indirect methods, that combine differential equations with root-finding, and focuses on direct methods, that combine differential equations with nonlinear optimization. He also explains that there are two major parts of a successful optimal control solution technique: the "optimization" method and the "differential equation" method.

In a direct collocation method, the continuous-time optimal control problem is discretized (transcribed or converted) into a polynomial approximation of the state/function and a sparse, finite nonlinear programming problem (NLP) is obtained [Rao and Darby, 2009]. Direct collocation methods guess controls and motion, discretize dynamics equations and iterate controls and motion guess to find motion and controls over all time frames simultaneously. They are called "collocation" methods because the function is discretized using mesh intervals, where different points are located and the equations are solved. Then, the mesh is refined until the desired accuracy is achieved. An *h*-method varies the number/width of mesh intervals and a *p*-method varies the degree of approximating polynomial within each mesh. A particular case is the *hp*-adaptive version of the Legendre-Gauss-Radau (LGR) orthogonal collocation method. (see Figure 3.6). Collocation is performed at LGR points, and a hybrid mesh refinement strategy between *h*- and *p*-methods is used to achieve the desired accuracy.

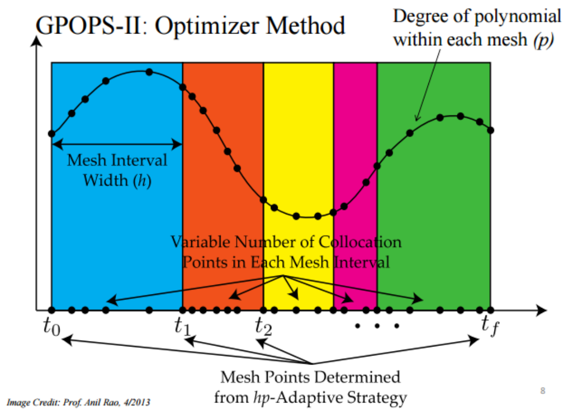

Figure 3. 6: Scheme of an hp-Adaptive Gaussian quadrature collocation method [Spangelo, 2012]

3.3.3. GPOPS-II

GPOPS-II is a general-purpose MATLAB-based software for solving multiple-phase optimal control problems developed by Michael Patterson and Anil V. Rao, from the University of Florida. It solves general nonlinear optimal control problems, where it is desired to optimize systems defined by differential-algebraic equations. GPOPS-II implements *hp*-adaptive version of the Legendre-Gauss-Radau (LGR) orthogonal collocation method. The NLP is then solved using an existing NLP solver. In Figure 3.7 the flowchart of the algorithm is presented.

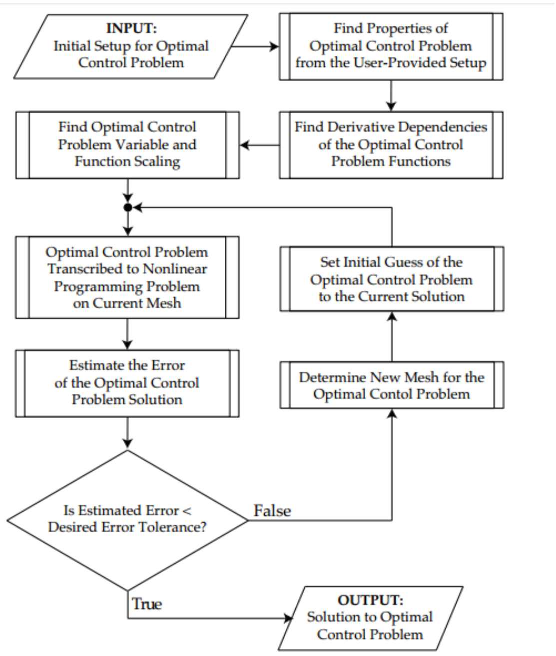

Figure 3. 7: Flowchart of the GPOPS−II Algorithm, from [Patterson and Rao, 2014]

The main advantage that offers this software is that the problem formulation is easy to understand, and it allows for an extremely general formulation of the optimal control problem and for inclusion of integral constraints and general boundary conditions. GPOPS-II has a user interface that enables the optimal control problem to be input in an intuitive yet compact manner. The key MATLAB programming element that makes this user interface possible is the structure.

User has to define the following:

- **States and controls:** The number of states and controls is defined by the size of bounds and guesses. States are constrained to follow the differential equations prescribed by the dynamics of the problem (defined inside the continuous function), whereas controls are not.

- **Cost function:** The integrand of the cost function is defined inside the continuous function, and the objective (cost) is defined inside the endpoint function.

- **Continuous function:** The continuous function defines the evolution of the dynamics in any phase of the problem, the integrands that are required to compute any integrals in any phase of the problem, and any path constraints in any phase of the problem. The input for the continuous function is a structure array that contains the values for time, states, controls, and static parameters for each *phase*. The output has three fields: dynamics, path and integrand. *Dynamics* is used to check if the derivatives obtained in the continuous function are the time derivatives of the states, and it checks if the integral of d*ynamics* is equal to the states. *Path* only is defined if there are path constraints. *Integrand* computes the integrand of the cost function for each frame time.

- **Endpoint function:** The endpoint function defines how the start and/or terminus in any of the phases in the problem, the integrals in any phase of the problem and the static parameters are related to each another. The endpoint function also defines the cost to be minimized. The inputs for the endpoint function are a structure array containing initial time, final time, initial state, final state and integral for each *phase*, a structure array containing auxiliary data (*auxdata*), and the static parameters. The output includes the *objective* that is minimized and, if any, endpoint constraints, which can only exist at the beginning and end of a phase.

- **Constraints (bounds, dynamics, path, events):** Bounds for time, states and controls (initial, final, and during the problem) are defined in a structure called *bounds*. If a value is desired to be fixed, upper and lower bounds must be equal. Dynamic and path constraints are defined inside the continuous function. They can be equations or functions. Event constraints are defined inside the endpoint function.

- **Initial guess:** A structure that contains a guess of the time, state, control, integrals, and static parameters in the problem.

- **Auxdata:** Optional structure containing auxiliary data that may be used by different functions in the problem. Including auxdata eliminates any need to specify global variables for use in the problem.

3.3.4. Optimal Control Formulation

The model described in section 3.1 is used to predict a squat movement. The problem is formulated as an optimal control problem, and is solved using GPOPS-II.

It is formulated using two phases, where the first phase takes the model from a static upright posture to a specified deep squatting posture in a specified amount of time, and the second phase takes the model from the specified deep squatting posture back to the original static upright posture in the same amount of time. It uses static optimization for solving the muscle force-sharing problem.

States
States in this problem are the generalized coordinates representing the ankle and knee angles (q_1 and q_2), the associated generalized speeds (u_1 and u_2), and the muscle activations (a_1 ... a_5).

The model has five generalized coordinates, but since the foot is fixed to the floor, it has only two independent coordinates or degrees of freedom (DOF).

Controls
Controls are the muscle excitations (e_1 ... e_5).

Cost function
The cost function is the integral of the sum of squares of the five muscle excitations.

Constraints

- Dynamic (continuous constraints)

$$\frac{da_i}{dt} = f_i\,(e_i, a_i, t_{act}, t_{deact}) \tag{3.11}$$

$$\frac{dq_j}{dt} = \frac{du_j}{dt} \tag{3.12}$$

$$\frac{du_j}{dt} = \; g_j(q_1, q_2, u_1, u_2, p) \tag{3.13}$$

Where:
i = 1,...,5 (for each muscle),
j = 1,2 (each degree of freedom),
f_i describes the activation dynamics
g_j describes the skeletal dynamics
p groups the geometrical and anthropometrical parameters

- Bounds

$$0 \leq a_i \leq 1 \tag{3.14}$$
$$0 \leq e_i \leq 1 \tag{3.15}$$

Where:
i = 1,...,5 (for each muscle),

- Initial and terminal

Phase 1 Initial: t = 0, q_1= 0, q_2, = 0, u_1 = 0, u_2 = 0
Phase 1 Terminal: $t = 0.5$, q_1 = 30°, q_2 = 60°, u_1= 0, u_2 = 0
Phase 2 Terminal: $t = 1$, $q1$ = 0, q_2= 0, u_1 = 0, u_2 = 0

These constraints are defined by imposing lower and upper initial and final bounds equal.

- Event (endpoint constraint)

Time and state at end of phase 1 are equal to time and state at beginning of phase 2.

3.4. Data Analysis

First of all, the problem has been explored carefully in order to understand each part of the algorithm, and has been solved with the initial values for all parameters. Secondly, some parameters have been changed, and has been studied how the output is affected. Finally, the captured motion has been used as reference motion to simulate a movement close to the experimental data.

3.4.1. Initial prediction

The squatting problem has been solved with an initial set of parameters, that can be classified in two groups:

- Movement parameters
- Anthropometric parameters

The parameters that define the squatting movement are the initial values for both ankle and knee angles, the values at half movement (squat position, via angle) and the final time for each phase (0.5 s duration each one). Initial values from are collected in Table 3.3.

Parameter	Initial angle [°]		Via angle [°]		Final time [s]	
	q1	q2	q1	q2	tf1	tf2
Value	0	0	30	60	0.5	1

Table 3. 3: Initial values for movement parameters

Anthropometric parameters are defined in Table 3.1.

3.4.2. Influence of parameter changes on the predicted motion

Some analyses have been performed to explore how the output (range of movement, muscle forces and joint torques) of the optimal control changes due to variations in some input parameters. In Tables 3.4 and 3.5 the new values for each parameter are detailed.

Parameter	**New value**	**Parameter**	**New value**
q1 initial	2.5°	q1 via	35°
	5°		40°
q2 initial	5°	q2 via	65°
	10°		70°

Table 3. 4: New values for movement parameters

%	**-20%**	**-15%**	**-10%**	**-5%**	**Initial**	**+5%**	**+10%**	**+15%**	**+20%**
Mass (kg)	60.856	64.6595	68.463	72.2665	76.07	79.8735	83.677	87.4805	91.284

Table 3. 5: New values for the model total mass

3.4.3. Minimize difference to captured motion

Using a captured motion as a reference movement tracked along the simulation is a common practice in human motion analysis and prediction [Fregly *et al.*, 2007; Thelen *et al.*, 2003].

For that purpose, the cost function has been modified, adding a term of the quadratic error of the angle coordinates along the motion.

$$J = \min\left[w_1 \sum_{i=1}^{5} e_i^2 + w_2 \sum_{j=1}^{2} \left(\frac{q\_ref_j - q_j}{q_{max}}\right)^2\right] \tag{3.16}$$

Where:
w_1 and w_2 correspond to the weight for each term in the cost function,
e_i are the muscle excitations (i=1,...5) ,
q_ref_j are the angles of the captured motion (j=1 for the ankle and j=2 for the knee),
q_j are the angles predicted (j=1 for the ankle and j=2 for the knee),
and $q_{max} = \frac{\pi}{2}$ is used to obtain the second term in a similar order of magnitude.

4. RESULTS AND DISCUSSION

In this chapter, the main results are presented and discussed. First of all, the squatting optimal control problem presented in section 3.3.4 has been solved for a set of initial parameters. Secondly, different parameters and initial and final conditions where chosen and simulated to explore how parameters affect the output of the optimal control problem. Finally, the captured motion was used as a reference motion, since the initial prediction did not match a real squatting movement. As it can be seen in Figure 4.1, both knee and ankle angles increase and decrease a little before reaching the target angle at 0.5 s, and this behavior was not found in the real capture.

4.1. Initial prediction

In this initial prediction, the target angles in squat position (at 0.5 s) are 30º (0.5236 rad) for ankle angle and 60º (1.0472 rad) for knee angle. The optimal motion in order to achieve minimum excitations seems to be like doing two squats, the first one reaches 5.8121º (0.1014 rad) for the ankle angle and 11.7060º (0.2043 rad) for the knee angle (see Figure 4.1). Then, it returns to the initial position (upright) and goes down to the defined target angles.

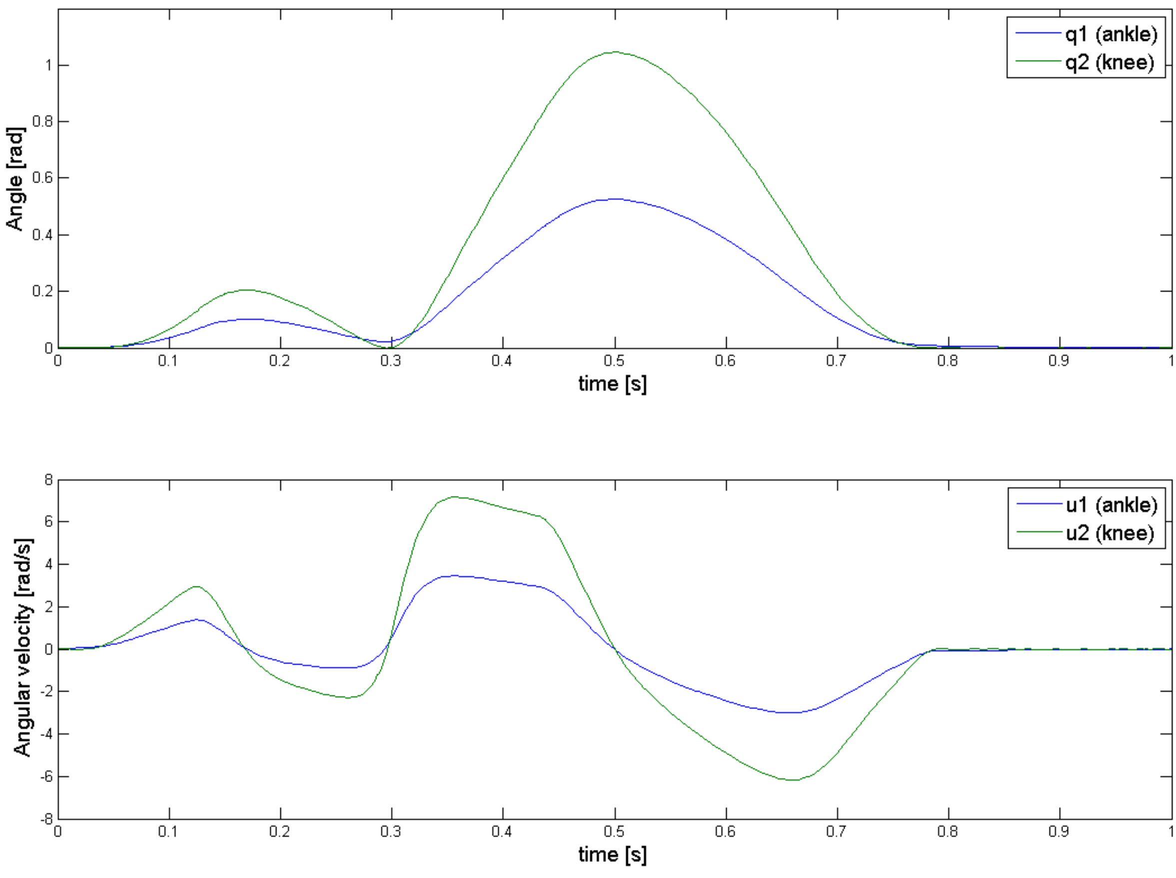

Figure 4. 1: Angles and angular velocities for initial prediction

Excitations, activations and muscle forces for this motion are shown in Figure 4.2. Muscles 1 and 5 do not have excitation, but they present forces along the movement, so we can conclude that correspond to passive forces (due to the elasticity of the muscle connective tissue). The maximum force for muscle 1 occurs at the squat position, where the muscle is more extended. For muscle 5, squatting position is the one that presents lower force, because the muscle is shortened. Muscle 4, simulating the quadriceps, is the muscle that stops the descending movement.

This is the optimal motion for reaching a knee angle of 60º and an ankle angle of 30º, when the excitations are minimized. One solution could be waiting in a static upright posture and then begin to go down, but it seems that it is better to go down and go up another time instead of staying in a static position which would require higher muscle excitations. As it can be seen, muscles 2 and 4 are the muscles that are working more during the first phase (descending phase).

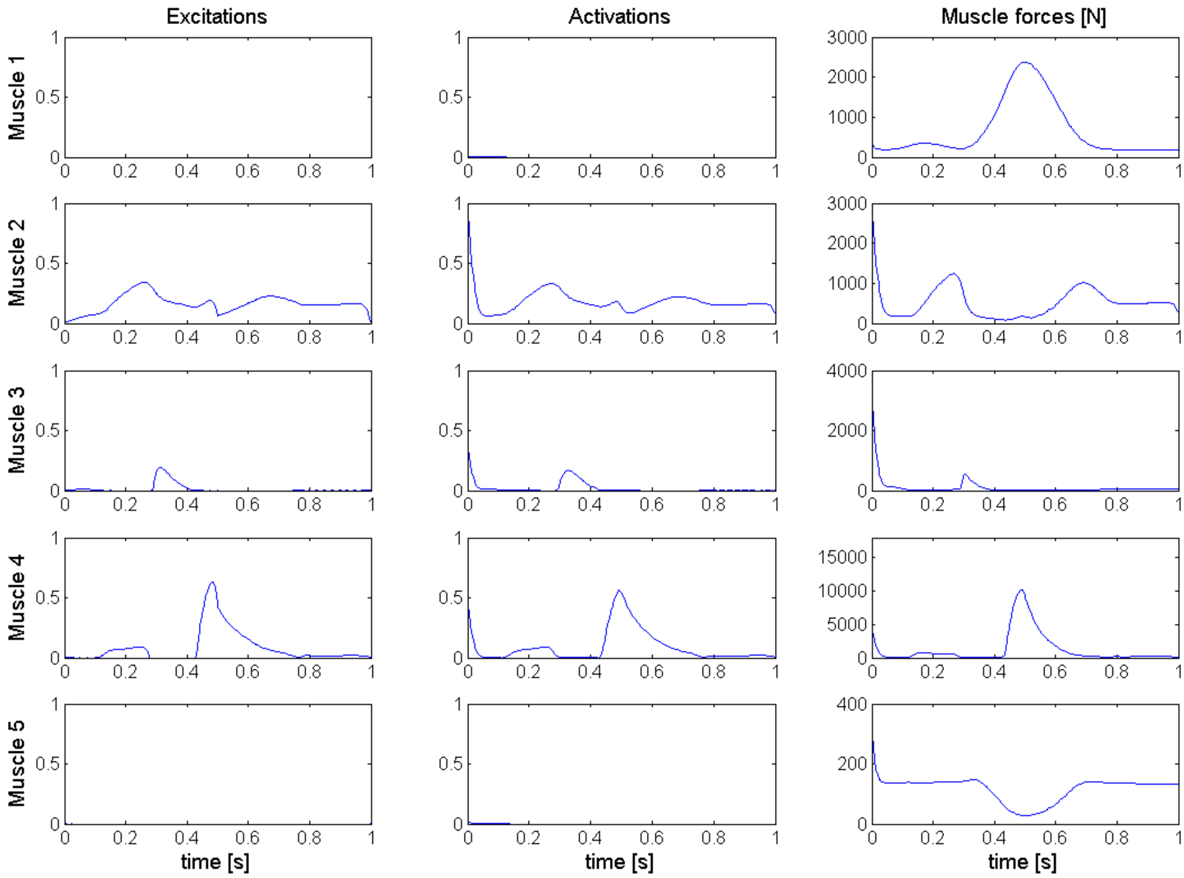

Figure 4. 2: Excitations, activations and muscle forces for initial prediction

Figure 4.3 shows the realtion between excitation and activation, which is modelled by the differential equation of activation dynamics. Activation presents a delay respect to excitation and has a lower value.

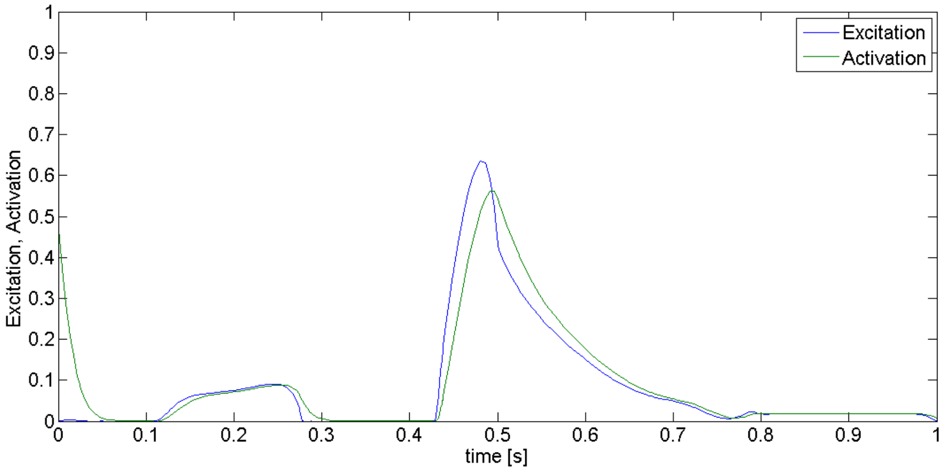

Figure 4. 3: Muscle 4 excitation and activation for initial prediction

Muscles 1, 2 and 5 actuate on ankle joint, and muscles 3, 4 and 5 actuate on knee joint. Moment arms for each muscle and joint have been computed, and joint torques have been calculated using Eqs. 4.1 and 4.2. Both negative peaks in ankle torque occur when muscle 2 is exerting large force, and positive peaks in the centre of the motion occur when muscle 1 and muscle 4 present also a peak.

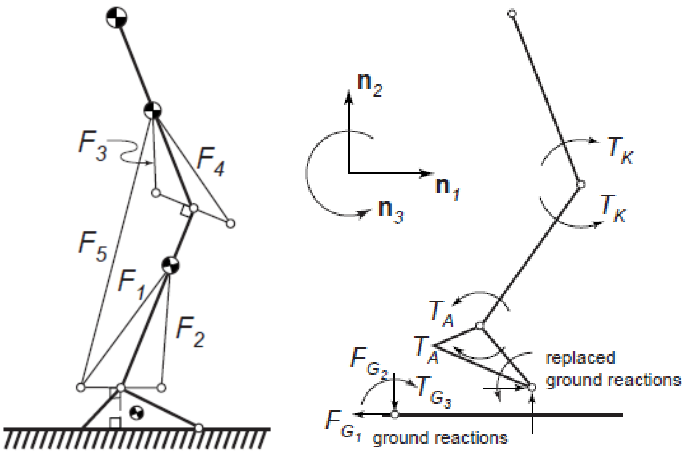

Figure 4. 4: Muscles and torques

$$T_A = F_1 * R_1^A - F_2 * R_2^A + F_5 * R_5^A \quad (4.1)$$

$$T_K = -F_3 * R_3^K + F_4 * R_4^K - F_5 * R_5^K \quad (4.2)$$

Where:

T_A, T_K is the ankle and knee torques, respectively,
F_i is the muscle force for each muscle i (i=1,…,5),
R_i^A, R_i^K is the ankle or knee moment arm for each muscle i (i=1,…,5)

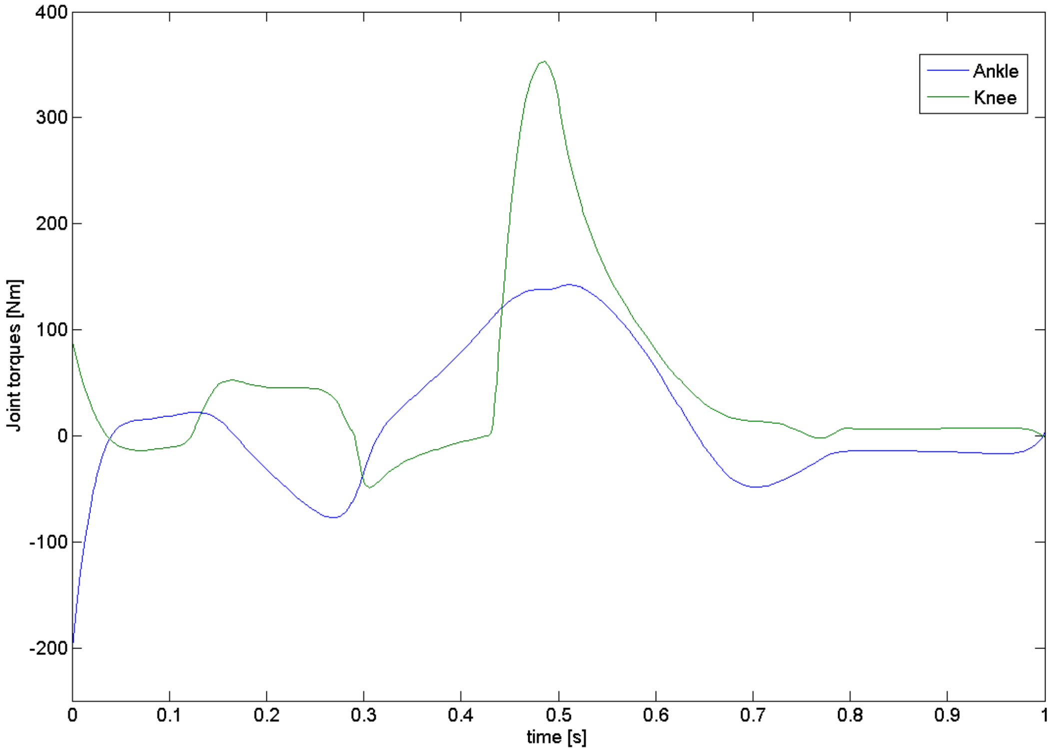

Figure 4. 5: Angle and knee torques for initial prediction

Once the first prediction has been analysed, some parameters have been changed in order to study how the solution is affected.

4.2. Influence of parameter changes on the predicted motion

Initial angle
When initial angle is changed, the predicted motion is similar to the first solution. It is also optimal to do first a small squat (see Figure 4.6), that varies a little bit among simulations, and then the second part of the movement is equal for the three simulations. Muscle forces are similar as well. In Figure 4.7, the activations and forces for muscle 2 are shown.

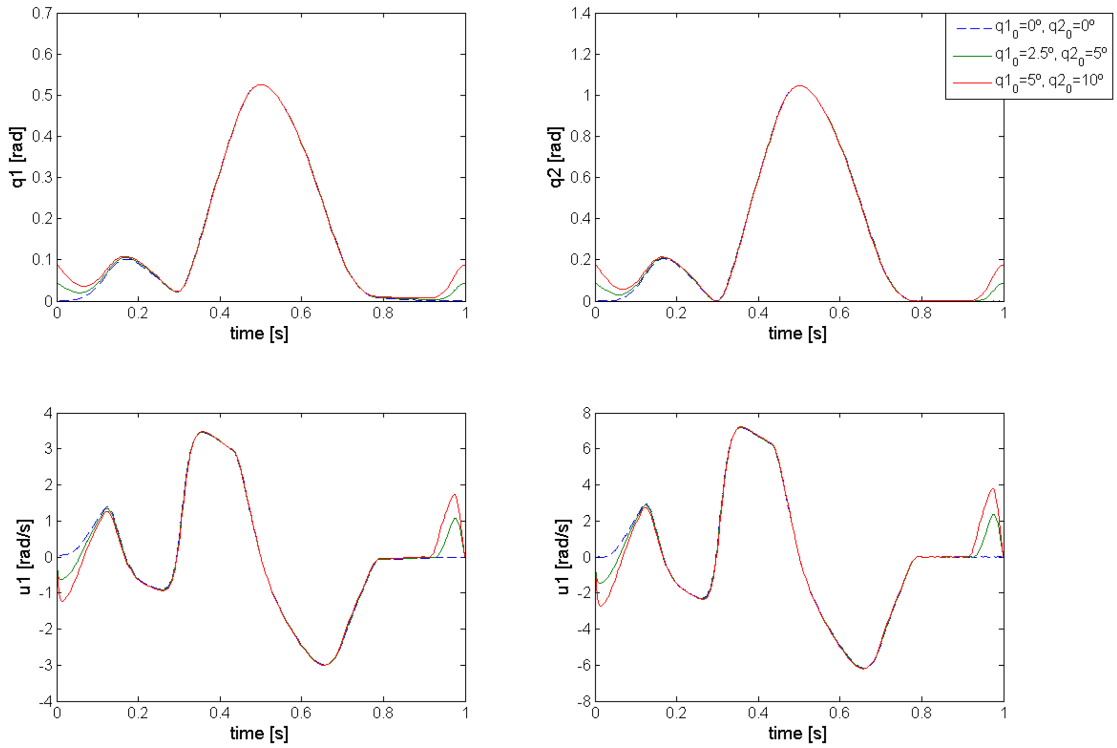

Figure 4. 6: Angles and angular velocities for different values of initial angles

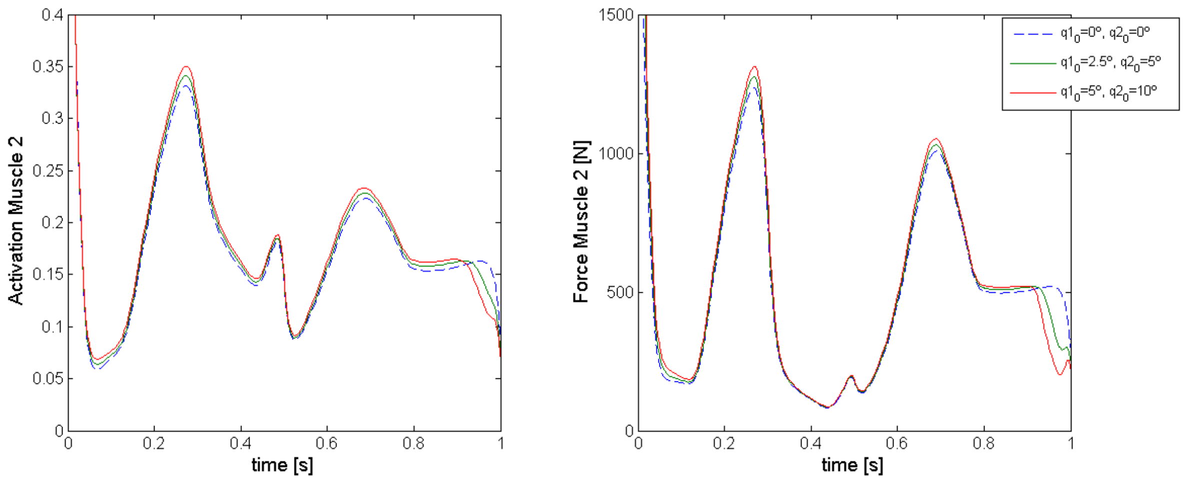

Figure 4. 7: Activation and force of muscle 2 for different values of initial angles

Via angle

Via angle is the target angle that defines the lowest position at half of the squatting movement. Initial $q_1$via is 30º and initial $q_2$via is 60º. Other values that have been studied are 35º and 40º for ankle angle, and 65º and 70º for knee angle.

The optimal control problem is solved correctly, and solution converges when via angle is modified. The algorithm has changed the motion in order to find the optimal.

The best option in terms of minimizing excitations is the combination of $q_1$via=35º and $q_2$via=60º (Table 4.1). In Figure 4.8, can be observed that the first small squat is not done for this combination.

q1 via	q2 via	objective
30	60	0.5942
30	65	1.3873
30	70	2.7968
35	60	0.5654
35	65	0.6497
35	70	1.4486
40	60	1.1701
40	65	0.9110
40	70	0.9322

Table 4. 1: Values for q1via and q2via and objective value for each simulation

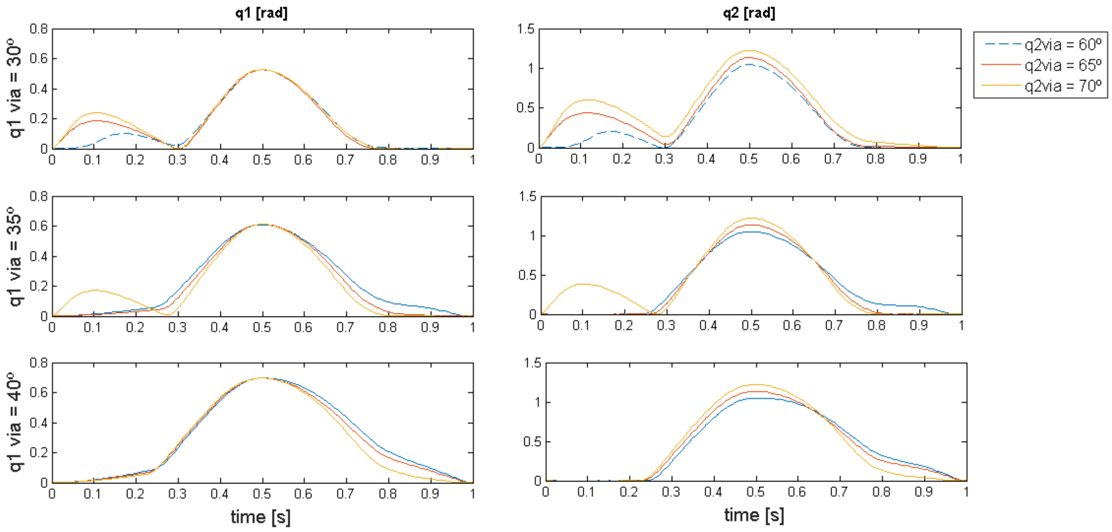

Figure 4.8: Ankle and knee angle for the nine combinations of $q_1$via and $q_2$via.

Muscle forces have also been evaluated. When $q_2$via is higher, the force of muscle 4 is bigger, because muscle 4 is the one that stops the model when is descending, so if the final position is lower, the moment of the weight (hip and thigh) is larger. It is interesting to see that for $q_1$via=35º and $q_2$via=60º (minimum excitations), the muscle 4 force in the middle of the movement is not the minimum, but for $q_1$via=40º and $q_2$via=60º, that correspond to the least peak force, the force increases at the last part of the movement.

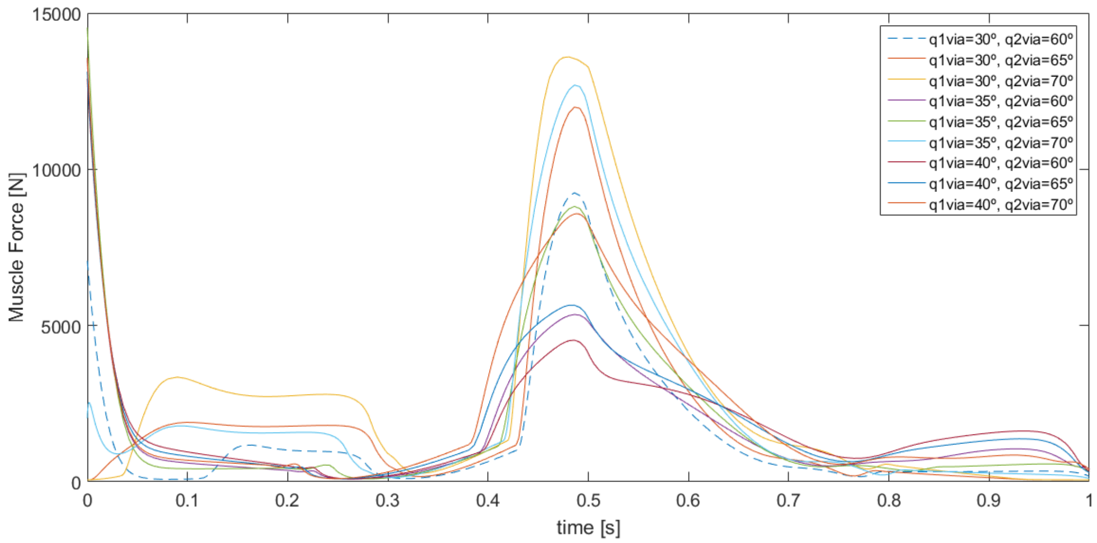

Figure 4. 9: Force muscle 4 for different changes of via angles

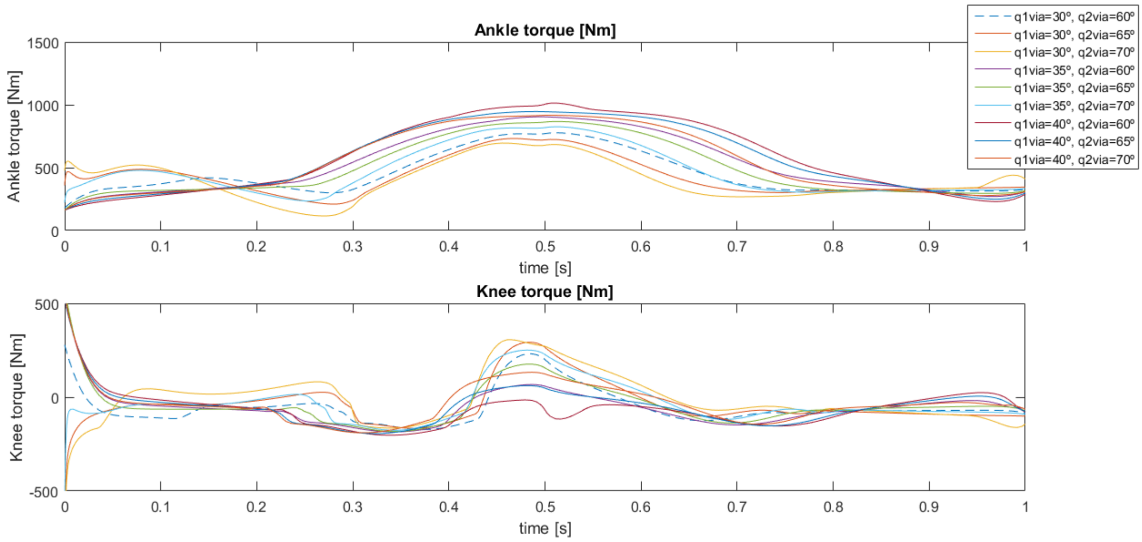

Figure 4. 10: Joint torques for different changes of via angles

Anthropometric parameters: mass

The maximum force and joint torque have been calculated when the subject mass is varied. Simulations have been performed for ±5%, ±10%, ±15% and ±20% variations of the initial mass. When mass is varied, the optimal motion is the same as the initial. But since each segment mass (as well as moments of inertia) has been changed, forces and torques vary. In Figures 4.11 and 4.12, results for the initial and the extreme values of mass (±20%) are shown.

Forces have been plotted for muscles 1, 2 and 4, that are the ones that present greater changes.

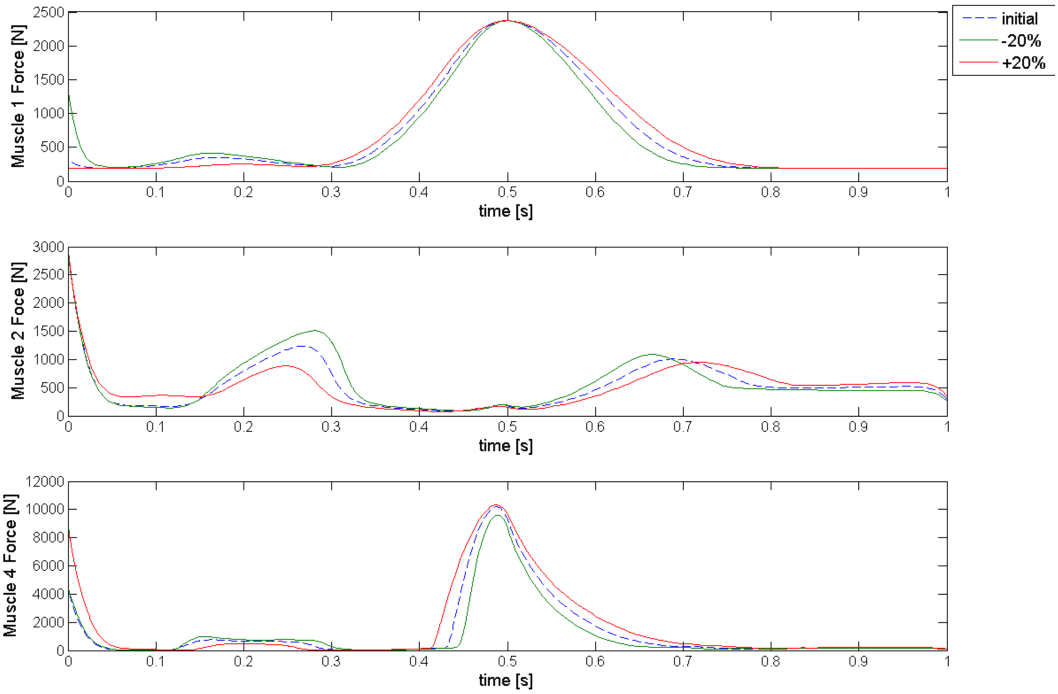

Figure 4. 11: Forces for muscles 1, 2 and 4 for mass changes

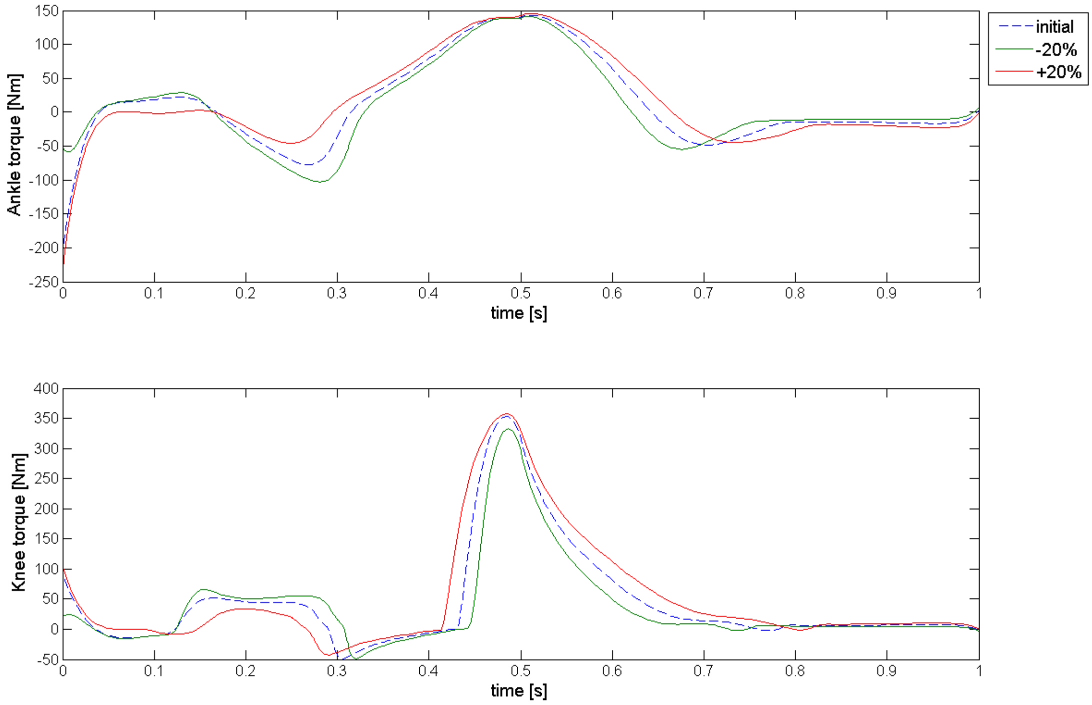

Figure 4. 12: Joint torques for mass changes

4.3. Minimize difference to captured motion

The duration of the squat motion capture has a been normalized to 1s to coincide with the predicted squat duration.

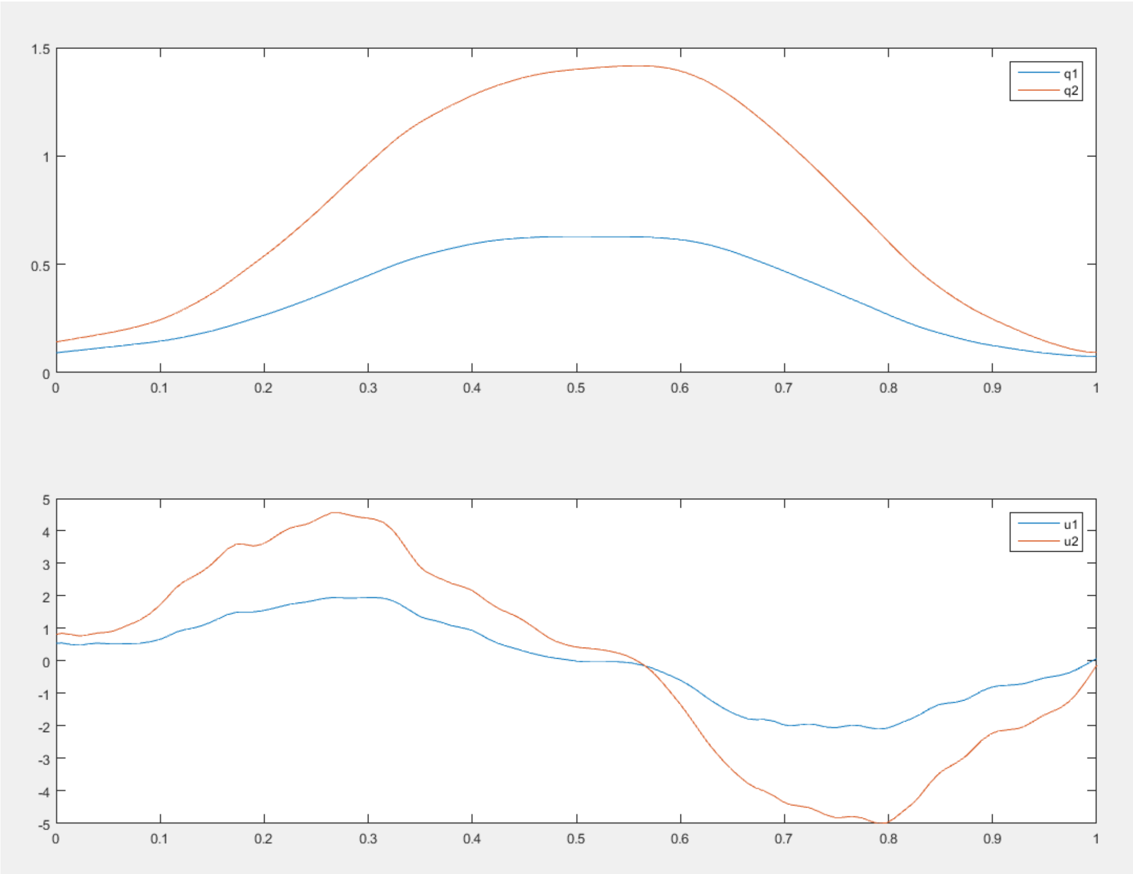

Figure 4. 13: Angles and angular velocities of the captured motion.

Different modifications have been done in order to use the captured motion as a tracking movement for prediction:

- In phase 1, angular velocities have been imposed to be positive
- No via angle has been defined (neither for ankle nor kneee angle)
- A new term has been added to the cost function (see Eq. 3.16)

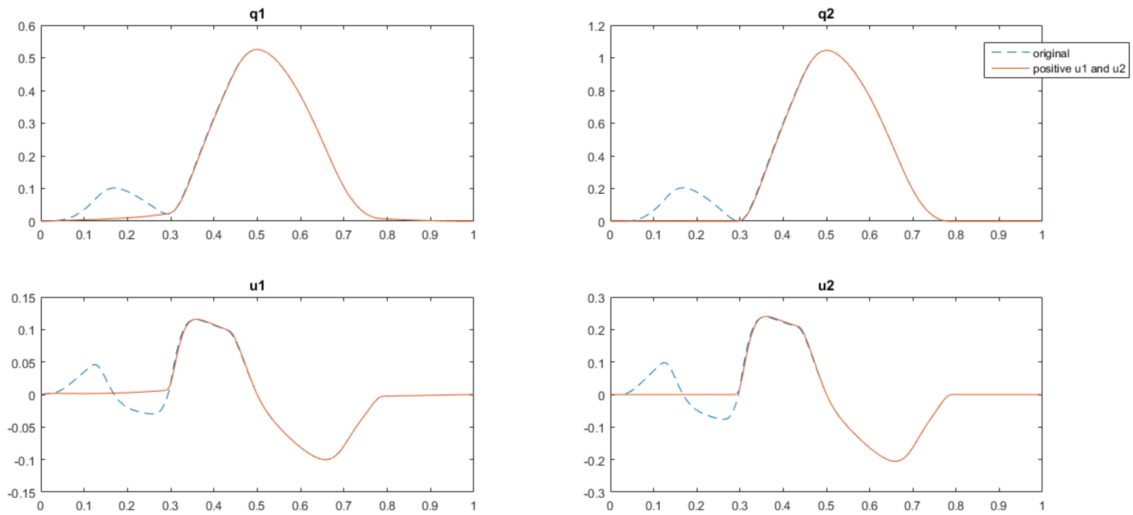

Figure 4. 14: Angles and angular velocities for the initial movement (original) and the movement with the condition of positive velocities

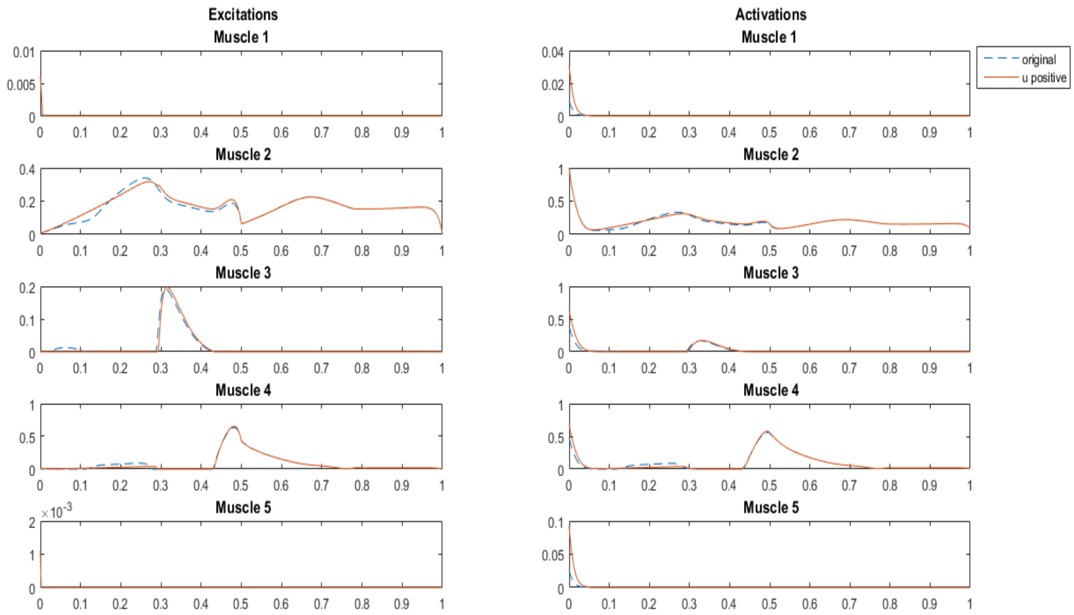

Figure 4. 15: Excitations and activations for the initial movement (original) and the movement with the condition of positive velocities

The objective value in the initial prediction is 0.5942, and when the condition of positive velocities is added, the new value is 0.6040, so there's not so much difference between one movement and the other in terms of muscle excitations, although the motion (see Figure 4.14) is quite different. In Figure 4.15 one can observe that is better in terms of excitations to activate muscles 3 and 4 and not activate so much muscle 2 at the beginning of the movement.

Different values for weights w_1 and w_2 have been tested:

- w_1=0.25, w_2=0.75
- w_1=0.50, w_2=0.50
- w_1=0.75, w_2=0.75

Results of the predicted motion (angles and velocities) in each case are shown in Figure 4.16. Results are coherent with the weigths used, and when excitations have greater weight, the squat position reached is smaller, and when motion error has greater weight, the motion predicted is similar to the captured motion.

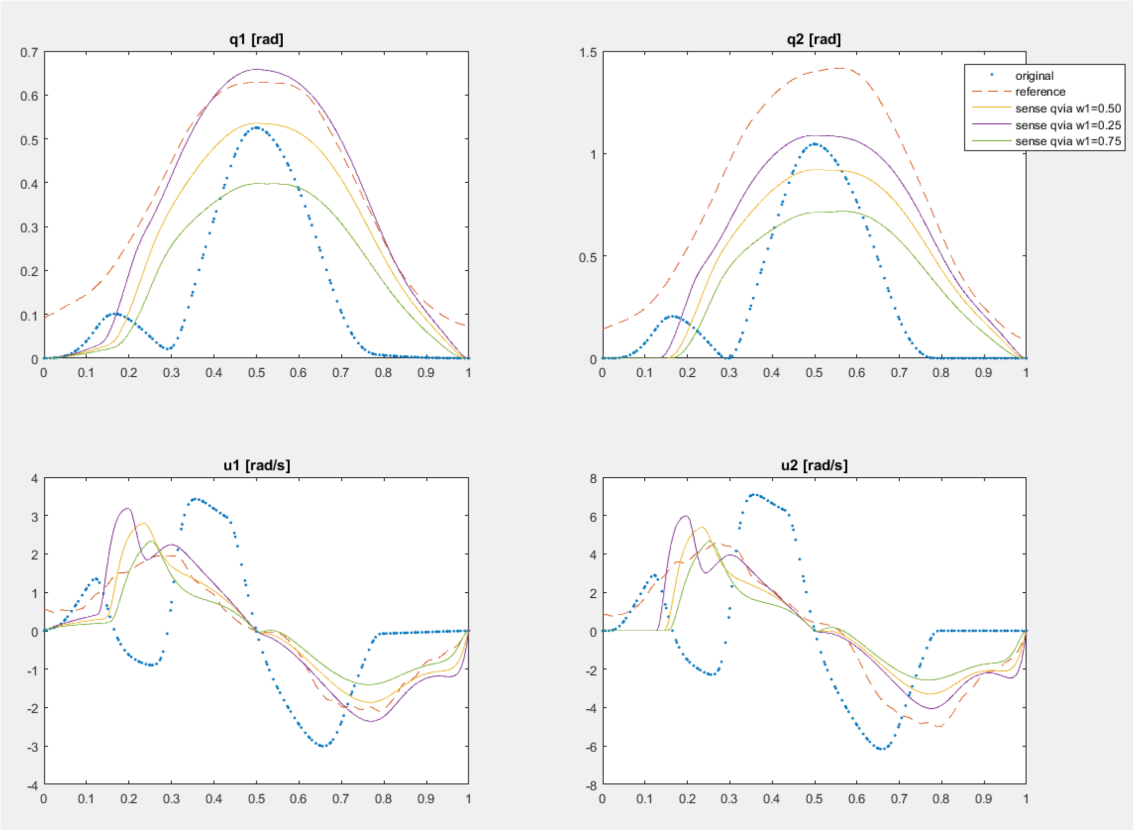

Figure 4. 16: Angles and angular velocities for the initial movement (original), the reference motion and three different combinations of w_1 and w_2

Forces for each muscle are compared between the initial prediction and the best squat predicted when tracking the captured motion. Muscle forces are different in this two predicted motions. In the squat predicted with the captured motion, muscle 2 works more at the beginning and final part of the squat, and at the squat position, muscle 1 does more force, and muscle 4 less.

Predictions in sections 4.1 and 4.2 were predictions without taking into account real movement, and in this last section they have an added value, because they have been performed tracking a real motion. GPOPS-II has been used to predict a movement with a reference motion. This ensures that motion equations are accomplished while the predicted motion is more similar to the real movement.

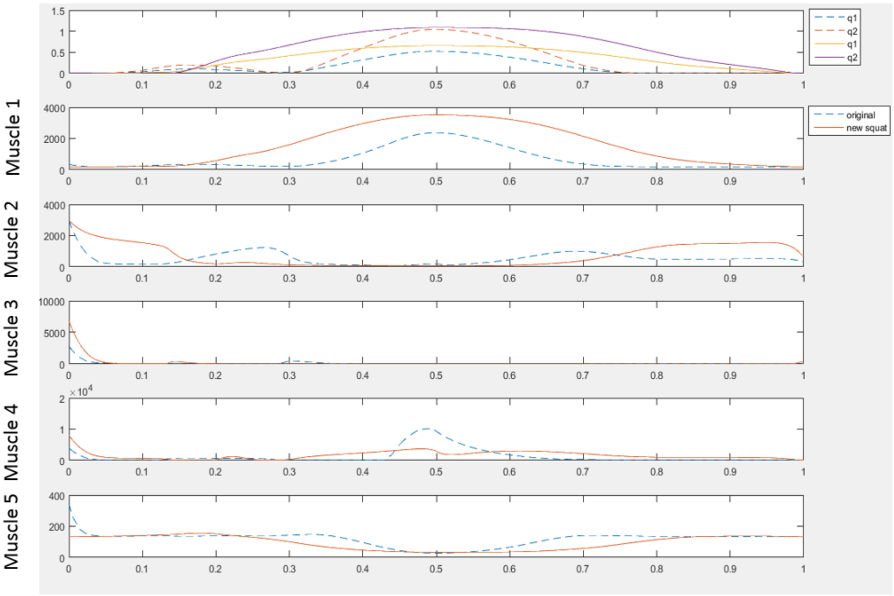

Figure 4. 17: q1, q2 and muscle forces for initial prediction (original) and prediction with w_1=0.25 i w_2=0.75

CONCLUSIONS

In this master thesis a review of the state of the art in multibody system dynamics, dynamic analysis, muscle force redundancy problem and simulation has been done. One of the most used methods in motion simulation nowadays is Optimal Control, which has been used for years in other fields, e.g., economics and aerospace, and is proved to give good results in motion simulation.

The principal basis of formulation for an optimal control problem have been studied and now are part of the knowledge of the group for current research projects. Specifically, it has been learnt how to state an optimal control problem in GPOPS-II (MATLAB).

The motion capture equipment has been calibrated experimentally. A squat exercise has been captured by optical cameras and passive markers, and an inverse kinematics analysis has been done in order to obtain the coordinates of the biomechanical model.

Different predictions have been done, in order to test the problem formulation and the tool. Some parameters have been changed and results have shown that the tool is able to predict motions in different conditions. When the captured data has been used for tracking, the motion prediction has improved. Therefore, GPOPS-II represents a suitable tool for solving optimal control problems that predict human motion.

This work can be considered a first research work in optimal control formulations to predict human motion. In future works, these formulations will be combined with a more complex model of a real patient wearing robotic orthoses in order to predict and anticipate the human-orthosis behaviour with the purpose of personalizing and optimizing the design of the orthosis to facilitate the patient adaptation process.

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